Cost Optimization for the Capacitated Railroad Blocking and Train Design Problem

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Terminology and Problem Definition

Wagons

Blocks: **same O-D**
Cannot be separated till the block reaches destination

Which blocks to be assigned to which trains?

Which trains to run?
Problem Objectives

- Blocking Problem
  - Combining shipments to form one unit (*block*)

- Train Design Problem
  - Decide trains origins, destinations and paths
  - Crew segment constraint is also considered

- Block-to-Train Assignment (BTA)
  - Determine which block is assigned to which train

- For our problem, the composition of the blocks are known.
Problem Description

- Model: directed graph
  - Nodes -> train stations
  - The graph is not necessarily complete.

Constraints
- Number of blocks per train
- Number of block swaps per block
- Number of work events per train
- Length and tonnage restrictions on arcs
- Number of trains per arc
- Crew segments

Cost Components
- Fixed setup and travel costs
- Marginal cost per wagon
- Work event cost
- Block swap cost
- Train imbalance cost
- Crew imbalance cost
- Unsatisfied demand
Toy Problem

<table>
<thead>
<tr>
<th>Block</th>
<th>Origin</th>
<th>Destination</th>
<th># of cars</th>
<th>Total length</th>
<th>Total tonnage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>C</td>
<td>50</td>
<td>3000</td>
<td>2500</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>D</td>
<td>25</td>
<td>1500</td>
<td>1250</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>D</td>
<td>40</td>
<td>2400</td>
<td>2000</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>A</td>
<td>28</td>
<td>1680</td>
<td>1400</td>
</tr>
<tr>
<td>5</td>
<td>D</td>
<td>B</td>
<td>16</td>
<td>960</td>
<td>800</td>
</tr>
</tbody>
</table>
Literature - Motivation

  - MIP formulation of the railroad blocking problem
  - Arc-based and path-based formulation of the block-to-train assignment problem

- Literature assumes that train design (with crew constraints) is given and blocking and BTA are solved separately.

- INFORMS RAS 2011 Competition Problem (with real data)
**Cost Breakup**

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train start cost</td>
<td>$400.00</td>
</tr>
<tr>
<td>Railcar travel cost (per mile)</td>
<td>$0.75</td>
</tr>
<tr>
<td>Work event cost</td>
<td>$350.00</td>
</tr>
<tr>
<td>Block swap cost</td>
<td>$40.00 – $100.00</td>
</tr>
<tr>
<td>Crew imbalance penalty</td>
<td>$600.00</td>
</tr>
<tr>
<td>Train imbalance penalty</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>Missed railcar penalty</td>
<td>$5,000.00</td>
</tr>
</tbody>
</table>

**IDEA:**
For each shipment, find path from origin to destination which minimizes travel cost. If no constraints exist, this is the shortest path.

**Constraints:**
- Length and tonnage restrictions on arcs
- Crew segments

*Has highest influence on cost, should be minimized with priority*

*Should be avoided if possible (isolated shipment and/or network capacity)*
Methodological framework

- Travel cost is the most discriminating cost component
- Crew constraints are the most complex
- Three-step process
  - Identify the shortest path for each block, under constraints
  - Guarantee feasibility wrt crew constraint with pre-preprocessing
  - Solve a MIP
Step 1: Constrained shortest path

Min \( \sum_{a \in A} \sum_{b \in B} c_{ab} \cdot Dist_{Arc_a} \cdot Num_{Car_b} \cdot x_{ab} \)

\[ + \sum_{a \in A} (c_{train} \cdot Dist_{Arc_a} + TrainStartCost) \cdot y_a \]  \hspace{1cm} (1)

subject to

\[ \sum_{a \in A_{b+}} x_{ab} = 1 \hspace{2cm} \forall b \in B \]  \hspace{1cm} (2)

\[ \sum_{a \in A_{b-}} x_{ab} = 0 \hspace{2cm} \forall b \in B \]  \hspace{1cm} (3)

\[ \sum_{a \in A_{b+}^{-}} x_{ab} = 1 \hspace{2cm} \forall b \in B \]  \hspace{1cm} (4)

\[ \sum_{a \in A_{n+}} x_{ab} = \sum_{a \in A_{n-}} x_{ab} \hspace{1cm} \forall n \in N \setminus \{D_b, O_b\}, \forall b \in B \]  \hspace{1cm} (5)

\[ \sum_{b \in B} Ton_b \cdot x_{ab} \leq y_a \cdot Max_{Ton_a} \hspace{2cm} \forall a \in A \]  \hspace{1cm} (6)

\[ \sum_{b \in B} Len_b \cdot x_{ab} \leq y_a \cdot Max_{Len_a} \hspace{2cm} \forall a \in A \]  \hspace{1cm} (7)

\[ y_a \leq Max_{Num_{Trains_a}} \hspace{1cm} \forall a \in A \]  \hspace{1cm} (8)

\[ x_{ab} \leq y_a \hspace{2cm} \forall a \in A, \forall b \in B \]  \hspace{1cm} (9)

\[ x_{ab} \in \{0, 1\}, y_a \in \mathbb{N} \hspace{2cm} \forall a \in A, \forall b \in B \]  \hspace{1cm} (10)
Step 1: Constrained shortest path

\[\text{Min } \sum_{a \in A} \sum_{b \in B} c_{\text{car}} \cdot \text{DistArc}_a \cdot \text{NumCar}_b \cdot x_{ab} + \sum_{a \in A} (c_{\text{train}} \cdot \text{DistArc}_a + \text{TrainStartCost}) \cdot y_a \]

if block \( b \) is carried on arc \( a \) or not

number of trains running on arc \( a \)

railcar travel cost + train travel cost + train start cost

\( \forall b \in B \)

shipment leaves its origin

\( \forall b \in B \)

subtour elimination constraint

\( \forall b \in B \)

shipment reaches its destination

\( \forall n \in N \setminus \{D_b, O_b\}, \forall b \in B \)

shipment leaves the node that it enters

\( \forall a \in A \)

maximum tonnage constraint on arcs

\( \forall a \in A \)

maximum length constraint on arcs

\( \forall a \in A \)

maximum number of trains constraint on arcs

\( x_{ab} \in \{0, 1\} \cdot y_a \in \mathbb{N} \)
Shipment and Train Path Generation

- **Step 1:** Resource constraint shortest path problem for shipments
  - Does not take care of crew segments
  - Find shortest crew segment covering for the paths if the shortest path is not on crew segments

- **Step 2:** Train Path Generation (preprocessing)
  - Assign one train per crew segment
  - It guarantees feasibility wrt crew constraints
  - Assign more trains to sequences of crew segments to increase flexibility
  - We duplicate trains to meet capacity constraints
  - Next, we decide which of these many trains will be actually operated.
Step 3: MIP

$$\begin{align*}
\text{Min} \sum_{t \in T} (\text{TrainStartCost} + \text{TrainTravelCostPerMile} \cdot \text{TrainPathLength}_t) \cdot z_t \\
+ \sum_{t \in T} \sum_{a \in A} \text{CostPerWorkEvent} \cdot w_t^a + \sum_{b \in B} \sum_{a \in A} \text{BlockSwapCost}_a \cdot w_b^a \\
+ \sum_{n \in N} \text{TrainImbalancePenalty} \cdot s_n + \sum_{i \in C} \text{CrewImbalancePenalty} \cdot c_i \\
+ \sum_{b \in B} (1 - k_b) \cdot \text{MissedRailcarPenalty} \cdot \text{NumCar}_b
\end{align*}$$

subject to

1. $$\sum_{\{t \in T : a \in I_{bt}\}} x_{bt}^a = k_b \quad \forall b \in B, \forall a \in R_b$$
2. $$x_{bt}^a \leq y_{bt} \quad \forall b \in B, \forall t \in T, \forall a \in I_{bt}$$
3. $$x_{bt}^a \leq z_t \quad \forall b \in B, \forall t \in T, \forall a \in I_{bt}$$
4. $$x_{bt}^a - x_{bt}^{N_t^a} \leq w_t^a, \quad x_{bt}^{N_t^a} - x_{bt}^a \leq w_t^a \quad \forall b \in B, \forall t \in T, \forall a \in I_{bt} \setminus \{a_{t1}, a_{t | P_t |}\}$$
5. $$\sum_{a \in P_t} w_t^a \leq \text{maxNbWorkEvents}_t \quad \forall t \in T$$
Mathematical Model (cont.)

\[
\begin{align*}
    x_{bt}^a - x_{bt}^{N_b^a} &\leq u_b^a, \quad x_{bt}^{N_b^a} - x_{bt}^a &\leq u_b^a &\forall b \in B, \forall t \in T, \forall a \in I_{bt} \setminus \{a_{b1}, a_{b|R_b}\} \\
    \sum_{a \in R_b} u_b^a &\leq \text{maxNbBlockSwaps}_b &\forall b \in B \\
    \sum_{t \in T : n = O_t} z_t - \sum_{t \in T : n = D_t} z_t &\leq s_n &\forall n \in N \\
    \sum_{t \in T : n = D_t} z_t - \sum_{t \in T : n = O_t} z_t &\leq s_n &\forall n \in N \\
    \sum_{t \in T} CrSeg_t \cdot z_t - \sum_{t \in T} OpCrSeg_t \cdot z_t &\leq c_i &\forall i \in C \\
    \sum_{t \in T} OpCrSeg_t \cdot z_t - \sum_{t \in T} CrSeg_t \cdot z_t &\leq c_i &\forall i \in C \\
    \sum_{b \in B} y_{bt} &\leq \text{maxNbBlocksPerTrain} &\forall t \in T, \forall b \in B \\
    x_{bt}^a, y_{bt}, z_t, w_t^a, u_b^a, k_b &\in \{0, 1\} &\forall b \in B, \forall t \in T, \forall a \in A \\
    s_n, c_i &\in \mathbb{N} &\forall n \in N, \forall i \in C
\end{align*}
\]
Mathematical Model

Min \sum_{t \in T} \left( \text{TrainStartCost} + \text{TrainTravelCostPerMile} \cdot \text{TrainPathLength}_t \right) \cdot z_t

+ \sum_{t \in T} \sum_{a \in A} \text{CostPerWorkEvent} \cdot w^a_t + \sum_{b \in B} \sum_{a \in A} \text{BlockSwapCost}_a \cdot w^b_a

+ \sum_{n \in N} \text{TrainImbalancePenalty} \cdot s_n + \sum_{i \in C} \text{CrewImbalancePenalty} \cdot c_i

+ \sum_{b \in B} (1 - k_b) \cdot \text{MissedRailcarPenalty} \cdot \text{NumCar}_b

subject to

\sum_{\{t \in T: a \in I_{bt}\}} x^a_{bt} = k_b

if block b is carried by train t on arc a or not

\forall b \in B, \forall a \in R_b

(2)

\forall b \in B, \forall t \in T, \forall a \in I_{bt}

(3)

connect variables

\forall b \in B, \forall t \in T, \forall a \in I_{bt}

(4)

work event

\forall b \in B, \forall t \in T, \forall a \in I_{bt} \setminus \{0_t, a_t | P_t\}

(5)

(1) all costs and penalties

\forall b \in B

x^a_{bt} \leq \gamma_{bt}

if block b is carried by train t at least once or not

x^a_{bt} \leq z_t

if train t is used or not

x^a_{bt} - \text{maxNbWorkEvents}_t \cdot w^a_t \leq x^a_{bt} - x^a_{bt} \leq w^a_t

if train t has a work event after arc a or not

\sum_{a \in P_t} \gamma_{a | P_t}

first arc of train t

\forall t \in T

\forall b \in B, \forall t \in T, \forall a \in I_{bt}

(6)
Mathematical Model (cont.)

\[
\begin{align*}
    \sum_{t \in T} x_{bt}^{a} - x_{bt}^{N_a} & \leq u_{bt}^a, \quad x_{bt}^{a} - x_{bt}^{N_a} \leq u_{bt}^a, & \forall b \in B, \forall t \in T, \forall a \in I_{bt} \setminus \{a_b \mid a_b \in R_d\} & (7) \\
    \sum_{a \in R_b} u_{b}^a & \leq \max N_b BlockSwaps_b & \forall b \in B & (8) \\
    \sum_{t \in T} z_t - \sum_{\{t \in T : n = O_t\}} z_t \leq s_n & \forall n \in N & (9) \\
    \sum_{\{t \in T : n = D_t\}} z_t \leq s_n & \forall n \in N & (10) \\
    \sum_{t \in T} OpCrSeg_t \cdot z_t - \sum_{t \in T} CrSeg_t \cdot z_t \leq c_i & \forall i \in C & (11) \\
    \sum_{t \in T} OpCrSeg_t \cdot z_t - \sum_{t \in T} CrSeg_t \cdot z_t \leq c_i & \forall i \in C & (12) \\
    \sum_{b \in B} y_{bt} & \leq \max N_b BlocksPerTrain & \forall t \in T, \forall b \in B & (13) \\
    x_{bt}^{a}, y_{bt}, z_t, w_{bt}^{a}, u_{bt}^a, k_b, s_n, c_i & \in \{0, 1\} & \forall b \in B, \forall t \in T, \forall a \in A & (14) \\
    s_n, c_i & \in \mathbb{N} & \forall n \in N, \forall i \in C & (15)
\end{align*}
\]
Data Sets: CSX - railroad company in the US

Data 1
- 134 arcs
- 94 nodes
- 239 shipments (1-12 arcs)
- 154 crew segments (1-4 arcs)
- 534 candidate trains

Data 2
- 294 arcs
- 221 nodes
- 369 shipments (1-17 arcs)
- 154 crew segments (1-7 arcs)
- 575 candidate trains
## Results

<table>
<thead>
<tr>
<th>Objective Function Component</th>
<th>Data 1</th>
<th></th>
<th>Data 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost ($)</td>
<td>%</td>
<td>Cost ($)</td>
<td>%</td>
</tr>
<tr>
<td>Train Start Cost</td>
<td>23600</td>
<td>1.2</td>
<td>33600</td>
<td>1.1</td>
</tr>
<tr>
<td>Train Travel Cost</td>
<td>238692</td>
<td>12.6</td>
<td>267989</td>
<td>8.9</td>
</tr>
<tr>
<td>Railcar Travel Cost</td>
<td>1566012.38</td>
<td>82.5</td>
<td>219517.5</td>
<td>72.8</td>
</tr>
<tr>
<td>Work Event Cost</td>
<td>47600</td>
<td>2.6</td>
<td>73150</td>
<td>2.4</td>
</tr>
<tr>
<td>Block Swap Cost</td>
<td>4050</td>
<td>0.2</td>
<td>5860</td>
<td>0.2</td>
</tr>
<tr>
<td>Crew Imbalance Cost</td>
<td>7200</td>
<td>0.4</td>
<td>13800</td>
<td>0.5</td>
</tr>
<tr>
<td>Train Imbalance Cost</td>
<td>12000</td>
<td>0.6</td>
<td>4000</td>
<td>0.1</td>
</tr>
<tr>
<td>Missed Railcars Cost</td>
<td>0</td>
<td>0</td>
<td>420000</td>
<td>14.0</td>
</tr>
<tr>
<td>Total Cost</td>
<td>1899154.38</td>
<td>100</td>
<td>3008556.5</td>
<td>100</td>
</tr>
</tbody>
</table>
Findings

- Integrated problem can be solved on real instances
- Crew constraints are the most complex → processed first
- We exploit the cost structure → shortest paths followed by MIP
Ongoing Research

- Relax the strong relation with the cost structure
- E.g. combining shipment path generation algorithm with the shipment to train assignment
  - Multioptional shipment paths (e.g. k-shortest paths)
  - Online train generation (column generation)
- Include uncertainty in the model (robust optimization, recovery)
- Include a time dimension (scheduling)