Optimization based control for target estimation and tracking via highly observable trajectories*

An application to motion control of autonomous robotic vehicles

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Abstract. This paper proposes a Model Predictive Control (MPC) scheme to solve the target estimation and tracking problem. The objective is to derive a feedback law that drives a follower vehicle to a target vehicle using an on-line estimate of the target's state. In this scenario, when the target is observed through a nonlinear observation model, e.g., bearing only or range only sensors, it is possible to show that solving the tracking problem independently from the estimation problem can lead to an unsatisfactory result where the follower-target system is driven by the controller through unobservable or weakly observable trajectories and, as result, the state of the target vehicle cannot be recovered or cannot be recovered with high accuracy leading to the failure of the control strategy. In this paper, we propose an optimization based scheme that embeds, in a seamless way, an index of observability in the design of the target tracking controller resulting in a closed loop behavior that balances the objective of target tracking with the competing objective of maintaining a good estimate of the state of the target. Numerical results are presented that illustrate this type of behavior.

Keywords: Target tracking, target estimation, model predictive control, highly observable trajectories, observability

1 Introduction

This paper addresses the design of a continuous time output feedback sample-data MPC controller for target estimation and tracking.

One of the main challenges in the design of a target tracking algorithm steams from the unavailability of the state of the target vehicle which has to be estimated

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on-line using measurements from the sensors with often highly nonlinear models. Such nonlinearities often define a set of unobservable state and input trajectories that should be avoided in order to maintain a good estimate of the state of the target vehicle, which is crucial for the success of the target tracking algorithm.

To face this problem, [4, 9] propose solutions based on suitable maneuvers designed to keep the state of the target vehicle observable are performed. Although, with this type of strategies, it may be difficult to asses the quality of the overall estimate obtained during the all vehicle trajectory, moreover the resulting system may result in artificial closed loop position trajectories.

In this work we exploit the potentiality of optimization based control (see, e.g., [5, 8, 7] and, more recently, [10, 6] for an overview) to design a control law that jointly minimizes the distance target-follower and an index of observability specifically designed to penalize weakly observable trajectories.

Related works are [2] and [3] where an economic index is embedded in a MPC controller to influence the transient and asymptotic closed loop behavior, respectively.

The organization of the paper is as follows. Section 2 contains the main result of the paper. In Section 3, we illustrate the potentials of the proposed method by solving the target estimation and tracking problem first in a decoupled manner, i.e., without introducing the observably index, and then using the proposed scheme. Section 4 closes the paper with some conclusions.

2 Follower-target MPC design

In this section a solution of the target estimation and tracking problem discussed above is proposed. We start with the problem statement description of the follower-target system. Then, a generic MPC control law for target tracking is presented in combination with an observability index to obtain the proposed control law strategy.

2.1 Follower-target system description

Let the follower and target vehicles be described by

$$\begin{aligned} \dot{x}_f(t) &= f_f(x_f(t), u_f(t), w_f(t)), & u_f(t) &\in \mathcal{U}_f, & t &\geq 0 \\ \dot{x}_t(t) &= f_t(x_t(t), u_t(t), w_t(t)), & u_t(t) &\in \mathcal{U}_t, & t &\geq 0 \end{aligned}$$

where $(x_f(t), u_f(t), w_f(t)) \in \mathbb{R}^{n_f} \times \mathbb{R}^{q_f} \times \mathbb{R}^{p_f}$ and $(x_t(t), u_t(t), w_t(t)) \in \mathbb{R}^{n_t} \times \mathbb{R}^{q_t} \times \mathbb{R}^{p_t}$ denote the triples of state, input, and disturbance vectors of the follower vehicle and target vehicle, respectively, evaluated at time t. For sake of simplicity, the dependence on time and parameters is dropped whenever clear from the context. We consider the follower and target input vectors to be constrained in the sets $\mathcal{U}_f$ and $\mathcal{U}_t$, respectively. The follower-target system is defined by

$$\dot{x} = f(x, u_f, u_t, w_f, w_t) := \begin{pmatrix} f_f(x_f, u_f, w_f) \\ f_t(x_t, u_t, w_t) \end{pmatrix}, \qquad x := \begin{pmatrix} x_f \\ x_t \end{pmatrix} \tag{1}$$

The state of the follower vehicle is measured through the observation model

$$y_f = h_f(x_f, v_f),$$

and, moreover the follower vehicle takes measurements of the target vehicle through the observation model described by

$$y_t = h_t(x_f, x_t, v_t)$$

where $(y_f, v_f) \in \mathbb{R}^{m_f} \times \mathbb{R}^{b_f}$ and $(y_t, v_t) \in \mathbb{R}^{m_t} \times \mathbb{R}^{b_t}$ are the pairs of measurement and measurement noise of the follower and target observation models, respectively. Thus, the observation model of the follower-target system is defined by

$$y = h(x, v) := \begin{pmatrix} h_f(x_f, v_f) \\ h_t(x_t, v_t) \end{pmatrix}, \qquad y := \begin{pmatrix} y_f \\ y_t \end{pmatrix} \qquad v := \begin{pmatrix} v_f \\ v_t \end{pmatrix}. \tag{2}$$

The function $h_f(\cdot)$ is related to the sensors on-board of the follower vehicle, e.g., Global Positioning System (GPS), Inertial Measurement Unit (IMU), and magnetic compass, and the function $h_t(\cdot)$ with the sensor used to observe the target vehicle, e.g., a bearing only sensor, like a camera, or a range only sensor, like a sonar.

In order to perform target tracking it is convenient to estimate the future control input signal of the target vehicle, and thus its future position. Toward this goal, we choose a finite dimensional smooth parameterization $p_u \in \mathbb{R}^{n_p}$ of such signal, with the associated time derivative $\dot{p}_u = f_u(p_u)$. The parameter p_u is then estimated, together with the state of the follower-target system, by the observer defined by

$$\dot{x}_o = f_o(x_o, u_f, y), \tag{3a}$$

$$\begin{pmatrix} \hat{x} \\ \hat{p}_u \end{pmatrix} = h_o(x_o) \tag{3b}$$

where $x_o \in \mathbb{R}^{n_o}$ denotes the internal state vector of the observer and $\hat{x} \in \mathbb{R}^{n_f+n_t}$ and $\hat{p}_u \in \mathbb{R}^{n_p}$ denote the estimates of the state vector x and parameter p_u, respectively. We denote with $\bar{u}_t(\cdot; p_u)$ the predicted input signal of the target associated with the parameter p_u.

As an example, a possible choice of nonlinear observer could be the well known Extended Kalman Filter (EKF) and a possible parameterization of the input of the target vehicle could be $p_u = u_f$ and $\dot{p}_u = f_u(p_u) = 0$, which captures the set of constant (and in practice slowly varying) target inputs, i.e., $\bar{u}_t(\tau; p_u) = p_u$, for all $\tau > 0$.

2.2 MPC for Target tracking

Using the notation introduced above, in this section we describe a generic MPC controller for target tracking that uses as input the estimate provided by the observer (3). For a generic trajectory $x(\cdot)$, $x([t_1, t_2])$ denotes the trajectory considered in the time interval $[t_1, t_2]$ and we use the notation $x(\cdot; z_1, \dots)$ whenever we would like to make explicit the dependence of the trajectory $x(\cdot)$ on the generic optimization problem parameters $z_1, \dots$.

The MPC optimization problem $\mathcal{P}(z, p)$, with $(z, p) \in \mathbb{R}^{n_f+n_t} \times \mathbb{R}^{n_p}$ is defined as follows:

Definition 1. *(MPC problem) Given the pair of vectors $(z, p) \in \mathbb{R}^{n_f+n_t} \times \mathbb{R}^{n_p}$, with the associated parametric target input signal $\bar{u}_t(\cdot; p)$, and a horizon length $T \in \mathbb{R}_{>0}$, the open loop MPC optimization problem $\mathcal{P}(z, p)$ consists of finding the optimal control signal $\bar{u}_f^*([0, T])$ that solves*

$$J_T^*(z,p) = \min_{\bar{u}_f([0,T])} \int_0^T l_{tt}(\bar{x}(\tau), \bar{u}_f(\tau), \bar{u}_t(\tau; p))d\tau + m_{tt}(\bar{x}(T)) \tag{4a}$$

$$\begin{aligned} s.t. \quad & \dot{\bar{x}} = f(\bar{x}, \bar{u}_f, \bar{u}_t(\tau; p), 0, 0) \\ & \bar{u}_f \in \mathcal{U}_f,\ \bar{x}(0) = z,\ \bar{x}(T) \in \mathcal{X}_f \end{aligned}$$

□

The *finite horizon cost* is composed of the *stage cost* $l_{tt} : \mathbb{R}^{n_f+n_t} \times \mathbb{R}^{q_t} \times \mathbb{R}^{n_p} \to \mathbb{R}_{\geq 0}$ and the *terminal cost* $m_{tt} : \mathbb{R}^{n_f+n_t} \times \mathbb{R}^{n_p} \to \mathbb{R}_{\geq 0}$, which is defined over the *terminal set* $\mathcal{X}_f \subseteq \mathbb{R}^{n_f+n_t}$. The subscript tt is used to emphasize that the stage and terminal costs are designed for target tracking.

In a sample-data receding horizon strategy, the control input is computed at discrete sample times $\mathcal{T} := \{t_0 = 0, t_1, \dots\}$, and the MPC control law is defined as

$$u_f(t) := \bar{u}^*(t - \lfloor t \rfloor; \hat{x}(\lfloor t \rfloor), \hat{p}_u(\lfloor t \rfloor)), \tag{5}$$

where $\lfloor t \rfloor$ is the maximum sampling time $t_i \in \mathcal{T}$ smaller or equal than t, i.e., $\lfloor t \rfloor = \max_{i \in \mathbb{N}_{\geq 0}} \{t_i \in \mathcal{T} : t_i \leq t\}$. Since the system is not time varying, the space of trajectories over which we optimize are considered, without loss of generality, in the interval $[0, T]$ and t_0 is chosen to be the time zero. Note that (4)-(5) is an output dynamic feedback control law since it uses the estimates of the state and target control parameter provided by the observer.

2.3 Observability index

In this work we propose a strategy to modify the MPC optimization problem of the previous section in order to avoid weakly observable/non observable closed loop trajectory resulting in an effective target estimation and tracking controller. In this section we propose an index of observability.

Consider the observability matrix of the system (1) associated to the output y defined in (2), both considered in the nominal case (i.e., $v_f = 0$, $v_t = 0$, $w_f = 0$, $w_t = 0$), i.e.,

$$\mathcal{O}(x,u_f,u_t) = \frac{\partial}{\partial x}\left(y' \; \dot{y}' \; \ddot{y}' \; \dots \; y^{\{r\}'}\right)'$$

where $y^{\{r\}}$ denotes the rth derivative of the output y with respect of time. From the properties of the observability matrix, given $r \in \mathbb{N}_{>0}$ the state of system (1) is locally observable at a given state and input pair $(\bar{x}, \bar{u}_f)$ if the matrix $\mathcal{O}(\bar{x}, \bar{u}_f, \bar{u}_t)$, for a given estimated target input vector $\bar{u}_t$, is full rank. For general nonlinear systems the number of derivatives r to be considered is not known a priori. An intuitive procedure to select r consists in increasing it until the observability matrix becomes full rank for some values of the state and input vectors. Then, driving the system through those values is enough to guarantee observability.

Let $\sigma_{min}(A)$ and $\sigma_{max}(A)$ denote the minimum and maximum singular value of a generic matrix A. To obtain a measure of the degree of observability, one possibility is to use the index $1/\sigma_{min}(\mathcal{O})$, which increases as $\mathcal{O}$ gets close to singularity and becomes infinity when $\mathcal{O}$ loses rank. Another index of interest is the condition number of $\mathcal{O}$, i.e., $\kappa(\mathcal{O}) := \sigma_{max}(\mathcal{O})/\sigma_{min}(\mathcal{O})$, which broadly speaking, provides a measure of the difference of the "quality" of observability of the state components, where $\kappa(\mathcal{O}) = 1$ if all the state components have the same "quality" of observability. Prompted by these observations, we select the following observability index:

$$l_o(x,u_f,u_t) = k \, \arctan\left(\frac{1}{k}\left(\frac{\alpha_1}{\sigma_{min}\mathcal{O}(x,u_f,u_t)} + \alpha_2(\kappa(\mathcal{O}(x,u_f,u_t)) - 1)^2\right)\right) \tag{6}$$

for some positive constants $\alpha_1 > 0$ and $\alpha_2 > 0$, where the positive constant $k > 0$ defines the width of the region where the nonlinearity $\arctan(\cdot)$, used as smooth saturation-like function, behaves almost linearly.

Note that the observability matrix is not the only method to define the index of observability and other mathematical tools, e.g., the determinant or the trace of the Fisher Information Matrix (FIM), can be exploited in a similar fashion.

2.4 Proposed controller

Using the observability index suggested in the previous section, the proposed controller is obtained from the controller of Section 2.2 but redefining the stage cost as

$$l(x,u_f,u_t) = l_{tt}(x,u_f,u_t) + l_o(x,u_f,u_t) \tag{7}$$

Note that from (6) the value of the function $l_o(\cdot)$, responsible to keep the closed loop trajectories observable, is saturated to ensure that it does not constantly dominate the costs $l_{tt}(\cdot)$ and $m_{tt}(\cdot)$, which are in charge of driving the follower toward the target.

Varying the value of k in (6) is possible to regulate the importance of the observability of the closed loop trajectories with the conflicting goal of perfect target tracking.

3 Simulation results

In this section we consider the target estimation and tracking problems for both the target and the follower unicycle-like vehicles.

Let $\{I\}$ be an inertial coordinate frame and $\{B_f\}$ a body coordinate frame attached to the follower vehicle. The pair $(p_f(t), R(\theta_f(t))) \in SE(2)$ denotes the configuration of the follower vehicle, position and orientation, where $R(\theta_f(t))$ is the rotation matrix, from body to inertial coordinates, associated with the angle $\theta_f(t)$. For a unicycle-type vehicle, the kinematic model of a vehicle satisfies

$$\dot{p}_f(t) = R(\theta_f(t)) \begin{pmatrix} v_f(t) \\ 0 \end{pmatrix}, \qquad \dot{\theta}_f(t) = \omega_f(t),$$

where $\omega_f(t)$ denotes the angular velocity express in the body frame. For this example we consider the case where the control input of the follower vehicle

$$u_f(t) = \begin{pmatrix} v_f(t) \; \omega_f(t) \end{pmatrix}' \in \mathcal{U}_f,$$

is constrained in the set

$$\mathcal{U}_f = \left\{ \begin{pmatrix} v_f \\ \omega_f \end{pmatrix} : -2 \leq v \leq 2, -\pi \leq \omega \leq \pi \right\}.$$

We assume that the target vehicle is of the same kind of the follower vehicle, but with unconstrained input. Thus, the follower-target system is defined as follows:

$$\dot{x} = \begin{pmatrix} \dot{p}_f \\ \dot{\theta}_f \\ \dot{p}_t \\ \dot{\theta}_t \end{pmatrix} = \begin{pmatrix} R(\theta_f(t)) \begin{pmatrix} v_f(t) \\ 0 \end{pmatrix} \\ \omega_f(t) \\ R(\theta_t(t)) \begin{pmatrix} v_t(t) \\ 0 \end{pmatrix} \\ \omega_t(t) \end{pmatrix}, \qquad x = \begin{pmatrix} p_f \\ \theta_f \\ p_t \\ \theta_t \end{pmatrix}.$$

The input signal of the target vehicle, which is unknown to the follower, is defined as $v_t(t) = 0.5$ and $\omega_t(t) = (\pi/200)\cos(0.05t)$.

The position and heading of the follower vehicle is continuously measured, i.e,

$$y_f(t) = h_f(x_f, v_f) = \begin{pmatrix} p_f(t) \\ \theta_f(t) \end{pmatrix}$$

and we consider that the position of the target vehicle is continuously observed by an omnidirectional camera centered at the position of the follower vehicle. In this case, the observation model can be described as

$$y_t(t) = h_t(x_f, x_t, v_t) = \frac{p_t(t) - p(t)}{\|p_t(t) - p(t)\|},$$

where $y_t(t) \in \mathbb{R}^2$ is a bearing only observation, which provides information about the relative direction of the vehicle but not about the distance (in fact, $\|y_t(t)\| = 1$).

The future input of the follower is parameterized by the parameter $p_u = u_t$, with associated time derivative $\dot{p}_u = 0$, and an Extended Kalman Filter is used to jointly estimate the state of the follower-target system and of the parameterization p_u using $x_o = (x', p_u')'$ and $h_o(x_o) = x_o$.

The target-tracking controller is designed using the MPC for trajectory-tracking proposed in [1]. In this last work, the trajectory to be followed is defined by specifying a desired position, its derivative, and a bound on the value of the derivative. In our case we wish to track the position of the target vehicle, thus we can use the target position, the target velocity, which are components of our state vector, and a bound on the target velocity, which is considered to be 0.5. Using the same notation of [1], the MPC controller parameters are $\epsilon = \begin{pmatrix} -0.2 \\ 0 \end{pmatrix}$, $K = 0.1 I_{2\times2}$, $O = 0.1 I_{2\times2}$, and $Q = 10 I_{2\times2}$, where $I_{2\times2}$ denotes an identity matrix of size 2×2. The resulting terminal set is $\mathcal{E}_f = \{e'e \leq 26.4^2\}$.

For simulation purposes the system is discretized with discretization step of 0.1, the set of sampling times is $\mathcal{T} = \{t_i = 0.1i,\ i \in \mathbb{N}_{\geq 0}\}$, and the horizon length is chosen to be $T = 0.3$ s.

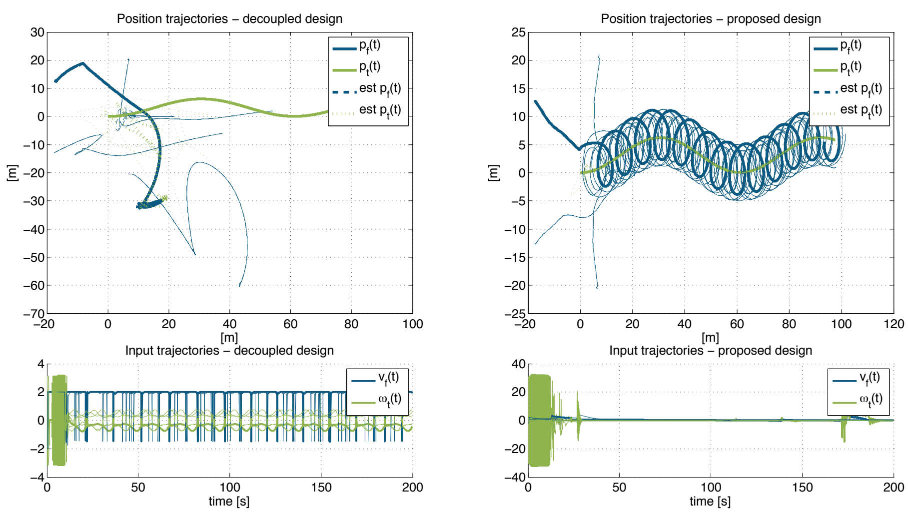

Fig. 1. The closed loop system trajectories associated with the decoupled (left) and coupled scheme (right) for different initial conditions. The ticker lines are associated to the same initial condition. The figure on the top displays the follower and target position trajectories, with solid blue and green lines, and the associated estimates, with dashed lines of the same color. The input signals of the target tracking controller is plotted in the bottom figures.

As expected using a decoupled design, i.e., using the target tracking controller in closed loop with the observer without embedding the observability index, the system is driven through unobservable/weakly observable trajectory and, as consequence, we have an increase of the estimation error along time and eventually a failure of the target tracking algorithm. Fig.1 displays the trajectories of the position of the follower and target together with the associated estimates provided by the Kalman filter (top) and the associated input signal (bottom) for the case of decoupled (left) and coupled (right) design. Fig.(2)(top) displays the observability index evaluated along the closed loop trajectories. It can be seen that for the decoupled design (left) the system is driven through unobservable/weakly observable state-input trajectories resulting in a bad estimate of the state of the follower-target system.

In order to avoid this problem, we use the proposed method adding in the stage cost of the MPC control the observability index (6) with $k = 10^5, \alpha_1 = 10^3, \alpha_2 = 10^2$. To reduce the computational complexity, the observability matrix, with $r = 3$, was computed considering the vehicles to be single integrators and using the real linear velocity to evaluate it. In this case we notice from Fig. 1 that the follower do not reach the vehicle but approaches it and start orbiting around it, so reducing the observability index. After an initial transient behavior, along the closed loop state and input trajectories the system is always observable, i.e., the observability index never saturates and, as effect, the estimation error converges to zero, Fig. 2 (right column). Note that, for one initial condition, the estimation error does not converge to zero, although it converges to an indistinguishable configuration (coherent with the observation) where the target moves backward (negative velocity).

4 Conclusion

A systematic procedure to design an optimization based target estimation and tracking controller was presented. The main idea consist in embedding in the stage cost of the MPC controller an observability index that reward highly observable trajectories resulting in this way a good estimate of the state of the follower-target system and, thus, to a successful and accurate tracking of the target.

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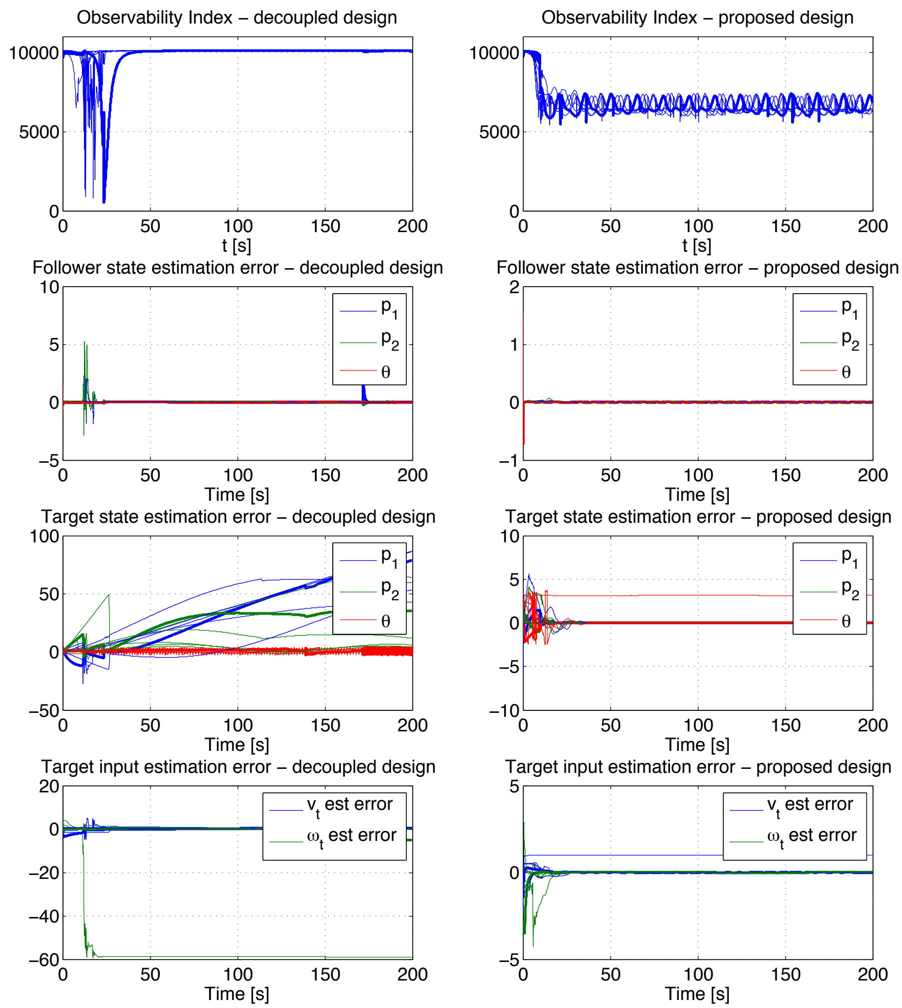

Fig. 2. The closed loop observability index (first row) and the estimation errors for the follower state (second row), target state (third row), and target inputs (bottom row) for the decoupled (left column) and coupled (right column) design.

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