# Integrated and Robust Planning of Bulk Port Operations

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# Contents



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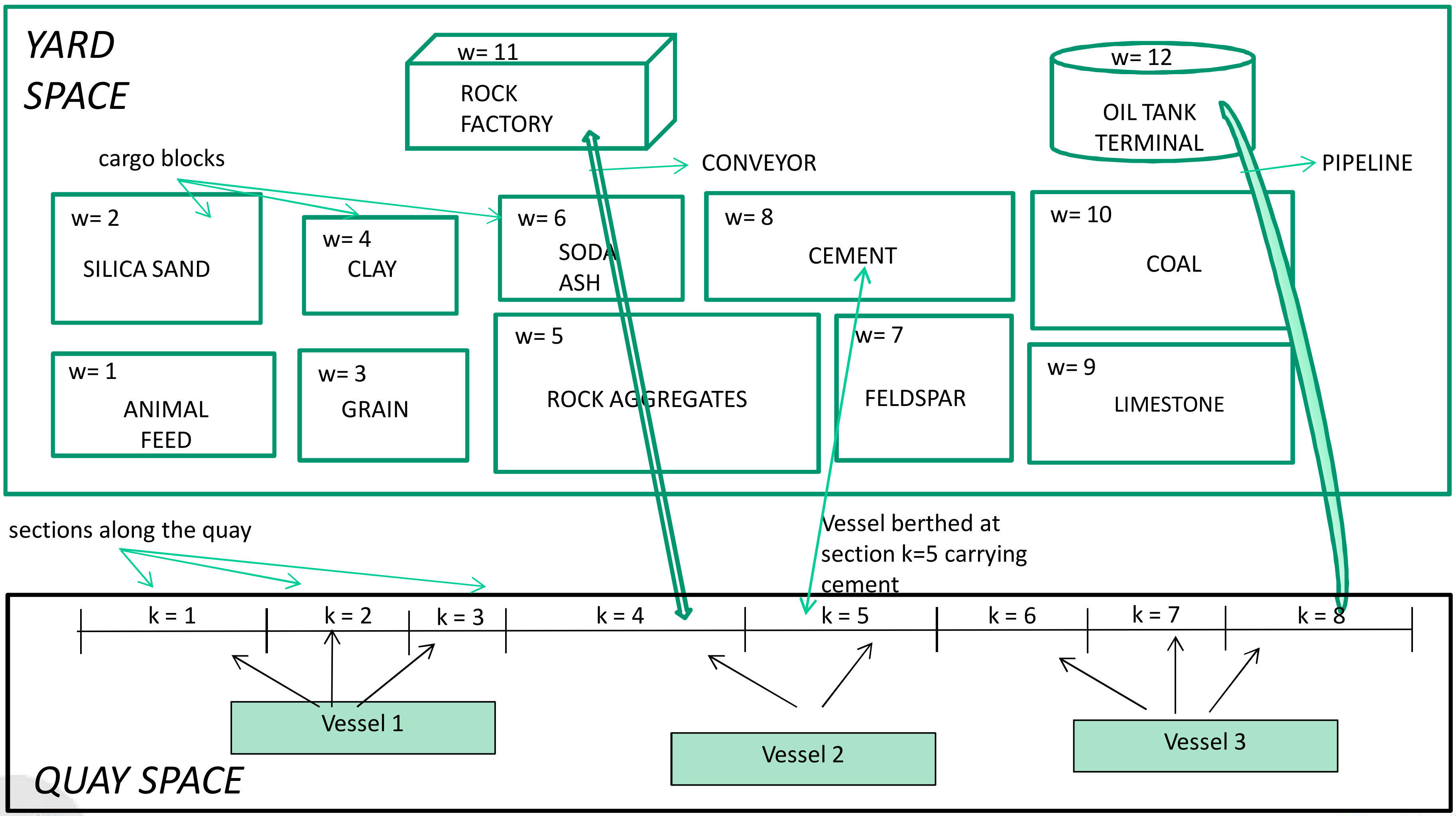
# Schematic Diagram of a Bulk Terminal

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# Motivation

- International shipping tonnage in solid bulk and liquid bulk trade has registered an increase by 52% and 48% respectively. The total volume of dry bulk cargoes loaded in 2008 stood at 5.4 billion tons, accounting for 66.3 per cent of total world goods loaded (UNCTAD, 2009)

- Bulk port terminals have received significantly less attention than container terminals in the field of large scale optimization

- High level of uncertainty in bulk port operations due to weather conditions, mechanical problems etc.

  - Disrupt the normal functioning of the port

  - Require quick real time action.

- In context of container terminals, comprehensive literature surveys can be found in Steenken et al. (2004), Stahlbock and Voss (2008), Bierwirth and Meisel (2010).

# Research Objectives

- Study the crucial problems of
  - **Berth Allocation** – scheduling and assignment of vessels to sections along the quay
  - **Yard Assignment** – assignment of vessels and cargo types to specific locations on the yard
- **Large Scale Integrated Planning:** Integration of the berth allocation and yard assignment for better coordination between berthing and yard activities
- Develop **real time** and **robust optimization algorithms** to account for uncertainties in arrival times and handling times of vessels, and other unforeseen disruptions and delays in operations.

# Deterministic BAP: Problem Definition

- **Find**
  - Optimal assignment and schedule of vessels along the quay (without accounting for any uncertainty in arrival information)

- **Given**

  - Expected arrival times of vessels

  - Handling times dependent on

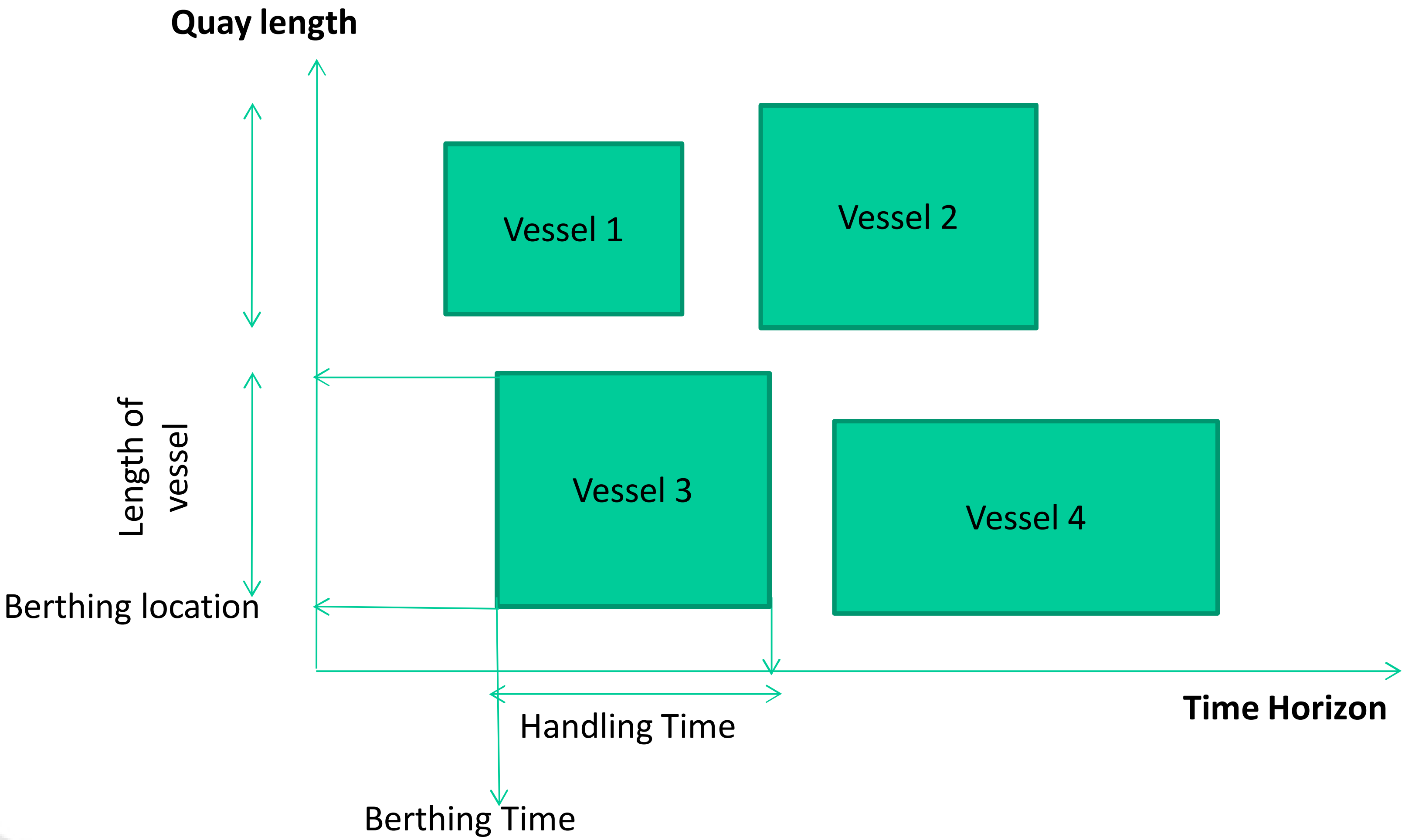
    - **Cargo type** on the vessel (the relative location of the vessel along the quay with respect to the cargo location on the yard)

    - Number of cranes operating on the vessel

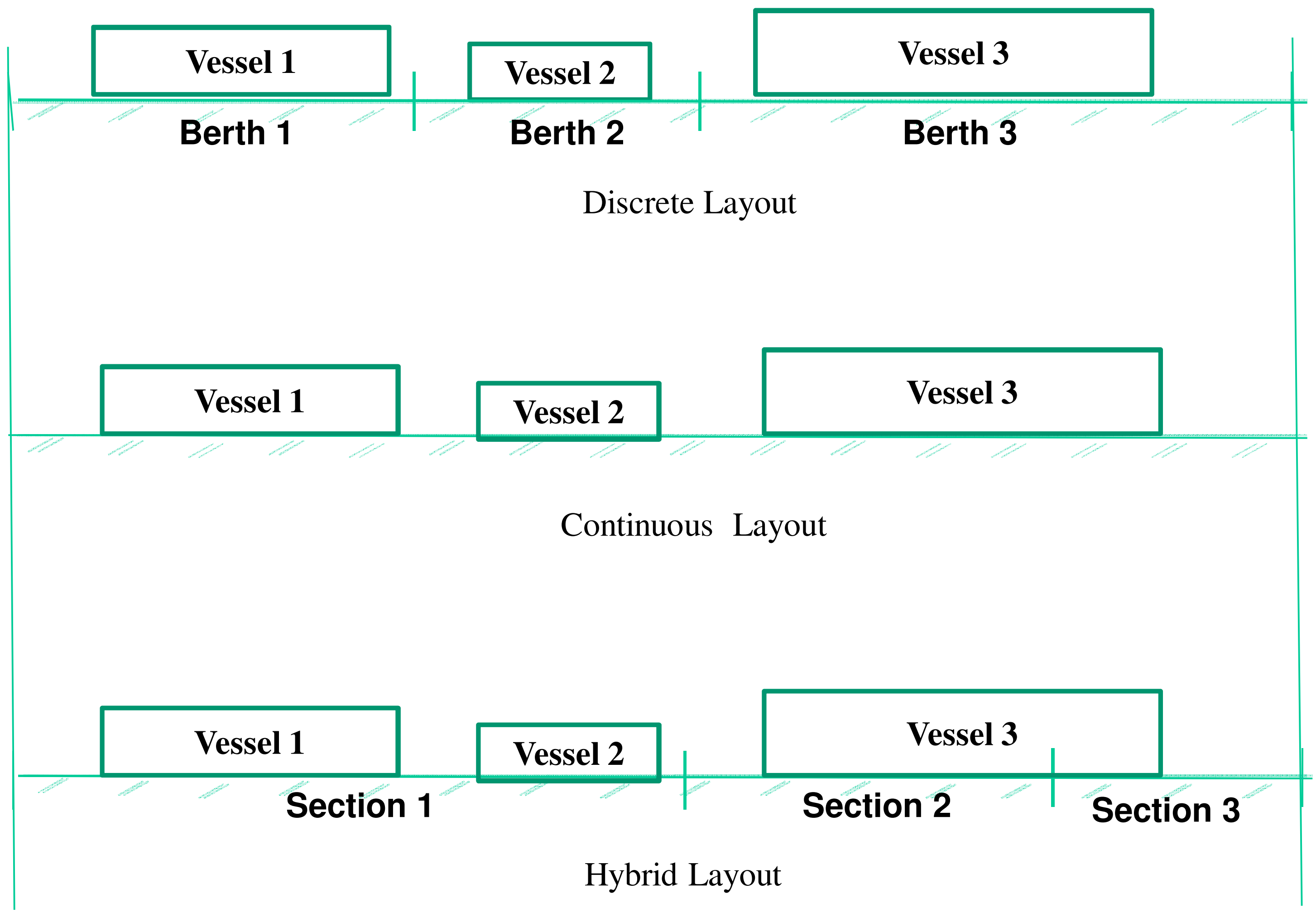
- **Objective**

  - Minimize total service times (waiting time + handling time) of vessels berthing at the port

# BAP Solution



# Discretization



# MILP Model

## Objective Function

$$\min \sum_{i \in N} (m_i - A_i + c_i)$$

*Decision variables:*

$m_i$ starting time of handling of vessel $i \in N$;

$c_i$ total handling time of vessel $i \in N$;

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# MILP Model

## Dynamic vessel arrival constraints

$$m_i - A_i \geq 0 \qquad \forall i \in N,$$

## Non overlapping constraints

$$\sum_{k \in M} (b_k s_k^j) + B(1 - y_{ij}) \geq \sum_{k \in M} (b_k s_k^i) + L_i \qquad \forall i, j \in N, i \neq j,$$

$$m_j + B(1 - z_{ij}) \geq m_i + c_i \qquad \forall i, j \in N, i \neq j,$$

$$y_{ij} + y_{ji} + z_{ij} + z_{ji} \geq 1 \qquad \forall i, j \in N, i \neq j,$$

# MILP Model

## Section covering constraints

$$\sum_{k \in M} s_k^i = 1 \qquad \forall i \in N,$$

$$\sum_{k \in M} (b_k s_i^k) + L_i \leq L \qquad \forall i \in N,$$

$$\sum_{l \in M} (d_{ilk} s_l^i) = x_{ik} \quad \forall i \in N, \forall k \in M,$$

## Draft Restrictions

$$(d_k - D_i) x_{ik} \geq 0 \qquad \forall i \in N, \forall k \in M,$$

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# MILP Model

## Determination of Handling Times

- Given an input vector of unit handling times for each combination of cargo type and section along the quay
- Specialized facilities (conveyors, pipelines etc.) also modeled as cargo types
- All sections occupied by the vessel are operated simultaneously

$$c_i \geq h_k^w p_{ilk} Q_i s_l^i \qquad \forall i \in N, \forall k \in M, \forall l \in M, \forall w \in W_i$$

$Q_i$ quantity of cargo to be loaded on or discharged from vessel *i*

$h_k^w$ handling time for unit quantity of cargo $w \in W$ and vessel berthed at section $k \in M$;

$p_{ilk}$ fraction of cargo handled at section $k \in M$ when vessel *i* is berthed at starting section $l \in M$

# GSPP Model

- Used in context of container terminals by Christensen and Holst (2008)

- Generate set P of columns, where each column $p \in P$ represents a feasible assignment of a single vessel in both space and time

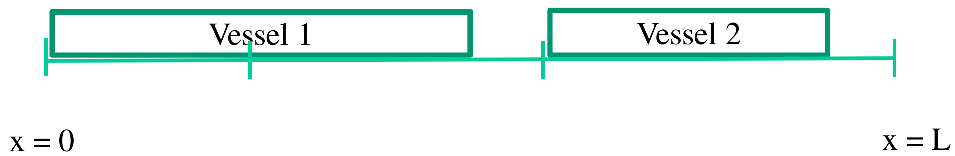
- Generate two matrices

  - Matrix A = $(A_{ip})$ ; equal to 1 if vessel $i \in N$ is the assigned vessel in the feasible assignment represented by column $p \in P$

  - Matrix B = $(b_p^{st})$ ; equal to 1 if section $s \in M$ is occupied at time $t \in H$ in column $p \in P$

*Note: Assume integer values for all time measurements*

# GSPP Formulation: A simple example




- $|N| = 2$, $|M| = 3$, $|H| = 3$
- Vessel 1 cannot occupy section 3 owing to spatial constraints (does not have conveyor facility), vessel 2 arrives at time t = 1
- Constraint matrix P has 4 feasible assignments:

| | | | | |
|---|---|---|---|---|
| **Vessel 1** | 1 | 1 | 0 | 0 |
| **Vessel 2** | 0 | 0 | 1 | 1 |
| **Section 1 , Time 1** | 1 | 0 | 0 | 0 |
| **Section 1, Time 2** | 1 | 1 | 1 | 0 |
| **Section 1, Time 3** | 0 | 1 | 1 | 0 |
| **Section 2, Time 1** | 1 | 0 | 0 | 0 |
| **Section 2, Time 2** | 1 | 1 | 1 | 1 |
| **Section 2, Time 3** | 0 | 1 | 1 | 1 |
| **Section 3, Time 1** | 0 | 0 | 0 | 0 |
| **Section 3, Time 2** | 0 | 0 | 0 | 1 |
| **Section 3, Time 3** | 0 | 0 | 0 | 1 |

# GSPP Model Formulation

**Objective Function:**

$$\min \sum_{p \in P} (d_p \lambda_p + h_p \lambda_p)$$

**Constraints:**

$$\sum_{p \in P} (A_{ip} \lambda_p) = 1 \qquad \forall i \in N$$

$$\sum_{p \in P} (b_p^{st} \lambda_p) \leq 1 \qquad \forall s \in M, \forall t \in H$$

$d_p$ : *delay in service associated with assignment* $p \in P$

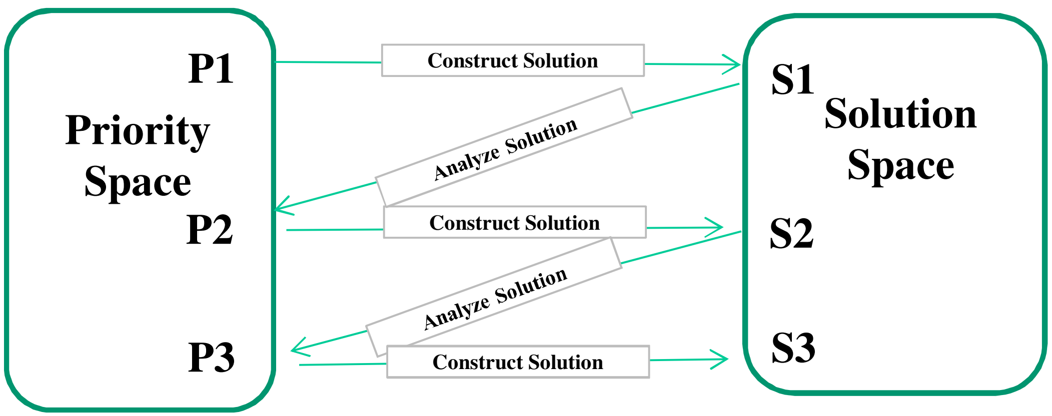
$h_p$ : *handling time associated with assignment* $p \in P$

$\lambda_p$ : binary parameter, equal to 1 if assignment $p \in P$ is part of the optimal solution

# SWO Heuristic Approach

- Introduced by Clements (1997), typically successful in problems where it is possible to quantify the contribution of each single problem element to the overall solution quality

- Construct/ Analyze/ Prioritize: Solution generated at each successive iteration is constructed and analyzed, results of analysis used to generate a new priority order




- Moves in search space are motivated by the weak performing elements and not the overall objective function value

# SWO Heuristic Approach

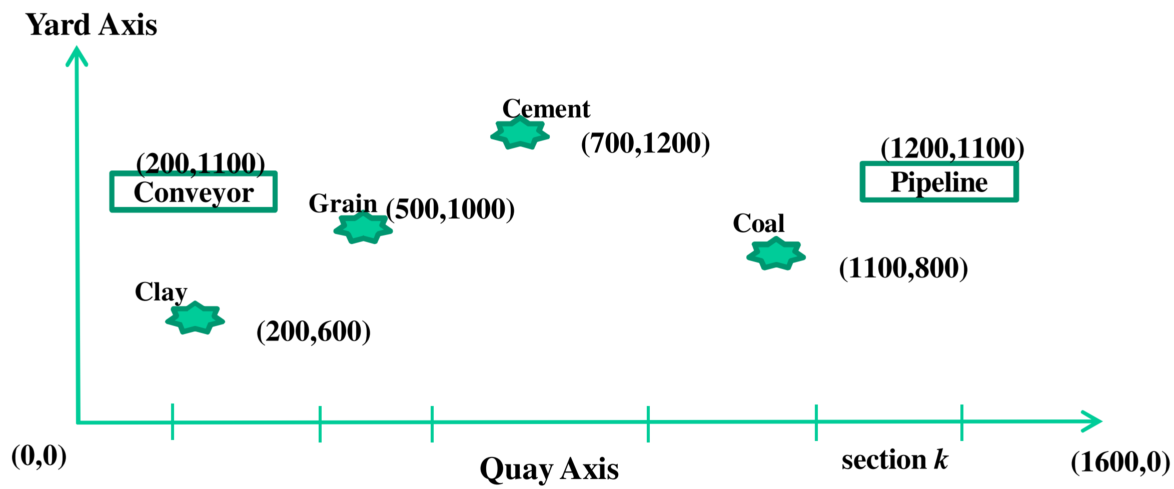
- **Construction heuristic**: Returns a feasible berthing assignment for given priority order of vessels
- **Initial Solution**: First-Cum-First-Served ordering based on arrival times of vessels
- **Algorithm**: In each successive iteration, a new priority order is constructed based on the service quality measure of each berthing vessel in the previous solution
  - Service time of the vessel in the solution found in the last iteration
  - Deviation of service time of vessel from the minimum service time possible for that vessel ( zero delay + minimum handling time )
  - Sum of service times of the vessel in all iterations completed so far!
- If a priority order is already evaluated, introduce randomization by swapping two or more vessels, until we obtained a priority order that has not been evaluated so far
- Algorithm terminates after a preset number of iterations and best solution is selected as the final solution

# Generation of Instances

- Instances based on data from SAQR port with quay length of 1600 meters and vessel lengths in the range 80-260 meters.

- Generate 6 instances sizes with $|N|$ = 10, 25 and 40 vessels, and $|M|$ = 10 and 30 sections, with 9 instances for each instance size.

- Handling times generated for 6 cargo types.

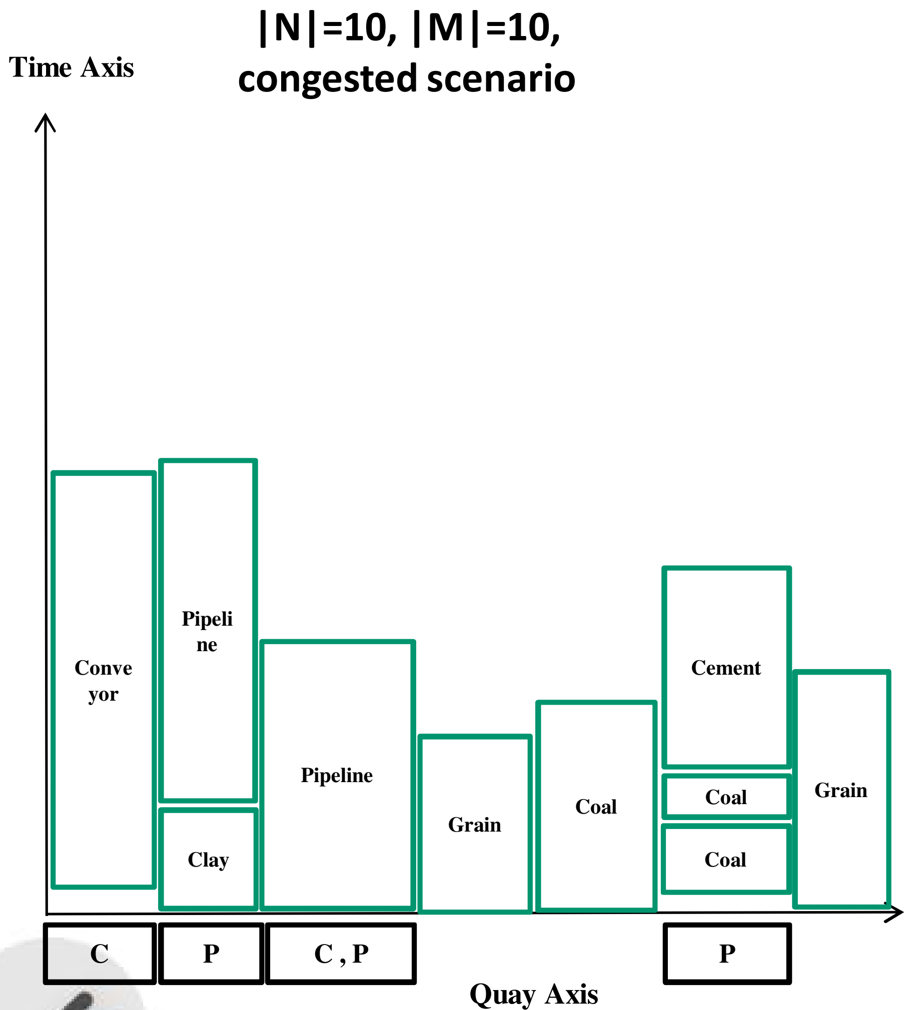


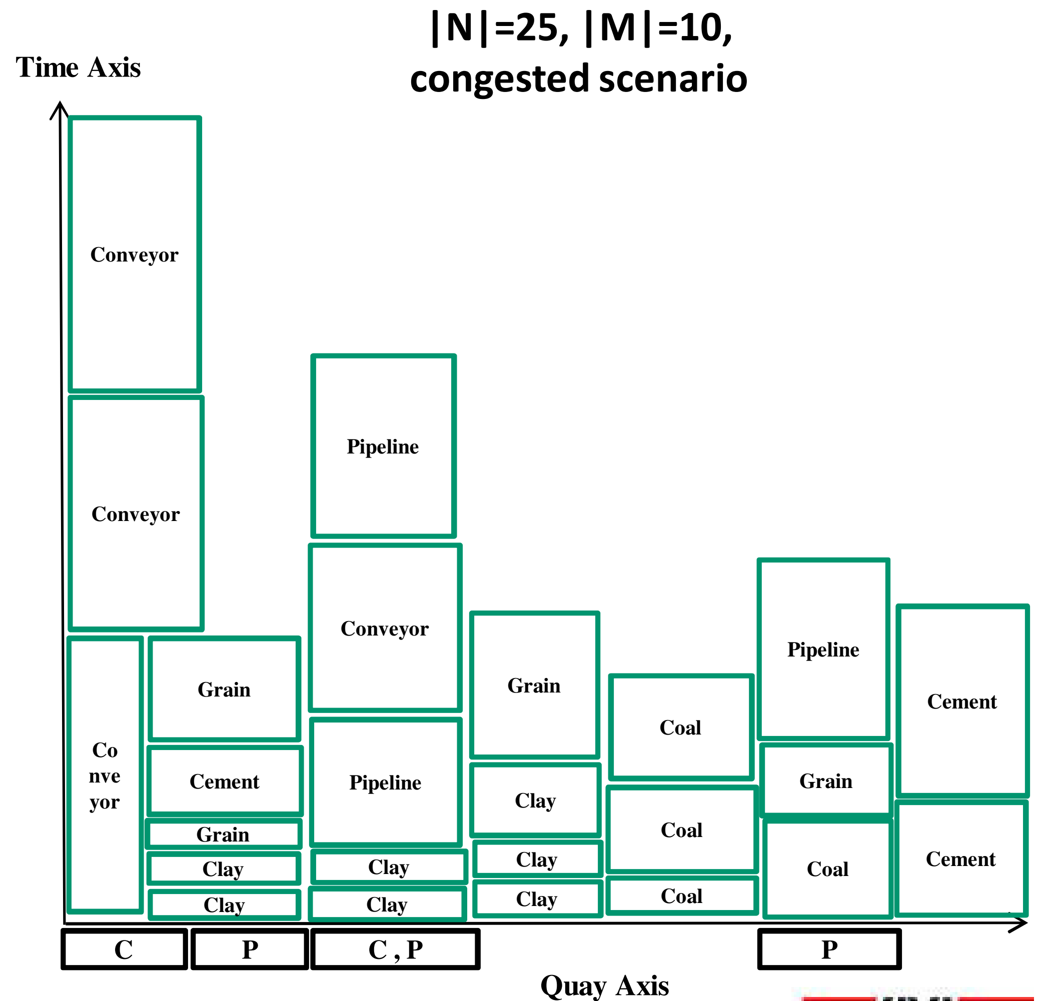

- Drafts of all vessels $D_i$ are less than the minimum draft along the quay.

# Computational Results

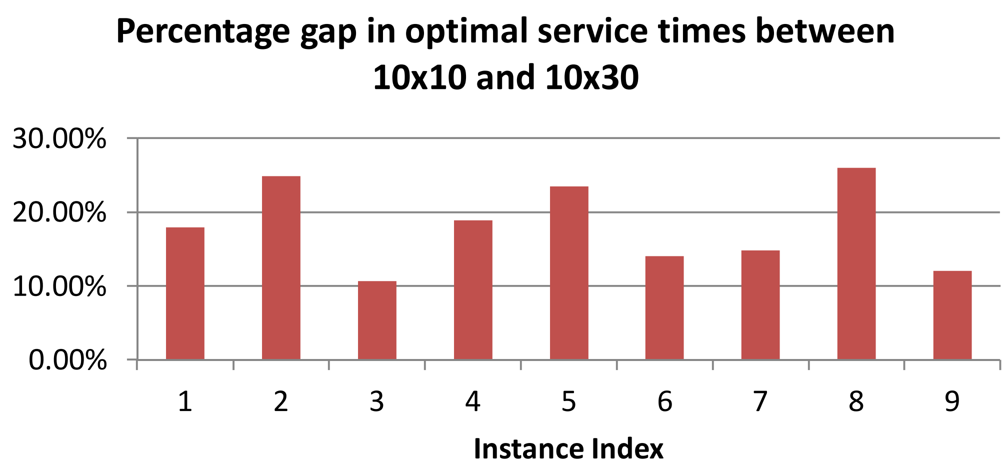
- Instances based on data from SAQR port
- All tests were run on an Intel Core i7 (2.80 GHz) processor and used a 32-bit version of CPLEX 12.2.
- Results inspired by port data show that the problem is complex !
- MILP formulation fails to produce optimal results for even small instances with $|N|$= 10 vessels within CPLEX time limit of 2 hours.
- The performance of the GSPP model is quite remarkable!
  - Can solve instances up to $|N|$ = 40 vessels
  - Limitations: For larger instances, or longer horizon $H$ solver runs out of memory (use dynamic column generation!)
- Alternate heuristic approach based on squeaky wheel optimization (SWO) performs reasonably well for not so large instances. Optimality gap is less than 10% (with respect to exact solution obtained from GSPP approach) averaged over all tested instances.

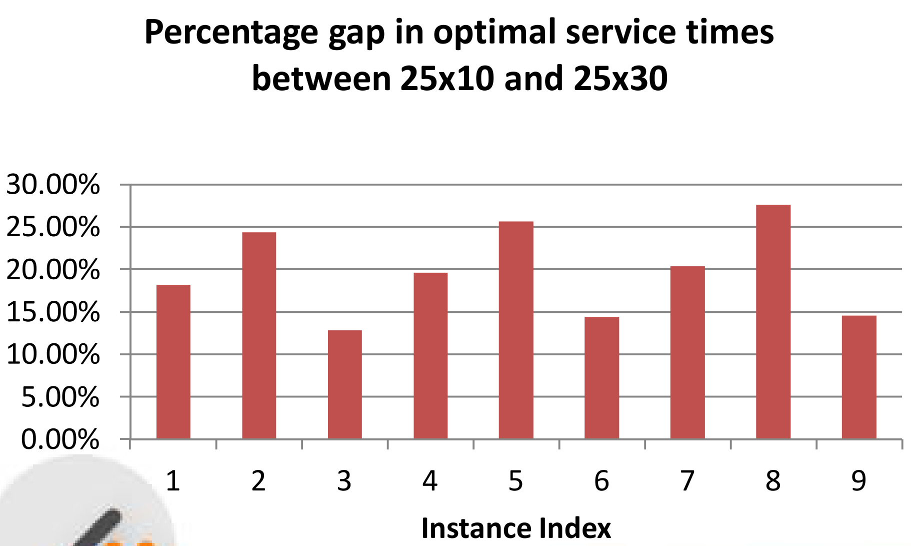
# Results Analysis

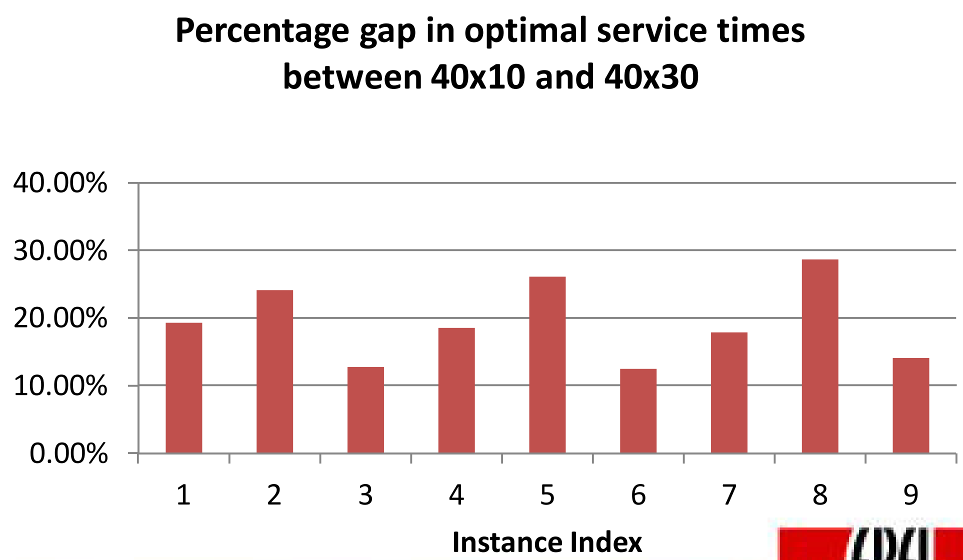


# Results Analysis

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# Integrated Berth Allocation and Yard Assignment Problem

# Literature Review

- To the best of our knowledge, there is almost no existing literature in bulk ports
- In container terminals
  - Berth Allocation: Imai et al. (1997), Imai et al. (2001), Li et al. (1998), Guan et al. (2002), Park and Kim (2003), Moorthy and Teo (2006), Dai et al. (2008), Nishimura et al. (2001)
  - Yard management: Cheung et al. (2002), Zhang et al. (2002), Ng and Mak (2005), Ng (2005)
  - Integration approaches: Major focus on integrated berth allocation and quay crane assignment or scheduling by Park and Kim (2003), Meisel and Bierwirth (2006), Imai et al. (2008a), Meisel and Bierwirth (2008) and others.
  - Other contributions in context of yard management include Moorthy and Teo (2006) ,Cordeau et al. (2007) and more recently Zhen et al.(2011).

# Integrated Berth and Yard Assignment Problem

- **Find**
  - Determine the berthing schedule of vessels
  - Assign cargo locations to specific cargo types on the yard
  - Assign cargo locations to vessels berthing at the port
- **Given**
  - Expected arrival times of vessels
- **Objective**
  - Minimize total service times (waiting time + handling time) of vessels berthing at the port

# Decision Variables

$m_i$ integer ≥ 0, represents the starting time of handling of vessel $i \in N$;

$c_i$ integer ≥ 0, represents the total handling time of handling of vessel $i \in N$;

$h_{ik}^{w}$ handling time for unit quantity of cargo type w $\in W_i$ for vessel $i \in N$ berthed at section k $\in M$

$\beta_{ik}^{w}$ variable component of handling time for unit quantity of cargo type w $\in W_i$ for vessel $i \in N$ berthed at section $k \in M$

$r_k^i$ weighted average distance between vessel $i$ occupying section $k$ and all cargo locations assigned to the vessel;

$\lambda_{ip}$ amount of cargo handled by vessel $i \in N$ at cargo location $p \in P$

# Decision Variables

| | |
|---|---|
| $s_i^k$ | binary, equals 1 if section $k \in M$ is the starting section of vessel $i \in N$, 0 else; |
| $x_{ik}$ | binary, equals 1 if vessel $i \in N$ occupies section $k \in M$ , 0 otherwise; |
| $y_{ij}$ | binary, equals 1 if vessel $i \in N$ is berthed to the left of vessel $j \in M$ , 0 otherwise; |
| $z_{ij}$ | binary, equals 1 if handling of vessel $i \in N$ finishes before the start of handling of vessel $j \in N$, 0 otherwise; |
| $\pi_w^p$ | binary, equals 1 if cargo type $w$ is stored at cargo location $p$; |
| $\omega_t^{ip}$ | binary, equals 1 if vessel $i$ is being handled at location $p$ at time $t$; |
| $\phi_{ip}$ | binary, equals 1 if vessel $i$ uses cargo location $p$; |
| $\theta_{it}$ | binary, equals 1 if vessel $i$ is being handled at time $t$; |

# Mixed Integer Program

- **Objective Function**

$$\min \sum_{i \in N} (m_i - A_i + c_i)$$

- **BAP Constraints**

- **Handling Time Constraints**

$$c_i \geq h_{ik}^{w} p_{ilk} Q_i - B(1 - s_l^i) \qquad \forall i \in N, \forall k \in M, \forall l \in M, w \in W_i,$$

$$h_{ik}^{w} = \alpha_{ik}^{w} + \beta_{ik}^{w} \qquad \forall i \in N, \forall k \in M, w \in W_i,$$

$$\alpha_{ik}^{w} = T / n_{ik}^{w} \qquad \forall i \in N, \forall k \in M, w \in W_i,$$

$$\beta_{ik}^{w} = V_w r_k^i \qquad \forall i \in N, \forall k \in M, w \in W_i,$$

$$r_k^i = \sum_{p \in P} (r_k^p \lambda_{ip}) / Q_i \qquad \forall i \in N, \forall k \in M,$$

# Mixed Integer Program

- **Number of cargo locations**

$$\sum_{p \in P} \phi_{ip} \leq F \quad \forall i \in N,$$

- **Cargo Storage Restrictions**

$$\pi_w^p + \pi_u^q \leq 1 \qquad \forall w \in W, \forall u \in \overline{W}(w), \forall p \in P, \forall q \in \overline{P}(p),$$

- **Cargo assignment and capacity constraints**

$$Q_i = \sum_{p \in P} \lambda_{ip} \qquad \forall i \in N,$$

$$\lambda_{ip} \leq \phi_{ip} Q_i \qquad \forall i \in N, \forall w \in W_i, p \in P,$$

$$\phi_{ip} \leq \lambda_{ip} \qquad \forall i \in N, \forall w \in W_i, p \in P,$$

$$\lambda_{ip} \leq \sum_{w \in W_i} \sum_{t \in H} (R_w \omega_t^{ip} + B(1 - \pi_w^p)) \qquad \forall i \in N, p \in P,$$

# Mixed Integer Program

- Cargo location can be assigned to a vessel only if it stores the cargo type on the vessel

$$\phi_{ip} \leq \pi_w^p \qquad \forall i \in N, \forall w \in W_i, p \in P,$$

- Given cargo location can be handled by at most one vessel at a given time

$$\sum_{i \in N} \omega_t^{ip} \leq 1 \qquad \forall p \in P, \forall t \in H,$$

- Given cargo location can be used to store at most one cargo type

$$\sum_{w \in W} \pi_w^p \leq 1 \qquad \forall p \in P,$$

# Mixed Integer Program

- Control values of binary variable $\theta_{it}$

$$\sum_{t \in H} \theta_{it} = c_i \qquad \forall i \in N,$$

$$t_i + B(1 - \theta_{it}) \geq m_i + 1 \qquad \forall i \in N, \forall t \in H,$$

$$t_i \leq m_i + c_i + + B(1 - \theta_{it}) \qquad \forall i \in N, \forall t \in H,$$

- Control values of binary variable $\omega_t^{ip}$

$$\omega_t^{ip} \geq \phi_{ip} + \theta_{it} - 1 \qquad \forall i \in N, p \in P, \forall t \in H,$$

$$\omega_t^{ip} \leq \phi_{ip} \qquad \forall i \in N, p \in P, \forall t \in H,$$

$$\omega_t^{ip} \leq \theta_{it} \qquad \forall i \in N, p \in P, \forall t \in H,$$

# MIP formulation

$$\min \sum_{i} (m_i - A_i + c_i) \tag{4}$$

$$\text{s.t. } m_i - A_i \geq 0 \qquad \forall i \epsilon N \tag{5}$$

$$\sum_{k \epsilon M} (s_k^j b_k) + B(1 - y_{ij}) \geq \sum_{k \epsilon M} (s_k^i b_k) + L_i \qquad \forall i, j \epsilon N, i \neq j \tag{6}$$

$$m_j + B(1 - z_{ij}) \geq m_i + c_i \qquad \forall i \epsilon N, \forall j \epsilon N, i \neq j \tag{7}$$

$$y_{ij} + y_{ji} + z_{ij} + z_{ji} \geq 1 \qquad \forall i \epsilon N, \forall j \epsilon N, i \neq j \tag{8}$$

$$\sum_{k \epsilon M} s_k^i = 1 \qquad \forall i \epsilon N \tag{9}$$

$$\sum_{k \epsilon M} (s_k^i b_k) + L_i \leq L \qquad \forall i \epsilon N \tag{10}$$

$$\sum_{p \epsilon M} (\delta_{i \ell k} s_\ell^i) = x_{ik} \qquad \forall i \epsilon N, \forall k \epsilon M \tag{11}$$

$$(d_k - D_i) x_{ik} \geq 0 \qquad \forall i \epsilon N, \forall k \epsilon M \tag{12}$$

$$c_i \geq h_{ik}^w \rho_{i \ell k} Q_i - B(1 - s_\ell^i) \qquad \forall i \epsilon N, \forall l \epsilon M, \forall k \epsilon M, \forall w \epsilon W_i \tag{13}$$

$$h_{ik}^w = \alpha_{ik}^w + \beta_{ik}^w \qquad \forall w \epsilon W_i, \forall k \epsilon M \tag{14}$$

$$\beta_{ik}^w = V_w r_k^i \qquad \forall i \epsilon N, \forall w \epsilon W_i, \forall k \epsilon M \tag{15}$$

$$r_k^i = \sum_{p \epsilon P} (r_k^p \lambda_{ip}) / Q_i \qquad \forall i \epsilon N, \forall k \epsilon M \tag{16}$$

$$\sum_{p \epsilon p} \phi_{ip} \leq F \qquad \forall i \epsilon N \tag{17}$$

$$\pi_w^p + \pi_u^q \leq 1 \qquad \forall w \epsilon W, \forall u \epsilon \bar{W}(w), \forall p \epsilon P, \forall q \epsilon \bar{P}(p) \tag{18}$$

$$\sum_{i \epsilon N} \omega_t^{ip} \leq 1 \qquad \forall p \epsilon P, \forall t \epsilon H \tag{19}$$

$$\sum_{w \epsilon W} \pi_w^p \leq 1 \qquad \forall p \epsilon P \tag{20}$$

$$\phi_{ip} \leq \pi_w^p \qquad \forall i \epsilon N, \forall w \epsilon W_i, \forall p \epsilon P \tag{21}$$

$$\omega_t^{ip} \geq \phi_{ip} + \theta_{it} - 1 \qquad \forall i \epsilon N, \forall p \epsilon P, \forall t \epsilon H \tag{22}$$

$$\omega_t^{ip} \leq \phi_{ip} \qquad \forall i \epsilon N, \forall p \epsilon P, \forall t \epsilon H \tag{23}$$

$$\omega_t^{ip} \leq \theta_{it} \qquad \forall i \epsilon N, \forall p \epsilon P, \forall t \epsilon H \tag{24}$$

$$\sum_{t \epsilon H} \theta_{it} = c_i \qquad \forall i \epsilon N \tag{25}$$

$$t + B(1 - \theta_{it}) \geq m_i + 1 \qquad \forall i \epsilon N, \forall t \epsilon H \tag{26}$$

$$t \leq m_i + c_i + + B(1 - \theta_{it}) \qquad \forall i \epsilon N, \forall t \epsilon H \tag{27}$$

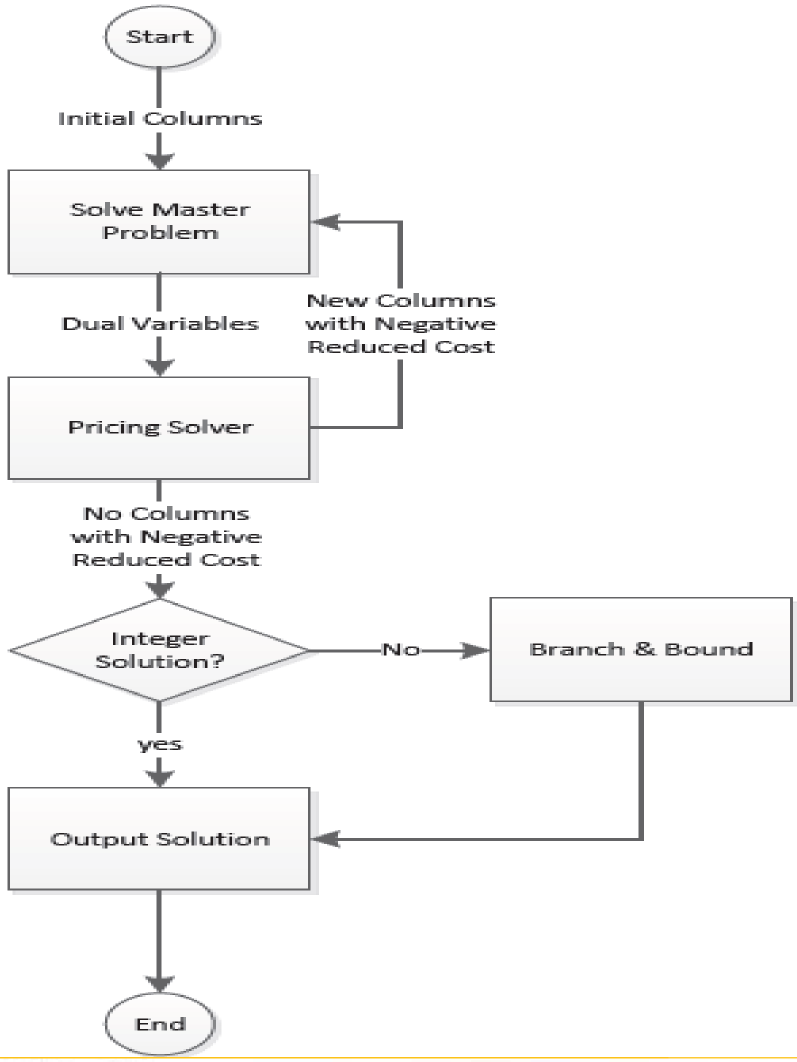
$$Q_i = \sum_{p \epsilon P} \lambda_{ip} \qquad \forall i \epsilon N \tag{28}$$

$$\lambda_{ip} \leq \phi_{ip} Q_i \qquad \forall i \epsilon N, \forall p \epsilon P \tag{29}$$

$$\phi_{ip} \leq \lambda_{ip} \qquad \forall p \epsilon P \tag{30}$$

$$\lambda_{ip} \leq \sum_{w \epsilon W_i} \sum_{t \epsilon H} (R_w \omega_t^{ip} + B(1 - \pi_w^p)) \qquad \forall i \epsilon N, \forall p \epsilon P \tag{31}$$

# Branch and Price Framework




- Initial Solution (using greedy heuristic)
- Column Generation: Lower Bound
- Branch and Bound: Optimal Integer Solution

# Master Problem

- Formulated and solved as a set-partitioning problem
- **Decision Variables**

$\lambda_a$ binary, equals 1 if assignment $a \in \Omega_1$ is part of optimal solution , 0 otherwise;

$\mu_w^l$ binary, equals 1 if cargo type *w* is stored at cargo location *l*, 0 otherwise;

- **Idea**

To obtain berth and yard schedule and assignment of

- Vessel to berth sections
- Vessel to cargo locations
- Cargo types to cargo locations

# Master Problem

- **Input Parameters**

$A_a^i$ binary , equal to 1 if vessel *i* is assigned in assignment *a,* 0 otherwise

$B_a^{kt}$ binary , equal to 1 if section *k* is occupied at time *t* in assignment *a,* 0 otherwise

$C_a^{lw}$ binary , equal to 1 if cargo type *w* is stored at cargo location *l* in assignment *a,* 0 otherwise

$D_a^{lt}$ binary , equal to 1 if cargo location handling assignment *a* at time *t,* 0 otherwise

$N_w$ number of vessels carrying cargo type *w*

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# Master Problem

$$\sum_{a \in \Omega_1} A_a^i \lambda_a = 1 \qquad \forall i \in N,$$

$$\sum_{a \in \Omega_1} B_a^{kt} \lambda_a \le 1 \qquad \forall k \in M, \forall t \in H,$$

$$\sum_{w \in W} \mu_w^l \le 1 \qquad \forall l \in L,$$

$$\sum_{a \in \Omega_1} C_a^{lw} \lambda_a - N_w \mu_w^l \le 0 \qquad \forall l \in L, \forall w \in W,$$

$$\mu_w^l + \mu_{\bar{w}}^{\bar{l}} \le 1 \qquad \forall l \in L, \forall \bar{l} \in \bar{L}, \forall w \in W, \forall \bar{w} \in \bar{W},$$

$$\sum_{a \in \Omega_1} D_a^{lt} \lambda_a \le 1 \qquad \forall l \in L, \forall t \in H,$$

$$\lambda_a \ge 0 \qquad \forall a \in \Omega_1,$$

$$\mu_w^l \ge 0 \qquad \forall l \in L, \forall w \in W$$

# Sub-Problem

- Price out the negative reduced cost columns and add them to the current pool of columns

- Solve /*N*/ sub-problems at every iteration of the column generation, one for each vessel

- Objective Function

$$\min\ z = (m - a + c) - (\alpha + \sum_{k\in M}\sum_{t\in H}\beta_{kt} beta_{kt} + \sum_{k\in M}\sum_{t\in H}\gamma_{lt} gamma_{lt} + \sum_{k\in M}\sum_{t\in H}\delta_{lw} delta_{lw})$$

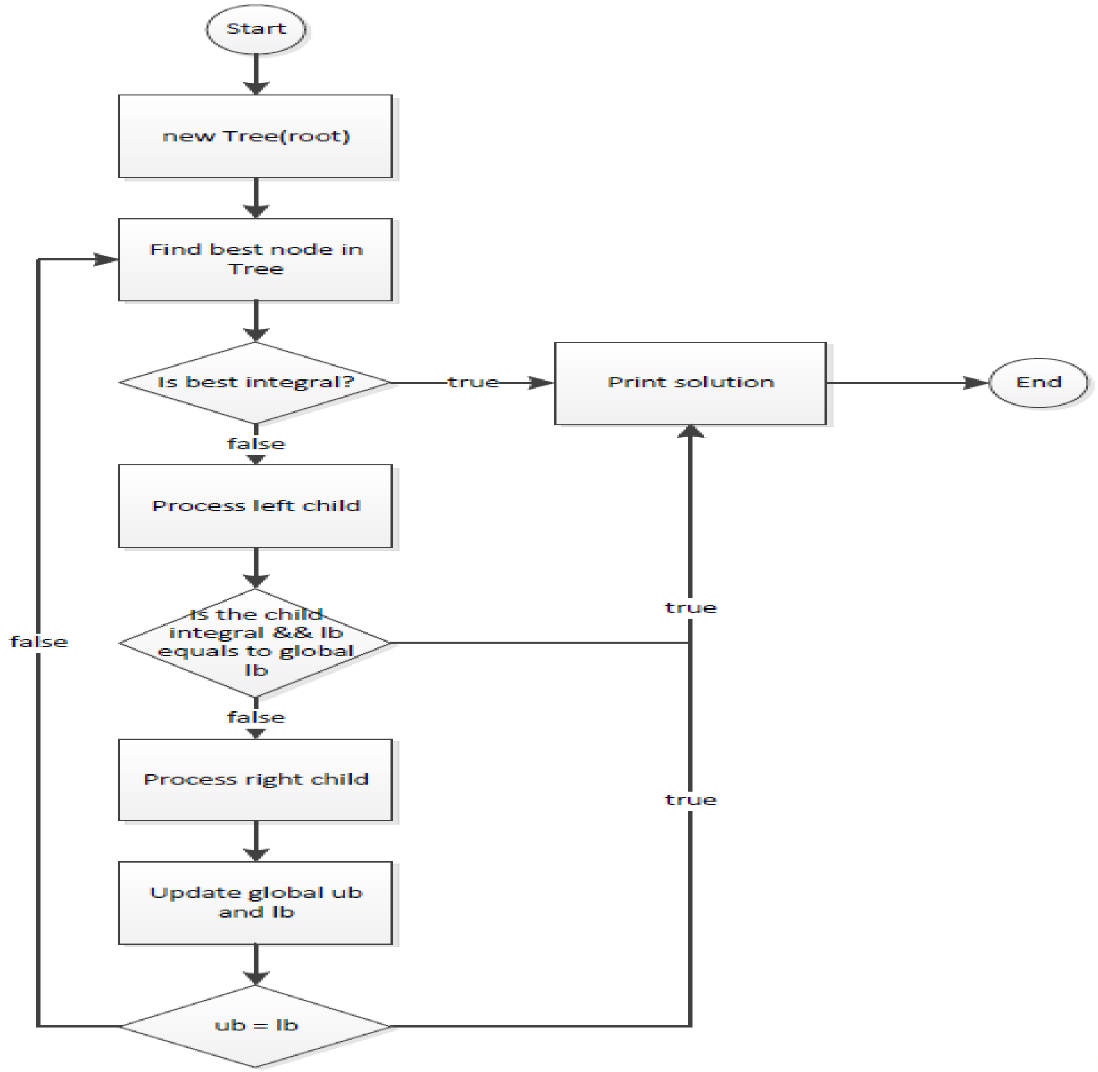
where $\alpha$, $\beta_{kt}$, $\gamma_{lt}$ and $\delta_{lw}$ are the duals associated with the constraints in the restricted master problem.

- Solved as mixed integer linear program using CPLEX solver

# Sub-Problem

$$
\begin{aligned}
s - a &\geq 0, && \\
c &\geq ht_k \cdot fraction_{jk} - M \cdot (1 - ss_j), && \forall j, k \in K, \\
\sum_{j \in K} ss_j &= 1, && \\
\sum_{j \in K} ss_j \cdot sc_j + length &\leq ql, && \\
\sum_{k \in K} o_{jk} \cdot ss_j &= x_j, && \forall j \in K, \\
\sum_{l \in L} split_l &\leq Z, && \\
split_l &\leq delta_{lw}, && \forall l \in L, \\
\sum_{l \in L} cs_l &= quantity, && \\
cs_l &\leq split_l \cdot quantity, && \forall l \in L, \\
split_l &\leq cs_l, && \forall l \in L, \\
td_k &= \left( \sum_{l \in L} d_{kl} \cdot cs_l \right) / quantity, && \forall k \in K, \\
ht_k &= F / cranes_k + V_w \cdot td_k, && \forall k \in K, \\
\sum_{t \in T} time_t &= c, && \\
t + M \cdot (1 - time_t) &\geq s + 1, && \forall t \in T, \\
t &\leq s + c + M \cdot (1 - time_t), && \forall t \in T, \\
beta_{kt} &\geq x_k + time_t - 1, && \forall k \in K, \forall t \in T, \\
beta_{kt} &\leq x_k, && \forall k \in K, \forall t \in T, \\
beta_{kt} &\leq time_t, && \forall k \in K, \forall t \in T, \\
gamma_{lt} &\geq split_l + time_t - 1, && \forall l \in L, \forall t \in T, \\
gamma_{lt} &\leq split_l, && \forall l \in L, \forall t \in T, \\
gamma_{lt} &\leq time_t, && \forall l \in L, \forall t \in T.
\end{aligned}
$$

# Branch and Bound



# Current Status of Work

- Column Generation
  - Tested instances containing up to 10 vessels, solved the LP relaxation solution to optimality, and implemented branch and bound to obtain integer solution
  - Acceleration Techniques
    - Select pool of negative reduced columns instead of single most negative reduced column
    - Dynamic Constraint Aggregation
    - Dual Stability

# Real Time Recovery in Berth Allocation Problem

# Problem Definition: Real time recovery in BAP

- **Objective:** For a given baseline berthing schedule, minimize the total realized costs including the total actual service costs and total cost of rescheduling in space and time

$$\min Z = \sum_{i \in N_u} (m_i - A_i + h_i) + \sum_{i \in N_u} (c_1 \mid b_i(k') - b_i(k) \mid + c_2 \mu_i \mid e'_i - e_i \mid)$$

*$N_u$ : set of unassigned vessels*

*$c_1$ : cost coefficient of shifting berthing location*

*$b_i(k')$ : actual berthing location of vessel i*

*$b_i(k)$ : estimated berthing location of vessel i*

*$c_2$: cost coefficient of departure delay*

*$\mu_i$ : service priority assigned to vessel i*

*$e_i'$ : actual departure time of vessel i*

*$e_i$: estimated departure time of vessel i*

# Motivation

- High level of uncertainty in bulk port operations due to weather conditions, mechanical problems etc.
  - Actual arrival times of vessels can deviate from expected values making the baseline schedule infeasible
  - Disrupt the normal functioning of the port and require quick real time action.
- Very few studies address the problem of real time recovery in port operations, while the problem has not been studied at all in context of bulk ports.
- Our research problem derives from the realistic requirements at the SAQR port, Ras Al Khaimah, UAE

# Research Objectives

- Develop real time algorithms for disruption recovery in berth allocation problem (BAP)

- For a given baseline berthing schedule, minimize the total realized costs of the updated schedule as actual arrival data is revealed. The total realized costs include

  - The total service cost of all vessels berthing at the port which is the sum total of the handling times and berthing delays of all vessels berthing in the planning horizon.

  - Inconsistent cost of rescheduling over space and time to account for the cost of re-allocating human labor, handling equipment and availability of cargo.

# Literature Review

- Very scarce literature on the use of operations research methods in context of bulk ports.
- Comprehensive literature surveys on BAP in container terminals can be found in Steenken et al. (2004), Stahlobock and Voss (2007), Bierwirth and Meisel (2010).
- OR literature related to BAP under uncertainty in container terminals
  - **Pro-active Robustness**
    - Stochastic programming approach used by Zhen et al. (2011), Han et al. (2010)
    - Define surrogate problems to define the stochastic nature of the problem: Moorthy and Teo (2006), Zhen and Chang (2012), Xu et al. (2012) and Hendriks et al. (2010)
  - **Reactive approach or disruption management**
    - Zeng et al.(2012) and Du et al. (2010) propose reactive strategies to minimize the impact of disruptions.

# Baseline Schedule

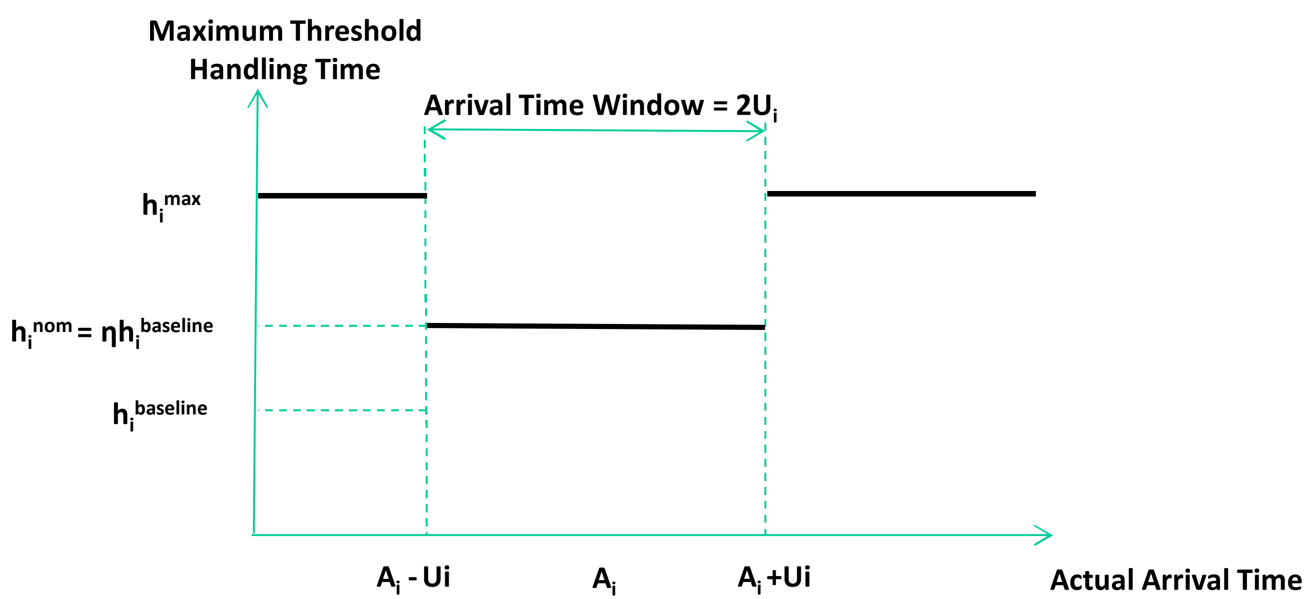
- Any feasible berthing assignment and schedule of vessels along the quay respecting the spatial and temporal constraints on the individual vessels

- Best case: Optimal solution of the deterministic berth allocation problem (without accounting for any uncertainty in arrival information)

# Problem Definition: Real time recovery in BAP

To maximize revenues earned by the port while guaranteeing a minimum level of service, we propose that the bulk terminal managers adopt and implement certain strategic measures

- **Handling Time Restrictions**: Impose an upper bound on the maximum handling time of a vessel $i \in N$ if it arrives within a pre-defined arrival time window $[A_i - U_i, A_i + U_i]$

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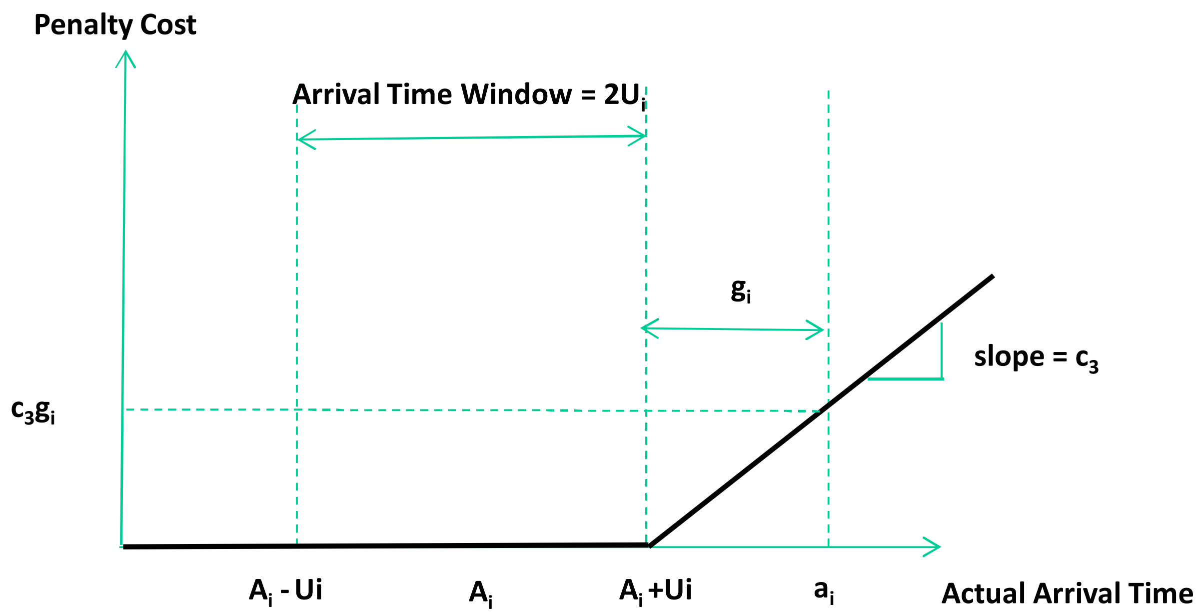
# Problem Definition: Real time recovery in BAP

- **Penalty Cost on late arriving vessels:** Impose a penalty fees on vessels arriving beyond the right end of the arrival window, $A_i+U_i$

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# Problem Definition: Real time recovery in BAP

- **Key Assumptions**
  - **Vessel Priorities:** In practice, if a vessel with higher priority arrives late, it may still be given preference over a vessel with low service priority.
  - **Release of information**: Each incoming vessel updates its exact arrival time a certain fixed time period $\tau$ before its actual arrival time, and once updated it does not change again.
  - **Future vessel arrivals:** At any time instant $t$, the arrival time of an unassigned vessel $i \in N_u$ that is not updated is assumed equal to the expected arrival time $A_i$ if current time $t$ is less than $A_i - \tau$, or otherwise assumed equal to $t + \tau$ . (The handling time restrictions are imposed accordingly.)

# Solution Algorithms

- **Optimization based recovery algorithm**
  - re-optimize the berthing schedule of all unassigned vessels using set-partitioning approach every time the arrival time of any vessel is updated and it deviates from its expected value.
  - the berthing assignment of a vessel determined after its arrival update is frozen and unchangeable
- **Heuristic based recovery algorithm**
  - If a vessel has arrived and current time in the planning horizon is greater than or equal to the estimated berthing time of the vessel (as per baseline schedule), assign it to the section(s) at which the total realized cost of all unassigned vessels at that instant is minimized
  - Assumption : All other unassigned vessels are assigned to the estimated berthing sections as per the baseline schedule

$$\min \; Z = \sum_{i \in N_u} (m_i - A_i + h_i) + \sum_{i \in N_u} \left(c_1 \,|\, b_i(k') - b_i(k) \,|\, + c_2 \mu_i \,|\, e'_i - e_i \,|\right)$$

# Optimization based Recovery Algorithm

**Require**: Baseline schedule of set $N$ of vessels, set $M$ of sections

    Initialize set $N_u$ of unassigned vessels to $N$
    Initialize boolean array arrivalUpdated of size $N$ = false *forall* $i \in N$
    Initialize counter = 0
    **while** $|N_u| > 0$ and counter $\leq |H|$ **do**
        Initialize boolean shouldOptimize = false
        **for** $i = 1$ to $N$ **do**
            **if** arrivalUpdated[$i$] = false and counter $\geq a_i - \tau$ and $a_i \neq A_i$ **then**
                Set arrivalUpdated[$i$] = true
                Set $A_i = a_i$
                Set shouldOptimize = true
            **end if**
        **end for**
        **if** shouldOptimize **then**
            Re-optimize *forall* $i \in N_u$
        **end if**
        **for** i = 1 to $N_u$ **do**
            **if** counter = latest updated start time $m'_i$ **then**
                Assign vessel $i$ to latest updated location $b_i$(k')
                Set $N_u$ to $N_u - \{i\}$
            **end if**
        **end for**
        counter++
    **end while**

# Heuristic based Recovery Algorithm

**Require**: Baseline schedule of set $N$ of vessels, set $M$ of sections

```
Initialize set N_u of unassigned vessels to N
Initialize boolean array arrivalUpdated of size N = false  forall i ϵ N
Initialize counter = 0
while | N_u | > 0 and counter ≤ |H| do
    for berthing Schedule: b do
        if b.hasArrived AND !b.isAssigned then
            Set boolean foundSection = false
            for k = 1 to M do
                if isStartSectionAvailable(b.vessel,k) then
                    foundSection = true;
                    break;
                end if
            end for
            if foundSection AND counter ≥ b.estimatedBerthingTime then
                Scan the entire quay and assign the vessel to the set of sections with minimum total
                cost forall i ϵ N_u
            end if
        end if
    end for
    counter++
end while
```

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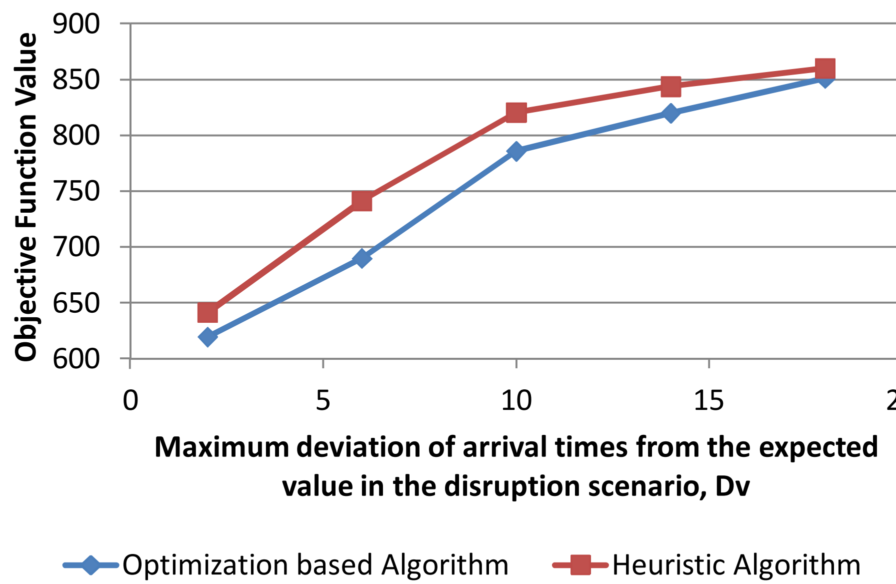
# Preliminary Results

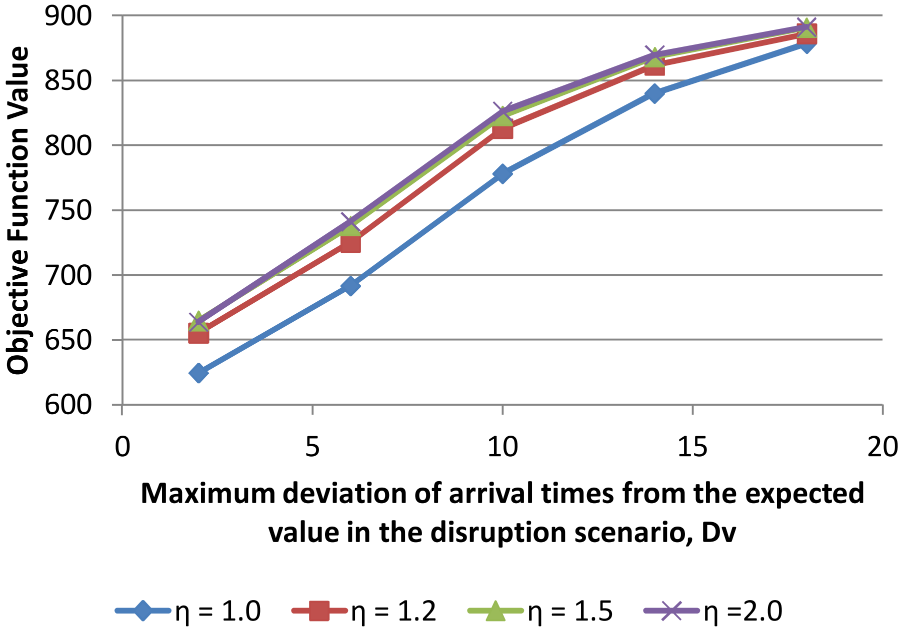
- $|N|$=25 vessels, $|M|$= 10 sections, $c_1$ = 1.0, $c_2$ = 0.02, $U_i$ = 8 hours, $\tau$ = 5 hours, $\eta$ = 1.2

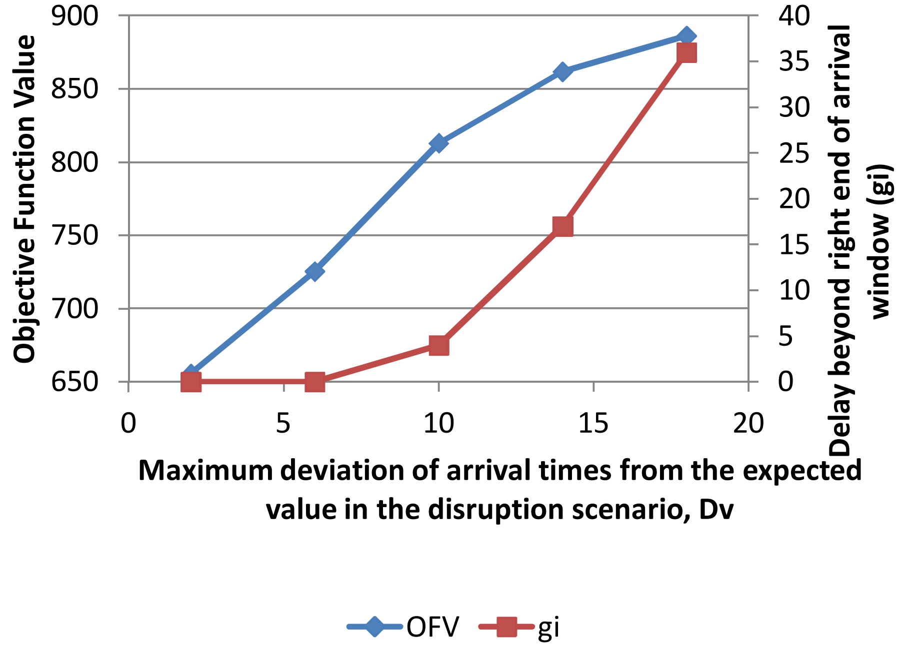
| $D_v$ | Optimization based algorithm | | Heuristic Algorithm |
|---|---|---|---|
| | Realized cost | Time (seconds) | Realized Cost |
| 0 | 534.0 | 0.1 | 534.0 |
| 2 | 619.6 | 148.0 | 641.5 |
| 6 | 689.9 | 159.3 | 741.3 |
| 10 | 786.0 | 158.7 | 820.7 |
| 14 | 820.0 | 214.5 | 843.8 |
| 18 | 851.2 | 181.6 | 860.2 |




- Results averaged over 10 arrival disruption scenarios
- Optimization based algorithm outperforms the heuristic based approach, but computationally much more expensive

# Preliminary Results




- Results averaged over 100 arrival scenarios for every instance
- Higher values of $\eta$ do not significantly increase the total realized costs of the berthing schedule for different delay scenarios
- Scope to earn more revenue from the late arriving vessels for arrival beyond the permissible arrival window of the vessels

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# Summary of Results

- Modeled and solved the dynamic, hybrid berth allocation problem in bulk ports
- Addressed the problem of recovering a baseline berthing schedule in bulk ports in real time as actual arrival data is revealed.
- Discussed strategies that the port can adopt and implement to maximize their revenues while ensuring a desired level of service
- Developed solution algorithms to solve the BAP in real time in bulk ports with the objective to minimize the total realized costs of the updated schedule.
- Conducted simple numerical experiments to validate the efficiency of the algorithms. Optimization based approach outperforms the heuristic approach, but is computationally much more expensive.

# Ongoing and Future Work

- More extensive numerical analysis to study the impact of
  - parameter values related to rescheduling of vessels including cost of shifting the vessel along the quay and cost of departure delay of a vessel
  - bounds on the maximum handling times for vessels arriving within the prescribed arrival window.
  - penalty cost function dependent on the late arriving vessels for arrival delay beyond the prescribed arrival window of the vessel

- Develop a robust formulation of the berth allocation problem in bulk ports with a certain degree of anticipation of variability in information.

# Thank you!

# Results and Analysis

## |*N*| = 10 vessels, and |*M*| = 10 sections

| Instance | MILP | | | GSPP | | FCFS | | SWO | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OFV | Gap | Time | OFV | Time (H=150, h=1) | OFV | RE | OFV | RE | Time |
| **A1** | 230.21 | 0.01% | 67.67 | 231.21 | 5.94 | 262.09 | 13.36% | 230.48 | -0.32% | 15.81 |
| **A2** | 237.35 | 0.01% | 15.31 | 238.49 | 5.54 | 250.44 | 5.01% | 239.08 | 0.25% | 16.66 |
| **A3** | 223.99 | 0.01% | 9.58 | 226.61 | 5.96 | 280.04 | 23.58% | 225.33 | -0.57% | 16.97 |
| **A4** | 227.12 | 0.01% | 10.31 | 227.22 | 5.68 | 240.91 | 6.03% | 228.00 | 0.35% | 16.60 |
| **A5** | 234.20 | 0.01% | 5.60 | 234.22 | 5.43 | 251.09 | 7.20% | 234.47 | 0.11% | 16.30 |
| **A6** | 233.12 | 0.01% | 11.06 | 234.06 | 6.85 | 262.61 | 12.20% | 233.12 | -0.40% | 16.90 |
| **A7** | 203.23 | 0.00% | 0.56 | 203.23 | 4.99 | 208.44 | 2.57% | 203.38 | 0.07% | 15.95 |
| **A8** | 218.87 | 0.00% | 0.56 | 219.99 | 5.29 | 220.90 | 0.41% | 218.87 | -0.51% | 16.72 |
| **A9** | 198.03 | 0.00% | 0.60 | 199.89 | 5.16 | 214.17 | 7.14% | 198.03 | -0.93% | 17.53 |
| **Mean** | | | | | | | **8.61%** | | **-0.22%** | |

# $|N|$ = 10 vessels, and $|M|$ = 30 sections

| Instance | MILP | | | GSPP | | FCFS | | SWO | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OFV | Gap | Time | OFV | Time (H=150, h=1) | OFV | RE | OFV | RE | Time |
| **B1** | 188.39 | 0.01% | 15.80 | 189.73 | 94.552 | 216.56 | 14.14% | 192.81 | 1.62% | 49.95 |
| **B2** | 178.07 | 0.01% | 15.78 | 179.10 | 86.08 | 186.41 | 4.08% | 178.08 | -0.57% | 48.42 |
| **B3** | 200.16 | 0.01% | 1094.61 | 202.33 | 101.93 | 230.14 | 13.74% | 216.14 | 6.82% | 49.79 |
| **B4** | 182.57 | 0.01% | 3.04 | 184.27 | 89.58 | 224.00 | 21.56% | 182.80 | -0.80% | 47.73 |
| **B5** | 178.48 | 0.01% | 10.97 | 179.23 | 85.01 | 185.60 | 3.55% | 179.01 | -0.12% | 48.37 |
| **B6** | 199.82 | 0.01% | 87.78 | 201.17 | 96.19 | 240.09 | 19.35% | 223.37 | 11.04% | 48.83 |
| **B7** | 173.02 | 0.01% | 1.30 | 173.02 | 86.00 | 175.72 | 1.56% | 175.30 | 1.32% | 48.61 |
| **B8** | 162.51 | 0.00% | 1.57 | 162.81 | 81.67 | 169.20 | 3.92% | 166.20 | 2.08% | 50.29 |
| **B9** | 175.29 | 0.00% | 1.39 | 175.81 | 95.74 | 192.41 | 9.44% | 191.26 | 8.79% | 50.27 |
| **Mean** | | | | | | | **10.15%** | | **3.35%** | |

## $|N|$ = 25 vessels, and $|M|$ = 10 sections

| Instance | MILP | | | GSPP | | FCFS | | SWO | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OFV | Gap | Time | OFV | Time (H=150, h=1) | OFV | RE | OFV | RE | Time |
| **C1** | 812.32 | 33.08% | - | 819.22 | 14.09 | 976.49 | 19.20% | 869.31 | 6.11% | 22.29 |
| **C2** | 783.45 | 30.27% | - | 781.72 | 11.70 | 924.35 | 18.25% | 825.92 | 5.65% | 22.58 |
| **C3** | 903.51 | 33.39% | - | 900.43 | 20.19 | 1107.59 | 23.01% | 929.32 | 3.21% | 22.24 |
| **C4** | 795.71 | 23.18% | - | 791.18 | 15.26 | 877.90 | 10.96% | 852.03 | 7.69% | 23.28 |
| **C5** | 751.19 | 27.47% | - | 747.88 | 10.41 | 846.26 | 13.15% | 774.17 | 3.51% | 22.16 |
| **C6** | 874.53 | 24.50% | - | 863.86 | 19.38 | 979.26 | 13.36% | 898.44 | 4.00% | 23.19 |
| **C7** | 735.13 | 19.72% | - | 741.16 | 15.91 | 840.85 | 13.45% | 806.23 | 8.78% | 22.54 |
| **C8** | 689.37 | 22.26% | - | 699.14 | 11.23 | 761.48 | 8.92% | 735.46 | 5.20% | 22.32 |
| **C9** | 800.00 | 22.76% | - | 793.24 | 12.82 | 936.55 | 18.07% | 872.76 | 10.03% | 23.56 |
| **Mean** | | | | | | | **15.37%** | | **6.02%** | |

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# $|N|$ = 25 vessels, and $|M|$ = 30 sections

| Instance | MILP | | | GSPP | | FCFS | | SWO | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OFV | Gap | Time | OFV | Time (H=150, h=1) | OFV | RE | OFV | RE | Time |
| **D1** | 690.79 | 23.14% | - | 670.42 | 219.04 | 857.99 | 27.98% | 785.91 | 17.23% | 105.36 |
| **D2** | 617.31 | 34.23% | - | 591.06 | 185.44 | 723.13 | 22.34% | 667.20 | 12.88% | 96.03 |
| **D3** | 809.55 | 30.75% | - | 784.94 | 387.40 | 984.28 | 25.40% | 866.82 | 10.43% | 105.66 |
| **D4** | 657.48 | 20.62% | - | 636.19 | 220.40 | 778.50 | 22.37% | 728.60 | 14.53% | 100.95 |
| **D5** | 560.65 | 25.96% | - | 556.37 | 172.09 | 622.14 | 11.82% | 614.72 | 10.49% | 91.35 |
| **D6** | 754.87 | 21.97% | - | 739.44 | 253.67 | 909.53 | 23.00% | 836.23 | 13.09% | 102.34 |
| **D7** | 581.54 | 8.36% | - | 590.24 | 194.80 | 731.55 | 23.94% | 706.82 | 19.75% | 100.82 |
| **D8** | 510.80 | 16.20% | - | 506.30 | 167.09 | 565.87 | 11.77% | 565.87 | 11.77% | 111.46 |
| **D9** | 704.76 | 19.18% | - | 677.97 | 200.63 | 848.97 | 25.22% | 778.67 | 14.85% | 99.87 |
| **Mean** | | | | | | | **21.54%** | | **13.89%** | |

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## $|N|$ = 40 vessels, and $|M|$ = 10 sections

| Instance | MILP | | | GSPP | | FCFS | | SWO | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OFV | Gap | Time | OFV | Time (H=150, h=1) | OFV | RE | OFV | RE | Time |
| **E1** | 1243.64 | 63.77% | - | 1140.60 | 41.73 | 1536.78 | 34.73% | 1289.88 | 13.09% | 28.24 |
| **E2** | 1193.05 | 59.69% | - | 1138.16 | 18.80 | 1571.07 | 38.04% | 1273.09 | 11.86% | 28.44 |
| **E3** | 1341.65 | 67.35% | - | 1249.06 | 139.47 | 1878.78 | 50.42% | 1416.54 | 13.41% | 31.50 |
| **E4** | 1113.36 | 59.53% | - | 1051.50 | 30.87 | 1408.95 | 33.99% | 1137.20 | 8.15% | 30.11 |
| **E5** | 1105.34 | 56.98% | - | 1063.85 | 19.06 | 1447.39 | 36.05% | 1202.50 | 13.03% | 32.06 |
| **E6** | 1361.62 | 68.15% | - | 1160.05 | 167.58 | 1903.39 | 64.08% | 1330.64 | 14.71% | 34.01 |
| **E7** | 1011.20 | 55.47% | - | 946.35 | 26.04 | 1291.11 | 36.43% | 1148.14 | 21.32% | 31.69 |
| **E8** | 1013.41 | 53.02% | - | 953.24 | 20.03 | 1183.57 | 24.16% | 1094.15 | 14.78% | 32.01 |
| **E9** | 1181.97 | 64.95% | - | 1071.46 | 94.88 | 1500.71 | 40.06% | 1296.79 | 21.03% | 35.64 |
| **Mean** | | | | | | | **39.77%** | | **14.60%** | |

## $|N|$ = 40 vessels, and $|M|$ = 30 sections

| Instance | MILP | | | GSPP | | FCFS | | SWO | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | OFV | Gap | Time | OFV | Time (H=150, h=2) | OFV | RE | OFV | RE | Time |
| **F1** | 1193.42 | 70.56% | - | 920.73 | 506.02 | 1278.04 | 38.81% | 1092.44 | 18.65% | 169.53 |
| **F2** | 913.59 | 62.66% | - | 863.43 | 127.91 | 1168.60 | 35.34% | 911.47 | 5.56% | 159.28 |
| **F3** | + | + | + | 1089.48 | 932.56 | 1784.98 | 63.84% | 1411.24 | 29.53% | 173.89 |
| **F4** | 902.74 | 59.15% | - | 856.41 | 3341.62 | 1143.59 | 33.53% | 1035.72 | 20.94% | 160.113 |
| **F5** | 881.37 | 61.20% | - | 786.27 | 137.91 | 973.94 | 23.87% | 857.47 | 9.06% | 163.41 |
| **F6** | 1121.14 | 66.39% | - | 1015.53 | 2281.78 | 1628.76 | 60.39% | 1286.73 | 26.71% | 265.29 |
| **F7** | 922.04 | 62.05% | - | 777.06 | 829.88 | 932.34 | 19.98% | 932.34 | 19.98% | 166.52 |
| **F8** | 728.48 | 52.93% | - | 679.58 | 131.81 | 774.12 | 13.91% | 745.00 | 9.63% | 160.49 |
| **F9** | 934.35 | 58.59% | - | 920.29 | 1767.57 | 1458.45 | 58.48% | 1214.66 | 31.99% | 171.97 |
| **Mean** | | | | | | | **38.68%** | | **19.12%** | |

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# Disruption: Arrival Delay Scenario

| Vessel | EAT | AAT |
|---|---|---|
| 0 | 3 | -1 |
| 1 | 3 | -3 |
| 2 | 3 | 1 |
| 3 | 2 | 4 |
| 4 | 2 | -2 |
| 5 | 2 | 4 |
| 6 | 2 | 4 |
| 7 | 1 | 0 |
| 8 | 4 | 12 |
| 9 | 2 | 1 |
| 10 | 2 | 4 |
| 11 | 0 | 4 |
| 12 | 1 | -2 |
| 13 | 0 | -2 |
| 14 | 1 | -8 |
| 15 | 4 | -4 |
| 16 | 4 | -2 |
| 17 | 5 | 1 |
| 18 | 1 | 2 |
| 19 | 0 | 9 |
| 20 | 3 | 9 |
| 21 | 3 | 5 |
| 22 | 5 | 1 |
| 23 | 0 | -2 |
| 24 | 1 | 10 |

# Master Problem

$$\min \sum_{a \in \Omega_1} c_a \lambda_a$$

$$\sum_{a \in \Omega_1} A_a^i \lambda_a = 1 \qquad \forall i \in N,$$

$$\sum_{a \in \Omega_1} B_a^{kt} \lambda_a \le 1 \qquad \forall k \in M, \forall t \in H,$$

$$\sum_{a \in \Omega_1} C_a^{lw} \lambda_a - N_w \mu_w^l \le 0 \qquad \forall l \in L, \forall w \in W,$$

$$\sum_{w \in W} \mu_w^l \le 1 \qquad \forall l \in L,$$

$$\mu_w^l + \mu_{\overline{w}}^{\overline{l}} \le 1 \qquad \forall l \in L, \forall \overline{l} \in \overline{L}, \forall w \in W, \forall \overline{w} \in \overline{W},$$

$$\sum_{a \in \Omega_1} D_a^{lt} \lambda_a \le 1 \qquad \forall l \in L, \forall t \in H,$$

$$\lambda_a \ge 0 \qquad \forall a \in \Omega_1,$$

$$\mu_w^l \ge 0 \qquad \forall l \in L, \forall w \in W$$

# Sub-Problem

$$
\begin{aligned}
s - a &\geq 0, \\
c \geq ht_k \cdot fraction_{jk} - M \cdot (1 - ss_j)\,, &\quad \forall j, k \in K, \\
\sum_{j \in K} ss_j &= 1, \\
\sum_{j \in K} ss_j \cdot sc_j + length &\leq ql, \\
\sum_{k \in K} o_{jk} \cdot ss_j &= x_j, &\quad \forall j \in K, \\
\sum_{l \in L} split_l &\leq Z, \\
split_l &\leq delta_{lw}, &\quad \forall l \in L, \\
\sum_{l \in L} cs_l &= quantity, \\
cs_l &\leq split_l \cdot quantity, &\quad \forall l \in L, \\
split_l &\leq cs_l, &\quad \forall l \in L, \\
td_k &= \left( \sum_{l \in L} d_{kl} \cdot cs_l \right) / quantity, &\quad \forall k \in K, \\
ht_k &= F / cranes_k + V_w \cdot td_k, &\quad \forall k \in K, \\
\sum_{t \in T} time_t &= c, \\
t + M \cdot (1 - time_t) &\geq s + 1, &\quad \forall t \in T, \\
t &\leq s + c + M \cdot (1 - time_t)\,, &\quad \forall t \in T, \\
beta_{kt} &\geq x_k + time_t - 1, &\quad \forall k \in K, \forall t \in T, \\
beta_{kt} &\leq x_k, &\quad \forall k \in K, \forall t \in T, \\
beta_{kt} &\leq time_t, &\quad \forall k \in K, \forall t \in T, \\
gamma_{lt} &\geq split_l + time_t - 1, &\quad \forall l \in L, \forall t \in T, \\
gamma_{lt} &\leq split_l, &\quad \forall l \in L, \forall t \in T, \\
gamma_{lt} &\leq time_t, &\quad \forall l \in L, \forall t \in T.
\end{aligned}
$$