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STRESS-STRAIN ANALYSIS OF A COILED COPPER PIPE FOR INNER PRESSURE LOADS

DAVID FERRAS[1,2], DÍDIA I.C. COVAS[1] & ANTON J. SCHLEISS[2]

[1] *Instituto Superior Técnico, Lisboa, Portugal*
david.ferras@ist.utl.pt, didia.covas@tecnico.ulisboa.pt

[2] *Laboratory of Hydraulic Constructions, LCH, École Polytechnique Fédérale de Lausanne, Switzerland*
david.ferrassegura@epfl.ch, anton.schleiss@epfl.ch

Abstract

This research aims to enhance the knowledge on stress-strain states of a copper pipe coil facility used for hydraulic transient experiments. The ultimate goal is the development of the pipe movement equations which will allow the implementation of Fluid-Structure Interaction (FSI) in the hydraulic transient model. The membrane theory of shells of revolution has been applied for the description of axial and circumferential strains while an inverse method has been used to analyze bending effects. Finally, the developed stress-strain model has been successfully validated for dynamic loading.

Keywords: Fluid-structure interaction; hydraulic transient; membrane theory of shells of revolution; stress-strain analysis.

1. Introduction

The aim of the present research is to achieve a better understanding of the stress-strain states, of a coiled copper pipe during hydraulic transient events. Classic theory of water-hammer assumes that the pipe does not move along its axis and that the circumferential deformation is incorporated, together with pipe deformability and fluid (liquid) compressibility, in the elastic wave speed. However, if the axial pipe movement is allowed then a Fluid-Structure Interaction analysis (FSI) has to be carried out in order to determine the effect of the structure inertia over the transient pressure wave. Pipe systems experience severe dynamic forces during water-hammer events. When these forces make the system move, significant FSI may occur, so that liquid and pipe systems cannot be treated separately in a theoretical analysis: interaction mechanisms must be taken into account (Tijsseling, 2007).

Coiled pipes have many industrial engineering applications. Due to their convenient geometry, pipe coils are used in most heat exchange systems, like cooling systems in power plants, industrial and commercial refrigerators, solar water heaters or radiators for auto-motion industry. To the knowledge of the authors, the incorporation of the pipe coil behavior in hydraulic transient analysis through FSI has never been carried out. This paper aims at finding out the relationship that describes the pipe coil deformation as a function of pressure changes, in order to incorporate it in the hydraulic transient solvers.

2. Background theory

A stress-strain analysis is a first step for FSI, the goal is to determine the pipe motion equations that will be used to couple with hydraulic transient equations. Torsion, bending, shear and axial stresses and strains are the structural responses that a piping system may experience during a water-hammer wave. In classic hydraulic transient theory FSI coupling is implicitly included applying thin-walled theory and considering a straight conduit with expansion joints throughout its length and a linear-elastic behavior of the pipe wall (Chaudhry, 1987). Circumferential strain in such system is described by the following relation and included in the conservation equations.

$$\epsilon_c = \frac{1}{E}\frac{pr}{e} \qquad [1]$$

where ϵ_c is the circumferential strain, E the Young's modulus of elasticity, p the inner pressure in the pipe, r the pipe radius, and e the wall thickness.

However in coil systems the structural behavior considerably differs from the one ins straight pipes, either in axial and circumferential directions, due to the toroid geometry and the cross-section shape, which becomes oval when the pipe is curved. Consequently, when classic water-hammer theory is applied in coiled pipes discrepancies generally arise changing the wave shape and overestimating computed pressures during peak transitions. Anderson & Johnson, 1990, analyzed the effect of tube ovalling on pressure wave propagation speed in the context of physiological flows, reaching the conclusion that water-hammer wave is very sensitive to the eccentricity of an ovalled cross-section.

The spiral system of a coil can be considered as a composition of toroids and be described as a thin shell of revolution. Membrane theory of shells of revolution is a suitable approach to solve circumferential and axial strains in an axisymmetrically loaded torus. However in a torus with ovalled or elliptic cross-section, when it is pressurized loads are not axisymmetric and bending moments are generated. Membrane theory of shells of revolution assumes that no bending moments, twisting moments and transverse shearing forces exist in the shell (Zingoni, 1997). To account for bending effects and to describe the complete state of stresses and strains, bending theory of shells must be applied. However, such theory is more general and, consequently, more difficult to be solved for complex geometries.

Clark & Reissner, 1950, proposed a methodology based on Boltzmann superposition principle to describe stress-strain states in Bourdon tubes. Such approach consisted essentially of the computation of axial and circumferential strains using the thin-walled assumption, and describing the bending effects using the thick-walled assumption and applying bending theory. In the context of hydraulic transients, Stephen *et al.*, 2001a, and Stephen *et al.*, 2001b, carried out FSI coupling in order to account for the Bourdon effect during water-hammer events. The structural constrain conditions of the pipe coil, though, do not comply with the Bourdon tube theory, as Bourdon tube is a disconnected torus with closed ends, while the pipe coil analyzed must be considered as a connected torus. However, a similar approach can be applied in order to determine its stress-strain states, combining thin and thick-walled assumptions by Boltzmann superposition principle as a function of the loads applied. Therefore, this paper approaches stress-strain states problem in coils by computing circumferential and axial strains using membrane theory of shells of revolution and the bending effects by using the thick-walled assumption.

3. Data collection

The experimental set up is composed of a copper coiled pipe of nominal diameter $D = 0.02\ m$, pipe-wall thickness $e = 0.001\ m$ and pipe length $L = 110\ m$. The torus radius is $R = 0.5\ m$ and 35 rings compose the entire coil. As shown in Fig. [1], seven strain gauges were installed in the middle section of the coil, three in the circumferential direction and four in the axial direction with the aim to get strain measurements in both directions for different positions of the cross-section. Young's modulus of elasticity and Poisson ratio were experimentally determined by measuring stress-strain states over a straight pipe sample for the experimental range of pressures. The obtained value for Young's modulus of elasticity is $E = 105\ GPa$ and for Poisson ratio is 0.33.

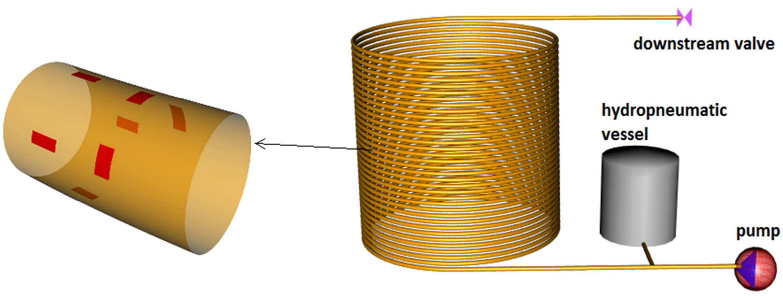

Figure 1. Scheme of the copper pipe coil facility used for the experimental data collection (right) and detail of strain-gauges installation (left).

Two different kinds of experiments were carried out in the coil facility. Firstly circumferential and axial stresses were measured for different quasi-steady pressure loads, and secondly dynamic loading was applied by producing water-hammer events for different flow rates. The results presented in the section [5] correspond to a steady pressure test of $6x10^5$ Pa and results in the section [6] correspond to a water-hammer wave produced for an initial flow rate of $1.4x10^4 m^3/s$. Fig. [2] shows the measurements obtained for static pressure tests, both for axial and circumferential strains.

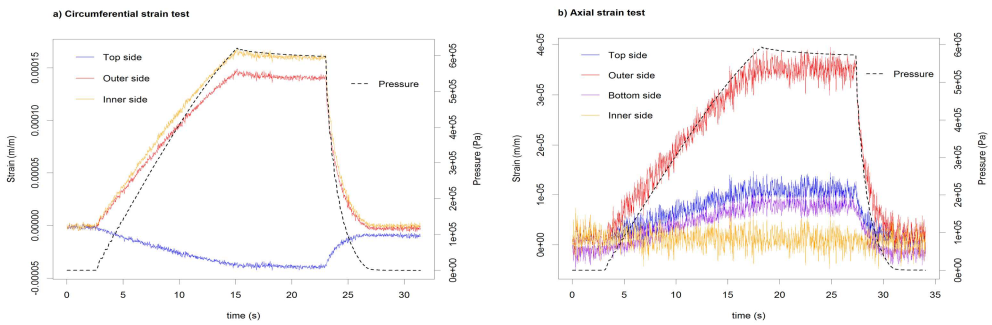

Figure 2. Strain and pressure measurements in the middle section of the pipe for circumferential direction (left) and for axial direction (right).

4. Model development

4.1 Membrane theory of shells of revolution

Two stress-strain models, one considering a torus with elliptic cross-section and the other for circular cross-section, were implemented following the development explained in Zingoni, 1997, from the general solution of the membrane theory of shells of revolution of axisymmetric loads.

4.1.1 Torus with elliptic cross-section

The governing equations for the description of circumferential and axial strains in a torus with elliptic cross-section are the following (Zingoni, 1997):

$$\sigma_c = \frac{pa^2}{e\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}} \left[\frac{R\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi + \frac{a^2}{2} \sin \varphi}}{R\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi} + a^2 \sin \varphi} \right] \quad [2]$$

$$\sigma_a = \frac{pa^2}{eb^2} \left[R \left(\frac{b^2 - a^2}{a^2} \right) \sin \varphi + \frac{b^2 - \frac{1}{2}(a^2 \sin^2 \varphi + b^2 \cos^2 \varphi)}{\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}} \right] \quad [3]$$

where σ_c is the circumferential stress, σ_a the axial stress, p is the inner pressure, a the minor semi-axis of the elliptic cross-section, b the major semi-axis, φ is the angular position within the cross-section, and R is the torus radius from the center of the toroid to the pipe axis.

For $R >> a$ or b, the term $R\left(\frac{b^2-a^2}{a^2}\right)\sin\varphi$ in Eq.[3] will be very sensitive to ellipse eccentricity. This high sensitivity and the uncertainty associated to the accurate measurement of ellipse semi-axes are the main reasons why eccentricity value is calibrated a posteriori from axial strain measurements.

4.1.2 Torus with circular cross-section

Circular torus equations are straightforwardly derived by simplifying Eq.[2] and [3] for $a=b=r$, resulting the following expressions (Zingoni, 1997):

$$\sigma_c = \frac{pr}{e} \left(\frac{R \pm \frac{r}{2} \sin \varphi}{R \pm r \sin \varphi} \right) \quad [4]$$

$$\sigma_a = \frac{pr}{2e} \quad [5]$$

For $R>>r$, Eq.[4] can be further simplified, canceling the second term and reaching the expression for straight pipes (Eq.[1]) used for classic water-hammer theory. Eq.[5] shows that axial stress is independent of coil radius and position angle φ. Hence, in contrast to elliptic torus, axial stress is constant along the pipe-wall for the circular cross-section.

Once stresses are defined, strains can be obtained by Hook's law for isotropic materials:

$$\epsilon_c = \frac{1}{E}(\sigma_c - \upsilon\sigma_a) \quad [6]$$

$$\epsilon_a = \frac{1}{E}(\sigma_a - \upsilon\sigma_c) \quad [7]$$

where ϵ_c and ϵ_a are the circumferential and axial strains, respectively, and υ is the Poisson ratio.

4.2 Cross-sectional bending analysis: inverse method

Membrane theory of shells of revolution assumes thin-walled shells, i.e. no bending moments are transmitted along the shell. However, due to the elliptic geometry of the pipe cross-section, when the fluid pressure changes, radial loads are not balanced in the coil (they are not axisymmetric any more) as its projection over the minor axis of the ellipse will not be equal to the projection over the major axis.

This unbalance of forces generates bending moments that for positive pressures will tend to reduce the eccentricity of the ellipse and vice verse for negative pressures. Hence, in the case of positive pressures, the outer fibers of the upper and lower generatrices of the cross-section will be compressed and the outer fibers of the lateral generatrices stretched. For negative pressures, the effect will be the opposite (see Fig. [3]).

Strain-gauges measurements give information from these outer fibers, so the real measurements will be actually a combination of circumferential strain (computable by membrane theory of shells of revolution) plus the extra deformation due to bending. Therefore, the effect of bending can be analyzed by an inverse method from computed circumferential strains in the central fibers of the pipe-wall and measured circumferential strain in the external fibers.

The assessment of this bending effect has two goals: first it will allow the comparison between measured and computed circumferential strains, and secondly it will provide information about the cross-sectional shape change, which could be important for fluid structure coupling during hydraulic transients in coils.

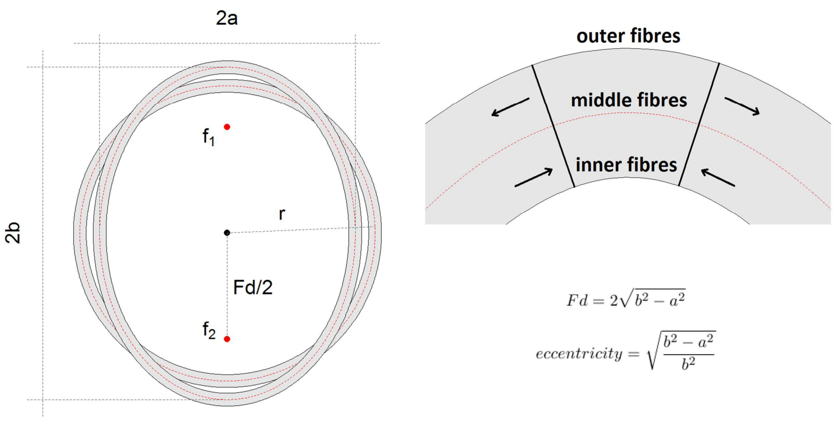

Figure 3. Schematics of circumferential and elliptic cross-sections, and detail of bending effect over circumferential strains.

5. Model application

5.1 Torus with elliptic cross-section

States of stresses and strains were computed by using Equations [2], [3], [6] and [7] for the static loading test with a pressure of 6x105 Pa. Obtained results are shown in Fig. [4].

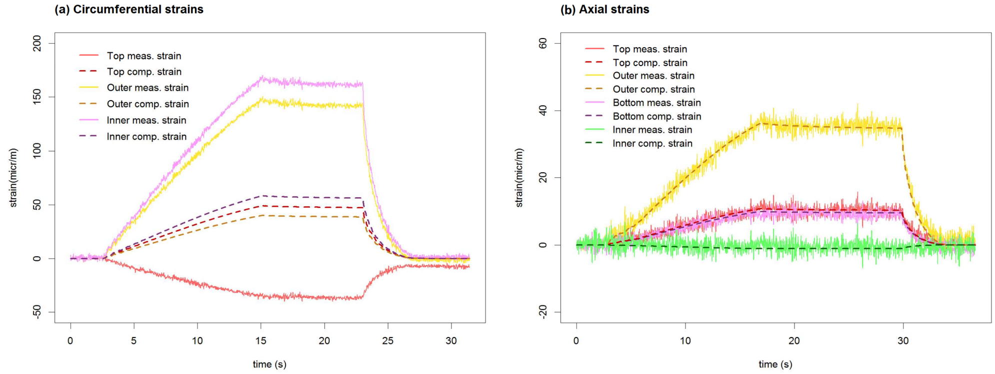

Figure 4. Measured circumferential and axial strains versus computed as elliptic torus.

After adjusting ellipse eccentricity to 0.0094 a good fitting of the computed axial strains versus measurements is observed. The consistency between the different positions in the cross-section strengthens the reliability of membrane theory of shells of revolution in regard to axial strains. Nonetheless, circumferential strain results do not present the same accuracy, as major discrepancies arise between circumferential strains in the different positions of the cross-section. The main reason of such discrepancies is the bending effect over the cross-sectional plane due to non-axisymmetry of loads, which at this stage has not been taken into account.

5.2 Torus with circular cross-section

Solving stress-strain states applying torus with circular cross-section for the same pressure loads as in the previous section, the results are presented in Fig. [5].

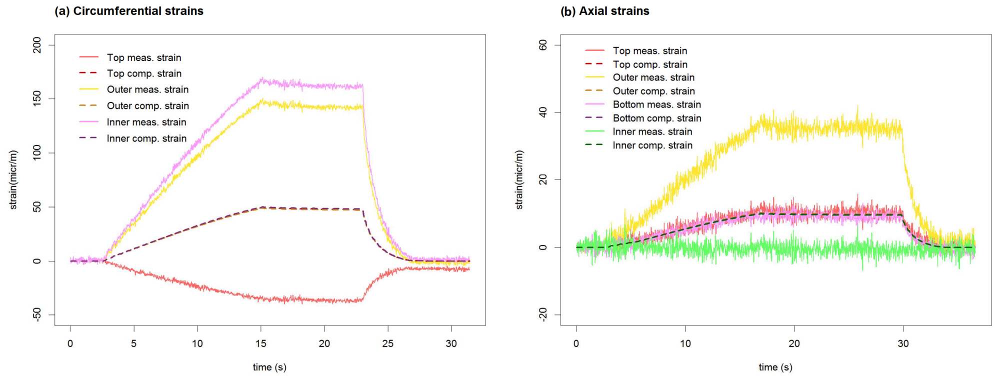

Figure 5. Measured circumferential and axial strains in the middle section of the pipe versus computed strains as circular torus.

It can be seen Fig. [5a] shows that strains computed by Equations [4] and [5] do not vary much with their relative position in the cross-section. This fact is in agreement with the assumption R>>r, indicating that circumferential strains computed by classic theory does not very much in comparison to the model for circular torus. However, as in the case of elliptic torus, discrepancies with measurements are evident.

In regard to axial strains, they are constant over the cross-section, and their magnitude seems to fit the measured axial strain in the top and in the bottom sides of the cross-section.

In general, the circular torus model can describe quite well average circumferential and axial strains. However, the capacity of elliptic torus to adjust a posteriori elliptic eccentricity of the pipe cross-section and its reliability shown in axial strains (Fig [3]) are good arguments to improve the accuracy of the model by carrying out the bending analysis of the cross-section, which will allow the correction of circumferential strains.

It should be highlighted that the differences between measurements and computed circumferential strains by membrane theory of shells of revolution, either considering elliptic or circumferential cross-section, are coherent with the phenomena explained in subsection [4.2]. ~

The bending of the cross-section when pressure is increased produces a compression of the external fibers of the top side of the cross-section and stretches the external fibers in the lateral sides. Hence, computed results overestimate circumferential strains in the top side and underestimate circumferential strains in the lateral sides.

5.3 Inverse approach for bending effects

As it was mentioned in the subsection [4.2] bending effects can be estimated by applying the following methodology:

- Determine the relation between strain and ellipse eccentricity.
- Define an empirical expression relating *Fd* (distance between the foci of the ellipse) and pressure from circumferential strain measurements.
- Compute strain due to bending around the external fibers of the pipe-wall using the two previous relations.

Figures [6] and [7] illustrate the application of the explained methodology. In Fig. [8] the final circumferential strains computed by membrane theory of shells of revolution are shown and corrected for bending effects in different positions of the cross-section.

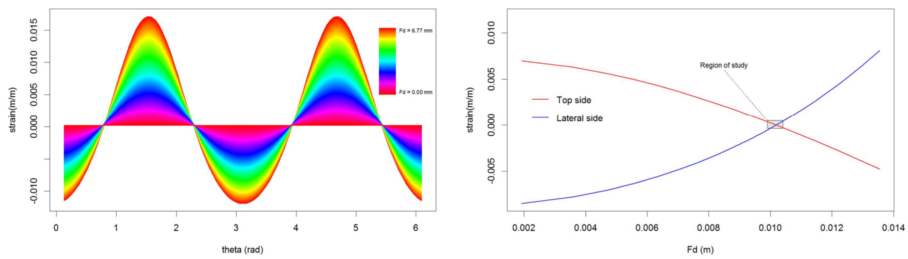

Figure 6. Strains of the outer fibers of the pipe-wall over the cross-section for different ellipticities when Fd is forced to 0 (left), and relation Fd-strain for the top and lateral sides of the cross-section (right).

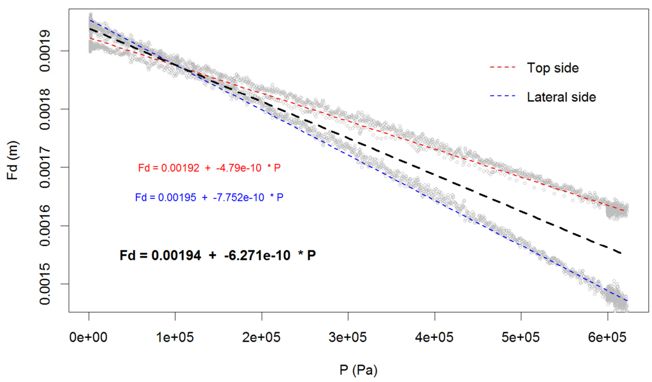

Figure 7. Empirical laws relating pressure to focal distance (*Fd*) of the elliptic cross section.

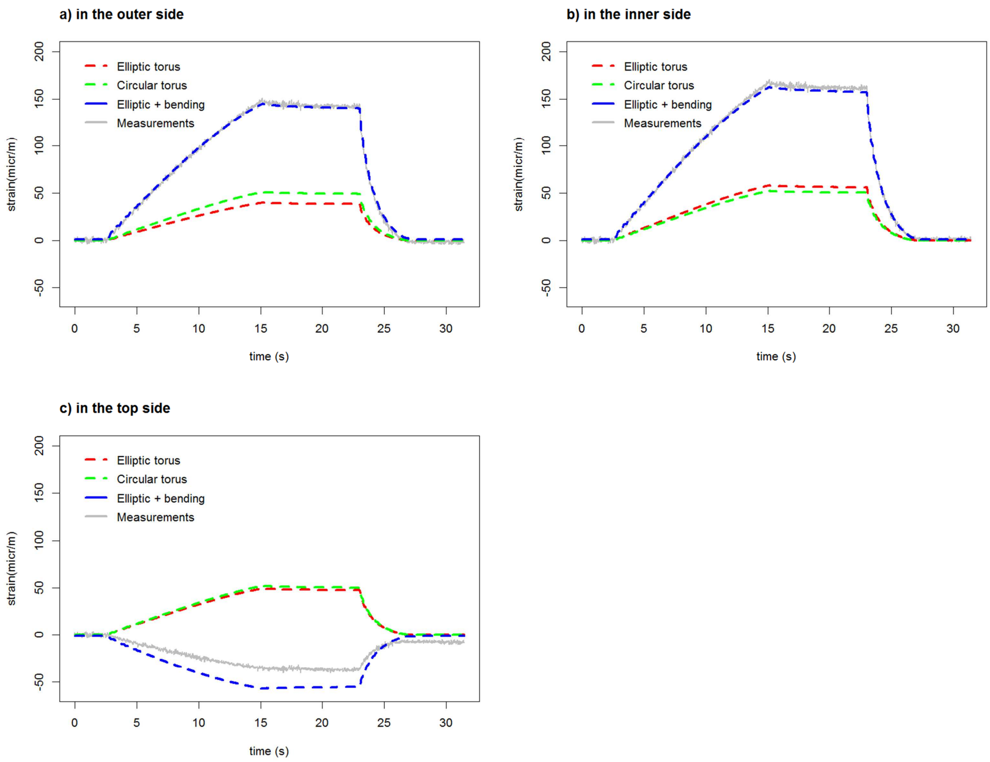

Figure 8. Circumferential strains corrected in strain gauges positions versus non-corrected and experimental data: a) in the outer side of the cross-section, b) in the inner side and c) in the top side.

Fig. [8] shows that once circumferential strains are corrected by taking into account bending effect, there is good agreement between measured and computed strains, particularly in the inner and outer sides of the cross-section. More discrepancy is observed in the top position of the cross-section where computed strains seem to be more sensitive to the inaccuracy of the method already depicted in Fig. [7]. However, the overall performance of the model after bending correction is quite satisfactory, allowing the assessment of pipe-wall displacements for the analyzed static loading range.

6. Model validation for dynamic loading

Dynamic loading tests were carried out in order to assess whether the calibrated stress-strain model is capable of accurately describe pipe-wall displacements during hydraulic transients. A hydraulic transient is characterized by a fast loading-unloading cycles over the pipe-wall. Consequently, other physical phenomena such as the non-elastic behavior of the copper material, not taken into account in a static analysis, may arise.

For this purpose, once the stress-strain model was defined, strains were computed for a hydraulic transient test and compared with measurements (Fig. [9]). Axial strains were computed by using the elliptic model developed by membrane theory of shells of revolution approach. Circumferential strains were computed by superposition of membrane theory and the correction for the bending effect. The test consisted of a hydraulic transient generated by the fast closure of a valve located at the downstream end of the pipe and for an initial discharge of $1.4x10^{-4}$ m^3/s.

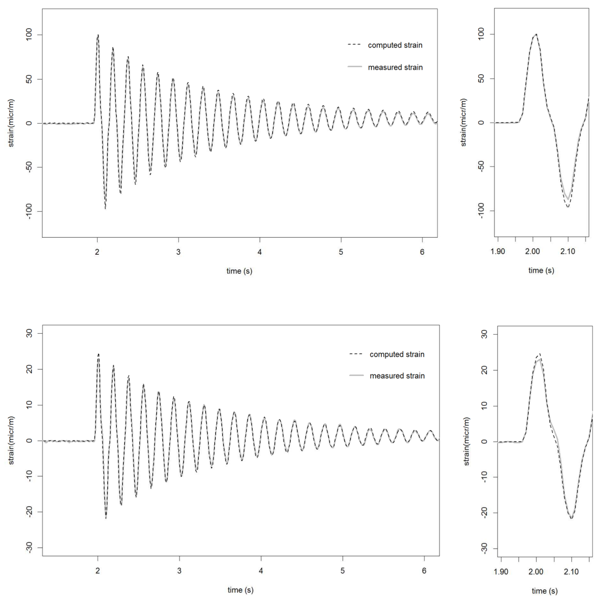

Figure 9. Circumferential (top) and axial (down) strains (in μm/m) during transient test at the middle section of the pipe and zoom of the first peaks (right).

Strains computed in the circumferential direction have an averaged sum of squares error equals to 2.1μm/m, while strains in axial direction have 0.25μm/m. The reason the stress-strain model presents a better performance on the axial direction is because computed strains in such direction do not need to be corrected, assumptions from membrane theory of shells of revolution are consistent with the physical phenomena because bending is only occurring in the cross-sectional plane, and not in the horizontal plane of the torus. However, strains in the circumferential direction are substantially affected by the bending of the elliptic cross-section, and therefore uncertainty arises from the introduced correction.

7. Conclusions

Two stress-strain models based on the theory of shells of revolution are presented to describe the structural response of a coiled pipe. A semi-empiric bending analysis has been carried out in order to complete and improve the accuracy of the models.

Membrane theory of shells of revolution applied for elliptic torus has been proven to be a good approach for axial strain description but inaccurate in regard to circumferential strain. The main reason is that circumferential strains are strongly affected by a common singularity of coiled pipes: cross-section is slightly elliptic and bending moments are generated over the cross-sectional plane. In order to assess the bending effect and correct circumferential strains, an inverse method has been proposed. The method enables the assessment of cross-sectional shape change and the correction of circumferential strains at the external fibers of the pipe-wall. Validation has been successfully carried out for dynamic loading with the aim to ensure the final purpose of the research, FSI during hydraulic transients.

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