

# Network dynamics of spiking neurons with adaptation

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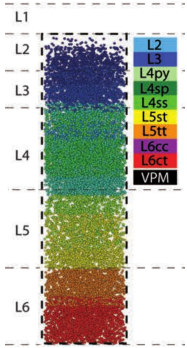


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## Circuit level: population dynamics



How to describe circuit level cortical dynamics?

- ▶ Cortex is layered.
- ▶ Few neuron types per layer.
- ▶ Each type in a layer forms a population.
- ▶ Populations have finite size.

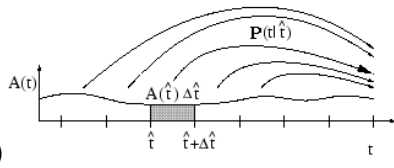
Here we derive a theory for such interacting populations, exploiting that neurons can be treated as quasi-renewal point processes, even in presence of spike-frequency adaptation.

Fig.: Oberlaender (2012) *Cereb Cortex*

## Stochastic population integral equation

For a population of  $N$  neurons with

- ▶ spike trains  $\mathbf{s}_i(\mathbf{t})$
- ▶ population activity  $\mathbf{A}(\mathbf{t}) = \frac{1}{N} \sum_i \mathbf{s}_i(\mathbf{t})$
- ▶ inter-spike-interval (ISI) probability density  $\mathbf{P}(\mathbf{t}|\hat{\mathbf{t}})$



we have (cf. [4]) 
$$\mathbf{A}(\mathbf{t}) = \int_{-\infty}^{\mathbf{t}} \underbrace{\mathbf{P}(\mathbf{t}|\hat{\mathbf{t}})\mathbf{A}(\hat{\mathbf{t}})}_{\mathbf{a}(\mathbf{t})} d\hat{\mathbf{t}} + \delta\mathbf{A}(\mathbf{t})$$

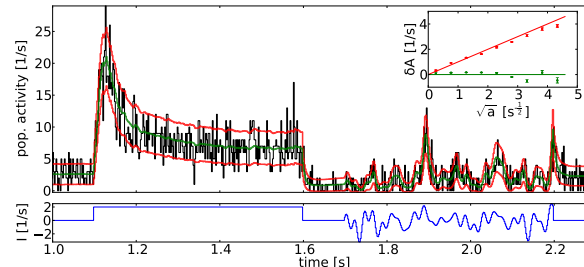
with fluctuations  $\delta\mathbf{A}(\mathbf{t})$  that obey (for  $\tau \geq 0$ , conditioned on  $\mathbf{A}(\hat{\mathbf{t}} < \mathbf{t})$ )

$$\langle \delta\mathbf{A}(\mathbf{t} + \tau) \delta\mathbf{A}(\mathbf{t}) \rangle =$$

$$\mathbf{N}^{-1} \mathbf{a}(\mathbf{t}) \delta(\tau) - \mathbf{N}^{-1} \int_{-\infty}^{\mathbf{t}} \mathbf{P}(\mathbf{t} + \tau|\hat{\mathbf{t}}) \mathbf{P}(\mathbf{t}|\hat{\mathbf{t}}) \mathbf{A}(\hat{\mathbf{t}}) d\hat{\mathbf{t}}$$

Because of synaptic coupling, external inputs and adaptation,  $\mathbf{P}(\mathbf{t}|\hat{\mathbf{t}})$  depends on the past activity and on time (inhomogeneous renewal point process).

## Network response to current injection

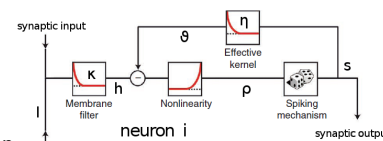


Population of 500 randomly connected excitatory neurons with adaptation.

## From neuronal dynamics to populations

Generalized linear model neuron:

$$\rho_i(\mathbf{t}) = c e^{h_i(\mathbf{t}) - \vartheta_i(\mathbf{t})}$$



Quasi-renewal approximation.

- mean-field [4]  $\mathbf{h}_i(\mathbf{t}, \{\mathbf{t}_j^f\}_{j,f}) \approx \mathbf{h}(\mathbf{t}) = (\kappa * [\mathbf{J}\mathbf{A} + \mathbf{I}])(\mathbf{t})$
- adaptation [3]  $\vartheta_i(\mathbf{t}, \{\mathbf{t}_j^f\}_f) \approx \vartheta(\mathbf{t}|\hat{\mathbf{t}}) = \eta(\mathbf{t} - \hat{\mathbf{t}}) + (\gamma * \mathbf{A})(\mathbf{t})$
- intensity  $\rho_i(\mathbf{t}, \{\mathbf{t}_j^f\}_{j,f}) \approx \rho(\mathbf{t}|\hat{\mathbf{t}}) = c e^{h(\mathbf{t}) - \vartheta(\mathbf{t}|\hat{\mathbf{t}})}$

Then follows survivor function  $\mathbf{S}(\mathbf{t}|\hat{\mathbf{t}}) = \exp(-\int_{\hat{\mathbf{t}}}^{\mathbf{t}} \rho(\mathbf{t}'|\hat{\mathbf{t}}) d\mathbf{t}')$   
interval density  $\mathbf{P}(\mathbf{t}|\hat{\mathbf{t}}) = \rho(\mathbf{t}|\hat{\mathbf{t}}) \mathbf{S}(\mathbf{t}|\hat{\mathbf{t}})$

## Linearized dynamics in the AI state

Assuming  $\mathbf{A}(\mathbf{t}) = \mathbf{A}_0 + \Delta\mathbf{A}(\mathbf{t})$ , with small  $\Delta\mathbf{A}$ , we linearize the population integral equation:

$$\mathbf{a}(\mathbf{t}) = \mathbf{A}_0 + (\mathbf{P}_0 * \Delta\mathbf{A})(\mathbf{t}) + \mathbf{A}_0 \frac{d}{dt} \{ \mathcal{L} * [(\kappa\mathbf{J} + \gamma) * \Delta\mathbf{A} + \kappa * \Delta\mathbf{I}] \}(\mathbf{t})$$

Then  $\mathbf{A}(\mathbf{t}) = \mathbf{a}(\mathbf{t}) + \sqrt{\frac{\mathbf{A}_0}{N}} \xi(\mathbf{t})$ , with colored noise  $\xi(\mathbf{t})$ .  
 $\langle \xi(\mathbf{t} + \tau) \xi(\mathbf{t}) \rangle = \delta(\tau) - \int_0^\infty \mathbf{P}_0(\mathbf{s} + \tau) \mathbf{P}_0(\mathbf{s}) d\mathbf{s}$

## Spectral density of population activity

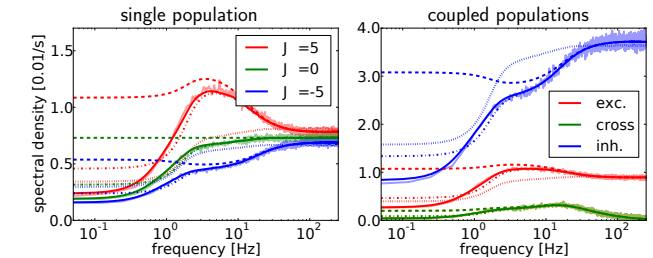
For renewal processes with ISI density  $\mathbf{P}_0$  and rate  $\mathbf{A}_0$ , the power spectrum is

$$\tilde{\mathbf{C}}_0(\omega) = \mathbf{A}_0 (\mathbf{1} - \tilde{\mathbf{P}}_0)^{-1} (\mathbf{1} - \tilde{\mathbf{P}}_0 \tilde{\mathbf{P}}_0^*) (\mathbf{1} - \tilde{\mathbf{P}}_0^*)^{-1}.$$

Using the linear response function  $\mathbf{R}(\mathbf{t})$  of the neurons to input currents, the spectral density of the population activity  $\mathbf{A}(\mathbf{t})$  is

$$\tilde{\mathbf{C}}(\omega) = \tilde{\mathbf{B}} \mathbf{N}^{-1} (\tilde{\mathbf{C}}_0 + \tilde{\mathbf{R}} \tilde{\mathbf{C}}_0 \tilde{\mathbf{R}}^*) \tilde{\mathbf{B}}^T, \quad \tilde{\mathbf{B}} = [\mathbf{1} - \tilde{\mathbf{R}}(\mathbf{J} + \tilde{\kappa}^{-1} \tilde{\gamma})]^{-1}$$

Here  $\tilde{\mathbf{C}}_i$  is the input spectrum, and  $\tilde{\mathbf{R}}(\omega) = (\mathbf{1} - \tilde{\mathbf{P}}_0)^{-1} i\omega \mathbf{A}_0 \tilde{\mathcal{L}} \tilde{\kappa}$ .



Special cases:

- $\mathbf{J} = \tilde{\gamma} = \tilde{\mathbf{C}}_i = \mathbf{0}$ : renewal process,  $\tilde{\mathbf{C}} = \tilde{\mathbf{C}}_0$  dotted
- $\tilde{\gamma} = \mathbf{0}$ : linear response theory for LIF [2] dash-dotted
- $\mathbf{P}_0 \sim \exp$  and  $\tilde{\gamma} = \mathbf{0}$ : Hawkes process (Hawkes, 1971) dashed

## Conclusions

New theory explains effects of adaptation at the circuit level:

- ▶ Finite-size fluctuations in coupled, randomly connected populations.
- ▶ Noise-shaping through adaptation: reduction in low-frequencies.

## References

1. Deger, Schwalger, Naud, Gerstner (in review) *arXiv:1311.4206*
2. Lindner, Doiron, Longtin (2005) *Phys Rev E*  
Trousdale, Hu, Shea-Brown, Josic (2012) *PLoS Comput Biol*
3. Naud & Gerstner (2012) *PLoS Comput Biol*
4. Gerstner (2000) *Neural Comput*