**Statistical structure of neural spiking under non-Poissonian stimulation**

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**Motivation**

- Can we understand the interspike interval (ISI) statistics of spontaneous neural activity?
- What is the relation between input and output statistics of a neuron? → important for understanding population activity.
- Most theoretical studies assume that neurons are driven by Poisson spike trains ("white noise" processes, i.e. uncorrelated in time).
- However, realistic synaptic inputs have temporal structure ("colored noise"), e.g. due to refractoriness, bursting, structured "signals", short-term synaptic plasticity, oscillatory activity, adaptation.

**Wanted:**

Spiking statistics for arbitrary input correlation function?

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**General theory**

**Perfect integrate-and-fire model**

\[ C_N V = I_N + I_{syn}(t), \]  if \( V = V_A : V \to 0. \]

- diffusion approximation: \( V = \mu + \sigma \eta(t) \)
- noise correlation function \( C(t) = \langle \eta(t)^2 \rangle - \langle \eta(t) \rangle^2 \)
- first-passage problem for non-Markovian process \( V(t) \)
- **Trick:** d-dimensional Ornstein-Uhlenbeck process

\[ \eta = a - Y, \quad Y = A \eta + B(\xi(t)), \]

**Fokker-Planck equation for \( p(v, y, t) \)**

\[
\frac{\partial p}{\partial t} = -\left( \mu \sum_{n} \frac{d}{dy} \frac{\partial p}{\partial y} \right) \frac{d}{dy} p(y) + \sum_{n} D_n \frac{d^2}{d^2y} p(y) \frac{d}{dy} \frac{\partial p}{\partial y} + \cdots
\]

initial condition: \( p(v, y, 0) = \delta(v) \frac{d}{dV} \exp \left( -\frac{1}{2} \sigma^2 \xi(V) \right) \)

- n-th-order interval density

\[
\rho_n(t) = \frac{1}{2^n \sqrt{\pi \sigma^2 \xi(t)}} \exp \left[ \frac{-(t - n \langle T \rangle)^2}{4 \langle T \rangle \xi(t)} \right] \times \left\{ \left[ \frac{(n - 1/2)^2}{4 \langle T \rangle \xi(t)} + (n-1)^2 \right] - \frac{\sigma^2}{2 \langle T \rangle \xi(t)} \right\}^{n-1} g(t)
\]

\( (\langle T \rangle) = V_0/\mu, \quad g(t) = \frac{2}{\sqrt{\pi \sigma^2 \xi(t)}} \exp \left( -\frac{t}{\langle T \rangle \xi(t)} \right) \)

**Analytical results**

- ISI cumulants

\[
\nu_{2,n} = 2 \langle T \rangle^n \sigma^2 \left( \frac{h_n}{h_0} \right) \quad g_n = \langle n(\langle T \rangle) \rangle \quad h_n = \langle (\langle T \rangle) \rangle
\]

- CV, skewness and ISI serial correlations

\[
C_{\xi} = 2 \langle T \rangle \sigma^2 \quad \text{and} \quad \text{for Gaussian input with arbitrary correlation function, we derived analytical formulas for ISI density, auto-correlogram, CV, skewness, ISI serial correlations and Fano factor.}
\]

- Short-term synaptic plasticity

**Powerlaw input**

- rate-modulated Poisson processes

**Non-Poissonian input**

- regular input spikes (refractoriness)

\[
\nu = 5 \text{ Hz}, \quad \text{CV}=0.5 \quad N_j = 400
\]

\[
\nu = 5 \text{ Hz}, \quad \text{CV}=0.5 \quad N_j = 200
\]

- irregular input spikes (burstiness)

\[
\nu = 5 \text{ Hz}, \quad \text{CV}=2.5 \quad N_j = 400
\]

\[
\nu = 5 \text{ Hz}, \quad \text{CV}=2.5 \quad N_j = 200
\]

**Conclusions**

- For Gaussian input with arbitrary correlation function, we derived analytical formulas for ISI density, auto-correlogram, CV, skewness, ISI serial correlations and Fano factor.
- works well for mean-driven regime with CV=0.5 or smaller
- regular input or synaptic depression => negative ISI correlations, small skew
- irregular input or facilitation => positive ISI correlations, large skew
- scaling behaviors for powerlaw input

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Lindner B., Phys. Rev. E 2004