Triangle Network Converse Proof

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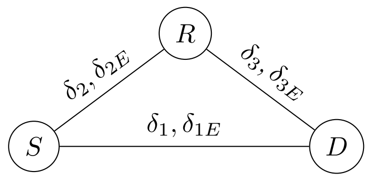
I. Scheme program



The linear program that describes the scheme is the following:

$$R \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{1}$$

$$m_1(1-\delta_1)\frac{1-\delta_{1E}}{1-\delta_1\delta_{1E}} \leq (k_1+c_1)\delta_{1E}(1-\delta_1) + r_3 + c_3(1-\delta_3) \tag{2}$$

$$m_2(1-\delta_2)\frac{1-\delta_{2E}}{1-\delta_2\delta_{2E}} \leq k_2\delta_{2E}(1-\delta_2) + k_1(1-\delta_1) \tag{3}$$

$$m_3(1-\delta_3)\frac{1-\delta_{3E}}{1-\delta_3\delta_{3E}} \leq k_3\delta_{3E}(1-\delta_3) + (k_1+c_1)(1-\delta_1) + c_3\delta_{3E}(1-\delta_3) + r_3\delta_{3E}\frac{1-\delta_3}{1-\delta_3\delta_{3E}} \tag{4}$$

$$1 \geq k_1 + m_1 + c_1 \tag{5}$$

$$1 \geq k_2 + m_2 \tag{6}$$

$$1 \geq k_3 + m_3 + c_3 + \frac{r_3}{1-\delta_3} \tag{7}$$

$$k_2(1-\delta_2) \geq c + r_3 \tag{8}$$

$$c \geq c_1(1-\delta_1\delta_{1E}) + c_3(1-\delta_3) \tag{9}$$

$$c \geq c_3(1-\delta_3\delta_{3E}) + c_1(1-\delta_1) \tag{10}$$

$$(1-\delta_3)m_3 \leq (1-\delta_2)m_2 + c_1(1-\delta_1) \tag{11}$$

II. Converse

When deriving our bounds we will use the assumption that the inputs X_{1i}, X_{2i}, X_{3i} of the different channels in the same time slot are generated from different independent random sources. It was shown in [1] that this assumption does not affect capacity. The proof trivially generalizes for the triangle network.

A. Notation

We use the following notation:

- $X_{k,i}$– the ith transmitted packet on channel k, $k \in \{1,2,3\}$
- $Y_{k,i}$– the ith received packet on channel k by the network node, $k \in \{1,2,3\}$
- $Z_{k,i}$– the ith received packet on channel k by the eavesdropper on channel k, $k \in \{1,2,3\}$
- X_k^i– $X_{k,1},\ldots,X_{k,i}$ we use the same shorthand for other vectors also

We assume that the sate of all channels including the eavesdroppers' channels is part of the channel outputs $Y_{k,i}, Z_{k,i}$ and it is available to S also. Note that knowing the channel state of the eavesdropper can only help, hence this assumption is valid for deriving an outer bound.

B. Constraints on the rate

$$nR \leq I(W;Y_1^nY_2^n) = \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) + (1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}) \tag{12}$$

$$nR \leq I(W;Y_1^nY_2^nZ_1^n) = \sum_{i=1}^{n}(1-\delta_1\delta_{1E})I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) + (1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{13}$$

$$nR \leq I(W;Y_1^nY_2^nZ_2^n) = \sum_{i=1}^{n}(1-\delta_2\delta_{2E})I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) + (1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{14}$$

$$nR \leq I(W;Y_1^nY_3^n) = \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}) + (1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}) \tag{15}$$

$$nR \leq I(W;Y_1^nY_3^nZ_1^n) = \sum_{i=1}^{n}(1-\delta_1\delta_{1E})I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) + (1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{16}$$

$$nR \leq I(W;Y_1^nY_3^nZ_3^n) = \sum_{i=1}^{n}(1-\delta_3\delta_{3E})I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) + (1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{17}$$

C. Bounds on key generation

We derive a key generation – key consumption type inequality for all eavesdroppers.

$$0 \leq H(Y_1^nY_2^n|Z_1^nW) = H(Y_1^{n-1}Y_2^{n-1}|Z_1^{n-1}W) - I(Y_1^{n-1}Y_2^{n-1};Z_{1n}|Z_1^{n-1}W) + H(Y_n|Y_1^{n-1}Y_2^{n-1}Z_1^nW) \tag{18}$$

$$= H(Y_1^{n-1}Y_2^{n-1}|Z_1^{n-1}W) - (1-\delta_{1E})I(Y_1^{n-1}Y_2^{n-1};X_{1n}|Z_1^{n-1}W) + H(Y_{1n}Y_{2n}|Y_1^{n-1}Y_2^{n-1}Z_1^nW) \tag{19}$$

$$= H(Y_1^{n-1}Y_2^{n-1}|Z_1^{n-1}W) - (1-\delta_{1E})I(Y_1^{n-1}Y_2^{n-1};X_{1n}|Z_1^{n-1}W) + H(Y_{1n}|Y_1^{n-1}Y_2^{n-1}Z_1^nW) \tag{20}$$

$$+ H(Y_{2n}|Y_1^{n-1}Y_2^{n-1}Y_{1n}Z_1^nW) \tag{21}$$

$$= H(Y_1^{n-1}Y_2^{n-1}|Z_1^{n-1}W) - (1-\delta_{1E})I(Y_1^{n-1}Y_2^{n-1};X_{1n}|Z_1^{n-1}W) \tag{22}$$

$$+ \delta_{1E}(1-\delta_1)H(X_{1n}|Y_1^{n-1}Y_2^{n-1}Z_1^{n-1}W) + (1-\delta_2)H(X_{2n}|Y_1^{n-1}Y_2^{n-1}Y_{1n}Z_1^nW) \tag{23}$$

$$= H(Y_1^{n-1}Y_2^{n-1}|Z_1^{n-1}W) - (1-\delta_{1E})I(Y_1^{n-1}Y_2^{n-1};X_{1n}|Z_1^{n-1}W) \tag{24}$$

$$+ \delta_{1E}(1-\delta_1)H(X_{1n}|Y_1^{n-1}Y_2^{n-1}Z_1^{n-1}W) \tag{25}$$

$$+ (1-\delta_2)H(X_{2n}|Y_1^{n-1}Y_2^{n-1}Z_1^{n-1}W) - (1-\delta_2)I(X_{2n};Y_{1n}Z_{1n}|Y_1^{n-1}Y_2^{n-1}Z_1^{n-1}W) \tag{26}$$

$$= H(Y_1^{n-1}Y_2^{n-1}|Z_1^{n-1}W) - (1-\delta_{1E})I(Y_1^{n-1}Y_2^{n-1};X_{1n}|Z_1^{n-1}W) \tag{27}$$

$$+ \delta_{1E}(1-\delta_1)H(X_{1n}|Y_1^{n-1}Y_2^{n-1}Z_1^{n-1}W) \tag{28}$$

$$+ (1-\delta_2)H(X_{2n}|Y_1^{n-1}Y_2^{n-1}Z_1^{n-1}W) \tag{29}$$

$$= \sum_{i=1}^{n} -(1-\delta_{1E})I(Y_1^{i-1}Y_2^{i-1};X_{1i}|Z_1^{i-1}W) + \delta_{1E}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{30}$$

$$+ (1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{31}$$

We have a similar inequality for the second eavesdropper.

$$0 \leq \sum_{i=1}^{n} -(1-\delta_{2E})I(Y_1^{i-1}Y_2^{i-1};X_{2i}|Z_2^{i-1}W) + \delta_{2E}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{32}$$

$$+ (1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{33}$$

We have two more inequalities of this kind.

$$0 \leq \sum_{i=1}^{n} -(1-\delta_{3E}) I(Y_1^{i-1} Y_3^{i-1}; X_{3i} | Z_3^{i-1} W) + \delta_{3E}(1-\delta_3) H(X_{3i} | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1} W) \quad (34)$$

$$+ (1-\delta_1) H(X_{1i} | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1} W) \quad (35)$$

$$0 \leq \sum_{i=1}^{n} -(1-\delta_{1E}) I(Y_1^{i-1} Y_3^{i-1}; X_{1i} | Z_1^{i-1} W) + \delta_{1E}(1-\delta_1) H(X_{1i} | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1} W) \quad (36)$$

$$+ (1-\delta_3) H(X_{3i} | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1} W) \quad (37)$$

D. Bounds from security

$$\sum_{i=1}^{n} I(X_{1i}; Y_1^{i-1} Y_2^{i-1} | Z_1^{i-1} W) = \sum_{i=1}^{n} I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1}) + I(X_{1i}; Y_1^{i-1} Y_2^{i-1} | Z_1^{i-1}) - I(X_{1i}; W | Z_1^{i-1}) \quad (38)$$

$$\geq \sum_{i=1}^{n} I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1}) - \epsilon \quad (39)$$

$$\sum_{i=1}^{n} I(X_{2i}; Y_1^{i-1} Y_2^{i-1} | Z_2^{i-1} W) \geq \sum_{i=1}^{n} I(X_{2i}; W | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1}) - \epsilon \quad (40)$$

$$\sum_{i=1}^{n} I(X_{1i}; Y_1^{i-1} Y_3^{i-1} | Z_1^{i-1} W) \geq \sum_{i=1}^{n} I(X_{1i}; W | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1}) - \epsilon \quad (41)$$

$$\sum_{i=1}^{n} I(X_{3i}; Y_1^{i-1} Y_3^{i-1} | Z_3^{i-1} W) \geq \sum_{i=1}^{n} I(X_{3i}; W | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1}) - \epsilon \quad (42)$$

(43)

E. Bounds on the number of transmissions

The following few bounds ensure that no more than n transmissions are needed.

$$n \geq \sum_{i=1}^{n} H(X_{1i}) \geq \sum_{i=1}^{n} H(X_{1i} | Y_1^{i-1} Y_2^{i-1}) = \sum_{i=1}^{n} H(X_{1i} | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1}) + I(X_{1i}; Z_2^{i-1} | Y_1^{i-1} Y_2^{i-1}) \quad (44)$$

$$= \sum_{i=1}^{n} H(X_{1i} | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1} W) + I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1}) + I(X_{1i}; Z_2^{i-1} | Y_1^{i-1} Y_2^{i-1}) \quad (45)$$

$$\geq \sum_{i=1}^{n} H(X_{1i} | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1} W) + I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1}) \quad (46)$$

$$n \geq \sum_{i=1}^{n} H(X_{2i} | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1} W) + I(X_{2i}; W | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1}) \quad (47)$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}) \geq \sum_{i=1}^{n} H(X_{1i} | Y_1^{i-1} Y_2^{i-1}) = \sum_{i=1}^{n} H(X_{1i} | Y_1^{i-1} Y_2^{i-1} W) + I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1}) \quad (48)$$

$$= \sum_{i=1}^{n} H(X_{1i} | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1} W) + I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1}) + I(X_{1i}; Z_1^{i-1} | Y_1^{i-1} Y_2^{i-1} W) \quad (49)$$

$$n \geq \sum_{i=1}^{n} H(X_{2i} | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1} W) + I(X_{2i}; W | Y_1^{i-1} Y_2^{i-1}) + I(X_{2i}; Z_1^{i-1} | Y_1^{i-1} Y_2^{i-1} W) \quad (50)$$

$$n \geq \sum_{i=1}^{n} H(X_{1i} | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1} W) + I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1}) \quad (51)$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}) \quad (52)$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \quad (53)$$

$$n \geq \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) + I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \quad (54)$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}) \quad (55)$$

$$n \geq \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) + I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}) \quad (56)$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}) \quad (57)$$

$$n \geq \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}) \quad (58)$$

F. Distinguishing keys

We distinguish keys that we use on channel 1 and on channel 3 based on where they were generated.

$$0 \leq H(Y_1^n|Z_1^nW) = \sum_{i=1}^{n} -(1-\delta_{1E})I(X_{1i};Y_1^{i-1}|Z_1^{i-1}W) + \delta_{1E}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Z_1^{i-1}W) \quad (59)$$

$$= \sum_{i=1}^{n} -(1-\delta_{1E})I(X_{1i};Y_1^{i-1}|Z_1^{i-1}W) + \delta_{1E}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) \quad (60)$$

$$+ \delta_{1E}(1-\delta_1)I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \quad (61)$$

A symmetric inequality holds for the other channel as well.

$$0 \leq \sum_{i=1}^{n} -(1-\delta_{3E})I(X_{3i};Y_3^{i-1}|Z_3^{i-1}W) + \delta_{3E}(1-\delta_3)H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \quad (62)$$

$$+ \delta_{3E}(1-\delta_3)I(X_{3i};Y_1^{i-1}|Y_3^{i-1}Z_3^{i-1}W) \quad (63)$$

Also,

$$\sum_{i=1}^{n} I(X_{1i};Y_1^{i-1}Y_3^{i-1}|Z_1^{i-1}W) = I(X_{1i};Y_1^{i-1}|Z_1^{i-1}W) + I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \quad (64)$$

$$\sum_{i=1}^{n} I(X_{3i};Y_1^{i-1}Y_3^{i-1}|Z_3^{i-1}W) = I(X_{3i};Y_3^{i-1}|Z_3^{i-1}W) + I(X_{3i};Y_1^{i-1}|Y_3^{i-1}Z_3^{i-1}W) \quad (65)$$

G. Connecting cuts

So far all our inequalities hold for either the first cut or the other cut. However, since we want to show that the cut values are not achievable we need to connect the two cuts through some more constraints.

$$0 \leq H(Y_2^n|Y_1^nY_3^nZ_1^nZ_3^nW) = H(Y_2^{n-1}|Y_1^nY_3^nZ_1^nZ_3^nW) + H(Y_{2n}|Y_1^nY_2^{n-1}Y_3^nZ_1^nZ_3^nW) \quad (66)$$

$$= H(Y_2^{n-1}|Y_1^{n-1}Y_3^{n-1}Z_1^{n-1}Z_3^{n-1}W) - I(Y_{1n}Z_{1n}Y_{3n}Z_{3n};Y_2^{n-1}|Y_1^{n-1}Y_3^{n-1}Z_1^{n-1}Z_3^{n-1}W) \quad (67)$$

$$+ H(Y_{2n}|Y_1^{n-1}Y_2^{n-1}Y_3^{n-1}Z_1^{n-1}Z_3^{n-1}W) \quad (68)$$

$$= \sum_{i=1}^{n} -(1-\delta_1\delta_{1E})I(X_{1i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \quad (69)$$

$$- (1-\delta_3\delta_{3E})I(X_{3i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) + (1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \quad (70)$$

Further,

$$0 \leq I(Z_1^n Y_1^n; Y_2^n | Y_3^n Z_3^n W) = I(Z_1^{n-1} Y_1^{n-1}; Y_2^n | Y_3^n Z_3^n W) + I(Z_{1n} Y_{1n}; Y_2^n | Y_1^{n-1} Y_3^n Z_1^{n-1} Z_3^n W) \tag{71}$$

$$= I(Z_1^{n-1} Y_1^{n-1}; Y_2^{n-1} | Y_3^n Z_3^n W) + I(Y_{2n}; Z_1^{n-1} Y_1^{n-1} | Y_2^{n-1} Y_3^n Z_3^n W) \tag{72}$$

$$+ I(Z_{1n} Y_{1n}; Y_2^{n-1} | Y_1^{n-1} Y_3^{n-1} Z_1^{n-1} Z_3^{n-1} W) \tag{73}$$

$$= I(Z_1^{n-1} Y_1^{n-1}; Y_2^{n-1} | Y_3^{n-1} Z_3^{n-1} W) - I(Y_{3n} Z_{3n}; Z_1^{n-1} Y_1^{n-1} | Y_3^{n-1} Z_3^{n-1} W) \tag{74}$$

$$+ I(Y_{2n}; Z_1^{n-1} Y_1^{n-1} | Y_2^{n-1} Y_3^{n-1} Z_3^{n-1} W) + I(Z_{1n} Y_{1n}; Y_2^{n-1} | Y_1^{n-1} Y_3^{n-1} Z_1^{n-1} Z_3^{n-1} W) \tag{75}$$

$$= \sum_{i=1}^{n} -(1 - \delta_3 \delta_{3E}) I(X_{3i}; Y_1^{i-1} | Y_3^{i-1} Z_3^{i-1} W) - (1 - \delta_3 \delta_{3E}) I(X_{3i}; Z_1^{i-1} | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1} W) \tag{76}$$

$$+ (1 - \delta_2) I(X_{2i}; Y_1^{i-1} | Y_2^{i-1} W) + (1 - \delta_2) I(X_{2i}; Z_1^{i-1} | Y_1^{i-1} Y_2^{i-1} W) \tag{77}$$

$$+ (1 - \delta_1 \delta_{1E}) I(X_{1i}; Y_2^{i-1} | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1} Z_3^{i-1} W) \tag{78}$$

In a similar way,

$$0 \leq I(Y_3^n; Y_2^n | Y_1^n Z_1^n W) = \sum_{i=1}^{n} -(1 - \delta_1 \delta_{1E}) I(X_{1i}; Y_3^{i-1} | Y_1^{i-1} Z_1^{i-1} W) \tag{79}$$

$$+ (1 - \delta_2) I(X_{2i}; Y_3^{i-1} | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1} W) + (1 - \delta_3) I(X_{3i}; Y_2^{i-1} | Y_3^{i-1} Y_1^{i-1} Z_1^{i-1} W) \tag{80}$$

$$= \sum_{i=1}^{n} -(1 - \delta_1 \delta_{1E}) I(X_{1i}; Y_3^{i-1} | Y_1^{i-1} Z_1^{i-1} W) \tag{81}$$

$$+ (1 - \delta_3) I(X_{3i}; Y_2^{i-1} | Y_3^{i-1} Y_1^{i-1} Z_1^{i-1} Z_3^{i-1} W) \tag{82}$$

$$+ (1 - \delta_3) I(X_{3i}; Z_3^{i-1} | Y_3^{i-1} Y_1^{i-1} Z_1^{i-1} W) - (1 - \delta_3) I(X_{3i}; Z_3^{i-1} | Y_2^{i-1} Y_3^{i-1} Y_1^{i-1} Z_1^{i-1} W) \tag{83}$$

We further have

$$H(Z_3^n | Y_3^n Y_1^n Z_1^n W) = \sum_{i=1}^{n} \delta_3 (1 - \delta_{3E}) H(X_{3i} | Y_3^{i-1} Y_1^{i-1} Z_3^{i-1} Z_1^{i-1} W) \tag{84}$$

$$- (1 - \delta_1 \delta_{1E}) I(X_{1i}; Z_3^{i-1} | Y_3^{i-1} Y_1^{i-1} Z_1^{i-1} W) - (1 - \delta_3) I(X_{3i}; Z_3^{i-1} | Y_3^{i-1} Y_1^{i-1} Z_1^{i-1} W) \tag{85}$$

$$H(Z_3^n | Y_3^n Y_2^n Y_1^n Z_1^n W) = \sum_{i=1}^{n} \delta_3 (1 - \delta_{3E}) H(X_{3i} | Y_1^{i-1} Y_2^{i-1} Y_3^{i-1} Z_3^{i-1} Z_1^{i-1} W) \tag{86}$$

$$- (1 - \delta_3) I(X_{3i}; Z_3^{i-1} | Y_1^{i-1} Y_2^{i-1} Y_3^{i-1} Z_1^{i-1} W) \tag{87}$$

And hence

$$0 \leq I(Z_3^n; Y_2^n | Y_1^n Y_3^n Z_1^n W) = \sum_{i=1}^{n} \delta_3 (1 - \delta_{3E}) I(X_{3i}; Y_2^{i-1} | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1} Z_1^{i-1} W) \tag{88}$$

$$- (1 - \delta_1 \delta_{1E}) I(X_{1i}; Z_3^{i-1} | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1} W) - (1 - \delta_3) I(X_{3i}; Z_3^{i-1} | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1} W) \tag{89}$$

$$+ (1 - \delta_3) I(X_{3i}; Z_3^{i-1} | Y_1^{i-1} Y_2^{i-1} Y_3^{i-1} Z_1^{i-1} W) \tag{90}$$

Also,

$$0 \leq H(Y_1^n | Y_2^n W) = \sum_{i=1}^{n} (1 - \delta_1) H(X_{1i} | Y_1^{i-1} Y_2^{i-1} W) - (1 - \delta_2) I(X_{2i}; Y_1^{i-1} | Y_2^{i-1} W) \tag{91}$$

$$= \sum_{i=1}^{n} (1 - \delta_1) H(X_{1i} | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1} W) + (1 - \delta_1) I(X_{1i}; Z_1^{i-1} | Y_1^{i-1} Y_2^{i-1} W) \tag{92}$$

$$- (1 - \delta_2) I(X_{2i}; Y_1^{i-1} | Y_2^{i-1} W) \tag{93}$$

Finally,

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) = \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \tag{94}$$

$$= \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) - I(X_{1i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \tag{95}$$

H. Trivial constraints

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) = \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) - I(X_{1i};Z_3^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) \tag{96}$$

$$\sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) = \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) - I(X_{3i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{97}$$

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) = \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) - I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{98}$$

$$\sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) = \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) - I(X_{3i};Z_3^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) \tag{99}$$

$$\sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \geq \sum_{i=1}^{n} I(X_{3i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \tag{100}$$

III. Reduction

We apply a series of reduction steps on the outer bound linear program until we reach the same program as the scheme program. We ensure that during reduction the optimal value of the program does not decrease, however the feasibility region might shrink. Our possible reduction steps are:

1) Renaming of variables. By any renaming we apply we make sure that the introduced new variables are also nonnegative.
2) Eliminating variables. We apply the well known Fourier-Motzkin elimination to reduce the number of variables.
3) Introducing constraints. We introduce constraints that do not follow from the inequalities in the linear program. By this we reduce the feasibility region but do not reduce the optimal value of the program. Our arguments show that if in an optimal point the given constraint is not satisfied, we can apply a transform on the variables without violating any constraints to arrive to another optimal point, where the constraint is satisfied. We conclude that introducing the constraint in question does not restrict the value of the program.
4) Dropping constraints. Obviously by dropping some constraints we cannot decrease the value of the program.
5) Deriving constraints. We add constraints that follow from existing constraints.

In general, we do not introduce naming on the variables, but from now we do not use any properties of the information terms except that they are nonnegative. We replace some terms with a named variable to help readability or when there is a corresponding variable in the scheme program. We omit the small ϵ terms that do not influence the asymptotic result. Our outer bound linear program is the following.

Summary of the outer bound program:

$$nR \leq \sum_{i=1}^{n} (1-\delta_1) I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1}) + (1-\delta_2) I(X_{2i}; W | Y_1^{i-1} Y_2^{i-1}) \tag{101}$$

$$nR \leq \sum_{i=1}^{n} (1-\delta_1\delta_{1E}) I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1}) + (1-\delta_2) I(X_{2i}; W | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1}) \tag{102}$$

$$nR \leq \sum_{i=1}^{n} (1-\delta_2\delta_{2E}) I(X_{2i}; W | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1}) + (1-\delta_1) I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1}) \tag{103}$$

$$nR \leq \sum_{i=1}^{n} (1-\delta_1) I(X_{1i}; W | Y_1^{i-1} Y_3^{i-1}) + (1-\delta_3) I(X_{3i}; W | Y_1^{i-1} Y_3^{i-1}) \tag{104}$$

$$nR \leq \sum_{i=1}^{n} (1-\delta_1\delta_{1E}) I(X_{1i}; W | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1}) + (1-\delta_3) I(X_{3i}; W | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1}) \tag{105}$$

$$nR \leq \sum_{i=1}^{n} (1-\delta_3\delta_{3E}) I(X_{3i}; W | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1}) + (1-\delta_1) I(X_{1i}; W | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1}) \tag{106}$$

$$0 \leq \sum_{i=1}^{n} -(1-\delta_{1E}) I(Y_1^{i-1} Y_2^{i-1}; X_{1i} | Z_1^{i-1} W) + \delta_{1E}(1-\delta_1) H(X_{1i} | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1} W)$$
$$+ (1-\delta_2) H(X_{2i} | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1} W) \tag{107}$$

$$0 \leq \sum_{i=1}^{n} -(1-\delta_{2E}) I(Y_1^{i-1} Y_2^{i-1}; X_{2i} | Z_2^{i-1} W) + \delta_{2E}(1-\delta_2) H(X_{2i} | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1} W)$$
$$+ (1-\delta_1) H(X_{1i} | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1} W) \tag{108}$$

$$0 \leq \sum_{i=1}^{n} -(1-\delta_{3E}) I(Y_1^{i-1} Y_3^{i-1}; X_{3i} | Z_3^{i-1} W) + \delta_{3E}(1-\delta_3) H(X_{3i} | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1} W)$$
$$+ (1-\delta_1) H(X_{1i} | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1} W) \tag{109}$$

$$0 \leq \sum_{i=1}^{n} -(1-\delta_{1E}) I(Y_1^{i-1} Y_3^{i-1}; X_{1i} | Z_1^{i-1} W) + \delta_{1E}(1-\delta_1) H(X_{1i} | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1} W)$$
$$+ (1-\delta_3) H(X_{3i} | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1} W) \tag{110}$$

$$\sum_{i=1}^{n} I(X_{1i}; Y_1^{i-1} Y_2^{i-1} | Z_1^{i-1} W) \geq \sum_{i=1}^{n} I(X_{1i}; W | Y_1^{i-1} Y_2^{i-1} Z_1^{i-1}) \tag{111}$$

$$\sum_{i=1}^{n} I(X_{2i}; Y_1^{i-1} Y_2^{i-1} | Z_2^{i-1} W) \geq \sum_{i=1}^{n} I(X_{2i}; W | Y_1^{i-1} Y_2^{i-1} Z_2^{i-1}) \tag{112}$$

$$\sum_{i=1}^{n} I(X_{1i}; Y_1^{i-1} Y_3^{i-1} | Z_1^{i-1} W) \geq \sum_{i=1}^{n} I(X_{1i}; W | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1}) \tag{113}$$

$$\sum_{i=1}^{n} I(X_{3i}; Y_1^{i-1} Y_3^{i-1} | Z_3^{i-1} W) \geq \sum_{i=1}^{n} I(X_{3i}; W | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1}) \tag{114}$$

$$\sum_{i=1}^{n} (1-\delta_{1E}) I(X_{1i}; Y_1^{i-1} | Z_1^{i-1} W) \leq \sum_{i=1}^{n} \delta_{1E}(1-\delta_1) H(X_{1i} | Y_1^{i-1} Y_3^{i-1} Z_1^{i-1} W)$$
$$+ \delta_{1E}(1-\delta_1) I(X_{1i}; Y_3^{i-1} | Y_1^{i-1} Z_1^{i-1} W) \tag{115}$$

$$\sum_{i=1}^{n} (1-\delta_{3E}) I(X_{3i}; Y_3^{i-1} | Z_3^{i-1} W) \leq \sum_{i=1}^{n} \delta_{3E}(1-\delta_3) H(X_{3i} | Y_1^{i-1} Y_3^{i-1} Z_3^{i-1} W)$$
$$+ \delta_{3E}(1-\delta_3) I(X_{3i}; Y_1^{i-1} | Y_3^{i-1} Z_3^{i-1} W) \tag{116}$$

$$\sum_{i=1}^{n} I(X_{1i};Y_1^{i-1}Y_3^{i-1}|Z_1^{i-1}W) = \sum_{i=1}^{n} I(X_{1i};Y_1^{i-1}|Z_1^{i-1}W) + I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \tag{117}$$

$$\sum_{i=1}^{n} I(X_{3i};Y_1^{i-1}Y_3^{i-1}|Z_3^{i-1}W) = \sum_{i=1}^{n} I(X_{3i};Y_3^{i-1}|Z_3^{i-1}W) + I(X_{3i};Y_1^{i-1}|Y_3^{i-1}Z_3^{i-1}W) \tag{118}$$

$$\begin{aligned} 0 \le \sum_{i=1}^{n} & -(1-\delta_1\delta_{1E})I(X_{1i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \\ & -(1-\delta_3\delta_{3E})I(X_{3i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) + (1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \end{aligned} \tag{119}$$

$$\begin{aligned} 0 \le \sum_{i=1}^{n} & -(1-\delta_3\delta_{3E})I(X_{3i};Y_1^{i-1}|Y_3^{i-1}Z_3^{i-1}W) - (1-\delta_3\delta_{3E})I(X_{3i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \\ & + (1-\delta_2)I(X_{2i};Y_1^{i-1}|Y_2^{i-1}W) + (1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \\ & + (1-\delta_1\delta_{1E})I(X_{1i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \end{aligned} \tag{120}$$

$$\begin{aligned} 0 \le \sum_{i=1}^{n} & -(1-\delta_1\delta_{1E})I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \\ & + (1-\delta_3)I(X_{3i};Y_2^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}Z_3^{i-1}W) \\ & + (1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) - (1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_2^{i-1}Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \end{aligned} \tag{121}$$

$$\begin{aligned} 0 \le \sum_{i=1}^{n} & \delta_3(1-\delta_{3E})I(X_{3i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}Z_1^{i-1}W) \\ & -(1-\delta_1\delta_{1E})I(X_{1i};Z_3^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) - (1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) \\ & + (1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_1^{i-1}Y_2^{i-1}Y_3^{i-1}Z_1^{i-1}W) \end{aligned} \tag{122}$$

$$\begin{aligned} 0 \le \sum_{i=1}^{n} & (1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + (1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \\ & -(1-\delta_2)I(X_{2i};Y_1^{i-1}|Y_2^{i-1}W) \end{aligned} \tag{123}$$

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) = \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) - I(X_{1i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \tag{124}$$

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) = \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) - I(X_{1i};Z_3^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) \tag{125}$$

$$\sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) = \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) - I(X_{3i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{126}$$

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) = \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) - I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{127}$$

$$\sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) = \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) - I(X_{3i};Z_3^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) \tag{128}$$

$$\sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \ge \sum_{i=1}^{n} I(X_{3i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \tag{129}$$

$$n \ge \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{130}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{131}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{132}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{133}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{134}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}) \tag{135}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{136}$$

$$n \geq \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) + I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{137}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}) \tag{138}$$

$$n \geq \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) + I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}) \tag{139}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}) \tag{140}$$

$$n \geq \sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}) \tag{141}$$

We apply (111)-(112) in (107)-(108) and keep the resulting inequalities while dropping (111)-(112) and (107)-(108).

We show that (113)-(114) both can be made equalities without reducing the value of the program. If (113) is not equality, we apply the following transform (for some $\Delta > 0$):

$$\sum_{i=1}^{n} I(X_{1i};Y_1^{i-1}Y_3^{i-1}|Z_1^{i-1}W) \downarrow \Delta \tag{142}$$

$$\sum_{i=1}^{n} I(X_{1i};Y_1^{i-1}|Z_1^{i-1}W) \downarrow \Delta. \tag{143}$$

This transform either makes (113) tight or $\sum_{i=1}^{n} I(X_{1i};Y_1^{i-1}|Z_1^{i-1}W)$ becomes 0. No constraints are violated. If the latter happens, we can do the following transform:

$$\sum_{i=1}^{n} I(X_{1i};Y_1^{i-1}Y_3^{i-1}|Z_1^{i-1}W) \downarrow \Delta \tag{144}$$

$$\sum_{i=1}^{n} I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \downarrow \Delta. \tag{145}$$

Note that (115) cannot be violated, since the RHS is already 0. Eventually, this transform makes (113) tight. A similar argument shows that also (114) can be made tight, which allows to eliminate the variables $\sum_{i=1}^{n} I(X_{1i};Y_1^{i-1}Y_3^{i-1}|Z_1^{i-1}W)$ and $\sum_{i=1}^{n} I(X_{3i};Y_1^{i-1}Y_3^{i-1}|Z_3^{i-1}W)$.

Using equalities (125)-(128) we eliminate $\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W)$, $\sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)$, $\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)$ and $\sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W)$.

Further, we introduce the following naming:

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \sim k_1 \tag{146}$$

$$\sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \sim k_2 \tag{147}$$

$$\sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}) \sim m_1 \tag{148}$$

$$\sum_{i=1}^{n} I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}) \sim m_2 \tag{149}$$

$$\sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}) \sim m_3 \tag{150}$$

$$\sum_{i=1}^{n} I(X_{1i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \sim \bar{s_1} \tag{151}$$

$$\sum_{i=1}^{n} I(X_{3i};Y_2^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \sim \bar{s_3} \tag{152}$$

$$\sum_{i=1}^{n} I(X_{1i};Z_3^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) \sim \bar{\ell_1} \tag{153}$$

$$\sum_{i=1}^{n} I(X_{3i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \sim \bar{\ell_3} \tag{154}$$

$$\sum_{i=1}^{n} H(X_{3i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \sim k_3 + \bar{s_3} \tag{155}$$

$$\sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \sim \bar{x}_{121} \tag{156}$$

$$\sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \sim \bar{x}_{131} \tag{157}$$

$$\sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \sim \bar{x}_{313} \tag{158}$$

$$\sum_{i=1}^{n} I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \sim \bar{x}_{22} \tag{159}$$

$$\sum_{i=1}^{n} I(X_{3i};Z_3^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}W) \sim \bar{z_3} \tag{160}$$

We distinguish the variable names used only in the outer bound program with overscore. Recall that in the following the unnamed terms are also treated as nonnegative variables.

We get the following:

$$nR \leq (1-\delta_2)m_2 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{161}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{162}$$

$$nR \leq (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{163}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{164}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{165}$$

$$nR \leq (1-\delta_3\delta_{3E})\bar{x}_{313} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{166}$$

$$0 \leq \delta_{1E}(1-\delta_1)k_1 - (1-\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{167}$$

$$0 \leq \delta_{2E}(1-\delta_2)k_2 - (1-\delta_{2E})\bar{x}_{22} + \sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{168}$$

$$0 \leq \delta_{3E}(1-\delta_3)\bar{\ell}_3 + \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} - (1-\delta_{3E})\bar{x}_{313} \tag{169}$$

$$+\sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) + (1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{170}$$

$$0 \leq \delta_{1E}(1-\delta_1)\bar{\ell}_1 + (1-\delta_3)k_3 + (1-\delta_3)\bar{s_3} - (1-\delta_{1E})\bar{x}_{131} + (1-\delta_3)\bar{z_3}+ \tag{171}$$

$$\sum_{i=1}^{n}\delta_{1E}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \tag{172}$$

$$\sum_{i=1}^{n}(1-\delta_{1E})I(X_{1i};Y_1^{i-1}|Z_1^{i-1}W) \leq \delta_{1E}(1-\delta_1)\bar{\ell}_1 + \sum_{i=1}^{n}\delta_{1E}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \tag{173}$$

$$+\delta_{1E}(1-\delta_1)I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \tag{174}$$

$$\sum_{i=1}^{n}(1-\delta_{3E})I(X_{3i};Y_3^{i-1}|Z_3^{i-1}W) \leq \delta_{3E}(1-\delta_3)\bar{\ell}_3 + \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} \tag{175}$$

$$+\sum_{i=1}^{n}\delta_{3E}(1-\delta_3)I(X_{3i};Y_1^{i-1}|Y_3^{i-1}Z_3^{i-1}W) \tag{176}$$

$$\bar{x}_{131} = \sum_{i=1}^{n}I(X_{1i};Y_1^{i-1}|Z_1^{i-1}W) + I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \tag{177}$$

$$\bar{x}_{313} = \sum_{i=1}^{n}I(X_{3i};Y_3^{i-1}|Z_3^{i-1}W) + I(X_{3i};Y_1^{i-1}|Y_3^{i-1}Z_3^{i-1}W) \tag{178}$$

$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{179}$$

$$(1-\delta_3\delta_{3E})\bar{\ell}_3 \leq (1-\delta_1\delta_{1E})\bar{s_1} + \sum_{i=1}^{n}-(1-\delta_3\delta_{3E})I(X_{3i};Y_1^{i-1}|Y_3^{i-1}Z_3^{i-1}W) \tag{180}$$

$$+(1-\delta_2)I(X_{2i};Y_1^{i-1}|Y_2^{i-1}W) + (1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{181}$$

$$0 \leq (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}-(1-\delta_1\delta_{1E})I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \tag{182}$$

$$+(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) - (1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_2^{i-1}Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{183}$$

$$(1-\delta_1\delta_{1E})\bar{\ell}_1 \leq \delta_3(1-\delta_{3E})\bar{s_3} - (1-\delta_3)\bar{z_3} \tag{184}$$

$$+\sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_1^{i-1}Y_2^{i-1}Y_3^{i-1}Z_1^{i-1}W) \tag{185}$$

$$0 \leq (1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) - (1-\delta_2)I(X_{2i};Y_1^{i-1}|Y_2^{i-1}W) \tag{186}$$

$$\bar{s_1} + k_1 = \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \tag{187}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{188}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{189}$$

$$n \geq k_1 + \sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{190}$$

$$n \geq m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{191}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{192}$$

$$n \geq m_2 + k_2 \tag{193}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{194}$$

$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{195}$$

$$n \geq m_1 + \bar{\ell_1} + \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) \tag{196}$$

$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{197}$$

$$n \geq m_1 + \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{198}$$

$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{199}$$

We use eqalities (177)-(178) and (187) to eliminate $\sum_{i=1}^{n} I(X_{1i};Y_1^{i-1}|Z_1^{i-1}W)$, $\sum_{i=1}^{n} I(X_{3i};Y_3^{i-1}|Z_3^{i-1}W)$ and $\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}Z_3^{i-1}W)$.

$$nR \leq (1-\delta_2)m_2 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{200}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{201}$$

$$nR \leq (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{202}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{203}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{204}$$

$$nR \leq (1-\delta_3\delta_{3E})\bar{x}_{313} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{205}$$

$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{206}$$

$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + \sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{207}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)\bar{\ell_3} + \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} \tag{208}$$
$$+ \sum_{i=1}^{n}(1-\delta_1)\bar{s_1} + (1-\delta_1)k_1 + (1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{209}$$
$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{210}$$
$$+ (1-\delta_3)k_3 + (1-\delta_3)\bar{s_3} + (1-\delta_3)\bar{z_3} \tag{211}$$
$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{212}$$
$$+ \sum_{i=1}^{n}(1-\delta_1\delta_{1E})I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \tag{213}$$
$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)\bar{\ell_3} + \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} \tag{214}$$
$$+ \sum_{i=1}^{n}(1-\delta_3\delta_{3E})I(X_{3i};Y_1^{i-1}|Y_3^{i-1}Z_3^{i-1}W) \tag{215}$$
$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{216}$$
$$(1-\delta_3\delta_{3E})\bar{\ell_3} \leq (1-\delta_1\delta_{1E})\bar{s_1} + \sum_{i=1}^{n} -(1-\delta_3\delta_{3E})I(X_{3i};Y_1^{i-1}|Y_3^{i-1}Z_3^{i-1}W) \tag{217}$$
$$+ (1-\delta_2)I(X_{2i};Y_1^{i-1}|Y_2^{i-1}W) + (1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{218}$$
$$0 \leq (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n} -(1-\delta_1\delta_{1E})I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \tag{219}$$
$$+ (1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) - (1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_2^{i-1}Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{220}$$
$$(1-\delta_1\delta_{1E})\bar{\ell_1} \leq \delta_3(1-\delta_{3E})\bar{s_3} - (1-\delta_3)\bar{z_3} \tag{221}$$
$$+ \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_1^{i-1}Y_2^{i-1}Y_3^{i-1}Z_1^{i-1}W) \tag{222}$$
$$0 \leq (1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) - (1-\delta_2)I(X_{2i};Y_1^{i-1}|Y_2^{i-1}W) \tag{223}$$
$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{224}$$
$$n \geq k_1 + \sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{225}$$
$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{226}$$
$$n \geq \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{227}$$
$$n \geq m_1 + \bar{\ell_1} + \bar{s_1} + k_1 \tag{228}$$
$$n \geq m_1 + \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{229}$$
$$n \geq m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{230}$$
$$n \geq m_2 + k_2 \tag{231}$$
$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{232}$$
$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{233}$$

$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \quad (234)$$

$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \quad (235)$$

We apply (218) in (215) and keep only the derived inequality.

$$nR \leq (1-\delta_2)m_2 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) \quad (236)$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \quad (237)$$

$$nR \leq (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \quad (238)$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \quad (239)$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \quad (240)$$

$$nR \leq (1-\delta_3\delta_{3E})\bar{x}_{313} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \quad (241)$$

$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \quad (242)$$

$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + \sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \quad (243)$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)\bar{\ell_3} + \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} \quad (244)$$

$$+ \sum_{i=1}^{n}(1-\delta_1)\bar{s_1} + (1-\delta_1)k_1 + (1-\delta_1)I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \quad (245)$$

$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \quad (246)$$

$$+ (1-\delta_3)k_3 + (1-\delta_3)\bar{s_3} + (1-\delta_3)\bar{z_3} \quad (247)$$

$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \quad (248)$$

$$+ \sum_{i=1}^{n}(1-\delta_1\delta_{1E})I(X_{1i}; Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \quad (249)$$

$$(1-\delta_{3E})\bar{x}_{313} \leq (1-\delta_1\delta_{1E})\bar{s_1} - (1-\delta_{3E})\bar{\ell_3} \quad (250)$$

$$+ \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i}; Y_1^{i-1}|Y_2^{i-1}W) \quad (251)$$

$$+ (1-\delta_2)I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \quad (252)$$

$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \quad (253)$$

$$0 \leq (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n} -(1-\delta_1\delta_{1E})I(X_{1i}; Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \quad (254)$$

$$+ (1-\delta_3)I(X_{3i}; Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) - (1-\delta_3)I(X_{3i}; Z_3^{i-1}|Y_2^{i-1}Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \quad (255)$$

$$(1-\delta_1\delta_{1E})\bar{\ell_1} \leq \delta_3(1-\delta_{3E})\bar{s_3} - (1-\delta_3)\bar{z_3} \quad (256)$$

$$+ \sum_{i=1}^{n}(1-\delta_3)I(X_{3i}; Z_3^{i-1}|Y_1^{i-1}Y_2^{i-1}Y_3^{i-1}Z_1^{i-1}W) \quad (257)$$

$$0 \leq (1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) - (1-\delta_2)I(X_{2i}; Y_1^{i-1}|Y_2^{i-1}W)) \quad (258)$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{259}$$

$$n \geq k_1 + \sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{260}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{261}$$

$$n \geq \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{262}$$

$$n \geq m_1 + \bar{\ell_1} + \bar{s_1} + k_1 \tag{263}$$

$$n \geq m_1 + \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{264}$$

$$n \geq m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{265}$$

$$n \geq m_2 + k_2 \tag{266}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{267}$$

$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{268}$$

$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{269}$$

$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{270}$$

We observe that (258) can be made tight by increasing $\sum_{i=1}^{n} I(X_{2i};Y_1^{i-1}|Y_2^{i-1}W))$. We apply the resulting equality in (252). At the same time we eliminate the variable $\sum_{i=1}^{n} I(X_{3i};Z_3^{i-1}|Y_2^{i-1}Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W)$ from (255)-(257).

$$nR \leq (1-\delta_2)m_2 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{271}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{272}$$

$$nR \leq (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{273}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{274}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{275}$$

$$nR \leq (1-\delta_3\delta_{3E})\bar{x}_{313} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{276}$$

$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{277}$$

$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + \sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{278}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)\bar{\ell_3} + \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} \tag{279}$$

$$\sum_{i=1}^{n} +(1-\delta_1)\bar{s_1} + (1-\delta_1)k_1 + (1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{280}$$

$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{281}$$
$$+ (1-\delta_3)k_3 + (1-\delta_3)\bar{s_3} + (1-\delta_3)\bar{z_3} \tag{282}$$
$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{283}$$
$$+ \sum_{i=1}^{n}(1-\delta_1\delta_{1E})I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \tag{284}$$
$$(1-\delta_{3E})\bar{x}_{313} \leq (1-\delta_1\delta_{1E})\bar{s_1} - (1-\delta_{3E})\bar{\ell_3} + (1-\delta_1)k_1 \tag{285}$$
$$+ \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{286}$$
$$+ (1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{287}$$
$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{288}$$
$$\sum_{i=1}^{n}(1-\delta_1\delta_{1E})I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \leq (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{289}$$
$$\sum_{i=1}^{n}(1-\delta_1\delta_{1E})I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W) \leq (1-\delta_3\delta_{3E})\bar{s_3} - (1-\delta_1\delta_{1E})\bar{\ell_1} \tag{290}$$
$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{291}$$
$$n \geq k_1 + \sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{292}$$
$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{293}$$
$$n \geq \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{294}$$
$$n \geq m_1 + \bar{\ell_1} + \bar{s_1} + k_1 \tag{295}$$
$$n \geq m_1 + \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{296}$$
$$n \geq m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{297}$$
$$n \geq m_2 + k_2 \tag{298}$$
$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{299}$$
$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{300}$$
$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{301}$$
$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{302}$$

We eliminate $\sum_{i=1}^{n} I(X_{1i};Y_3^{i-1}|Y_1^{i-1}Z_1^{i-1}W)$. (282) becomes redundant.

$$nR \leq (1-\delta_2)m_2 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{303}$$
$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{304}$$

$$nR \leq (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{305}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{306}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{307}$$

$$nR \leq (1-\delta_3\delta_{3E})\bar{x}_{313} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{308}$$

$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{309}$$

$$(1-\delta_{1E})\bar{x}_{131} \leq -(1-\delta_{1E})\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{310}$$

$$+ (1-\delta_3\delta_{3E})\bar{s_3} \tag{311}$$

$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{312}$$

$$+ (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{313}$$

$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + \sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{314}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)\bar{\ell_3} + (1-\delta_1)\bar{s_1} + (1-\delta_1)k_1 \tag{315}$$

$$+ \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{316}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq (1-\delta_1\delta_{1E})\bar{s_1} - (1-\delta_{3E})\bar{\ell_3} + (1-\delta_1)k_1 \tag{317}$$

$$+ \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{318}$$

$$+ (1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{319}$$

$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{320}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \tag{321}$$

$$n \geq k_1 + \sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{322}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{323}$$

$$n \geq \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{324}$$

$$n \geq m_1 + \bar{\ell_1} + \bar{s_1} + k_1 \tag{325}$$

$$n \geq m_1 + \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{326}$$

$$n \geq m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{327}$$

$$n \geq m_2 + k_2 \tag{328}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{329}$$

$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{330}$$

$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{331}$$

$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{332}$$

We show that the we can assume that the RHS of (313) can be made smaller than the RHS of (311). If this assumption is not true at an optimal point and $\bar{\ell_1} > 0$ we can decrease $\bar{\ell_1}$ until the assumption becomes true or $\bar{\ell_1} = 0$ without violating any constraints. If $\bar{\ell_1} = 0$, then we can decrease the value of $\sum_{i=1}^{n} I(X_{3i}; Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W)$ until the assumption holds. Note that we reach equality between the RHS of (313) and the RHS of (311) before $\sum_{i=1}^{n} I(X_{3i}; Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W)$ reaches 0. Thus,

$$(1-\delta_1\delta_{1E})\bar{\ell_1} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i}; Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \leq \delta_3(1-\delta_{3E})\bar{s_3} \tag{333}$$

can be assumed. Given this (311) can be dropped. We can apply the same argument on (316)-(319) with reducing first $\bar{\ell_3}$ and then if needed $\sum_{i=1}^{n} I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)$. We thus replace (319) with the following inequality:

$$(1-\delta_3\delta_{3E})\bar{\ell_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \leq \tag{334}$$

$$\delta_1(1-\delta_{1E})\bar{s_1} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) + (1-\delta_2)I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W). \tag{335}$$

Independently of this transform, we observe that $\sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1})$ and $\sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1})$ can be assumed to be equal. If at an optimal point $\sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) > \sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1})$, then $\sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1})$ can be increased until equality holds without violating any constraints. In case $\sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) < \sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1})$ then we can do the following transform for some $\Delta > 0$, until equality holds:

$$\sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}) \downarrow \frac{\Delta}{1-\delta_1} \tag{336}$$

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \uparrow \frac{\Delta}{1-\delta_1} \tag{337}$$

$$\bar{x}_{22} \uparrow \frac{\Delta}{1-\delta_{2E}}. \tag{338}$$

No constraints are violated by this transform. Note that if (323) becomes equality then $\sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) \geq \sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1})$ follows, while the RHS of (305) is increasing by the transform.

$$nR \leq (1-\delta_2)m_2 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) \tag{339}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{340}$$

$$nR \leq (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) \tag{341}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{342}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{343}$$

$$nR \leq (1-\delta_3\delta_{3E})\bar{x}_{313} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{344}$$

$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{345}$$

$$(1-\delta_1\delta_{1E})\bar{\ell_1} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \leq \delta_3(1-\delta_{3E})\bar{s_3} \tag{346}$$

$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{347}$$

$$+ (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{348}$$

$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + \sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{349}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)\bar{\ell_3} + (1-\delta_1)\bar{s_1} + (1-\delta_1)k_1 \tag{350}$$

$$+ \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{351}$$

$$(1-\delta_3\delta_{3E})\bar{\ell_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \leq \tag{352}$$

$$\delta_1(1-\delta_{1E})\bar{s_1} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) + (1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{353}$$

$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{354}$$

$$n \geq k_1 + \sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{355}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{356}$$

$$n \geq \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{357}$$

$$n \geq m_1 + \bar{\ell_1} + \bar{s_1} + k_1 \tag{358}$$

$$n \geq m_1 + \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{359}$$

$$n \geq m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{360}$$

$$n \geq m_2 + k_2 \tag{361}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{362}$$

$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{363}$$

$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{364}$$

$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{365}$$

We observe that

$$m_1 \leq \sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \tag{366}$$

$$m_3 \leq \sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{367}$$

can be assumed. If these are not true at an optimal point then we could increase $\sum_{i=1}^n I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1})$ and/or $\sum_{i=1}^n I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1})$ until they both hold. No constraints can be violated by this increase.

We next show that (366) can be made equality. We do the following transform (T1) ($\Delta > 0$):

$$\bar{\ell}_1 \downarrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{368}$$

$$\bar{s_3} \downarrow \frac{\Delta}{\delta_3(1-\delta_{3E})} \tag{369}$$

$$\bar{s_1} \uparrow \frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})} \tag{370}$$

$$\sum_{i=1}^n I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \uparrow \frac{\Delta(1-\delta_3\delta_{3E})\delta_1(1-\delta_{1E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})(1-\delta_1)} \tag{371}$$

$$\bar{x}_{131} \downarrow \frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})} \tag{372}$$

$$\bar{x}_{313} \uparrow \frac{\Delta}{\delta_3(1-\delta_{3E})} \tag{373}$$

$$\sum_{i=1}^n I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \downarrow \frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1)} \tag{374}$$

$$m_1 \downarrow \frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})(1-\delta_1)} \tag{375}$$

$$m_3 \uparrow \frac{\Delta}{\delta_3(1-\delta_{3E})} \tag{376}$$

$$\sum_{i=1}^n I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \uparrow \frac{\Delta}{\delta_3(1-\delta_{3E})} \tag{377}$$

We give a side-calculation in Appendix A-A to help verifying that the transform does not violate any constraints. If (366) is not yet equality, we can perform this transform unless any of the variables the transform decreases already equals 0 or (359) is equality. In the latter case $m_1 \geq \sum_{i=1}^n I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1})$ follows from (357) which implies that (366) is equality. Otherwise, we have the following cases:

1) $\bar{\ell}_1 = 0$. In this case m_1 can be increased until (366) is equality without violating any constraints.
2) $\bar{s_3} = 0$. In this case (346) implies $\bar{\ell}_1 = 0$ and the first case applies.
3) $\bar{x}_{131} = 0$. In this case $\bar{\ell}_1$ can be decreased until it equals 0 without violating any constraints. Then, the first case applies.
4) $\sum_{i=1}^n I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) = 0$. In this case (366) implies that $m_1 = 0$ and hence (366) is equality.
5) $m_1 = 0$. In this case from (367) it follows that the RHS of (342) is strictly smaller than the RHS of (343) unless $\bar{x}_{131} = 0$. Hence, $\bar{x}_{131}$ can be decreased to 0 without violating any constraints and thus case 3 applies.

We have shown that assuming $m_1 = \sum_{i=1}^n I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1})$ does not restrict the value of the program. We now have the following program:

$$nR \leq (1-\delta_2)m_2 + \sum_{i=1}^n (1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{378}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^n (1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{379}$$

$$nR \leq (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^n (1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{380}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{381}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{382}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{383}$$

$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{384}$$

$$(1-\delta_1\delta_{1E})\bar{\ell_1} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \leq \delta_3(1-\delta_{3E})\bar{s_3} \tag{385}$$

$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{386}$$

$$+ (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{387}$$

$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + \sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{388}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)\bar{\ell_3} + (1-\delta_1)\bar{s_1} + (1-\delta_1)k_1 \tag{389}$$

$$+ \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{390}$$

$$(1-\delta_3\delta_{3E})\bar{\ell_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \leq \tag{391}$$

$$\delta_1(1-\delta_{1E})\bar{s_1} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) + (1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{392}$$

$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{393}$$

$$m_3 \leq \sum_{i=1}^{n}I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{394}$$

$$n \geq k_1 + \sum_{i=1}^{n}I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{395}$$

$$n \geq \sum_{i=1}^{n}H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{396}$$

$$n \geq m_1 + \bar{s_1} + k_1 + \sum_{i=1}^{n}I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{397}$$

$$n \geq m_1 + \bar{\ell_1} + \bar{s_1} + k_1 \tag{398}$$

$$n \geq m_2 + \sum_{i=1}^{n}H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{399}$$

$$n \geq m_2 + k_2 \tag{400}$$

$$n \geq \sum_{i=1}^{n}H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{401}$$

$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{402}$$

$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n}I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{403}$$

$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{404}$$

We show that $\bar{\ell_1} \le \sum_{i=1}^n I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)$ can be assumed. Do the following transform:

$$\bar{\ell_1} \downarrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{405}$$

$$\bar{s_3} \downarrow \frac{\Delta}{\delta_3(1-\delta_{3E})} \tag{406}$$

$$\bar{s_1} \uparrow \frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})} \tag{407}$$

$$\sum_{i=1}^n I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \uparrow \frac{\Delta(1-\delta_3\delta_{3E})\delta_1(1-\delta_{1E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})(1-\delta_1)} \tag{408}$$

$$\bar{x}_{131} \downarrow \frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})} \tag{409}$$

$$\bar{x}_{313} \uparrow \frac{\Delta}{\delta_3(1-\delta_{3E})} \tag{410}$$

$$m_1 \downarrow \frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})(1-\delta_1)} \tag{411}$$

$$m_3 \uparrow \frac{\Delta}{\delta_3(1-\delta_{3E})} \tag{412}$$

$$\sum_{i=1}^n I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \uparrow \frac{\Delta}{\delta_3(1-\delta_{3E})} \tag{413}$$

Observe, that this transform is the same as T1, we need to recheck only the changed constraint (383), which is straightforward to verify. We cannot do this transform in the following cases:

1) (397) is equality. Then, (398) implies that $\bar{\ell_1} \le \sum_{i=1}^n I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)$.
2) $\bar{\ell_1} = 0$. $\bar{\ell_1} \le \sum_{i=1}^n I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)$ is immediate.
3) $\bar{s_3} = 0$. From (385) $\bar{\ell_1} = 0$ follows, hence $\bar{\ell_1} \le \sum_{i=1}^n I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)$.
4) $\bar{x}_{131} = 0$. Then, $\bar{\ell_1}$ can be reduced to 0 without violating any constraints.
5) $m_1 = 0$. $\bar{x}_{131} = 0$ can be reduced to 0 without violating any constraints, because (394) implies that the RHS of (382) is no larger than that of (381). Then, the previous case applies.

We can add the constraint

$$\bar{\ell_1} \le \sum_{i=1}^n I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{414}$$

and then (398) becomes redundant.

$$nR \le (1-\delta_2)m_2 + \sum_{i=1}^n (1-\delta_1)I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) \tag{415}$$

$$nR \le \sum_{i=1}^n (1-\delta_1\delta_{1E})\bar{x}_{121} + (1-\delta_2)I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{416}$$

$$nR \le (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^n (1-\delta_1)I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) \tag{417}$$

$$nR \le (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{418}$$

$$nR \le (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^n (1-\delta_3)I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{419}$$

$$nR \le (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{420}$$

$$(1-\delta_{1E})\bar{x}_{121} \le \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^n (1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{421}$$

$$(1-\delta_1\delta_{1E})\bar{\ell_1} + \sum_{i=1}^n (1-\delta_3)I(X_{3i}; Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \le \delta_3(1-\delta_{3E})\bar{s_3} \tag{422}$$

$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{423}$$

$$+ (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i}; Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{424}$$

$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + \sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{425}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)\bar{\ell_3} + (1-\delta_1)\bar{s_1} + (1-\delta_1)k_1 \tag{426}$$

$$+ \delta_{3E}(1-\delta_3)k_3 + \delta_{3E}(1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{427}$$

$$(1-\delta_3\delta_{3E})\bar{\ell_3} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \leq \tag{428}$$

$$\delta_1(1-\delta_{1E})\bar{s_1} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) + (1-\delta_2)I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{429}$$

$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{430}$$

$$m_3 \leq \sum_{i=1}^{n} I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{431}$$

$$\bar{\ell_1} \leq \sum_{i=1}^{n} I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{432}$$

$$n \geq k_1 + \sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{433}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) \tag{434}$$

$$n \geq m_1 + \bar{s_1} + k_1 + \sum_{i=1}^{n} I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \tag{435}$$

$$n \geq m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{436}$$

$$n \geq m_2 + k_2 \tag{437}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{438}$$

$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{439}$$

$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{440}$$

$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{441}$$

In this system (432) can be made equality. In case it is not equality, do the following transform:

$$\sum_{i=1}^{n} I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W) \downarrow \frac{\Delta}{1-\delta_1} \tag{442}$$

$$m_1 \uparrow \frac{\Delta}{1-\delta_1} \tag{443}$$

$$m_3 \downarrow \frac{\Delta}{1-\delta_3} \tag{444}$$

$$\bar{\ell_3} \uparrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{445}$$

$$\bar{x}_{313} \downarrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{446}$$

We verify that (427) is not violated. Other constraints are straightforward to check. Change of LHS of (427):

$$\frac{-\Delta(1-\delta_{3E})}{1-\delta_3\delta_{3E}} \tag{447}$$

Change of RHS:

$$\underbrace{\frac{\Delta\delta_{3E}(1-\delta_3)}{1-\delta_3\delta_{3E}}}_{\text{from } \delta_{3E}(1-\delta_3)\bar{\ell}_3} \underbrace{-\Delta}_{\text{from } (1-\delta_1)\sum_{i=1}^n I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)} = \frac{-\Delta(1-\delta_{3E})}{1-\delta_3\delta_{3E}} \tag{448}$$

We have two cases when we cannot do this transform:

1) $\bar{x}_{313} = 0$. In this case decreasing $\sum_{i=1}^n I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)$ until equality holds does not violate any constraints.
2) $m_3 = 0$. In this case decreasing $\bar{x}_{313}$ to 0 does not violate any constraints and the first case applies.

$$nR \leq (1-\delta_2)m_2 + \sum_{i=1}^n (1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{449}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^n (1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{450}$$

$$nR \leq (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^n (1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{451}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{452}$$

$$nR \leq (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^n (1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{453}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{454}$$

$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^n (1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{455}$$

$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell}_1 + \delta_{1E}(1-\delta_1)\bar{s}_1 + \delta_{1E}(1-\delta_1)k_1 \tag{456}$$

$$+ (1-\delta_3)\bar{s}_3 + \sum_{i=1}^n (1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{457}$$

$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + \sum_{i=1}^n (1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{458}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)(\bar{\ell}_3 + k_3 + \bar{s}_3) + (1-\delta_1)(\bar{s}_1 + k_1 + \bar{\ell}_1) \tag{459}$$

$$(1-\delta_1\delta_{1E})\bar{\ell}_1 + \sum_{i=1}^n (1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \leq \delta_3(1-\delta_{3E})\bar{s}_3 \tag{460}$$

$$(1-\delta_3\delta_{3E})\bar{\ell}_3 + (1-\delta_1)\bar{\ell}_1 \leq \delta_1(1-\delta_{1E})\bar{s}_1 \tag{461}$$

$$+ \sum_{i=1}^n (1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) + (1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{462}$$

$$(1-\delta_1\delta_{1E})\bar{s}_1 + (1-\delta_3\delta_{3E})\bar{s}_3 \leq \sum_{i=1}^n (1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{463}$$

$$m_3 \leq \sum_{i=1}^n I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{464}$$

$$n \geq k_1 + \sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{465}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) \tag{466}$$

$$n \geq m_1 + \bar{s_1} + k_1 + \bar{\ell_1} \tag{467}$$

$$n \geq m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{468}$$

$$n \geq m_2 + k_2 \tag{469}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{470}$$

$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{471}$$

$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{472}$$

$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{473}$$

In this system one can increase the value of $\sum_{i=1}^{n} I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)$, $\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W)$ and m_1 until (465)-(467) all become equality. Then, it follows that we can assume

$$m_1 + \bar{s_1} + \bar{\ell_1} \geq \sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) + I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W). \tag{474}$$

We can also increase $\bar{x}_{313}$ until (459) is equality. From this, we can assume

$$(1 - \delta_3\delta_{3E})\bar{x}_{313} \geq (1 - \delta_1)(\bar{s_1} + \bar{\ell_1}). \tag{475}$$

We show that $\sum_{i=1}^{n} I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) = 0$ can be assumed. We do the following transform:

$$\sum_{i=1}^{n} I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \downarrow \frac{\Delta}{1 - \delta_1} \tag{476}$$

$$\sum_{i=1}^{n} I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \uparrow \frac{\Delta}{1 - \delta_2} \tag{477}$$

$$m_2 \downarrow \frac{\Delta}{1 - \delta_2} \tag{478}$$

$$k_2 \uparrow \frac{\Delta}{1 - \delta_2} \tag{479}$$

$$\sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_2^{i-1}) \uparrow \frac{\Delta}{1 - \delta_1} \tag{480}$$

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \downarrow \frac{\Delta}{1 - \delta_1} \tag{481}$$

$$\bar{x}_{22} \downarrow \frac{\Delta}{1 - \delta_2\delta_{2E}}. \tag{482}$$

It is straightforward to verify that this transform does not violate any constraints. Note that $\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) = 0$ implies $\sum_{i=1}^{n} I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) = 0$, so there are two cases when we cannot do this transform:

1) $m_2 = 0$. In this case we can decrease $\bar{x}_{22}$ to 0 without violating any constraints (due to (449)).
2) $\bar{x}_{22} = 0$. Then, we can decrease m_2 to 0 without violating any constraints (due to (451)).

Hence, if $\sum_{i=1}^{n} I(X_{1i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \neq 0$ we can assume that $0 = m_2 = \bar{x}_{22}$. We observe that in this case (475) and (474) together imply that (454) cannot be equality. Thus, a transform that reduces the RHS of (454) while maintaining (474) and the equality of (459) does not violate (454). Apply the following transform of this kind:

$$\bar{\ell_3} \downarrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{483}$$

$$\sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \downarrow \frac{\Delta}{1-\delta_1} \tag{484}$$

$$\bar{x}_{313} \downarrow \frac{\Delta\delta_{3E}(1-\delta_3)}{(1-\delta_{3E})(1-\delta_3\delta_{3E})} \tag{485}$$

$$\sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \uparrow \frac{\Delta}{1-\delta_1} \tag{486}$$

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \downarrow \frac{\Delta}{1-\delta_1}. \tag{487}$$

It is straightforward to verify that all constraints are respected. After this either $\sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) = 0$, or $\bar{\ell_3} = 0$. Note that in case $\bar{x}_{313} = 0$, since equality of (459) is maintained, $\bar{\ell_3} = 0$ follows. We do yet another transform:

$$\bar{\ell_1} \downarrow \frac{\Delta}{1-\delta_1} \tag{488}$$

$$\sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \downarrow \frac{\Delta}{1-\delta_1} \tag{489}$$

$$k_1 \uparrow \frac{\Delta}{1-\delta_1}. \tag{490}$$

$$\tag{491}$$

We can do this transform unless $\bar{\ell_1} = 0$. In this case, since $\bar{\ell_3} = 0$, reducing $\sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)$ to 0 does not violate any constraints. We have shown that $\sum_{i=1}^{n} I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) = 0$ can be assumed.

$$nR \le (1-\delta_2)m_2 + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{492}$$

$$nR \le (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{493}$$

$$nR \le (1-\delta_2\delta_{2E}) \sim \bar{x}_{22} + \sum_{i=1}^{n}(1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{494}$$

$$nR \le (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{495}$$

$$nR \le (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{496}$$

$$nR \le (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{497}$$

$$(1-\delta_{1E})\bar{x}_{121} \le \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{498}$$

$$(1-\delta_{1E})\bar{x}_{131} \le \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{499}$$

$$+ (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{500}$$

$$(1-\delta_{2E})\bar{x}_{22} \le \delta_{2E}(1-\delta_2)k_2 + \sum_{i=1}^{n}(1-\delta_1)H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \tag{501}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)(\bar{\ell}_3 + k_3 + \bar{s_3}) + (1-\delta_1)(\bar{s_1} + k_1 + \bar{\ell}_1) \tag{502}$$

$$(1-\delta_1\delta_{1E})\bar{\ell}_1 + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \leq \delta_3(1-\delta_{3E})\bar{s_3} \tag{503}$$

$$(1-\delta_3\delta_{3E})\bar{\ell}_3 + (1-\delta_1)\bar{\ell}_1 \leq \delta_1(1-\delta_{1E})\bar{s_1} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{504}$$

$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{505}$$

$$m_3 \leq \sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{506}$$

$$n \geq k_1 + \sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{507}$$

$$n \geq \sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) + I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \tag{508}$$

$$n \geq m_1 + \bar{s_1} + k_1 + \bar{\ell}_1 \tag{509}$$

$$n \geq m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{510}$$

$$n \geq m_2 + k_2 \tag{511}$$

$$n \geq \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{512}$$

$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{513}$$

$$n \geq k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{514}$$

$$n \geq m_3 + \bar{\ell}_3 + k_3 + \bar{s_3} \tag{515}$$

We show that $\sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) = m_1 + \bar{s_1} + \bar{\ell}_1$ can be assumed. It is enough to show that (507)-(509) are all equalities. By increasing $\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W)$ and m_1 we can easily make (508) and (509) equality. If (507) is not equality, we do the following transform:

$$\sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \uparrow \frac{\Delta}{1-\delta_1} \tag{516}$$

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \downarrow \frac{\Delta}{1-\delta_1} \tag{517}$$

$$k_2 \uparrow \frac{\Delta}{1-\delta_2} \tag{518}$$

$$m_2 \downarrow \frac{\Delta}{1-\delta_2} \tag{519}$$

$$\bar{x}_{22} \downarrow \frac{\Delta}{1-\delta_2\delta_{2E}} \tag{520}$$

There are three cases when we cannot do this transform:

1) $\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) = 0$. This implies $\sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) = n$, i.e. (507) is equality.
2) $\bar{x}_{22} = 0$. The following transform does not violate any constraints:

$$\sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) \uparrow \frac{\Delta}{1-\delta_1} \tag{521}$$

$$\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) \downarrow \frac{\Delta}{1-\delta_1} \tag{522}$$

If $\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W)$ reaches 0, the first case applies.

3) $m_2 = 0$. In this case reducing $\bar{x}_{22}$ to 0 does not violate any constraints. Then, the second case applies.

From $\sum_{i=1}^{n} I(X_{1i};W|Y_1^{i-1}Y_2^{i-1}) = m_1 + \bar{s_1} + \bar{\ell_1}$ and the equalities (507)-(509) it also follows that $\sum_{i=1}^{n} H(X_{1i}|Y_1^{i-1}Y_2^{i-1}Z_2^{i-1}W) = k_1$ can be assumed.

$$nR \le (1-\delta_2)m_2 + (1-\delta_1)(m_1 + \bar{s_1} + \bar{\ell_1}) \tag{523}$$

$$nR \le (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{524}$$

$$nR \le (1-\delta_1)(m_1 + \bar{s_1} + \bar{\ell_1}) + (1-\delta_2\delta_{2E})\bar{x}_{22} \tag{525}$$

$$nR \le (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{526}$$

$$nR \le (1-\delta_1\delta_{1E})\bar{x}_{131} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{527}$$

$$nR \le (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{528}$$

$$(1-\delta_{1E})\bar{x}_{121} \le \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{529}$$

$$(1-\delta_{1E})\bar{x}_{131} \le \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{530}$$

$$+ (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{531}$$

$$(1-\delta_{2E})\bar{x}_{22} \le \delta_{2E}(1-\delta_2)k_2 + (1-\delta_1)k_1 \tag{532}$$

$$(1-\delta_{3E})\bar{x}_{313} \le \delta_{3E}(1-\delta_3)(\bar{\ell_3} + k_3 + \bar{s_3}) + (1-\delta_1)(\bar{s_1} + k_1 + \bar{\ell_1}) \tag{533}$$

$$(1-\delta_1\delta_{1E})\bar{\ell_1} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \le \delta_3(1-\delta_{3E})\bar{s_3} \tag{534}$$

$$(1-\delta_3\delta_{3E})\bar{\ell_3} + (1-\delta_1)\bar{\ell_1} \le \delta_1(1-\delta_{1E})\bar{s_1} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{535}$$

$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \le \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{536}$$

$$m_3 \le \sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{537}$$

$$n \ge m_1 + \bar{s_1} + \bar{\ell_1} + k_1 \tag{538}$$

$$n \ge m_2 + \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{539}$$

$$n \ge m_2 + k_2 \tag{540}$$

$$n \ge \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) + I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{541}$$

$$n \ge m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{542}$$

$$n \ge k_3 + \bar{s_3} + \bar{z_3} + \sum_{i=1}^{n} I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \tag{543}$$

$$n \ge m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{544}$$

In this system we can see that one can increase the value of $\sum_{i=1}^{n} I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)$, k_2 and $\sum_{i=1}^{n} I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1})$ until (539)-(541) all become equalities. We next show that (537) can also be made equality. We do the following transform (T2):

$$\bar{\ell_3} \downarrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{545}$$

$$\bar{s_1} \downarrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{546}$$

$$\bar{s_3} \uparrow \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})} \tag{547}$$

$$\bar{z_3} \uparrow \frac{\Delta(1-\delta_1\delta_{1E})\delta_3(1-\delta_{3E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})(1-\delta_3)} \tag{548}$$

$$\bar{x}_{131} \uparrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{549}$$

$$\sum_{i=1}^{n} I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \downarrow \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3)} \tag{550}$$

$$m_3 \downarrow \frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3)} \tag{551}$$

$$m_1 \uparrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{552}$$

$$\bar{x}_{313} \downarrow \frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})} \tag{553}$$

$$\sum_{i=1}^{n} I(X_{1i}; W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}) \uparrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{554}$$

The side-calculations that verify that transform T2 respects all constraints are found in Appendix A-B. If (537) is not yet equality we can do this transform unless any variable the transform decreases is 0 or (542) becomes equality. The latter already implies that (537) is equality, so we have the following cases:

1) $\bar{\ell_3} = 0$. In this case we can increase m_3 without violating any constraints until (537) is equality.
2) $\sum_{i=1}^{n} I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) = 0$. This implies $m_3 = 0$, hence equality follows.
3) $\bar{x}_{313} = 0$. In this case we can decrease $\bar{\ell_3}$ to 0 without violating any constraints and then the first case applies.
4) $m_3 = 0$. In this case decreasing $\bar{x}_{313}$ to 0 does not violate any constraints, hence the previous case applies.
5) $\bar{s_1} = 0$. In this case we do the following transform:

$$\sum_{i=1}^{n} I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \downarrow \frac{\Delta}{1-\delta_2} \tag{555}$$

$$\sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \uparrow \frac{\Delta}{1-\delta_2} \tag{556}$$

$$\bar{\ell_1} \downarrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{557}$$

$$\bar{s_1} \uparrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{558}$$

$$\sum_{i=1}^{n} I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \downarrow \frac{\Delta}{1-\delta_2} \tag{559}$$

$$\bar{x}_{121} \uparrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{560}$$

It is straightforward to verify that we can do this transform unless any of the decreased variables equal 0. There are three cases:

a) $\sum_{i=1}^{n} I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) = 0$. Since $\bar{s_1} = 0$, (535) implies that $\bar{\ell_3} = 0$ and then case 1) applies.
b) $\sum_{i=1}^{n} I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) = 0$. Since the transform maintains the equalities (539)-(541), $m_2 = \sum_{i=1}^{n} I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) = 0$ also follows, hence the previous case applies.
c) $\bar{\ell_1} = 0$. In this case, we do the following transform:

$$\sum_{i=1}^{n} I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \downarrow \frac{\Delta}{1-\delta_2} \tag{561}$$

$$\sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \uparrow \frac{\Delta}{1-\delta_2} \tag{562}$$

$$\bar{\ell_3} \downarrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{563}$$

$$\bar{s_3} \uparrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{564}$$

$$\sum_{i=1}^{n} I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \downarrow \frac{\Delta}{1-\delta_2} \tag{565}$$

$$\bar{x}_{121} \uparrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{566}$$

$$\sum_{i=1}^{n} I(X_{3i}; W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1}) \downarrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{567}$$

$$\bar{x}_{131} \uparrow \frac{\Delta(1-\delta_3)}{(1-\delta_3\delta_{3E})(1-\delta_1\delta_{1E})} \tag{568}$$

Again, it is straightforward to verify the correctness of this transform. There are four cases if (537) is not equality:

i) $\sum_{i=1}^{n} I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) = 0$. Case 5a) applies.
ii) $\sum_{i=1}^{n} I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) = 0$. Since the transform maintains the equalities (539)-(541), $m_2 = \sum_{i=1}^{n} I(X_{2i}; Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) = 0$ also follows and the previous case holds.
iii) $\bar{\ell_3} = 0$. Case 1) holds.
iv) $\sum_{i=1}^{n} I(X_{3i}; W|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) = 0$. Case 2) holds.

$$nR \le (1-\delta_2)m_2 + (1-\delta_1)(m_1 + \bar{s_1} + \bar{\ell_1}) \tag{569}$$

$$nR \le (1-\delta_1\delta_{1E})\bar{x}_{121} + \sum_{i=1}^{n}(1-\delta_2)I(X_{2i}; W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{570}$$

$$nR \le (1-\delta_1)(m_1 + \bar{s_1} + \bar{\ell_1}) + (1-\delta_2\delta_{2E})\bar{x}_{22} \tag{571}$$

$$nR \le (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{572}$$

$$nR \le (1-\delta_3)m_3 + (1-\delta_1\delta_{1E})\bar{x}_{131} \tag{573}$$

$$nR \le (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{574}$$

$$(1-\delta_{1E})\bar{x}_{121} \le \delta_{1E}(1-\delta_1)k_1 + \sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{575}$$

$$(1-\delta_{1E})\bar{x}_{131} \le \delta_{1E}(1-\delta_1)\bar{\ell_1} + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{576}$$

$$+ (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i}; Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{577}$$

$$(1-\delta_{2E})\bar{x}_{22} \le \delta_{2E}(1-\delta_2)k_2 + (1-\delta_1)k_1 \tag{578}$$

$$(1-\delta_{3E})\bar{x}_{313} \le \delta_{3E}(1-\delta_3)(\bar{\ell_3} + k_3 + \bar{s_3}) + (1-\delta_1)(\bar{s_1} + k_1 + \bar{\ell_1}) \tag{579}$$

$$(1-\delta_1\delta_{1E})\bar{\ell_1}+\sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W)\le\delta_3(1-\delta_{3E})\bar{s_3} \tag{580}$$

$$(1-\delta_3\delta_{3E})\bar{\ell_3}+(1-\delta_1)\bar{\ell_1}\le\delta_1(1-\delta_{1E})\bar{s_1}+\sum_{i=1}^{n}(1-\delta_2)I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{581}$$

$$(1-\delta_1\delta_{1E})\bar{s_1}+(1-\delta_3\delta_{3E})\bar{s_3}\le\sum_{i=1}^{n}(1-\delta_2)H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \tag{582}$$

$$n\ge m_1+\bar{s_1}+\bar{\ell_1}+k_1 \tag{583}$$

$$n\ge m_2+\sum_{i=1}^{n}H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W)+I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \tag{584}$$

$$n\ge m_2+k_2 \tag{585}$$

$$n\ge\sum_{i=1}^{n}H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W)+I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \tag{586}$$

$$n\ge m_3+k_3+\bar{s_3}+\bar{z_3} \tag{587}$$

$$n\ge m_3+\bar{\ell_3}+k_3+\bar{s_3} \tag{588}$$

We next show that $\sum_{i=1}^{n}I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)=0$ can be assumed in this system. Again, we can increase the value of $\sum_{i=1}^{n}I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)$, k_2 and $\sum_{i=1}^{n}I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1})$ until (584)-(586) all become equalities. We do the following transform:

$$\sum_{i=1}^{n}I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)\downarrow\frac{\Delta}{1-\delta_2} \tag{589}$$

$$\sum_{i=1}^{n}H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W)\uparrow\frac{\Delta}{1-\delta_2} \tag{590}$$

$$\bar{\ell_1}\downarrow\frac{\Delta}{1-\delta_1\delta_{1E}} \tag{591}$$

$$\bar{s_1}\uparrow\frac{\Delta}{1-\delta_1\delta_{1E}} \tag{592}$$

$$\sum_{i=1}^{n}I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1})\downarrow\frac{\Delta}{1-\delta_2} \tag{593}$$

$$\bar{x}_{121}\uparrow\frac{\Delta}{1-\delta_1\delta_{1E}} \tag{594}$$

The transform maintains all the inequalities, thus we cannot do this transform in the following two cases:

1) $\sum_{i=1}^{n}I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1})=0$. Since the transform maintains the equalities (539)-(541), $m_2=\sum_{i=1}^{n}I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)=0$ also follows.
2) $\bar{\ell_1}=0$. In this case we do the following transform:

$$\sum_{i=1}^{n}I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)\downarrow\frac{\Delta}{1-\delta_2} \tag{595}$$

$$\sum_{i=1}^{n}H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W)\uparrow\frac{\Delta}{1-\delta_2} \tag{596}$$

$$\bar{\ell_3}\downarrow\frac{\Delta}{1-\delta_3\delta_{3E}} \tag{597}$$

$$\bar{s_3}\uparrow\frac{\Delta}{1-\delta_3\delta_{3E}} \tag{598}$$

$$\sum_{i=1}^{n}I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1})\downarrow\frac{\Delta}{1-\delta_2} \tag{599}$$

$$\bar{x}_{121} \uparrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{600}$$

$$\tag{601}$$

This transform maintains all inequalities except (587), which case is considered as case 2c below. Hence, there are three cases if $\sum_{i=1}^{n} I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)$ is not 0:

a) $\sum_{i=1}^{n} I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) = 0$. Since the transform maintains the equalities (539)-(541), $m_2 = \sum_{i=1}^{n} I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) = 0$ also follows.

b) $\bar{\ell}_3 = 0$. Since $\bar{\ell}_1 = 0$ we can reduce $\sum_{i=1}^{n} I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W)$ to 0 without violating any constraints.

c) (587) is equality. In this case we know that $\bar{z_3} \geq \bar{\ell}_3$. We do yet another transform:

$$\sum_{i=1}^{n} I(X_{2i};Z_1^{i-1}|Y_1^{i-1}Y_2^{i-1}W) \downarrow \frac{\Delta}{1-\delta_2} \tag{602}$$

$$\sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W) \uparrow \frac{\Delta}{1-\delta_2} \tag{603}$$

$$\bar{\ell}_3 \downarrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{604}$$

$$\bar{s_3} \uparrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{605}$$

$$\sum_{i=1}^{n} I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}) \downarrow \frac{\Delta}{1-\delta_2} \tag{606}$$

$$\bar{x}_{121} \uparrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{607}$$

$$\bar{z_3} \downarrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{608}$$

After this transform either case 2a or case 2b occurs.

Since all transforms maintain equalities (584)-(586), $m_2 = \sum_{i=1}^{n} I(X_{2i};W|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1})$ and $k_2 = \sum_{i=1}^{n} H(X_{2i}|Y_1^{i-1}Y_2^{i-1}Z_1^{i-1}W)$ can also be assumed.

$$nR \leq (1-\delta_2)m_2 + (1-\delta_1)(m_1+\bar{s_1}+\bar{\ell}_1) \tag{609}$$

$$nR \leq (1-\delta_2)m_2 + (1-\delta_1\delta_{1E})\bar{x}_{121} \tag{610}$$

$$nR \leq (1-\delta_1)(m_1+\bar{s_1}+\bar{\ell}_1) + (1-\delta_2\delta_{2E})\bar{x}_{22} \tag{611}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{612}$$

$$nR \leq (1-\delta_3)m_3 + (1-\delta_1\delta_{1E})\bar{x}_{131} \tag{613}$$

$$nR \leq (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{614}$$

$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + (1-\delta_2)k_2 \tag{615}$$

$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)\bar{\ell}_1 + \delta_{1E}(1-\delta_1)\bar{s_1} + \delta_{1E}(1-\delta_1)k_1 \tag{616}$$

$$+ (1-\delta_3)\bar{s_3} + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \tag{617}$$

$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + (1-\delta_1)k_1 \tag{618}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)(\bar{\ell}_3+k_3+\bar{s_3}) + (1-\delta_1)(\bar{s_1}+k_1+\bar{\ell}_1) \tag{619}$$

$$(1-\delta_1\delta_{1E})\bar{\ell}_1 + \sum_{i=1}^{n}(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W) \leq \delta_3(1-\delta_{3E})\bar{s_3} \tag{620}$$

$$(1-\delta_3\delta_{3E})\bar{\ell}_3 + (1-\delta_1)\bar{\ell}_1 \leq \delta_1(1-\delta_{1E})\bar{s_1} \tag{621}$$

$$(1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} \leq (1-\delta_2)k_2 \tag{622}$$

$$n \geq m_1 + \bar{s_1} + \bar{\ell}_1 + k_1 \tag{623}$$

$$n \geq m_2 + k_2 \tag{624}$$

$$n \geq m_3 + k_3 + \bar{s_3} + \bar{z_3} \tag{625}$$
$$n \geq m_3 + \bar{\ell_3} + k_3 + \bar{s_3} \tag{626}$$

We show that $\bar{\ell_3} \leq \bar{z_3}$ can be assumed. If in an optimal point this inequality does not hold, do the following transform (T3):

$$\bar{\ell_3} \downarrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{627}$$
$$\bar{s_1} \downarrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{628}$$
$$\bar{s_3} \uparrow \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})} \tag{629}$$
$$\bar{z_3} \uparrow \frac{\Delta\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})(1-\delta_3)} \tag{630}$$
$$m_1 \uparrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{631}$$
$$\bar{x}_{131} \uparrow \frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_1\delta_{1E})} \tag{632}$$
$$m_3 \downarrow \frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3)} \tag{633}$$
$$\bar{x}_{313} \downarrow \frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})} \tag{634}$$

We validate this transform in Appendix A-C.

We can do this transform unless one of the following cases occurs:

1) (625) is equality. In this case $\bar{\ell_3} \leq \bar{z_3}$ already follows from (626).
2) $\bar{\ell_3} = 0$. The inequality in question is immediate.
3) $\bar{s_1} = 0$. In this case (621) implies $\bar{\ell_3} = 0$ and the inequality follows.
4) $\bar{x}_{313} = 0$. Then $\bar{\ell_3}$ can be reduced to 0 without violating any constraints.
5) $m_3 = 0$. In this case $\bar{x}_{313}$ can be reduced to 0 without violating any constraints and the previous case applies.

We introduce the following new variables replacing some of the existing ones:

$$r_3 \sim \left(\bar{z_3} - \bar{\ell_3}\right) \frac{(1-\delta_3\delta_{3E})(1-\delta_3)}{\delta_3(1-\delta_{3E})} \tag{635}$$
$$c_1 \sim \bar{s_1} + \bar{\ell_1} \tag{636}$$
$$c_3 \sim \bar{\ell_3} + \bar{s_3} - \frac{r_3}{1-\delta_3\delta_{3E}} \tag{637}$$
$$c \sim (1-\delta_1\delta_{1E})\bar{s_1} + (1-\delta_3\delta_{3E})\bar{s_3} - r_3 \tag{638}$$

The non-negativity of the introduced variables is the consequence of $\bar{\ell_3} \leq \bar{z_3}$ and (620). The new system is

$$nR \leq (1-\delta_2)m_2 + (1-\delta_1)(m_1+c_1) \tag{639}$$
$$nR \leq (1-\delta_2)m_2 + (1-\delta_1\delta_{1E})\bar{x}_{121} \tag{640}$$
$$nR \leq (1-\delta_1)(m_1+c_1) + (1-\delta_2\delta_{2E})\bar{x}_{22} \tag{641}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{642}$$
$$nR \leq (1-\delta_3)m_3 + (1-\delta_1\delta_{1E})\bar{x}_{131} \tag{643}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{644}$$
$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + (1-\delta_2)k_2 \tag{645}$$
$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)(c_1+k_1) + (1-\delta_3)c_3 + r_3 \tag{646}$$
$$(1-\delta_{2E})\bar{x}_{22} \leq \delta_{2E}(1-\delta_2)k_2 + (1-\delta_1)k_1 \tag{647}$$

$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)(c_3+k_3) + r_3\frac{\delta_{3E}(1-\delta_3)}{1-\delta_3\delta_{3E}} + (1-\delta_1)(c_1+k_1) \tag{648}$$
$$(1-\delta_1\delta_{1E})c_1 + (1-\delta_3)c_3 \leq c \tag{649}$$
$$(1-\delta_3\delta_{3E})c_3 + (1-\delta_1)c_1 \leq c \tag{650}$$
$$c + r_3 \leq (1-\delta_2)k_2 \tag{651}$$
$$n \geq m_1 + c_1 + k_1 \tag{652}$$
$$n \geq m_2 + k_2 \tag{653}$$
$$n \geq m_3 + k_3 + c_3 + \frac{r_3}{1-\delta_3} \tag{654}$$

We show that $(1-\delta_2)m_2 = (1-\delta_2\delta_{2E})\bar{x}_{22}$ can be assumed. If equality does not hold, we have two cases:

1) $(1-\delta_2)m_2 < (1-\delta_2\delta_{2E})\bar{x}_{22}$. In this case the value of $\bar{x}_{22}$ can be decreased until equality holds without violating any constraints.
2) $(1-\delta_2)m_2 > (1-\delta_2\delta_{2E})\bar{x}_{22}$. Do the following transform:

$$m_2 \downarrow \frac{\Delta}{1-\delta_2} \tag{655}$$
$$k_2 \uparrow \frac{\Delta}{1-\delta_2} \tag{656}$$
$$\bar{x}_{121} \uparrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{657}$$

Since (639) cannot be equality by assumption, we can always do this transform until equality holds.

$$nR \leq (1-\delta_2)m_2 + (1-\delta_1)(m_1+c_1) \tag{658}$$
$$nR \leq (1-\delta_2)m_2 + (1-\delta_1\delta_{1E})\bar{x}_{121} \tag{659}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{660}$$
$$nR \leq (1-\delta_3)m_3 + (1-\delta_1\delta_{1E})\bar{x}_{131} \tag{661}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{662}$$
$$(1-\delta_{1E})\bar{x}_{121} \leq \delta_{1E}(1-\delta_1)k_1 + (1-\delta_2)k_2 \tag{663}$$
$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)(c_1+k_1) + (1-\delta_3)c_3 + r_3 \tag{664}$$
$$\frac{(1-\delta_2)(1-\delta_{2E})}{1-\delta_2\delta_{2E}}m_2 \leq \delta_{2E}(1-\delta_2)k_2 + (1-\delta_1)k_1 \tag{665}$$
$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)(c_3+k_3) + r_3\frac{\delta_{3E}(1-\delta_3)}{1-\delta_3\delta_{3E}} + (1-\delta_1)(c_1+k_1) \tag{666}$$
$$(1-\delta_1\delta_{1E})c_1 + (1-\delta_3)c_3 \leq c \tag{667}$$
$$(1-\delta_3\delta_{3E})c_3 + (1-\delta_1)c_1 \leq c \tag{668}$$
$$c + r_3 \leq (1-\delta_2)k_2 \tag{669}$$
$$n \geq m_1 + c_1 + k_1 \tag{670}$$
$$n \geq m_2 + k_2 \tag{671}$$
$$n \geq m_3 + k_3 + c_3 + \frac{r_3}{1-\delta_3} \tag{672}$$

We show that $(1-\delta_1)(m_1+c_1) = (1-\delta_1\delta_{1E})\bar{x}_{121}$ can be assumed. If equality does not hold, there are two cases:

1) $(1-\delta_1)(m_1+c_1) < (1-\delta_1\delta_{1E})\bar{x}_{121}$. In this case the value of $\bar{x}_{121}$ can be decreased until equality holds without violating any constraints.

2) $(1-\delta_1)(m_1+c_1) > (1-\delta_1\delta_{1E})\bar{x}_{121}$. Increase $\bar{x}_{121}$ until (663) is equality. From this we know that

$$(1-\delta_{1E})\bar{x}_{121} - (1-\delta_{1E})\bar{x}_{131} \geq \tag{673}$$
$$\geq (1-\delta_2)k_2 - \delta_{1E}(1-\delta_1)c_1 - (1-\delta_3)c_3 - r_3 \geq c - \delta_{1E}(1-\delta_1)c_1 - (1-\delta_3)c_3 \geq (1-\delta_{1E})c_1 \tag{674}$$

and hence

$$(1-\delta_1\delta_{1E})\bar{x}_{121} - (1-\delta_1\delta_{1E})\bar{x}_{131} \geq (1-\delta_1)c_1 \tag{675}$$

Do the following transform:

$$m_2 \downarrow \frac{\Delta}{1-\delta_2} \tag{676}$$
$$k_2 \uparrow \frac{\Delta}{1-\delta_2} \tag{677}$$
$$\bar{x}_{121} \uparrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{678}$$

We can do this transform, unless $m_2 = 0$. In this case first decrease $\bar{x}_{131}$ until it is 0 or (661) is equality. We then know that

$$(1-\delta_1)m_1 \geq (1-\delta_1\delta_{1E})\bar{x}_{131} \tag{679}$$

Then, increase $\bar{x}_{313}$ until (666) is equality. We then know that

$$(1-\delta_3\delta_{3E})\bar{x}_{313} \geq (1-\delta_1)c_1. \tag{680}$$

From these inequalities it follows that decreasing m_1 does not violate any constraints unless (679) is equality. In this latter case however, (675) implies that

$$(1-\delta_1\delta_{1E})\bar{x}_{121} \geq (1-\delta_1)(m_1+c_1). \tag{681}$$

Thus, we can always decrease m_1 until $(1-\delta_1)(m_1+c_1) = (1-\delta_1\delta_{1E})\bar{x}_{121}$.

$$nR \leq (1-\delta_1)(m_1+c_1) + (1-\delta_2)m_2 \tag{682}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{683}$$
$$nR \leq (1-\delta_3)m_3 + (1-\delta_1\delta_{1E})\bar{x}_{131} \tag{684}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{685}$$
$$\frac{(1-\delta_{1E})(1-\delta_1)}{1-\delta_1\delta_{1E}}(m_1+c_1) \leq \delta_{1E}(1-\delta_1)k_1 + (1-\delta_2)k_2 \tag{686}$$
$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)(c_1+k_1) + (1-\delta_3)c_3 + r_3 \tag{687}$$
$$\frac{(1-\delta_2)(1-\delta_{2E})}{1-\delta_2\delta_{2E}}m_2 \leq \delta_{2E}(1-\delta_2)k_2 + (1-\delta_1)k_1 \tag{688}$$
$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)(c_3+k_3) + r_3\frac{\delta_{3E}(1-\delta_3)}{1-\delta_3\delta_{3E}} + (1-\delta_1)(c_1+k_1) \tag{689}$$
$$(1-\delta_1\delta_{1E})c_1 + (1-\delta_3)c_3 \leq c \tag{690}$$
$$(1-\delta_3\delta_{3E})c_3 + (1-\delta_1)c_1 \leq c \tag{691}$$
$$c + r_3 \leq (1-\delta_2)k_2 \tag{692}$$
$$n \geq m_1 + c_1 + k_1 \tag{693}$$
$$n \geq m_2 + k_2 \tag{694}$$
$$n \geq m_3 + k_3 + c_3 + \frac{r_3}{1-\delta_3} \tag{695}$$

We show that $(1-\delta_3)m_3 \leq (1-\delta_1)c_1 + (1-\delta_2)m_2$ can be assumed. Assume the contrary. Then, we know that (683) is not equality.

Do the following transform (T4):

$$c_1 \downarrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{696}$$

$$m_1 \uparrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{697}$$

$$\bar{x}_{131} \uparrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{698}$$

$$\bar{x}_{313} \downarrow \frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})} \tag{699}$$

$$m_3 \downarrow \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3)} \tag{700}$$

$$r_3 \uparrow \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})} + \frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})} \tag{701}$$

$$c_3 \downarrow \frac{\Delta}{\delta_3(1-\delta_{3E})} \tag{702}$$

$$c \downarrow \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})} + \frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})} \tag{703}$$

$$\tag{704}$$

The side-calculation in A-D shows that the transform respects all inequalities. We have the following cases:

1) $m_3 = 0$. $(1-\delta_3)m_3 \leq (1-\delta_1)c_1 + (1-\delta_2)m_2$ already holds.
2) $\bar{x}_{313} = 0$. If this variable cannot be increased, then (689) is equality and thus $c_1 = 0$ follows, and case 3 applies.
3) $c_1 = 0$. In this case we know that (690) is not unless $c_3 = 0$. We either can do the following transform or $c_3 = 0$ (case 3c below):

$$c_1 \uparrow \frac{\Delta}{1-\delta_1} \tag{705}$$

$$m_1 \downarrow \frac{\Delta}{1-\delta_1} \tag{706}$$

$$k_2 \uparrow \frac{\Delta}{1-\delta_2} \tag{707}$$

$$c \uparrow \Delta \tag{708}$$

$$\bar{x}_{313} \uparrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{709}$$

$$m_2 \downarrow \frac{\Delta}{1-\delta_2} \tag{710}$$

It is straightforward to verify that the transform does not violate any constraints. We have thus three cases:

a) $m_1 = 0$. In this case we know that (684) cannot be equality, thus m_3 can be decreased until $(1-\delta_3)m_3 \leq (1-\delta_1)c_1 + (1-\delta_2)m_2$ holds.
b) $m_2 = 0$. In this case we can decrease $\bar{x}_{313}$ to 0 without violating any constraints. Do the following transform:

$$m_1 \downarrow \frac{\Delta}{1-\delta_1} \tag{711}$$

$$c_1 \uparrow \frac{\Delta}{1-\delta_1} \tag{712}$$

$$c_3 \downarrow \frac{\Delta}{1-\delta_3} \tag{713}$$

$$m_3 \uparrow \frac{\Delta}{1-\delta_3} \tag{714}$$

$$\bar{x}_{131} \downarrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{715}$$

$$\bar{x}_{313} \uparrow \frac{\Delta}{1-\delta_3\delta_{3E}} \tag{716}$$

We have the following cases:

i) $m_1 = 0$. Then (684) cannot be equality ($R = 0$), hence we can decrease m_3 to 0 and then $(1-\delta_3)m_3 \leq (1-\delta_1)c_1 + (1-\delta_2)m_2$ holds.

ii) $c_3 = 0$. Case 3c.

iii) $\bar{x}_{131} = 0$. If this variable cannot be increased then (687) is equality, hence $c_3 = 0$ and case 3c applies.

iv) (689) is equality. Then $c_3 = 0$ follows and case 3c applies.

c) $c_3 = 0$. Do the following transform:

$$c_3 \uparrow \frac{\Delta}{1-\delta_3} \tag{717}$$

$$m_3 \downarrow \frac{\Delta}{1-\delta_3} \tag{718}$$

$$\bar{x}_{131} \uparrow \frac{\Delta}{1-\delta_1\delta_{1E}} \tag{719}$$

We can do this transform unless $c = 0$. (Note that if $m_3 = 0$, then $(1-\delta_3)m_3 \leq (1-\delta_1)c_1 + (1-\delta_2)m_2$ already holds.) If $c = 0$, we can either increase c and do the transform or (692) is equality. Thus we can assume $c = 0$ and (692) is equality. In this case increase $\bar{x}_{131}$ until (687) is equality. Then from (686) and (687):

$$(1-\delta_1)m_1 \leq \frac{1-\delta_1\delta_{1E}}{1-\delta_{1E}}\left(\delta_{1E}(1-\delta_1)k_1 + (1-\delta_2)k_2\right) = \frac{1-\delta_1\delta_{1E}}{1-\delta_{1E}}\left(\delta_{1E}(1-\delta_1)k_1 + r_3\right) \tag{720}$$

$$= (1-\delta_1\delta_{1E})\bar{x}_{131}. \tag{721}$$

This means that (684) cannot be equality, and hence m_3 can be decreased until $(1-\delta_3)m_3 \leq (1-\delta_1)c_1 + (1-\delta_2)m_2$ holds.

4) $c_3 = 0$. In this case we know that (691) is not equality otherwise we have case 3. Do the following transform:

$$c_1 \downarrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{722}$$

$$m_1 \uparrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{723}$$

$$\bar{x}_{131} \uparrow \frac{\Delta}{\delta_1(1-\delta_{1E})} \tag{724}$$

$$\bar{x}_{313} \downarrow \frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})} \tag{725}$$

$$m_3 \downarrow \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3)} \tag{726}$$

$$r_3 \uparrow \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})} \tag{727}$$

$$c \downarrow \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})} \tag{728}$$

It is straightforward to verify that the transform does not violate any constraints. After this transform either of the previous three cases occurs. We add the constraint $(1-\delta_3)m_3 \leq (1-\delta_1)c_1 + (1-\delta_2)m_2$ and drop the constraint (686).

$$(1-\delta_3)m_3 \leq (1-\delta_1)c_1 + (1-\delta_2)m_2 \tag{729}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{730}$$
$$nR \leq (1-\delta_3)m_3 + (1-\delta_1\delta_{1E})\bar{x}_{131} \tag{731}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{732}$$
$$(1-\delta_{1E})\bar{x}_{131} \leq \delta_{1E}(1-\delta_1)(c_1+k_1) + (1-\delta_3)c_3 + r_3 \tag{733}$$
$$\frac{(1-\delta_2)(1-\delta_{2E})}{1-\delta_2\delta_{2E}}m_2 \leq \delta_{2E}(1-\delta_2)k_2 + (1-\delta_1)k_1 \tag{734}$$
$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)(c_3+k_3) + r_3\frac{\delta_{3E}(1-\delta_3)}{1-\delta_3\delta_{3E}} + (1-\delta_1)(c_1+k_1) \tag{735}$$
$$(1-\delta_1\delta_{1E})c_1 + (1-\delta_3)c_3 \leq c \tag{736}$$
$$(1-\delta_3\delta_{3E})c_3 + (1-\delta_1)c_1 \leq c \tag{737}$$
$$c + r_3 \leq (1-\delta_2)k_2 \tag{738}$$
$$n \geq m_1 + c_1 + k_1 \tag{739}$$
$$n \geq m_2 + k_2 \tag{740}$$
$$n \geq m_3 + k_3 + c_3 + \frac{r_3}{1-\delta_3} \tag{741}$$

We show that $(1-\delta_1)m_1 = (1-\delta_1\delta_{1E})\bar{x}_{131}$ can be assumed. If $(1-\delta_1)m_1 < (1-\delta_1\delta_{1E})\bar{x}_{131}$, then we can decrease $\bar{x}_{131}$ until equality holds without violating any constraint. Assume that $(1-\delta_1)m_1 > (1-\delta_1\delta_{1E})\bar{x}_{131}$. Do the following transform:

$$m_1 \downarrow \Delta \tag{742}$$
$$k_1 \uparrow \Delta \tag{743}$$
$$\bar{x}_{313} \uparrow \frac{\Delta(1-\delta_1)}{1-\delta_3\delta_{3E}} \tag{744}$$

This transform does not violate any constraints ((730) cannot be equality), thus m_1 decreases until $(1-\delta_1)m_1 = (1-\delta_1\delta_{1E})\bar{x}_{131}$.

$$(1-\delta_3)m_3 \leq (1-\delta_1)c_1 + (1-\delta_2)m_2 \tag{745}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{746}$$
$$nR \leq (1-\delta_1)m_1 + (1-\delta_3\delta_{3E})\bar{x}_{313} \tag{747}$$
$$\frac{(1-\delta_1)(1-\delta_{1E})}{1-\delta_1\delta_{1E}}m_1 \leq \delta_{1E}(1-\delta_1)(c_1+k_1) + (1-\delta_3)c_3 + r_3 \tag{748}$$
$$\frac{(1-\delta_2)(1-\delta_{2E})}{1-\delta_2\delta_{2E}}m_2 \leq \delta_{2E}(1-\delta_2)k_2 + (1-\delta_1)k_1 \tag{749}$$
$$(1-\delta_{3E})\bar{x}_{313} \leq \delta_{3E}(1-\delta_3)(c_3+k_3) + r_3\frac{\delta_{3E}(1-\delta_3)}{1-\delta_3\delta_{3E}} + (1-\delta_1)(c_1+k_1) \tag{750}$$
$$(1-\delta_1\delta_{1E})c_1 + (1-\delta_3)c_3 \leq c \tag{751}$$
$$(1-\delta_3\delta_{3E})c_3 + (1-\delta_1)c_1 \leq c \tag{752}$$
$$c + r_3 \leq (1-\delta_2)k_2 \tag{753}$$
$$n \geq m_1 + c_1 + k_1 \tag{754}$$
$$n \geq m_2 + k_2 \tag{755}$$
$$n \geq m_3 + k_3 + c_3 + \frac{r_3}{1-\delta_3} \tag{756}$$

Lastly, we show that $(1-\delta_3)m_3 = (1-\delta_3\delta_{3E})\bar{x}_{313}$ can be assumed. If equality does not hold, then we can always decrease the variable on the larger side of the inequality without violating any constraints until equality holds. After dividing every variable by n, the resulting linear program is the same as the linear program of the scheme:

$$(1-\delta_3)m_3 \le (1-\delta_1)c_1 + (1-\delta_2)m_2 \tag{757}$$
$$R \le (1-\delta_1)m_1 + (1-\delta_3)m_3 \tag{758}$$
$$\frac{(1-\delta_1)(1-\delta_{1E})}{1-\delta_1\delta_{1E}}m_1 \le \delta_{1E}(1-\delta_1)(c_1+k_1) + (1-\delta_3)c_3 + r_3 \tag{759}$$
$$\frac{(1-\delta_2)(1-\delta_{2E})}{1-\delta_2\delta_{2E}}m_2 \le \delta_{2E}(1-\delta_2)k_2 + (1-\delta_1)k_1 \tag{760}$$
$$\frac{(1-\delta_3)(1-\delta_{3E})}{1-\delta_3\delta_{3E}}m_3 \le \delta_{3E}(1-\delta_3)(c_3+k_3) + r_3\frac{\delta_{3E}(1-\delta_3)}{1-\delta_3\delta_{3E}} + (1-\delta_1)(c_1+k_1) \tag{761}$$
$$(1-\delta_1\delta_{1E})c_1 + (1-\delta_3)c_3 \le c \tag{762}$$
$$(1-\delta_3\delta_{3E})c_3 + (1-\delta_1)c_1 \le c \tag{763}$$
$$c + r_3 \le (1-\delta_2)k_2 \tag{764}$$
$$1 \ge m_1 + c_1 + k_1 \tag{765}$$
$$1 \ge m_2 + k_2 \tag{766}$$
$$1 \ge m_3 + k_3 + c_3 + \frac{r_3}{1-\delta_3} \tag{767}$$

APPENDIX A
SIDE-CALCULATIONS

A. Transform T1

Constraints (339)-(341) are not affected. Change of RHS of (342):

$$\underbrace{-\frac{1-\delta_3}{\delta_3(1-\delta_{3E})}\Delta}_{\text{from } (1-\delta_1)m_1} + \underbrace{\frac{1-\delta_3}{\delta_3(1-\delta_{3E})}\Delta}_{\text{from } (1-\delta_3)m_3} = 0. \tag{768}$$

Change of RHS of (343):

$$\underbrace{-\frac{1-\delta_3}{\delta_3(1-\delta_{3E})}\Delta}_{\text{from } (1-\delta_1\delta_{1E})\bar{x}_{131}} + \underbrace{\frac{1-\delta_3}{\delta_3(1-\delta_{3E})}\Delta}_{\text{from } \sum_{i=1}^n (1-\delta_3)I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1})} = 0. \tag{769}$$

Change of RHS of (344):

$$\underbrace{-\frac{1-\delta_3\delta_{3E}}{\delta_3(1-\delta_{3E})}\Delta}_{\text{from } (1-\delta_3\delta_{3E})\bar{x}_{313}} + \underbrace{\frac{1-\delta_3\delta_{3E}}{\delta_3(1-\delta_{3E})}\Delta}_{\text{from } \sum_{i=1}^n (1-\delta_1)I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1})} = 0. \tag{770}$$

Constraint (345) is not affected. Change of LHS of (346):

$$\underbrace{-\Delta}_{\text{from } (1-\delta_1\delta_{1E})\bar{\ell_1}} \tag{771}$$

Change of RHS of (346):

$$\underbrace{-\Delta}_{\text{from } \bar{s_3}} \tag{772}$$

Change of LHS of (348):

$$\underbrace{-\frac{\Delta(1-\delta_3)(1-\delta_{1E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}}_{\text{from } (1-\delta_{1E})\bar{x}_{131}} \tag{773}$$

Change of RHS of (348):

$$-\underbrace{\frac{\Delta\delta_{1E}(1-\delta_{1E})}{1-\delta_1\delta_{1E}}}_{\text{from } \delta_{1E}(1-\delta_1)\bar{\ell_1}}+\underbrace{\frac{\Delta\delta_{1E}(1-\delta_1)(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}}_{\text{from } \delta_{1E}(1-\delta_1)\bar{s_1}}-\underbrace{\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}}_{\text{from } (1-\delta_3)\bar{s_3}} \tag{774}$$

$$=\frac{\Delta\delta_{1E}(1-\delta_1)(1-\delta_3)}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}-\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}=-\frac{\Delta(1-\delta_3)(1-\delta_{1E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})} \tag{775}$$

Constraint (349) is not affected. Change of LHS of (351):

$$\underbrace{\frac{\Delta}{\delta_3}}_{\text{from } (1-\delta_{3E})\bar{x}_{313}} \tag{776}$$

Change of RHS of (351):

$$\underbrace{\frac{\Delta(1-\delta_1)(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}}_{\text{from } (1-\delta_1)\bar{s_1}}-\underbrace{\frac{\Delta\delta_{3E}(1-\delta_3)}{\delta_3(1-\delta_{3E})}}_{\text{from } \delta_{3E}(1-\delta_3)\bar{s_3}}+\underbrace{\frac{\Delta(1-\delta_3\delta_{3E})\delta_1(1-\delta_{1E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}}_{\text{from } \sum_{i=1}^n(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)} \tag{777}$$

$$=-\frac{\Delta\delta_{3E}(1-\delta_3)}{\delta_3(1-\delta_{3E})}+\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})}=\frac{\Delta}{\delta_3} \tag{778}$$

Change of LHS of (353):

$$\underbrace{\frac{\Delta(1-\delta_3\delta_{3E})\delta_1(1-\delta_{1E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}}_{\text{from } \sum_{i=1}^n(1-\delta_1)I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)} \tag{779}$$

Change of RHS of (353):

$$\underbrace{\frac{\Delta(1-\delta_3\delta_{3E})\delta_1(1-\delta_{1E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}}_{\text{from } \delta_1(1-\delta_{1E})\bar{s_1}} \tag{780}$$

Change of LHS of (354):

$$\underbrace{\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})}}_{\text{from } (1-\delta_1\delta_{1E})\bar{s_1}}-\underbrace{\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})}}_{\text{from } (1-\delta_3\delta_{3E})\bar{s_3}}=0 \tag{781}$$

Constraints (355)-(356) are not affected. Change of RHS of (357):

$$\underbrace{\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}}_{\text{from } \bar{s_1}}+\underbrace{\frac{\Delta(1-\delta_3\delta_{3E})\delta_1(1-\delta_{1E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})(1-\delta_1)}}_{\text{from } \sum_{i=1}^n I(X_{1i};Z_1^{i-1}|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1}W)}-\underbrace{\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1)}}_{\text{from } \sum_{i=1}^n I(X_{1i};W|Y_1^{i-1}Y_3^{i-1}Z_3^{i-1})} \tag{782}$$

$$=\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1)}-\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1)}=0 \tag{783}$$

Change of RHS of (358):

$$-\underbrace{\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})(1-\delta_1)}}_{\text{from } m_1}-\underbrace{\frac{\Delta}{1-\delta_1\delta_{1E}}}_{\text{from } \bar{\ell_1}}+\underbrace{\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}}_{\text{from } \bar{s_1}} \tag{784}$$

$$=-\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})(1-\delta_1)}+\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}\leq 0 \tag{785}$$

Constraint (359) cannot be violated by assumption of the transform. Constraints (360)-(362) are not affected. Change of RHS of (363):

$$\underbrace{\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } m_3} - \underbrace{\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } \bar{s_3}} = 0 \tag{786}$$

Change of RHS of (364):

$$\underbrace{\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } \sum_{i=1}^n I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1})} \underbrace{-\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } \bar{s_3}} = 0 \tag{787}$$

Change of RHS of (365):

$$\underbrace{\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } \sum_{i=1}^n I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1})} \underbrace{-\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } \bar{s_3}} = 0 \tag{788}$$

Change of LHS of (367):

$$\underbrace{\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } m_3} \tag{789}$$

Change of RHS of (367):

$$\underbrace{\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } \sum_{i=1}^n I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1})} \tag{790}$$

B. Transform T2

Change of RHS of (523):

$$\underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)m_1} - \underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)\bar{s_1}} = 0 \tag{791}$$

Constraint (524) is not affected. Change of RHS of (525) is 0, for the same reason as (523). Change of RHS of (526):

$$\underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)m_1} - \underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_3)m_3} = 0 \tag{792}$$

Change of RHS of (527):

$$\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1\delta_{1E})\bar{x}_{131}} \underbrace{-\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}}_{\text{from } \sum_{i=1}^n (1-\delta_3) I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1})} = 0 \tag{793}$$

Change of RHS of (528):

$$\underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)m_1} \underbrace{-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_3\delta_{3E})\bar{x}_{313}} = 0 \tag{794}$$

Constraint (529) is not affected. Change of LHS of (531):

$$\underbrace{\frac{\Delta}{\delta_1}}_{\text{from } (1-\delta_{1E})\bar{x}_{131}} \tag{795}$$

Change of RHS of (531):

$$\underbrace{-\frac{\Delta\delta_{1E}(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } \delta_{1E}(1-\delta_1)\bar{s_1}} + \underbrace{\frac{\Delta(1-\delta_3)(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } (1-\delta_3)\bar{s_3}} + \underbrace{\frac{\Delta(1-\delta_1\delta_{1E})\delta_3(1-\delta_{3E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } (1-\delta_3)\bar{z_3}} \tag{796}$$

$$= -\frac{\Delta\delta_{1E}(1-\delta_1)}{\delta_1(1-\delta_{1E})} + \frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})} = \frac{\Delta}{\delta_1} \tag{797}$$

Constraint (532) is not affected. Change of LHS of (533):

$$\underbrace{-\frac{\Delta(1-\delta_1)(1-\delta_{3E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } (1-\delta_{3E})\bar{x}_{313}} \tag{798}$$

Change of RHS of (533):

$$\underbrace{-\frac{\Delta\delta_{3E}(1-\delta_3)}{1-\delta_3\delta_{3E}}}_{\text{from } \delta_{3E}(1-\delta_3)\bar{\ell_3}} + \underbrace{\frac{\Delta\delta_{3E}(1-\delta_3)(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } \delta_{3E}(1-\delta_3)\bar{s_3}} \underbrace{-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)\bar{s_2}} \tag{799}$$

$$= \frac{\Delta\delta_{3E}(1-\delta_3)(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})} - \frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})} = -\frac{\Delta(1-\delta_1)(1-\delta_{3E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})} \tag{800}$$

Change of LHS of (534):

$$\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})\delta_3(1-\delta_{3E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } (1-\delta_3)\bar{z_3}} \tag{801}$$

Change of RHS of (534):

$$\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})\delta_3(1-\delta_{3E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } \delta_3(1-\delta_{3E})\bar{s_3}} \tag{802}$$

Change of LHS of (535):

$$\underbrace{-\Delta}_{\text{from } (1-\delta_3\delta_{3E})\bar{l_3}} \tag{803}$$

Change of RHS of (535):

$$\underbrace{-\Delta}_{\text{from } \delta_1(1-\delta_{1E})\bar{s_1}} \tag{804}$$

Change of LHS of (536):

$$\underbrace{-\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1\delta_{1E})\bar{s_1}} + \underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_3\delta_{3E})\bar{s_3}} = 0 \tag{805}$$

Constraint (537) is not violated by assumption. Change of RHS of (538):

$$\underbrace{\frac{\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } m_1} \underbrace{-\frac{\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } \bar{s_1}} = 0 \tag{806}$$

Constraints (539)-(541) are not affected, (542) is not violated by assumption. Change of RHS of (543):

$$\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } \bar{s_3}}+\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})\delta_3(1-\delta_{3E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})(1-\delta_3)}}_{\text{from } \bar{z_3}} \quad -\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3)}}_{\text{from } \sum_{i=1}^n I(X_{3i};W|Y_1^{i-1}Y_3^{i-1}Z_1^{i-1})} \tag{807}$$

$$\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3)}-\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3)}=0 \tag{808}$$

Change of RHS of (544):

$$-\underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3)}}_{\text{from } m_3}-\underbrace{\frac{\Delta}{1-\delta_3\delta_{3E}}}_{\text{from } \bar{\ell}_3}+\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } \bar{s_3}} \tag{809}$$

$$=-\underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3)}}+\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}\leq 0 \tag{810}$$

C. Transform T3

Change of RHS of (609):

$$-\underbrace{\frac{(1-\delta_1)\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)\bar{s_1}}+\underbrace{\frac{(1-\delta_1)\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)m_1}=0 \tag{811}$$

Constraint (610) is not affected, while the RHS of (611) changes the same way as (609). Change of RHS of (612):

$$\underbrace{\frac{(1-\delta_1)\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)m_1}-\underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_3)m_3}=0 \tag{812}$$

Change of RHS of (613):

$$-\underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_3)m_3}\ +\underbrace{\frac{(1-\delta_1)\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1\delta_{1E})\bar{x}_{131}}\ =0 \tag{813}$$

Change of RHS of (614):

$$\underbrace{\frac{(1-\delta_1)\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)m_1}\ -\underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_3\delta_{3E})\bar{x}_{313}}\ =0 \tag{814}$$

Constraint (615) is not affected. Change of LHS of (617):

$$\underbrace{\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_1\delta_{1E})}}_{\text{from } (1-\delta_{1E})\bar{x}_{131}} \tag{815}$$

Change of RHS of (617):

$$-\underbrace{\frac{\Delta\delta_{1E}(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } \delta_{1E}(1-\delta_1)\bar{s_1}}+\underbrace{\frac{\Delta(1-\delta_3)(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } (1-\delta_3)\bar{s_3}} \quad +\underbrace{\frac{\Delta\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } \sum_{i=1}^n(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W)} \tag{816}$$

$$=-\frac{\Delta\delta_{1E}(1-\delta_1)}{\delta_1(1-\delta_{1E})}+\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}=\frac{\Delta}{\delta_1}\geq\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_1\delta_{1E})} \tag{817}$$

Constraint (618) is not affected. Change of LHS of (619):

$$\underbrace{-\frac{\Delta(1-\delta_{3E})(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } (1-\delta_{3E})\bar{x}_{313}} \tag{818}$$

Change of LHS of (619):

$$\underbrace{-\frac{\Delta\delta_{3E}(1-\delta_3)}{1-\delta_3\delta_{3E}}}_{\text{from } \delta_{3E}(1-\delta_3)\bar{l_3}}\underbrace{+\frac{\Delta\delta_{3E}(1-\delta_3)(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } \delta_{3E}(1-\delta_3)\bar{s_3}}\underbrace{-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1)\bar{s_1}} \tag{819}$$

$$=\frac{\Delta\delta_{3E}(1-\delta_3)(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}=-\frac{\Delta(1-\delta_{3E})(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})} \tag{820}$$

Change of LHS of (620):

$$\underbrace{\frac{\Delta\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } \sum_{i=1}^n(1-\delta_3)I(X_{3i};Z_3^{i-1}|Y_3^{i-1}Y_1^{i-1}Z_1^{i-1}W)} \tag{821}$$

Change of RHS of (620):

$$\underbrace{\frac{\Delta\delta_3(1-\delta_{3E})(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } \delta_3(1-\delta_{3E})\bar{s_3}} \tag{822}$$

Change of LHS of (621):

$$\underbrace{-\Delta}_{\text{from } (1-\delta_3\delta_{3E})\bar{\ell_3}} \tag{823}$$

Change of RHS of (621):

$$\underbrace{-\Delta}_{\text{from } \delta_1(1-\delta_{1E})\bar{s_1}} \tag{824}$$

Change of LHS of (622):

$$\underbrace{-\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_1\delta_{1E})\bar{s_1}}\underbrace{+\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}}_{\text{from } (1-\delta_3\delta_{3E})\bar{s_3}}=0 \tag{825}$$

Change of RHS of (623):

$$\underbrace{\frac{\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } m_1}\underbrace{-\frac{\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } s_1}=0 \tag{826}$$

Inequality (624) is not affected and (625) is not violated by assumption. Change of RHS of (626):

$$\underbrace{-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_3)(1-\delta_{1E})}}_{\text{from } m_3}\underbrace{-\frac{\Delta}{(1-\delta_3\delta_{3E})}}_{\text{from } \bar{\ell_3}}\underbrace{+\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}}_{\text{from } \bar{s_3}} \tag{827}$$

$$=-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_3)(1-\delta_{1E})}+\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})(1-\delta_3\delta_{3E})}\le 0 \tag{828}$$

D. Transform T4

Change of RHS of (682):

$$\underbrace{\frac{(1-\delta_1)\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from }(1-\delta_1)m_1} \underbrace{-\frac{(1-\delta_1)\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from }(1-\delta_1)c_1} = 0 \tag{829}$$

Inequality (683) is not violated by assumption. Change of RHS of (684):

$$\underbrace{-\frac{(1-\delta_1\delta_{1E})\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from }(1-\delta_3)m_3} \underbrace{+\frac{(1-\delta_1\delta_{1E})\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from }(1-\delta_1\delta_{1E})\bar{x}_{131}} = 0 \tag{830}$$

Change of RHS of (685):

$$\underbrace{\frac{(1-\delta_1)\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from }(1-\delta_1)m_1} \underbrace{-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from }(1-\delta_3\delta_{3E})\bar{x}_{313}} = 0 \tag{831}$$

Change of LHS of (686):

$$\frac{(1-\delta_{1E})(1-\delta_1)}{1-\delta_1\delta_{1E}}\left(\underbrace{\frac{\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from }m_1} \underbrace{-\frac{\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from }c_1}\right) = 0 \tag{832}$$

Change of LHS of (687):

$$\underbrace{\frac{\Delta}{\delta_1}}_{\text{from }(1-\delta_{1E})\bar{x}_{131}} \tag{833}$$

Change of RHS of (687):

$$\underbrace{-\frac{\Delta\delta_{1E}(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from }\delta_{1E}(1-\delta_1)c_1} \underbrace{-\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}}_{\text{from }(1-\delta_3)c_3} \underbrace{+\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})} + \frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}}_{\text{from }r_3} = \frac{\Delta}{\delta_1} \tag{834}$$

Change of LHS of (690):

$$\underbrace{-\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}}_{\text{from }(1-\delta_1\delta_{1E})c_1} \underbrace{-\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}}_{\text{from }(1-\delta_3)c_3} \tag{835}$$

Change of RHS of (690):

$$\underbrace{-\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})} - \frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}}_{\text{from }c} \tag{836}$$

Change of LHS of (691):

$$\underbrace{-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}}_{\text{from }(1-\delta_1)c_1} \underbrace{-\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})}}_{\text{from }(1-\delta_3\delta_{3E})c_3} \tag{837}$$

Change of RHS of (691):

$$-\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}-\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}}_{\text{from } c} \tag{838}$$

$$= -\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}+\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}-\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}+\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})}-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}-\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})} \tag{839}$$

$$= -1+1-\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}-\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})} = -\frac{\Delta(1-\delta_1)}{\delta_1(1-\delta_{1E})}-\frac{\Delta(1-\delta_3\delta_{3E})}{\delta_3(1-\delta_{3E})} \tag{840}$$

Change of LHS of (692):

$$-\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}-\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}}_{\text{from } c}+\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})}-\frac{\Delta(1-\delta_3)}{\delta_3(1-\delta_{3E})}}_{\text{from } r_3} = 0 \tag{841}$$

Change of LHS of (693):

$$\underbrace{\frac{\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } m_1}-\underbrace{\frac{\Delta}{\delta_1(1-\delta_{1E})}}_{\text{from } c_1} = 0 \tag{842}$$

Constraint (694) is not affected. Change of LHS of (695):

$$-\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3)}}_{\text{from } m_3}-\underbrace{\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } c_3}+\underbrace{\frac{\Delta(1-\delta_1\delta_{1E})}{\delta_1(1-\delta_{1E})(1-\delta_3)}-\frac{\Delta}{\delta_3(1-\delta_{3E})}}_{\text{from } \frac{r_3}{1-\delta_3}} = 0 \tag{843}$$

References

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