# ACCURATE GYROTRON MAGNETIC AXIS DETERMINATION 

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#### Abstract

A way to determine the magnetic axis of gyrotron cryomagnets, and more generally magnets composed of coils wound around a single mandrel is presented. The method is based on measurements of the longitudinal component of the magnetic field performed at suitable locations with a high accuracy ( 100 ppm ) Hall probe [1]. The tilt and the shift of the magnetic axis with respect to the mechanical axis can be determined with a typical accuracy of 0.01 deg and 0.1 mm respectively.


## Introduction

In order to maximise the interaction between the hollow electron beam and the cavity resonant mode of a gyrotron, an accurate alignment of the magnetic and the mechanical axis of the magnet is required. It is usually specified that the mechanical axis should remain within a cylinder of diameter 0.3 mm surrounding the magnetic axis. In the method described here, measurements of the longitudinal field are performed on a circular path of radius $\mathrm{R}_{0}$ with respect to the mechanical axis, with the probe parallel to it. The assumption that the magnetic axis is linear is made. It is reasonable since the coils are usually wounded around a common mandrel.

## Principle

The geometry is described on Fig. 1. The magnetic axis $e_{z}$ is taken as a reference. The tilt is described by the angles $\lambda_{\text {tilt }}$ and $\theta_{\text {tilt, }}$, whereas the shift is given by DR in direction $\theta_{\text {shift }}$. Note that DR and $\theta_{\text {shift }}$ are $z$-dependent, whereas $\lambda_{\text {tilt }}$ and $\theta_{\text {tilt }}$ are not.


Fig. 1 Relative orientation of the magnetic axis and the mechanical axis.

The 1D Hall probe used measures the magnetic field component parallel to the mechanical axis, on a circle with radius $\mathrm{R}_{0}$. The variation of the measure along the probe path is due to the change in longitudinal position, but is also related to the radial component of the magnetic field.

By projecting the local magnetic field on a direction parallel to the probe, and by keeping first order terms in DR and in $\lambda_{\text {tilt }}$, on can estimate the field as a function of the misalignment parameters and the measure angle $\phi$ :

$$
\begin{align*}
& \mathrm{B}_{\text {mes }}\left(\mathrm{R}_{0}, \theta_{\text {tilt }}, \lambda_{\text {tilt }}, \theta_{\text {shift }}, \mathrm{DR}, \phi\right)= \\
& \mathrm{B}_{\mathrm{Z}}\left(\mathrm{R}_{0}, \mathrm{z}_{0}\right)  \tag{1}\\
&+\left.\frac{3}{2} \mathrm{R}_{0} \frac{\partial \mathrm{~B}_{\mathrm{Z}}}{\partial \mathrm{z}}\right)_{\mathrm{z}_{0}, \mathrm{r}=0} \sin \lambda_{\text {tilt }} \sin \left(\phi-\theta_{\text {tilt }}\right) \\
&-\left.\frac{\mathrm{R}_{0}}{2} \frac{\partial^{2} \mathrm{~B}_{\mathrm{Z}}}{\partial \mathrm{z}^{2}}\right)_{\mathrm{z}_{0}, \mathrm{r}=0} \operatorname{DR} \cos \left(\phi-\theta_{\text {shift }}\right)
\end{align*}
$$

where the longitudinal field and its derivatives are estimated on-axis, using analytical expansions [2,3]. A similar expression can be obtained if elliptical integrals are used:

$$
\begin{align*}
& \mathrm{B}_{\text {mes }}\left(\mathrm{R}_{0}, \theta_{\text {tilt }}, \lambda_{\text {tilt }}, \theta_{\text {shift }}, \mathrm{DR}, \phi\right)= \\
& \quad \mathrm{B}_{\mathrm{Z}}\left(\mathrm{R}_{0}, \mathrm{z}_{0}\right)  \tag{2}\\
& \left.\quad+\left(-\mathrm{B}_{\mathrm{r}}\left(\mathrm{R}_{0}, \mathrm{z}_{0}\right)+\frac{\partial \mathrm{B}_{\mathrm{z}}}{\partial \mathrm{z}}\right)_{\mathrm{z}_{0}, \mathrm{R}_{0}} \mathrm{R}_{0}\right) \sin \lambda_{\text {tilt }} \sin \left(\phi-\theta_{\text {tilt }}\right) \\
& \\
& \left.\quad+\frac{\partial \mathrm{B}_{\mathrm{Z}}}{\partial \mathrm{r}}\right)_{\mathrm{z}_{0}, R_{0}} \mathrm{DR} \cos \left(\phi-\theta_{\text {shift }}\right)
\end{align*}
$$

The formula (2) is more accurate than (1) in the absolute, but equally useful in practice since relative variations only are considered.

## Determination of the tilt and shift parameters

Looking at (1), it appears that the tilt is related to the first longitudinal derivative of the field, and the shift to the second derivative. It is then natural to perform measurements at longitudinal locations where one or the other of these quantities is maximal. The sequence of measurements that are usually performed is:

1. Measurement of the on-axis magnetic field profile, in case a calibration of the magnet power supplies is needed.
2. Measurement of the tilt angle by locating the probe at positions where $\partial \mathrm{B}_{\mathrm{z}} / \partial \mathrm{z}$ is maximal and $\partial^{2} \mathrm{~B}_{\mathrm{z}} / \partial \mathrm{z}^{2}=0$. The tilt is directly found by considering the maximal variation of the measured field:

$$
\begin{aligned}
& \left.\Delta \mathrm{B}_{\text {max }}=\mathrm{B}_{\operatorname{mes} \max }-\mathrm{B}_{\operatorname{mesmin}}=3 \mathrm{R}_{0} \frac{\partial \mathrm{~B}_{\mathrm{z}}}{\partial \mathrm{z}}\right)_{z_{0}, r=0} \sin \lambda_{\text {tilt }} \\
& \lambda_{\text {tilt }}=\sin ^{-1}\left(\frac{\Delta \mathrm{~B}_{\max }}{\left.3 \mathrm{R}_{0} \frac{\partial \mathrm{~B}_{\mathrm{z}}}{\partial \mathrm{z}}\right)_{z_{0}, r=0}}\right)
\end{aligned}
$$

The second tilt parameter, the direction $\theta_{\text {tilt }}$ is found by fitting a sinusoidal curve to the measurement.
3. Estimation of the shift by locating the probe where $\partial^{2} \mathrm{~B}_{\mathrm{z}} / \partial \mathrm{z}^{2}$ is maximal, and by explicitely taking into account the tilt contribution. Unfortunately, in the case of our magnets, the position with $\partial \mathrm{B}_{\mathrm{z}} / \partial \mathrm{z}$ is close to the gyrotron cavity and corresponds to a minimum of $\partial^{2} \mathrm{~B}_{\mathrm{z}} / \partial \mathrm{z}^{2}$ too. In order to extract the contribution from the shift alone, the following object is constructed:

$$
\begin{aligned}
\mathrm{B}_{\text {shift }}= & \mathrm{B}_{\text {mes }}(\phi)-\mathrm{B}_{\mathrm{Z}}\left(\mathrm{R}_{0}, \mathrm{z}_{0}\right) \\
& \left.-\frac{3}{2} \mathrm{R}_{0} \frac{\partial \mathrm{~B}_{\mathrm{Z}}}{\partial \mathrm{z}}\right)_{\mathrm{z}_{0}, \mathrm{r}=0} \sin \lambda_{\text {tilt }} \sin \left(\phi-\theta_{\text {tilt }}\right) \\
\cong & \left.-\frac{\mathrm{R}_{0}}{2} \frac{\partial^{2} \mathrm{~B}_{\mathrm{Z}}}{\partial \mathrm{z}^{2}}\right)_{\mathrm{z}_{0}, \mathrm{r}=0} \mathrm{DR} \cos \left(\phi-\theta_{\text {shift }}\right)
\end{aligned}
$$

with the tilt parameters determined above. The local shift amplitude is then obtained with:

$$
\mathrm{DR}\left(\mathrm{z}=\mathrm{z}_{0}\right)=\left|\frac{\Delta \mathrm{B}_{\text {shift max }}}{\left.\mathrm{R}_{0} \frac{\partial^{2} \mathrm{~B}_{\mathrm{z}}}{\partial z^{2}}\right)_{\mathrm{z}_{0}, \mathrm{r}=0}}\right|
$$

and the shift direction is obtained by fitting a cosine function to the measurements.

In principle, two sets of measurements are sufficient to determine all the tilt parameters, but five measures ( 2 tilts and 3 shifts) are performed.

## Application to a 118 GHz gyrotron magnet

The above procedure was applied to the magnet described in [1], which has a nominal magnetic field of 4.6T. Represented on Figures 2 and 3 are the second derivative and the first


Fig. 2 Second derivative of the longitudinal magnetic field and optimal locations for the tilt (squares) and shift (circles) determination.


Fig. 3: First derivative of the longitudinal magnetic field and optimal locations for the tilt (squares) and shift (circles) determination.
derivative of the magnetic field, with the most appropriate positions for the tilt (squares) and shift (circles) measurements. The probe was fixed at a radius $\mathrm{R}_{0}=25 \mathrm{~mm}$. The results of the tilt measurements are shown on Fig. 4, where the squares represent experimental data and the solid curves are sinusoidal fits. The error bars correspond to the Hall probe accuracy (100ppm). The typical maximal difference is $10-20 \mathrm{G}$, which is well above the probe accuracy.


Fig. 4: Measurement of the tilt at two different locations. The symbols are experimental data and the solid curves fits.

The estimated tilt angle is $\lambda_{\text {tilt }}=0.049 \mathrm{deg}$, indicating that this particular magnet is slightly out of specifications. The two curves on Fig. 4 should be in phase opposition since the sign of $\partial \mathrm{B}_{\mathrm{z}} / \partial \mathrm{z}$ is not the same at the two longitudinal positions. The
discrepancy gives the accuracy in determining the tilt orientation, i.e. 20-30 deg.

The estimation of the shift uses $\lambda_{\text {tilt }}$ and $\theta_{\text {tilt }}$ and is thus slightly more delicate. Depending on the longitudinal position, the shift DR is of the order of 0.2 mm , which again indicates that this magnet should be realigned.

The alignement of 4 magnets hosting $500 \mathrm{~kW}, 118 \mathrm{GHz}$ gyrotron tubes was measured, and, even though all of them were found to be slightly out of specifications, no realignment was necessary since all tubes could still operate at their nominal output power.

## Conclusion

A simple method to estimate the magnetic axis of a gyrotron magnet has been presented. It relies on the possibility to measure the longitudinal magnetic field component with a high accuracy, at suitable locations. The accuracy in the determination of the tilt parameters is improved with respect to that described in [1].

## References

[1] C. Schott et al., "High Accuracy magnetic field measurements with a Hall probe", Review of Scientific Instruments, 70(6), 1999, 2703-2707.
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