OPTIMIZATION OF AN ACOUSTIC LEAKY-WAVE ANTENNA BASED ON ACOUSTIC METAMATERIAL

S. H. S. Esfahlani, S. Karkar and H. Lissek

Ecole Polytechnique Fédérale de Lausanne, EPFL-STI-IEL-LEMA, Station 11, CH-1015 Lausanne, Switzerland

e-mail: hussein.esfahlani@epfl.ch

In recent years, an increasing number of pioneering studies have been carried out in the field of acoustic metamaterials, following the path of electromagnetic metamaterials. These artificial engineered materials are designed in such a way so as to achieve new macroscopic properties, like negative refraction, that are not readily present in nature. While the design and the fabrication of these artificial materials is a hot topic among scientists in different fields of physics such as photonic, electromagnetic, acoustic and recently mechanic, an important part of the scientific research is now oriented towards the identification of actual applications for these structures. As the novel idea of metamaterial was first developed in the electromagnetic realm and for the microwave frequency range, it is somehow more mature in these fields than in acoustics. Metamaterial applications are now widely developed in electromagnetics especially for the design of new antenna. Among other examples, metamaterial concepts are aiming at reducing the coupling between two adjacent radiating elements of the array and increasing the operating bandwidth of radiating elements. It is also used for phase compensation in microwave transistors, and many more applications are rising in the recent literature. In year 2009, in analogy with electromagnetic transmission line metamaterial, our group proposed a concept of acoustic transmission line metamaterial, consisting of a waveguide periodically loaded with membranes along the duct, and transverse open channels (denoted "stubs"). Based on our proposed structure and in analogy with applications of transmission line electromagnetic metamaterials, researchers proposed the idea of an acoustic counterpart to the "backward wave antenna". These antennas or radiating devices have a very special property such that the radiation angle or the directivity changes with the frequency. In this article, a comprehensive, step by step, design methodology for acoustic backward wave antenna is presented. For this purpose we use the model proposed in our 2009 publication for acoustic transmission line metamaterial, but we focus the discussion on the optimization of the antenna performance. We also propose some closed form formulas for the practical design of such devices, and a formal validation of the structure is proposed using Comsol Multiphysics®.

1. Introduction

Metamaterials are artificial engineered materials designed to achieve some macroscopic properties that are not available in nature. In 1968 Russian physicist Victor Veselago studied the electromagnetic properties of a hypothetical medium in which both the permittivity and permeability were simultaneously negative [1] and three decades later Pendry and Smith created the first prototype of such material and called it Electromagnetic Left-Handed Metamaterial (LHM). Later, Electromagnetic Transmission Line (TL) metamaterials were proposed to overcome the problem of narrow bandwidth of the resonant type LHM [2] and very soon they found many applications such as Leaky Wave Antenna (LWA).

Leaky Wave (LW) is a term that refers to a mechanism to leak power all along a waveguide. It was first developed in 1940 [3] as an Electromagnetic LWA. Owing to its high directivity and frequency scanning capability, these antennas are very attractive in electromagnetics, but conventional TL-LW antennas use positive wavenumbers and as a consequence, they have the drawback of scanning only the half-space from broadside to endfire. After the advent of composite right/left handed (CRLH) TL-metamaterials, which were supporting both positive and negative wavenumbers, the backfire to endfire electromagnetic antenna was designed and fabricated [4].

In 2004 the analogue of electromagnetic LHM was proposed for acoustics [5]. In 2009 our group reported a transmission line acoustic metamaterial with a considerable bandwidth compared to resonant type acoustic metamaterials, and in the same article the leaky wave nature of this structure was discussed in [6] and [7]. Later, the same prototype was fabricated and measured [8].

In this paper, we present aspects of acoustic leaky wave structures that have not been covered up to date and try to develop a methodology for the design of acoustic leaky wave structures. First, we introduce a method for the design of these structures. Then, we use that method to design a structure that satisfies the needs of a LW antenna. Last, we show the simulated results of our proposed structure and give some guidelines to improve the structure.

2. Theory of CRLH Metamaterials

An incremental section of a conventional fluid (with positive refractive index) can be modelled as in Figure 1 (a), where $m_a = (\rho / S)dz$ is an acoustic mass and $C_a = (S/K)dz$ is an acoustic compliance and ρ and K are the density and bulk modulus of the medium, respectively. Figure 1 (b) shows the dual topology of the conventional TL, which is often referred to as the dual TL. Such a structure is known to exhibit a negative refractive index. Combining these two structures, as in Figure 1 (c), results in the transmission line model of a CRLH metamaterial, the response of which is dominated by m_{ap} and C_{as} in the low frequencies, resulting in a left-handed (LH) behaviour (or negative refractive index). At higher frequencies, m_{as} and C_{ap} are predominant, resulting in a right-handed (RH) behaviour (or positive refractive index).

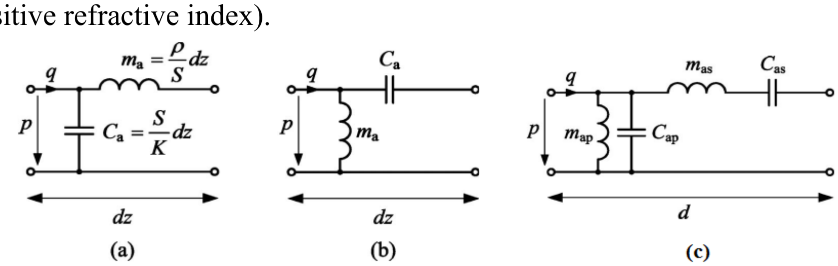
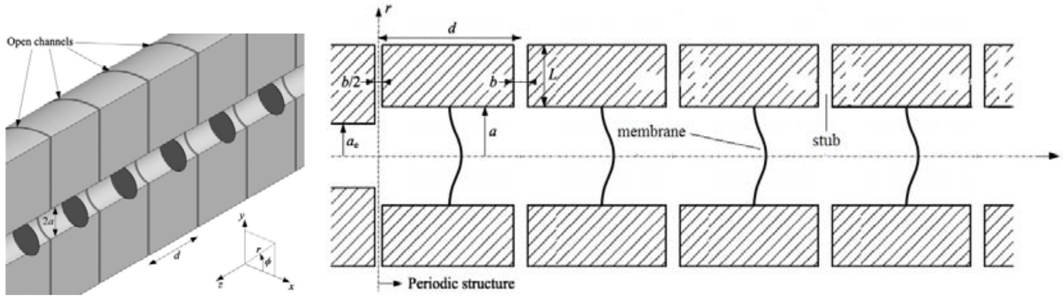


Figure 1. Lumped element model for (a) a conventional medium (right-hand) (b) a dual medium (left-hand) (c) a CRLH metamaterial.

On the basis of circuit modelling of CRLH metamaterials *Bongard et al* [4] implemented the CRLH acoustic metamaterial using acoustic waveguides, membranes and stubs (see Figure 2). Here the host "medium" is an acoustic waveguide with circular cross section and perfectly rigid walls operating as series acoustic mass and shunt acoustic compliance. By using membranes, a series mass and compliance are introduced, and a shunt acoustic masse is simply achieved with transversally connected open channels. Detailed explanation about the design of this structure is available in [6] and [7].



(a) **(b)**

Figure 2. CRLH TL combining membrane and radial open stub. (a) 3D view (yz-plane cut) (b) 2D axisymmetric view.

3. Designing Acoustic Leaky Wave Antenna (LWA)

The radiating formulas of acoustic LWA, such as radiation angle and radiation pattern were discussed in [8] so here we will discuss about the leaky wave trait and the efficiency of the antenna that are the most important factors in LWA. The leaky-mode (LM) time-space harmonic dependence takes the form $e^{-j(\omega t - k_z z)}$, where k_z is the complex longitudinal propagation constant. It is well known that k_z is related to the pointing angle θ_{RAD} and the radiation efficiency η_{RAD} through the following expressions [9]:

$$k_z = \beta_z - j\alpha_z , \tag{1}$$

$$sin(\theta_{RAD}) = \frac{\beta_z}{k_0} , \tag{2}$$

$$\eta_{RAD} = 1 - e^{-2\alpha_z L_A} , \tag{3}$$

where z is the longitudinal direction of the LWA (see Figure 2), β_z is the LM phase constant, α_z is the LM leakage rate, k_0 is the free-space wavenumber, L_A is the LWA length. The independent control of the LM phase β_z and leakage constants α_z is of key importance for the synthesis and the flexible adjustment of the radiation pattern of a practical LWA [9]. In order to make our analysis and synthesis easier we do not consider losses in our model so the total input power is divided between transmitted, reflected and radiated power:

$$P_{in} = P_{reflection} + P_{transmission} + P_{radiation} . \tag{4}$$

It is important to note that designing a structure to have a specific leakage constant α_z is difficult. Thus, L_A is taken as the free parameter in view of optimizing η_{RAD} by means of equation (3). The value of α_z is fixed by the internal structure of the cells and can be determined by using the following equation:

$$P_2 = P_1 e^{-2\alpha_z l_{12}} , \tag{5}$$

where P_2 and P_1 are the average power measured in two different cross sections of the transmission line that are separated by length l_{12}.

As numerical simulations of these structures are time consuming, the following steps can be followed to decrease the difficulty of design:

- First, the internal structure of the CRLH cell should be designed (here, using COMSOL Multiphysics).
- Then, using equation (5), the leakage constant α_z is calculated. The value of α_z is frequency dependent, so a mean value will be used in the next steps.
- Last, using equation (3), the appropriate length of L_A is calculated for any value of radiation efficiency (usually *90%*).

4. Results

We now present a practical example, using the same cell structure as in [6]. The dimensions and material properties of this structure are listed in Table 1. Its Bloch diagram is depicted in Figure 3, highlighting the radiation region.

Table 1. Geometric sizes and material properties of the structure

Physical quantity	Value/unit	Physical quantity	Value/unit
ρ (mass density of air)	1.188 kg/m^3	a_e (radius matching waveguide)	5.54 mm
K (bulk modulus of air)	137.4 kPa	d (length of each cell)	34 mm
C (celerity of air)	340 m/s	b (width of stub)	1 mm
E (membrane Young's modulus)	2.758 GPa	L (length of stub)	43.5 mm
v (Poisson ratio)	0.34	h (thickness of the membrane)	125 µm
ρ_m (mass density of membrane)	1420 kg/m^3	a (waveguide/membrane radius)	9.06 mm

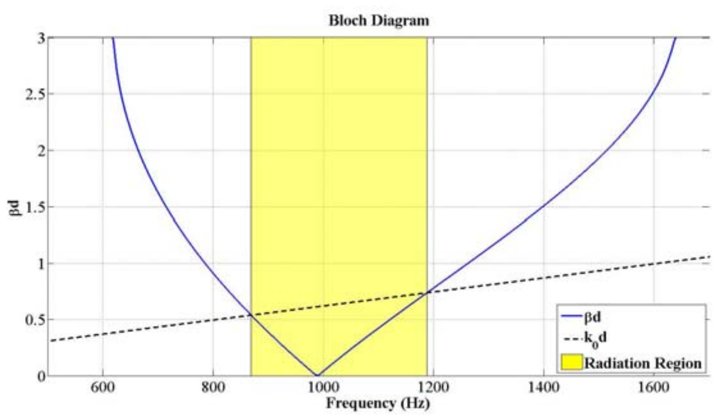


Figure 3. Dispersion diagram

A short length structure is simulated to derive the value of α_z. Here four different structures with *10, 19, 29* and *80-cell* are shown to validate the consistency of the results but usually a short length structure like *10-cell* is enough for defining the value of α_z. According to Figure 4, an average value of *0.5* is assigned for α_z.

Using equation (3) and assigning an efficiency of *0.9* as our goal, L_A should be approximately *230* cm or *67 cells.* Because α_z has values lower than *0.5* at certain frequencies, equation (3) implies that L_A should be higher than *67,* in order to ensure the efficiency of *90%* in most of the bandwidth. In the present case, an *80-cell* prototype has been chosen and simulated using Comsol.

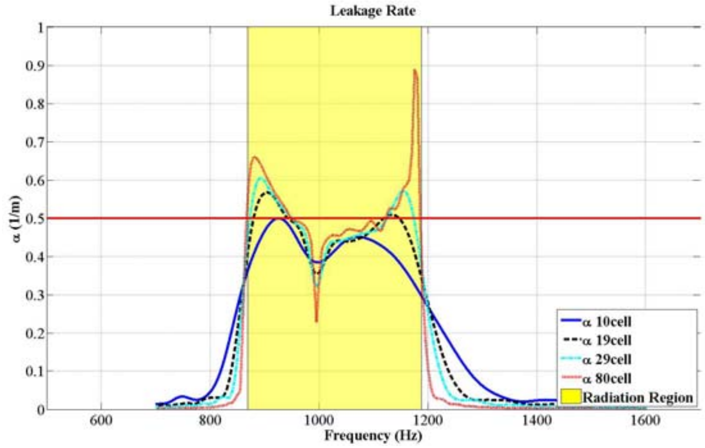


Figure 4. Leakage constant α as a function of frequency

Figure 5 shows the reflection and transmission coefficient diagram. The impedance match in the radiating region seems satisfying as S_{11} is on average below *-20dB* over the whole radiation region, except for a very sharp peak at the transition frequency.

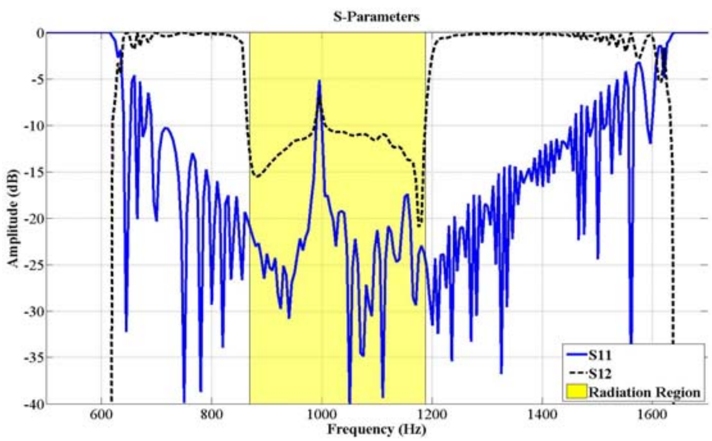


Figure 5. Transmission and Reflection coefficients for 80-cell leaky wave structure

Figure 6 shows the radiation efficiency of the antenna that is calculated using equation (4). Clearly an efficiency of above *90%* is guaranteed for the entire radiation bandwidth, except for a very sharp deep at the transition frequency.

Last, Figure 7 shows the radiating pattern of this antenna. Sweeping the frequency, the antenna pattern is scanning from back-fire to end-fire.

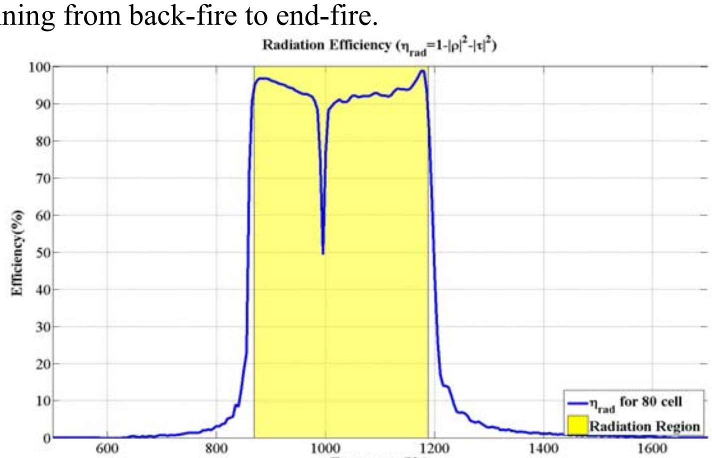


Figure 6. Radiation efficiency for 80-cell leaky wave structure

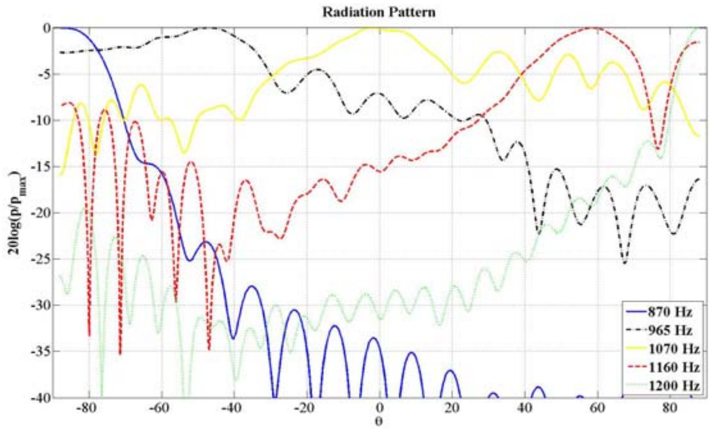


Figure 7. Radiation pattern of leaky wave structure normalized to maximum

5. Conclusion

As acoustic leaky wave structures are new in acoustics and have not yet been fully covered, a certain amount of research is still to be done in the field. Among the different topics to be addressed, we can cite a few such as increasing the directivity, optimizing side lobe level, controlling leakage constant or modelling the lossy structures. Also, using it in real life will motivate more research in this field. It is worth noting that our structure is axisymmetrical and radiates into whole 3D. But antenna structures or sensors are more likely to radiate or detect in only one hemisphere thus, another interesting topic would be to design a leaky wave structure that would radiate in only one hemisphere.

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