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PUNCHING STRENGTH OF ACTUAL TWO-WAY SLABS

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Abstract

Many building codes rely on empirical formulations with respect to the punching shear design provisions. Such empirical equations are typically calibrated on the basis of laboratory tests performed on simplified geometries (usually isolated specimens) with usually statically-determined loading conditions. On the contrary, actual structures, particularly two-way slabs, are continuous and highly redundant systems. The isolated test specimens are thus unfortunately not always suitable to reproduce the actual behaviour of flat slabs and empirical models calibrated on their basis may not be applicable for their design.

In this paper, the application of the Critical Shear Crack Theory (theoretical support of Model Code 2010 provisions) is shown for some unusual test specimens aiming at investigating actual inner bays of continuous flat slabs. This analysis shows that moment redistribution between the column region and mid-span develop after concrete cracking and that accounting for this behaviour is required to obtain good predictions of the actual flat slab response. It is also shown that the Critical Shear Crack Theory leads to consistent results in terms of failure mode, punching shear strength and deformation capacity as it allows accounting for this behaviour. On the contrary, empirical design formulas fail to predict the influence of continuous flat slabs on the punching strength.

Keywords: Critical Shear Crack Theory, Moment Redistribution, Numerical Analysis, Punching Shear Strength, Two-way Slabs

1 Introduction

Punching shear design provisions in most common codes of practice like ACI 318-11 (2011) and EC2 (2004) are empirical formulas fitted on the basis of the results of laboratory tests. Tests are usually conducted on isolated slab specimens supported in the centre and loaded close to the perimeter, as shown in Fig. 1. The size of the specimens is chosen to represent negative moment area around the support. The line of contraflexure of radial bending moments is determined by linear analysis and is usually assumed to be located at a constant distance $r_s \approx 0.22\,L$ from the centre of the column, where L is the distance between the columns (span length). The punching strength of these specimens is assumed to be representative of the punching strength of actual flat slabs. However, actual two-way slabs are continuous and highly redundant systems, where redistributions of internal force (particularly bending moments) develop after cracking and yielding near the supported areas. This influences the location of the line of contraflexure of radial bending moments, which may consequently influence the punching shear strength. This is justified by the fact that for larger values of parameter r_s, bending moments associated to a given level of shear action are larger. Thus, crack widths close to the column region (shear-critical area) which depend on the acting bending moments are larger for larger values of r_s, potentially reducing the ability of cracks to transfer shear forces and thus reducing the shear strength of the system. Such behaviour and the influence of the redistribution of bending moments on the punching shear strength has recently been shown by tests on continuous slabs (reproduced by restraining the specimen's edges) reported by Choi and Kim (2012).

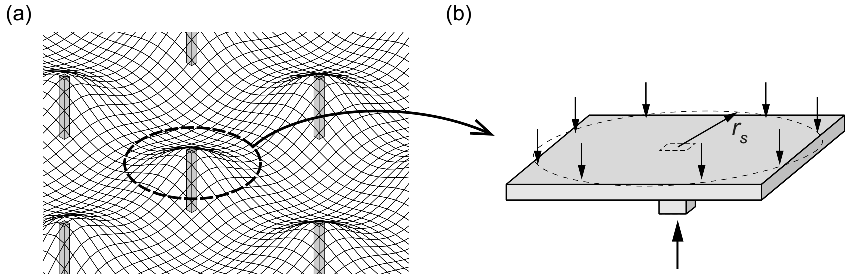

Fig. 1 Continuous flat slab (a) and a typical punching specimen (b).

Muttoni (2008) demonstrated in his critical shear crack theory (CSCT) that the punching strength of a slab-column connection depends on flexural deformations of the slab near the support. The current paper presents a method of application of CSCT to take into account the potential increased flexural stiffness and strength of continuous flat slabs.

2 Behaviour of continuous slabs

In order to assess the moment redistribution in actual flat slabs, a numerical approach was developed by Guandalini (2005). An axis-symmetric slab portion is divided into sector elements. By taking into account the equilibrium of the elements, compatibility of deformations and moment-curvature relationship of reinforced concrete elements, diagrams of internal forces along the radius of the slab can be developed for a given load and the curvature of the first element close to the column face. This curvature can then be determined by applying a boundary condition to the outermost element, for example $m_R = 0$ for isolated elements or $\psi = 0$ for continuous slabs. For every load V, moment, curvature, rotation and deflection diagrams can be obtained, as shown in the examples at Fig. 2.

Fig. 3(a) shows, on the basis of this approach, the influence of slab continuity on the load-rotation relationship of a slab. The effect is more pronounced once the reinforcement in the tangential direction starts to yield. It can also be noted that the increase on the punching strength due to continuity is higher for slabs with low ratio of flexural reinforcement on the support or with shear reinforcement. Fig. 3(b) shows the location of the line of contraflexure during loading of a continuous slab. It can be noted that the elastic assumption of $r_s = 0.22\ L$ is only valid for load levels that do not require significant redistribution of moment from the support to the span. Thereafter, the location of this line is closer to the column (redistribution of moments to the mid-span), with variable value depending mostly on the actual load level and on the ratio between the column-to-mid-span reinforcement ratios.

It can be noted that punching failure of test specimens with large amounts of shear reinforcement often occurs when the slab is close to a flexural failure or is even governed by the bending strength of the member. Therefore, the maximum punching strength determined on an isolated element may actually be limited by the flexural strength of specimens (Stein et al. 2007). In order to avoid such flexural failures during testing, higher longitudinal reinforcement ratios or smaller column sizes have traditionally been used for isolated test specimens. However, this might be unrealistic compared to practical cases. An alternative approach is to test continuous slabs (or slabs with restrained rotations at the edges). This leads to a more realistic behaviour and allows investigating the influence on punching of the reinforcement over column and at mid-span. In the following a recent test series (Choi and Kim 2012) following this testing layout will be investigated.

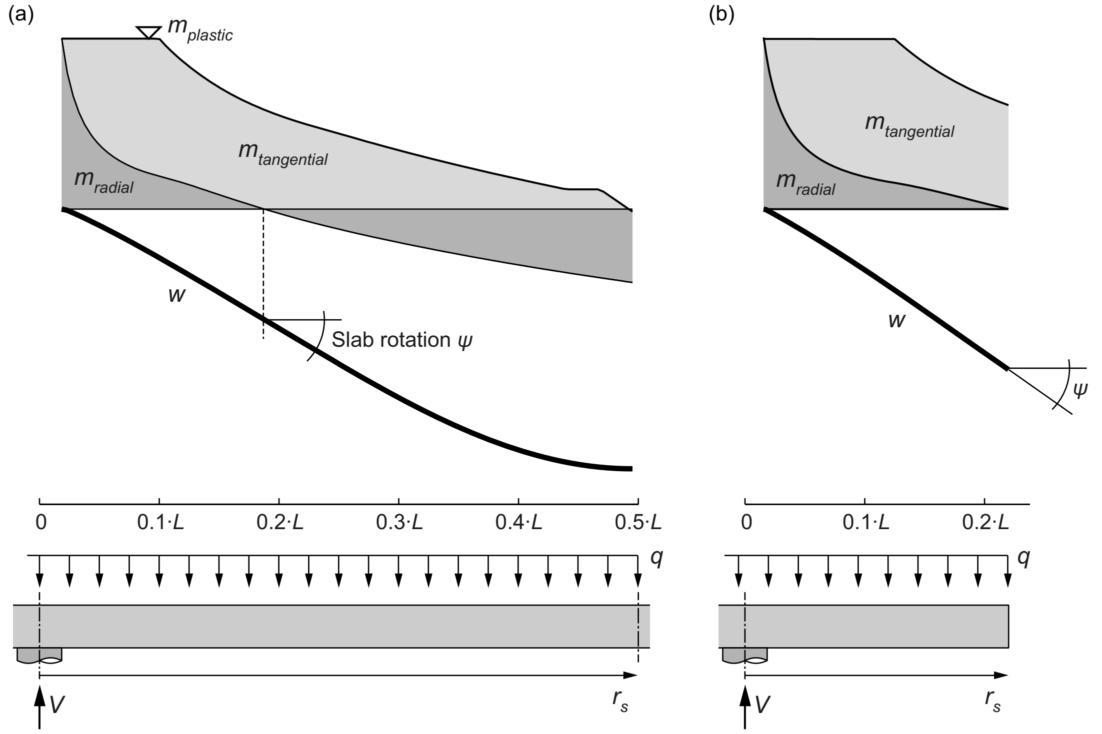

Fig. 2 Radial and tangential moments and deformed shape at load $V = 1800$ kN for (a) a continuous flat slab and (b) an isolated specimen ($f_c = 35$ MPa; $f_y = 550$ MPa; $h = 250$ mm; $d = 210$ mm; $L = 7$ m; $\rho_{top} = 1.33\%$; $\rho_{bot} = 0.67\%$).

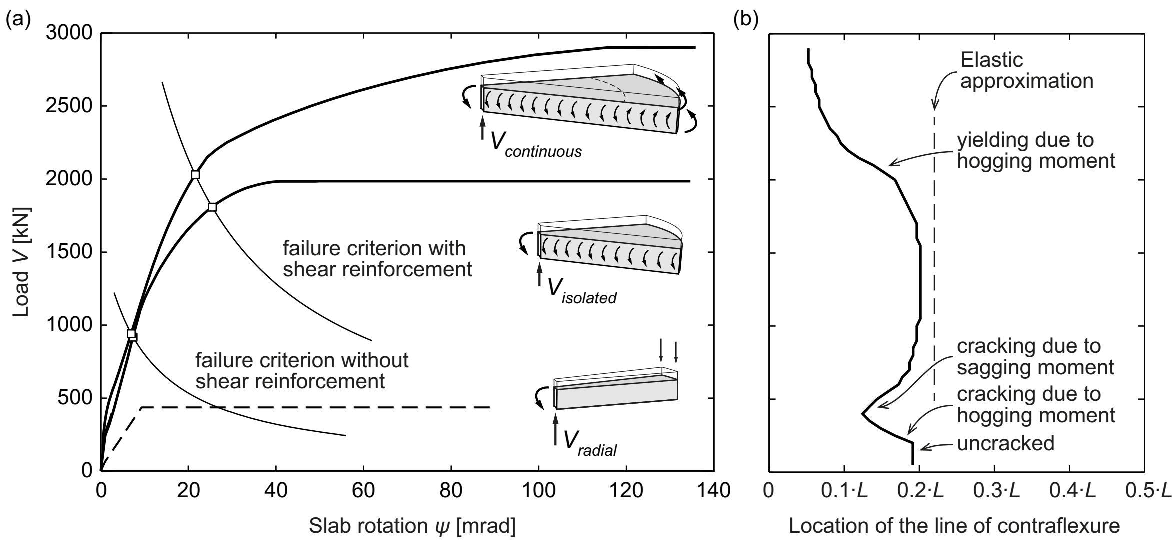

Fig. 3 (a) Load-rotation curve of an isolated specimen and a continuous slab; (b) Distance from the center to the line of contraflexure.

3 Analysis of experimental results

In order to investigate more in detail the behaviour of continuous slabs, the approach of Guandalini (2005) in combination to the punching strength prediction of the CSCT has been applied to analyse the results of three tests on slabs with restrained edge rotations (Choi and Kim 2012). All slabs were 4.2 x 4.2 m in plan dimensions and 152 mm thick supported on a 356 mm square column. The rotations of the edges of the slabs were restrained by a steel structure with a constant stiffness. The slabs differed on their flexural reinforcement ratios. All slabs were designed for the same flexural strength but varied the placement of reinforcement: MRA was designed according to elastic distribution of bending moments, MRB needed some redistribution of moments from support to mid-span to reach the flexural limit and MRC was designed with the same reinforcement for hogging and sagging moments.

Table 1: Main parameters and results of the test campaign (Choi and Kim 2012)

Specimen	Concrete strength f_c [N/mm^2]	Top reinf. ratio ρ_{top} [%]	Bottom reinf. ratio ρ_{bot} [%]	Measured max. load V_{test} [kN]	EC2 prediction V_{EC2} [kN]		CSCT prediction V_{CSCT} [kN]: Neglecting sagging moment		CSCT prediction V_{CSCT} [kN]: Considering continuous slab behaviour	
MRA	37.0	1.059	0.306	458	420	1.09	490	0.93	432	1.06
MRB	30.5	0.832	0.433	394	358	1.10	397	0.99	403	0.98
MRC	34.6	0.575	0.573	430	326	1.32	280	1.54	389	1.11

Table 1 presents the main parameters of the slabs, the failure loads and predictions of EC2 (2004) and CSCT with and without taking into account the redistribution of bending moments. It can be seen that the predictions of EC2, which take into account the ratio of flexural reinforcement only at support, are overly conservative for lower flexural reinforcement ratios over column. The same is true for CSCT without moment redistribution. In fact, specimen MRC is predicted to fail in bending at 280 kN. However, the predictions of CSCT that consider moment redistribution according to the analyses based on the flexural behaviour of Guandalini (2005) are in good agreement with the test results.

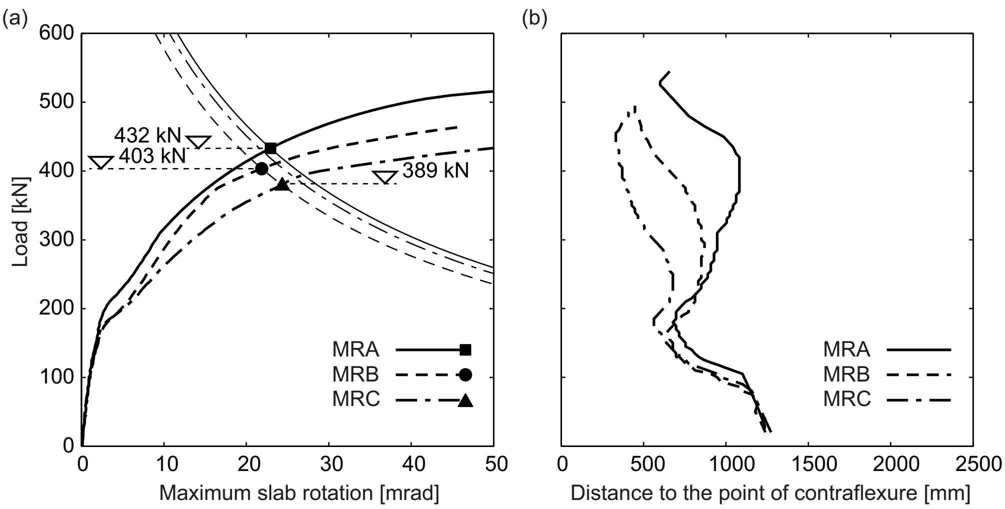

Fig. 4 (a) Load-rotation curves and failure criteria for the analyzed specimens; (b) Distance to the line of contraflexure for depending on the load.

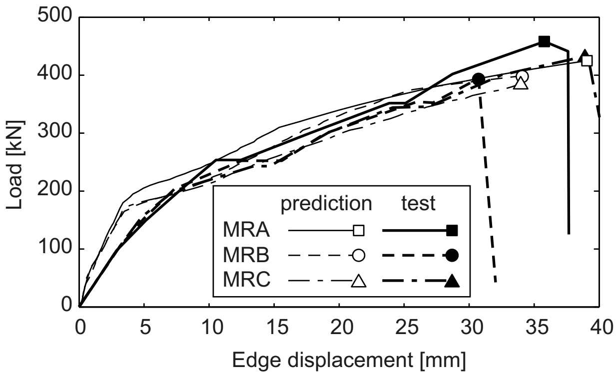

Fig. 5 Load-edge deflection

Fig. 4(a) shows the calculated load-rotation curves for the analysed specimens. The curves are similar, although the flexural reinforcement ratios on support vary between 1.06% and 0.58%. The effect of moment redistribution can be seen in Fig. 4(b), where the distance to the line of contraflexure (size of negative moment are) decreases quickly for specimen MRC.

Fig. 5 compares the reported deflections at slab edge to the ones calculated using the model. A very good agreement can be seen between the predicted and measured values

4 Conclusions

This paper presents a consistent approach to predict the punching behaviour of continuous flat slabs. The failure criterion of the Critical Shear Crack Theory (CSCT) was applied to predict strength and deformation capacity of three specimens with restrained edged in combination with a suitable bending model to simulate moment redistribution in flat slabs.

The main conclusions of this paper are:

1) Significant moment redistribution from support to mid-span may occur before punching in slabs with low amount of flexural reinforcement on supports due to cracking of concrete and by yielding of the reinforcement.
2) If moment redistribution is accounted, the failure criterion of the CSCT for slabs without shear reinforcement correctly predicts the punching strength of the analysed slabs. Without accounting for such redistributions, the results may differ depending on the amount of redistribution of bending forces.
3) Design codes usually neglect the phenomenon of bending moment redistribution. This seems to be a necessary future improvement for consistent punching design of flat slabs

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