Integrated airline schedule planning with supply-demand interactions

for a new generation of aircrafts

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Transport and mobility laboratory

EPFL

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Motivation

- Increased air travel demand

- Demand responsiveness
  - Flexible supply capacity
  - Improved demand management

- Sustainability
Motivation

Results

Heuristic method

Conclusions

Clip-Air concept

**Flexibility in transportation...**

- Modular capacity with detachable capsules
  - security, maintenance, storage and crew costs
- Multi-modality for passenger and cargo
- Robustness
- Demand management

**Sustainable transportation**

- Gas emissions, noise, accident rates

- Exists in a simulated environment
Objectives

- Comparative analysis between standard fleet and Clip-Air
- Development of integrated schedule design and fleet assignment model
  - integration of supply-demand interactions
    - logit demand model $\Rightarrow$ pricing
    - spill and recapture effects
  - Fare-class segmentation
    - demand model for each segment
    - seat allocation for business and economy
- Solution techniques for the resulting mixed integer nonlinear problem
Demand model for itinerary choice

- Utility of itinerary $i$, class $h$:

$$V^h_i = \beta_{\text{fare}}^h p^h_i + \beta_{\text{time}}^h time_i + \beta_{\text{stops}}^h nonstop_i$$

- $p^h_i$ is the price of itinerary $i$ for class $h$.
- $time_i$, binary variable, 1 if departure time is between 07:00-11:00.
- $nonstop_i$, binary variable, 1 if it is a non-stop itinerary.

- Demand for class $h$ for each itinerary $i$ in market segment $s$:

$$\tilde{d}_i^h = D_s^h \frac{\exp(V_i^h)}{\sum_{j \in I_s} \exp(V_j^h)}$$

- $D_s^h$ is the total expected demand for class $h$ and segment $s$.
- $\tilde{d}_i^h$ serves as an upper bound for the actual demand.
Spill and recapture effects

- In case of capacity shortage some passengers may not fly on their desired itineraries.
- They may accept to fly on other available itineraries in the same market segment.
- Recapture ratio is given by:

\[ b_{i,j}^h = \frac{\exp(V_{j}^h)}{\sum_{k \in I'_s \setminus i} \exp(V_{k}^h)} \]

- No-revenue represented by the subset \( I'_s \in I_s \) for segment \( s \).
Max \( \sum_{s \in S} \sum_{h \in H} \sum_{i \in (I_s \setminus I_s')} (d_i^h - \sum_{j \in I_s} t_{i,j}^h + \sum_{j \in (I_s \setminus I_s')} t_{j,i}^h b_{j,i}^h) \delta_{i,f}^h - \sum_{k \in K} \sum_{f \in F} C_{k,f} x_{k,f} \): revenue - cost \hspace{1cm} (1)

s.t. \( \sum_{k \in K} x_{k,f} = 1 \): mandatory flights \hspace{1cm} \forall f \in F^M \hspace{1cm} (2)

\( \sum_{k \in K} x_{k,f} \leq 1 \): optional flights \hspace{1cm} \forall f \in F^O \hspace{1cm} (3)

\( y_{k,a,t}^{-} + \sum_{f \in \text{In}(k,a,t)} x_{k,f} = y_{k,a,t}^{+} + \sum_{f \in \text{Out}(k,a,t)} x_{k,f} \): flow conservation \hspace{1cm} \forall [k,a,t] \in N \hspace{1cm} (4)

\( \sum_{a \in A} y_{k,a,t}^{n} + \sum_{f \in \text{CT}} x_{k,f} \leq R_k \): fleet availability \hspace{1cm} \forall k \in K \hspace{1cm} (5)

\( y_{k,a,minE_{a}^{-}} = y_{k,a,maxE_{a}^{+}} \): cyclic schedule \hspace{1cm} \forall k \in K, a \in A \hspace{1cm} (6)

\( \sum_{s \in S} \sum_{i \in (I_s \setminus I_s')} \delta_{i,f}^h d_i^h - \sum_{j \in I_s} \delta_{i,f}^h t_{i,j}^h + \sum_{j \in (I_s \setminus I_s')} \delta_{i,f}^h t_{j,i}^h b_{j,i}^h \leq \sum_{k \in K} \pi_{k,f}^h \): capacity \hspace{1cm} \forall h \in H, f \in F \hspace{1cm} (7)

\( \sum_{h \in H} \pi_{k,f}^h = Q_k x_{k,f} \): seat capacity \hspace{1cm} \forall f \in F, k \in K \hspace{1cm} (8)

\( x_{k,f} \in \{0,1\} \) \hspace{1cm} \forall k \in K, f \in F \hspace{1cm} (9)

\( y_{k,a,t} \geq 0 \) \hspace{1cm} \forall [k,a,t] \in N \hspace{1cm} (10)

\( \pi_{k,f}^h \geq 0 \) \hspace{1cm} \forall h \in H, k \in K, f \in F \hspace{1cm} (11)
Motivation

Integrated schedule planning

Results

Heuristic method

Conclusions

Integrated model - Demand part

\[
\sum_{j \in I_s, i \neq j} t_{i,j}^h \leq d_i^h : \text{total spill} \quad \forall s \in S, h \in H, i \in (I_s \setminus I_s') \tag{12}
\]

\[
\tilde{d}_i^h = D_s^h \frac{\exp (V_i^h)}{\sum_{j \in I_s} \exp (V_j^h)} : \text{logit demand} \quad \forall s \in S, h \in H, i \in I_s \tag{13}
\]

\[
b_{i,j}^h = \frac{\exp (V_j^h)}{\sum_{k \in I_s \setminus i} \exp (V_k^h)} : \text{recapture ratio} \quad \forall s \in S, h \in H, i \in (I_s \setminus I_s') \setminus j \in I_s \tag{14}
\]

\[
d_i^h \leq \tilde{d}_i^h \leq D_i^h : \text{realized demand} \quad \forall h \in H, i \in I \tag{15}
\]

\[
0 \leq p_{i}^h \leq UB_{i}^h : \text{upper bound on price} \quad \forall h \in H, i \in I \tag{16}
\]

\[
t_{i,j}^h \geq 0 \quad \forall s \in S, h \in H, i \in (I_s \setminus I_s'), j \in I_s \tag{17}
\]

\[
b_{i,j}^h \geq 0 \quad \forall s \in S, h \in H, i \in (I_s \setminus I_s'), j \in I_s \tag{18}
\]
Model extension for Clip-Air

- Decision variables for the assignment of wing and capsules:
  \[ x^w_f \in \{0,1\} \]
  \[ x_{k,f} \in \{0,1\} \text{ for } k \in \{1,2,3\} \]

- Operating cost:
  \[
  \sum_{f \in F} C^w_f x^w_f + \sum_{k \in K} C_{k,f} x_{k,f}
  \]

- Constraints:
  \[
  \sum_{k \in K} x_{k,f} = 1 \quad \forall f \in F^M: \text{ mandatory flights}
  \]
  \[
  \sum_{k \in K} x_{k,f} \leq x^w_f \quad \forall f \in F: \text{ capsule - wing}
  \]
Results

- Dataset from a major European airline
- Other inputs:
  - Cost figures for Clip-Air
    - Weight differences $\Rightarrow$ adjustment of fuel cost and airport and air navigation charges
    - Capsule wing separation $\Rightarrow$ adjustment of crew cost
  - Parameters of the demand model
- Model is implemented in AMPL and solved with BONMIN
- Results provide the schedule design, fleet assignment, seat allocation and pricing.
Demand model parameters

- Estimation of logit model parameters by maximum likelihood estimation using BIOGEME
- Booking data does not have the non-chosen alternatives ⇒ lack of variability
- Adjusted parameters to have enough elasticity

<table>
<thead>
<tr>
<th></th>
<th>Business demand</th>
<th>Economy demand</th>
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<tbody>
<tr>
<td>$\beta_{\text{fare}}$</td>
<td>-0.025</td>
<td>-0.050</td>
</tr>
<tr>
<td>$\beta_{\text{time}}$</td>
<td>0.323</td>
<td>0.139</td>
</tr>
<tr>
<td>$\beta_{\text{nonstop}}$</td>
<td>1.150</td>
<td>0.900</td>
</tr>
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</table>
## Standard planes vs Clip-Air

An instance with 18 flights and 1096 passengers:

<table>
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<tr>
<th></th>
<th>Standard Fleet</th>
<th>Clip-Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating cost</td>
<td>107,560</td>
<td>89,512</td>
</tr>
<tr>
<td>Revenue</td>
<td>185,835</td>
<td>200,199</td>
</tr>
<tr>
<td><strong>Profit</strong></td>
<td><strong>78,275</strong></td>
<td><strong>110,687</strong></td>
</tr>
<tr>
<td>Transported pax.</td>
<td>817</td>
<td>909</td>
</tr>
<tr>
<td></td>
<td>184 B, 633 E</td>
<td>192 B, 717 E</td>
</tr>
<tr>
<td>Flight count</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td><strong>Average pax/flight</strong></td>
<td><strong>51</strong></td>
<td><strong>57</strong></td>
</tr>
<tr>
<td>Total Flight Hours (min)</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td>Used fleet</td>
<td>2 A319, 1 ERJ135</td>
<td>5 wings</td>
</tr>
<tr>
<td></td>
<td>3 ERJ145</td>
<td>8 capsules</td>
</tr>
<tr>
<td><strong>Used aircraft</strong></td>
<td><strong>6</strong></td>
<td><strong>5</strong></td>
</tr>
<tr>
<td>Used capacity (seats)</td>
<td>345</td>
<td>400</td>
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<tr>
<td>Running time (min)</td>
<td>33.89</td>
<td>31.72</td>
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- More passengers
- Less aircraft ⇒ less flight crew
## Impacts of the demand model - Different scenarios

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*When competing itineraries are cheaper, integrated model keeps the prices low to attract passengers.*

*When elasticity is lower, integrated model results with higher prices and less transported passengers.*
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- When elasticity is lower, integrated model results with higher prices and less transported passengers.
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- When competing itineraries are cheaper, integrated model keeps the prices low to attract passengers.
- When elasticity is lower, integrated model results with higher prices and less transported passengers.
The resulting mixed integer nonlinear problem is highly complex.

We propose a heuristic method based on Lagrangian relaxation, sub-gradient optimization and a Lagrangian heuristic.

Capacity constraint is relaxed.

Problem is decomposed into 2 subproblems: revenue maximization and fleet assignment:

$$z_{REV}(\lambda) = \max \sum_{h \in H} \sum_{f \in F} \sum_{s \in S} \sum_{i \in (I_s \setminus I_s')} \delta_{i,f} (p_{i}^{h} - \lambda_{f}^{h}) \left( d_{i}^{h} - \sum_{j \in I_s, i \neq j} t_{i,j}^{h} + \sum_{j \in (I_s \setminus I_s')} t_{j,i}^{h} b_{j,i}^{h} \right)$$

$$z_{FAM}(\lambda) = \min \sum_{k \in K} \sum_{f \in F} \left( C_{k,f} x_{k,f} - \sum_{h \in H} \lambda_{f}^{h} \pi_{k,f}^{h} \right)$$
Lagrangian procedure

Require: $z_{LB}$, $\bar{k}$, $\bar{j}$, $\varepsilon$

\[
\lambda^0 := 0, \ k := 0, \ z_{UB} := \infty
\]

repeat
\[
\{\bar{d}, \bar{t}, \bar{b}\} := \text{solve } z_{REV}(\lambda^k), \ \{\bar{x}, \bar{y}, \bar{\pi}\} := \text{solve } z_{FAM}(\lambda^k)
\]
\[
z_{UB}(\lambda^k) := z_{REV}(\lambda^k) - z_{FAM}(\lambda^k)
\]
\[
z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k))
\]
loop
\[
\{\bar{x}, \bar{\pi}\} := \text{Local search}(\{\bar{x}, \bar{\pi}\})
\]
\[
lb := \text{Lagrangian heuristic } (\{\bar{x}, \bar{\pi}\})
\]
end loop
\[
z_{LB} := \max(z_{LB}, lb)
\]
\[
G := \text{compute sub-gradient}(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\})
\]
\[
T := \text{compute step}(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\})
\]
\[
\lambda^{k+1} := \max(0, \lambda^k - TG)
\]
\[
k := k + 1
\]
until $\|TG\|^2 \leq \varepsilon$ or $k \geq \bar{k}$
Lagrangian procedure

Require: $z_{LB}$, $k$, $j$, $\varepsilon$

\begin{align*}
\lambda^0 &:= 0, \ k := 0, \ z_{UB} := \infty \\
\text{repeat} & \\
\{\bar{d}, \bar{t}, \bar{b}\} &:= \text{solve } z_{REV}(\lambda^k) \\
\{\bar{x}, \bar{y}, \bar{\pi}\} &:= \text{solve } z_{FAM}(\lambda^k) \\
z_{UB}(\lambda^k) &:= z_{REV}(\lambda^k) - z_{FAM}(\lambda^k) \\
z_{UB} &:= \min(z_{UB}, z_{UB}(\lambda^k)) \textbf{ update UPPER BOUND} \\
\text{loop} & \\
\{\bar{x}, \bar{\pi}\} &:= \text{Local search(\{\bar{x}, \bar{\pi}\})} \\
lb &:= \text{Lagrangian heuristic (\{\bar{x}, \bar{\pi}\})} \\
\text{end loop} & \\
z_{LB} &:= \max(z_{LB}, lb) \\
G &:= \text{compute sub-gradient}(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\}) \\
T &:= \text{compute step}(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\}) \\
\lambda^{k+1} &:= \max(0, \lambda^k - TG) \\
k &:= k + 1 \\
\text{until } ||TG||^2 &\leq \varepsilon \textbf{ or } k \geq \bar{k}
\end{align*}
Lagrangian procedure

Require: \( z_{LB}, \bar{k}, \bar{j}, \varepsilon \)
\( \lambda^0 := 0, \ k := 0, \ z_{UB} := \infty \)

repeat
\( \{\bar{d}, \bar{t}, \bar{b}\} := \text{solve } z_{REV}(\lambda^k) \), \( \{\bar{x}, \bar{y}, \bar{\pi}\} := \text{solve } z_{FAM}(\lambda^k) \)
\( z_{UB}(\lambda^k) := z_{REV}(\lambda^k) - z_{FAM}(\lambda^k) \)
\( z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k)) \)
end loop

\( \{\bar{x}, \bar{\pi}\} := \text{Local search}(\{\bar{x}, \bar{\pi}\}) \) \textbf{based on } \lambda's \textbf{ under a Tabu mechanism}
\( lb := \text{Lagrangian heuristic}(\{\bar{x}, \bar{\pi}\}) \)

end loop
\( z_{LB} := \max(z_{LB}, lb) \)
\( G := \text{compute sub-gradient}(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\}) \)
\( T := \text{compute step}(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\}) \)
\( \lambda^{k+1} := \max(0, \lambda^k - TG) \)
\( k := k + 1 \)
until \( \|TG\|^2 \leq \varepsilon \) or \( k \geq \bar{k} \)
Lagrangian procedure

**Require:** $z_{LB}, \bar{k}, j, \varepsilon$

\[ \lambda^0 := 0, \; k := 0, \; z_{UB} := \infty \]

repeat

\[ \{\tilde{d}, \tilde{t}, \tilde{b}\} := \text{solve } z_{REV}(\lambda^k), \; \{\tilde{x}, \tilde{y}, \tilde{\pi}\} := \text{solve } z_{FAM}(\lambda^k) \]

\[ z_{UB}(\lambda^k) := z_{REV}(\lambda^k) - z_{FAM}(\lambda^k) \]

\[ z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k)) \]

loop

\[ \{\tilde{x}, \tilde{\pi}\} := \text{Local search}(\{\tilde{x}, \tilde{\pi}\}) \]

\[ lb := \text{Lagrangian heuristic}(\{\tilde{x}, \tilde{\pi}\}) \text{ a feasible solution} \]

end loop

\[ z_{LB} := \max(z_{LB}, lb) \]

\[ G := \text{compute sub-gradient}(z_{UB}, z_{LB}, \{\tilde{d}, \tilde{t}, \tilde{b}, \tilde{x}, \tilde{y}, \tilde{\pi}\}) \]

\[ T := \text{compute step}(z_{UB}, z_{LB}, \{\tilde{d}, \tilde{t}, \tilde{b}, \tilde{x}, \tilde{y}, \tilde{\pi}\}) \]

\[ \lambda^{k+1} := \max(0, \lambda^k - TG) \]

\[ k := k + 1 \]

until $||TG||^2 \leq \varepsilon$ or $k \geq \bar{k}$
Lagrangian procedure

**Require:** $z_{LB}$, $k$, $j$, $\varepsilon$

\[
\lambda^0 := 0, \quad k := 0, \quad z_{UB} := \infty
\]

**repeat**

\[
\{\tilde{d}, \tilde{t}, \tilde{b}\} := \text{solve } z_{REV}(\lambda^k), \quad \{\bar{x}, \bar{y}, \bar{\pi}\} := \text{solve } z_{FAM}(\lambda^k)
\]

\[
z_{UB}(\lambda^k) := z_{REV}(\lambda^k) - z_{FAM}(\lambda^k)
\]

\[
z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k))
\]

**loop**

\[
\{\bar{x}, \bar{\pi}\} := \text{Local search}(\{\bar{x}, \bar{\pi}\})
\]

$lb :=$ Lagrangian heuristic $(\{\bar{x}, \bar{\pi}\})$

**end loop**

\[
z_{LB} := \max(z_{LB}, lb) \quad \text{update LOWER BOUND}
\]

$G :=$ compute sub-gradient$(z_{UB}, z_{LB}, \{\tilde{d}, \tilde{t}, \tilde{b}, \bar{x}, \bar{y}, \bar{\pi}\})$

$T :=$ compute step$(z_{UB}, z_{LB}, \{\tilde{d}, \tilde{t}, \tilde{b}, \bar{x}, \bar{y}, \bar{\pi}\})$

\[
\lambda^{k+1} := \max(0, \lambda^k - TG)
\]

$k := k + 1$

**until** $\|TG\|^2 \leq \varepsilon$ or $k \geq \bar{k}$
Lagrangian procedure

**Require:** $z_{LB}$, $k$, $j$, $\varepsilon$

$\lambda^0 := 0$, $k := 0$, $z_{UB} := \infty$

repeat

$\{\bar{d}, \bar{t}, \bar{b}\} := \text{solve } z_{REV}(\lambda^k)$, $\{\bar{x}, \bar{y}, \bar{\pi}\} := \text{solve } z_{FAM}(\lambda^k)$

$z_{UB}(\lambda^k) := z_{REV}(\lambda^k) - z_{FAM}(\lambda^k)$

$z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k))$

loop

$\{\bar{x}, \bar{\pi}\} := \text{Local search}\{\bar{x}, \bar{\pi}\}$

$lb := \text{Lagrangian heuristic}\{\bar{x}, \bar{\pi}\}$

end loop

$z_{LB} := \max(z_{LB}, lb)$

$G := \text{compute sub-gradient}(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\})$

$T := \text{compute step}(z_{UB}, z_{LB}, \{\bar{d}, \bar{t}, \bar{b}, \bar{x}, \bar{y}, \bar{\pi}\})$

$\lambda^{k+1} := \max(0, \lambda^k - TG)$ update $\lambda$'s

$k := k + 1$

until $||TG||^2 \leq \varepsilon$ or $k \geq \bar{k}$
## Performance of the heuristic

<table>
<thead>
<tr>
<th>Instances</th>
<th>BONMIN solver</th>
<th>Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>opt solution</td>
<td>best solution</td>
</tr>
<tr>
<td>9 flights. 800 pax.</td>
<td>52,876</td>
<td>52,876</td>
</tr>
<tr>
<td></td>
<td>time(min)</td>
<td>GAP 0%</td>
</tr>
<tr>
<td>18 flights 1096 pax.</td>
<td>78,275</td>
<td>77,126</td>
</tr>
<tr>
<td></td>
<td>time(min)</td>
<td>1.47%</td>
</tr>
<tr>
<td>26 flights 2329 pax.</td>
<td>176,995</td>
<td>169,913</td>
</tr>
<tr>
<td></td>
<td>time(min)</td>
<td>4.00%</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Instances</th>
<th>BONMIN solver</th>
<th>Heuristic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>best solution</td>
<td>best solution</td>
</tr>
<tr>
<td>41 flights 3430 pax.</td>
<td>300,949</td>
<td>278,375</td>
</tr>
<tr>
<td></td>
<td>GAP 3.33%</td>
<td>GAP 10.48%</td>
</tr>
<tr>
<td></td>
<td>time(h) 15.01</td>
<td>time(h) 5.51</td>
</tr>
</tbody>
</table>
Conclusions and future work

- **Clip-Air**
  - Potential increase in transportation capacity and profit
  - A system level consideration
    - Repositioning of Clip-Air capsules

- **Integrated scheduling model**
  - Further investigation of the effects of the demand model

- **Heuristic method**
  - Improvement of the solutions
  - Test of the heuristic on a comprehensive test set
Thank you for your attention!

bilge.kucuk@epfl.ch
**Spill and recapture effects - Illustration**

Information regarding the itineraries in segment ORY-NCE:

<table>
<thead>
<tr>
<th>OD</th>
<th>fare</th>
<th>nonstop</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORY-NCE(_1)</td>
<td>220</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ORY-NCE(_2)</td>
<td>218</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>ORY-NCE(_3)</td>
<td>214</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>ORY-NCE(_')</td>
<td>250</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Resulting recapture ratios:

<table>
<thead>
<tr>
<th></th>
<th>ORY-NCE(_1)</th>
<th>ORY-NCE(_2)</th>
<th>ORY-NCE(_3)</th>
<th>ORY-NCE(_')</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORY-NCE(_1)</td>
<td>0</td>
<td>0.401</td>
<td>0.503</td>
<td>0.096</td>
</tr>
<tr>
<td>ORY-NCE(_2)</td>
<td>0.417</td>
<td>0</td>
<td>0.490</td>
<td>0.093</td>
</tr>
<tr>
<td>ORY-NCE(_3)</td>
<td>0.463</td>
<td>0.434</td>
<td>0</td>
<td>0.103</td>
</tr>
</tbody>
</table>
Price elasticity of demand

- Price elasticity of logit:

\[(1 - P^h(i))p^h_i \beta^h_{fare}\]

- When \(\beta_{fare}\) is \(-0.05\) and \(-0.025\) is for economy and business demand, the elasticities are around \(-3\) and \(-2\).

- When we decrease them to \(-0.03\) and \(-0.015\) elasticity values become \(-2\) and \(-1.3\).