Models and algorithms for integrated schedule planning and revenue management

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Motivation

- Demand responsive transportation systems
  - Better representation of demand ⇒ Appropriate demand models
  - Flexibility in supply ⇒ New concept: Clip-Air
  - Integration of supply-demand interactions in transportation models

Itinerary choice model DCA

- Market segments, s, defined by the class and each OD pair
- Itinerary choice among the set of alternatives, I_s, for each segment s
- For each itinerary $i \in I_s$ the utility is defined by:

$$\mathrm{V}_i = \mathrm{ASC}_i + \beta_p \cdot \ln(p_i) + \beta_{time} \cdot \mathrm{time}_i + \beta_{morning} \cdot \mathrm{morning}_i$$
$$\mathrm{V}_i = \mathrm{V}_i(p_i, z_i, \beta)$$

  - ASC_i : alternative specific constant
  - p is a policy variable and included as log
  - p and time are interacted with non-stop/stop
  - $\mathrm{morning}$ is 1 if the itinerary is a morning itinerary
- *No-revenue* represented by the subset $I_s' \in I_s$ for segment s.

Itinerary choice model

- Demand for class h for each itinerary i in market segment s:

$$\tilde{d}_i = D_s \frac{\exp\left(V_i(p_i, z_i, \beta)\right)}{\sum_{j \in I_s} \exp\left(V_j(p_j, z_j, \beta)\right)}$$

  - D_s is the total expected demand for market segment s.

- **Spill and recapture effects**: Capacity shortage $\Rightarrow$ passengers may be recaptured by other itineraries (instead of their desired itineraries)
- Recapture ratio is given by:

$$b_{i,j} = \frac{\exp\left(V_j(p_j, z_j, \beta)\right)}{\sum_{k \in I_s \setminus \{i\}} \exp\left(V_k(p_k, z_k, \beta)\right)}$$

Estimation

- **Revealed preferences (RP) data:** Booking data from a major European airline
  - Lack of variability
  - Price inelastic demand
- RP data is combined with a **stated preferences (SP) data**
- Time, cost and morning parameters are **fixed** to be the same for the two datasets.
- A **scale** parameter is introduced for SP to capture the differences in variance.

Estimation results

	β_{fare}		β_{time}		
	non-stop	one-stop	non-stop	one-stop	$\beta_{morning}$
economy	-2.23	-2.17	-0.102	-0.0762	0.0283
business	-1.97	-1.97	-0.104	-0.0821	0.079

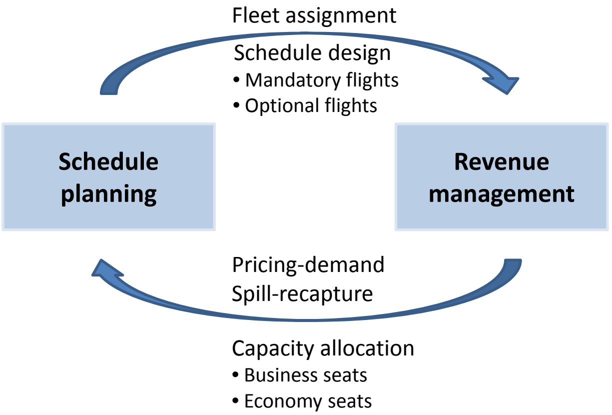
- **Price elasticity** of demand:

$$E^{P_i}_{price_i} = \frac{\partial P_i}{\partial price_i} \cdot \frac{price_i}{P_i}$$

An example

- for a non-stop itinerary
  - price elasticity for economy is -2.03 and -1.86 for business
- for a one-stop itinerary
  - price elasticity for economy is -2.14 and -1.95 for business

Integrated schedule planning and revenue management



Integrated model - Schedule planning

$$Max \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} (d_i - \sum_{j \in I_s} t_{i,j} + \sum_{j \in (I_s \setminus I'_s)} t_{j,i} b_{j,i}) p_i - \sum_{\substack{k \in K \\ f \in F}} C_{k,f} x_{k,f} : \text{ revenue - cost} \quad (1)$$

$$s.t. \sum_{k \in K} x_{k,f} = 1 : \text{ mandatory flights} \qquad \forall f \in F^M \quad (2)$$

$$\sum_{k \in K} x_{k,f} \leq 1 : \text{ optional flights} \qquad \forall f \in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f \in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in Out(k,a,t)} x_{k,f} : \text{ flow conservation} \qquad \forall [k,a,t] \in N \quad (4)$$

$$\sum_{a \in A} y_{k,a,minE_a^-} + \sum_{f \in CT} x_{k,f} \leq R_k : \text{ fleet availability} \qquad \forall k \in K \quad (5)$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+} : \text{ cyclic schedule} \qquad \forall k \in K, a \in A \quad (6)$$

$$\sum_{h \in H} \pi^h_{k,f} = Q_k x_{k,f} : \text{ seat capacity} \qquad \forall f \in F, k \in K \quad (7)$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \quad (8)$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \quad (9)$$

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Integrated model - Revenue management

$$\sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s^{'})} \delta_{i,f} d_i - \sum_{j \in I_s} \delta_{i,f} t_{i,j} + \sum_{\substack{j \in (I_s \setminus I_s^{'}) \\ i \neq j}} \delta_{i,f} t_{j,i} b_{j,i} \leq \sum_{k \in K} \pi_{k,f} : \text{ capacity} \qquad \forall h \in H, f \in F \qquad (10)$$

$$\sum_{\substack{j \in I_s \\ i \neq j}} t_{i,j} \leq d_i : \text{ total spill} \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}) \qquad (11)$$

$$\tilde{d}_i = D_s \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))} : \text{ logit demand} \qquad \forall h \in H, s \in S^h, i \in I_s \qquad (12)$$

$$b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))} : \text{ recapture ratio} \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}), j \in I_s \qquad (13)$$

$$d_i \leq \tilde{d}_i : \text{ realized demand} \qquad \forall h \in H, s \in S^h, i \in I_s \qquad (14)$$

$$0 \leq p_i \leq UB_i : \text{ upper bound on price} \qquad \forall h \in H, s \in S^h, i \in I_s \qquad (15)$$

$$t_{i,j} \geq 0 \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}), j \in I_s \qquad (16)$$

$$b_{i,j} \geq 0 \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}), j \in I_s \qquad (17)$$

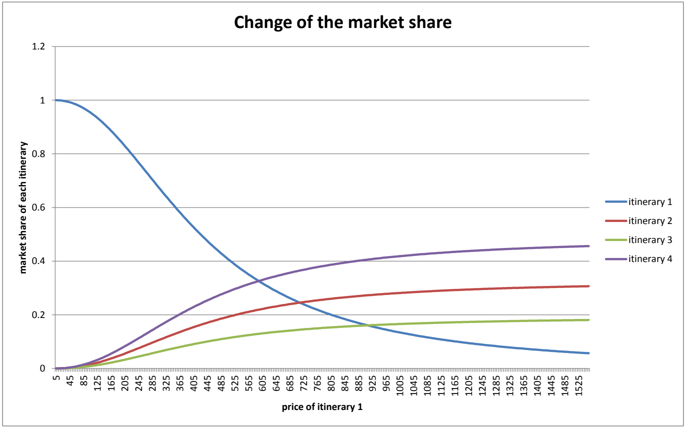
$$\pi^h_{k,f} \geq 0 \qquad \forall h \in H, k \in K, f \in F \qquad (18)$$

Integrated model

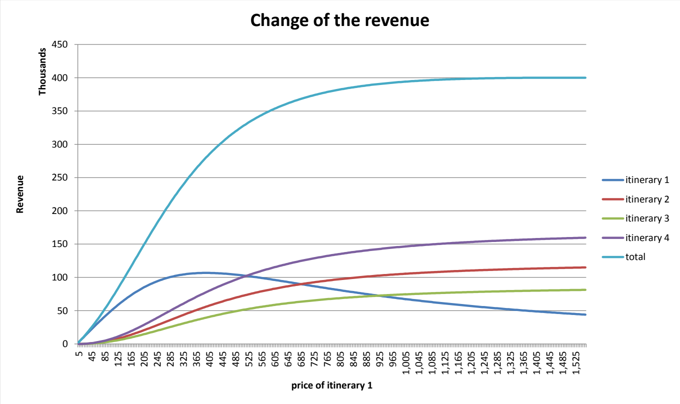
- We consider reference models to evaluate the integrated model
  - **Price-inleastic schedule planning**: M. Lohatepanont and C. Barnhart (2004)
  - **Sequential approach**: Revenue management considers fixed supply capacity

- The resulting model is a mixed integer nonlinear problem
- Nonlinearity is due to the explicit supply-demand interactions
- The model is implemented in AMPL and BONMIN solver is used
- BONMIN does not guarantee optimality

Illustration



Illustration



Impact of the integrated model

Number of airports:	3
Number of flights:	26
Average demand:	56.12 passengers per flight
Cabin classes:	Economy and business
Level of service:	All itineraries are nonstop
Available fleet:	3 types of aircraft (100, 50 and 37 seats)

	Price-inelastic schedule planning model	**Integrated model - limited prices**	**Integrated model**
Revenue	204,553	214,380	244,924
Operating costs	150,603	160,003	173,349
Profit	**53,949**	**54,377 (+ 0.8%)**	**71,575 (+ 32.7%)**
Number of flights	**22**	**22**	**24**
Transported passengers	**943**	**1031 (+ 9.3%)**	**1064 (+ 12.7%)**
Economy-Business	882 E - 61 B	970 E - 61 B	997 E - 67 B
Allocated seats	**274**	**324**	**324**

Sequential versus integrated

	Sequential approach				Integrated model - % Improvement			
No	Profit	Pax.	Flights	Seats	Profit	Pax.	Flights	Seats
1	15,091	284	8	124	-	-	-	-
2	35,372	400	8	**150**	**5.55%**	**33.50%**	8	**217**
3	50,149	859	10	300	-	-	-	-
4	69,901	931	**22**	**274**	**1.43%**	**14.18%**	**24**	**324**
5	82,311	1145	16	333	-	-	-	-
6	904,054	1448	10	**1148**	**0.30%**	-	10	**1312**
7	135,656	1814	32	498	-	-	-	-
8	115,983	2236	26	691	-	-	-	-
9	854,902	1270	10	**1016**	**0.43%**	**5.83%**	10	**1090**
10	137,428	1517	34	**391**	**0.83%**	**4.94%**	34	**476**
11	93,347	1144	20	**387**	**3.36%**	**1.40%**	20	**457**
12	49,448	1050	12	370	-	-	-	-
13	27,076	448	10	207	-	-	-	-
14	52,369	599	**10**	267	**1.45%**	**16.69%**	**12**	267
15	26,486	504	6	185	-	-	-	-

Heuristic method

- We are limited in terms of the computational time
- A heuristic based on two simplified versions of the model:
  - FAM^{LS}: price-inelastic schedule planning model ⇒ MILP
    - Explores new fleet assignment solutions based on a local search
    - Price sampling
    - Variable neighborhood search
  - REV^{LS}: Revenue management with fixed capacity ⇒ NLP
    - Optimizes the revenue for the explored fleet assignment solution

Heuristic method

Require: $\bar{x}_0$, $\bar{y}_0$, $\bar{d}_0$, $\bar{p}_0$, $\bar{t}_0$, $\bar{b}_0$, $\bar{\pi}_0$, z^*, z_{opt}, k_{max}, ε, n_{min}, n_{max}

  $k := 0$, $n_{fixed} := n_{min}$

  repeat

    $\bar{p}_k$:= Price sampling

    $\{\bar{d}_k, \bar{b}_k\}$:= Demand model($\bar{p}_k$)

    $\{\bar{x}_k, \bar{y}_k, \bar{\pi}_k, \bar{t}_k\}$:= solve $z_{\mathrm{FAM}^{\mathrm{LS}}(\bar{d}_k, \bar{b}_k, n_{fixed})}$

    $\{\bar{p}_k, \bar{d}_k, \bar{b}_k, \bar{\pi}_k, \bar{t}_k\}$:= solve $z_{\mathrm{REV}^{\mathrm{LS}}(\bar{x}_k, \bar{y}_k)}$

    if improvement($z_{\mathrm{REV}^{\mathrm{LS}}}$) **then**

      Update z^*

      Intensification: $n_{fixed} := n_{fixed} + 1$ when $n_{fixed} < n_{max}$

    else

      Diversification: $n_{fixed} := n_{fixed} - 1$ when $n_{fixed} > n_{min}$

    end if

    $k := k + 1$

  until $||z_{opt} - z^*||^2 \leq \varepsilon$ **or** $k \geq k_{max}$

Performance of the heuristic

The omitted instances are the ones where the sequential approach has the same solution as the integrated model.

		Best solution reported by Bonmin		**Sequential approach**		**Heuristic results** Average over 5 replications			
	Flights	Profit	Time (sec)	Profit	% dev.	Profit	%dev.	Time (sec)	Time red.
2	11	37,335	27	35,372	5.26%	37,335	0.00%	13	53.33%
4	26	70,904	2,479	69,901	1.41%	70,679	0.32%	6	99.75%
6	12	906,791	12,964	904,054	0.30%	906,791	0.00%	2	99.98%
9	11	858,544	7,343	854,902	0.42%	858,545	0.00%	1	99.99%
10	39	138,575	37,177	137,428	0.83%	138,575	0.00%	173	99.54%
11	23	96,486	17,142	93,347	3.25%	96,486	0.00%	89	99.48%
14	14	53,128	141	52,369	1.43%	53,128	0.00%	1	99.53%
16	77	194,598	42,360	208,561	-7.18%	210,395	-8.12%	791	98.13%
17	61	227,364	22,174	226,615	0.33%	227,284	0.04%	1283	94.21%
18	48	153,789	4,387	163,114	-6.06%	163,393	-6.24%	126.4	97.12%
			max 43200					*max 3600*	

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Conclusions and future work

- Heuristic
  - Inclusion of larger instances
- Further solution methods for the resulting mixed integer nonlinear problem
  - Convex approximation of the nonlinearity
  - Decomposition methods $\Rightarrow$ FAM and REV models
  - Subgradient optimization

Thank you for your attention!

Discrete choice analysis

- Finite and discrete set of alternatives
  - Choice of transportation mode: car, bus, etc.
  - Choice of brand: Leonidas, Lindt, Suchard, Toblerone, etc.
  - Choice of flight: GVA-NCE 10:00, GVA-NCE 06:30, etc.
- Individual n associates a utility to alternative i
- Represented by a random function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}$$

Discrete choice analysis

▸ Choice Model

- Individual n chooses alternative i if $U_{in} \geq U_{jn}$, for all j.
- Utility is random, so we have a probabilistic model

$$P_n(i|C_n) = Pr(U_{in} \geq U_{jn}) = Pr(V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn})$$

- Concrete models require
  - specification of V_{in}
  - assumptions about ε_{in}
  - estimation of the parameters from data