Models and algorithms for integrated schedule planning and revenue management

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Motivation

- Demand responsive transportation systems
  - Better representation of demand ⇒ Appropriate demand models
  - Flexibility in supply ⇒ New concept: Clip-Air
  - Integration of supply-demand interactions in transportation models
Itinerary choice model

- Market segments, $s$, defined by the class and each OD pair
- Itinerary choice among the set of alternatives, $I_s$, for each segment $s$
- For each itinerary $i \in I_s$ the utility is defined by:

$$V_i = ASC_i + \beta_p \cdot \ln(p_i) + \beta_{\text{time}} \cdot \text{time}_i + \beta_{\text{morning}} \cdot \text{morning}_i$$

$$V_i = V_i(p_i, z_i, \beta)$$

- $ASC_i$: alternative specific constant
- $p$ is a policy variable and included as log
- $p$ and time are interacted with non-stop/stop
- morning is 1 if the itinerary is a morning itinerary

- No-revenue represented by the subset $I'_s \subseteq I_s$ for segment $s$. 


Itinerary choice model

- Demand for class \( h \) for each itinerary \( i \) in market segment \( s \):
  \[
  \tilde{d}_i = D_s \frac{\exp (V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp (V_j(p_j, z_j, \beta))}
  \]
  - \( D_s \) is the total expected demand for market segment \( s \).

- **Spill and recapture effects**: Capacity shortage \( \Rightarrow \) passengers may be recaptured by other itineraries (instead of their desired itineraries)
  - Recapture ratio is given by:
    \[
    b_{i,j} = \frac{\exp (V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp (V_k(p_k, z_k, \beta))}
    \]
Estimation

- **Revealed preferences (RP) data**: Booking data from a major European airline
  - Lack of variability
  - Price inelastic demand
- RP data is combined with a **stated preferences (SP) data**
- Time, cost and morning parameters are **fixed** to be the same for the two datasets.
- A **scale** parameter is introduced for SP to capture the differences in variance.
Estimation results

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{fare}$</th>
<th></th>
<th>$\beta_{time}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>non-stop</td>
<td>one-stop</td>
<td>non-stop</td>
<td>one-stop</td>
</tr>
<tr>
<td>economy</td>
<td>-2.23</td>
<td>-2.17</td>
<td>-0.102</td>
<td>-0.0762</td>
</tr>
<tr>
<td>business</td>
<td>-1.97</td>
<td>-1.97</td>
<td>-0.104</td>
<td>-0.0821</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\beta_{morning}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.0283</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.079</td>
<td></td>
</tr>
</tbody>
</table>

- **Price elasticity** of demand:

$$E^{P_i}_{price_i} = \frac{\partial P_i}{\partial price_i} \cdot \frac{price_i}{P_i}$$

An example

- for a non-stop itinerary
  - price elasticity for economy is $-2.03$ and $-1.86$ for business
- for a one-stop itinerary
  - price elasticity for economy is $-2.14$ and $-1.95$ for business
Integrated schedule planning and revenue management

Schedule planning
- Mandatory flights
- Optional flights

Revenue management
- Pricing-demand
  - Spill-recapture
- Capacity allocation
  - Business seats
  - Economy seats

Fleet assignment

Schedule design
- Mandatory flights
- Optional flights
Integrated model - Schedule planning

Max \[
\sum_{h \in H} \sum_{s \in S^h} \left( \sum_{i \in (I_s \setminus I'_s)} (d_i - \sum_{j \in I_s} t_{i,j} + \sum_{j \in (I_s \setminus I'_s)} t_{j,i} b_{j,i}) p_i - \sum_{k \in K} C_{k,f} x_{k,f} \right) : revenue - cost
\]

\text{s.t. } \sum_{k \in K} x_{k,f} = 1: \text{ mandatory flights} \\
\sum_{k \in K} x_{k,f} \leq 1: \text{ optional flights} \\
y_{k,a,t^-} + \sum_{f \in \text{ln}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in \text{out}(k,a,t)} x_{k,f}: \text{ flow conservation} \\
\sum_{a \in A} y_{k,a,minE_a^-} + \sum_{f \in \text{CT}} x_{k,f} \leq R_k: \text{ fleet availability} \\
y_{k,a,minE_a^-} = y_{k,a,maxE_a^+}: \text{ cyclic schedule} \\
\sum_{h \in H} \pi_{k,f}^h = Q_k x_{k,f}: \text{ seat capacity} \\
x_{k,f} \in \{0,1\} \\
y_{k,a,t} \geq 0
## Integrated model - Revenue management

\[
\sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)^f} \delta_{i,f} d_i - \sum_{j \in I_s} \delta_{i,f} t_{i,j} + \sum_{j \in (I_s \setminus I'_s)^f} \delta_{j,f} t_{j,i} b_{j,i} \leq \sum_{k \in K} \pi_{k,f}^h : \text{capacity} \\
\forall h \in H, f \in F \tag{10}
\]

\[
\sum_{j \in I_s} t_{i,j} \leq d_i : \text{total spill} \\
\forall h \in H, s \in S^h, i \in (I_s \setminus I'_s) \tag{11}
\]

\[
\tilde{d}_i = D_s \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))} : \text{logit demand} \\
\forall h \in H, s \in S^h, i \in I_s \tag{12}
\]

\[
b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))} : \text{recapture ratio} \\
\forall h \in H, s \in S^h, i \in (I_s \setminus I'_s), j \in I_s \tag{13}
\]

\[
d_i \leq \tilde{d}_i : \text{realized demand} \\
\forall h \in H, s \in S^h, i \in I_s \tag{14}
\]

\[
0 \leq p_i \leq UB_i : \text{upper bound on price} \\
\forall h \in H, s \in S^h, i \in I_s \tag{15}
\]

\[
t_{i,j} \geq 0 \\
\forall h \in H, s \in S^h, i \in (I_s \setminus I'_s), j \in I_s \tag{16}
\]

\[
b_{i,j} \geq 0 \\
\forall h \in H, s \in S^h, i \in (I_s \setminus I'_s), j \in I_s \tag{17}
\]

\[
\pi_{k,f}^h \geq 0 \\
\forall h \in H, k \in K, f \in F \tag{18}
\]
Integrated model

- We consider reference models to evaluate the integrated model
  - **Price-inelastic schedule planning**: M. Lohatepanont and C. Barnhart (2004)
  - **Sequential approach**: Revenue management considers fixed supply capacity

- The resulting model is a mixed integer nonlinear problem
- Nonlinearity is due to the explicit supply-demand interactions
- The model is implemented in AMPL and BONMIN solver is used
- BONMIN does not guarantee optimality
Illustration

Change of the market share

market share of each itinerary

price of itinerary 1

- itinerary 1
- itinerary 2
- itinerary 3
- itinerary 4
Illustration

Change of the revenue

- itineraries 1 to 4
- total revenue
# Impact of the integrated model

<table>
<thead>
<tr>
<th>Number of airports:</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of flights:</td>
<td>26</td>
</tr>
<tr>
<td>Average demand:</td>
<td>56.12 passengers per flight</td>
</tr>
<tr>
<td>Cabin classes:</td>
<td>Economy and business</td>
</tr>
<tr>
<td>Level of service:</td>
<td>All itineraries are nonstop</td>
</tr>
<tr>
<td>Available fleet:</td>
<td>3 types of aircraft (100, 50 and 37 seats)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Price-inelastic schedule planning model</th>
<th>Integrated model - limited prices</th>
<th>Integrated model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>204,553</td>
<td>214,380</td>
<td>244,924</td>
</tr>
<tr>
<td>Operating costs</td>
<td>150,603</td>
<td>160,003</td>
<td>173,349</td>
</tr>
<tr>
<td>Profit</td>
<td>53,949</td>
<td>54,377 (+ 0.8%)</td>
<td>71,575 (+ 32.7%)</td>
</tr>
<tr>
<td>Number of flights</td>
<td>22</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>Transported passengers</td>
<td>943</td>
<td>1031 (+ 9.3%)</td>
<td>1064 (+ 12.7%)</td>
</tr>
<tr>
<td>Economy-Business</td>
<td>882 E - 61 B</td>
<td>970 E - 61 B</td>
<td>997 E - 67 B</td>
</tr>
<tr>
<td>Allocated seats</td>
<td>274</td>
<td>324</td>
<td>324</td>
</tr>
</tbody>
</table>
## Sequential versus integrated

<table>
<thead>
<tr>
<th>No</th>
<th>Profit</th>
<th>Pax.</th>
<th>Flights</th>
<th>Seats</th>
<th>Sequential approach</th>
<th>No</th>
<th>Profit</th>
<th>Pax.</th>
<th>Flights</th>
<th>Seats</th>
<th>Integrated model - % Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15,091</td>
<td>284</td>
<td>8</td>
<td>124</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>35,372</td>
<td>400</td>
<td>8</td>
<td>150</td>
<td>5.55%</td>
<td>3</td>
<td>50,149</td>
<td>859</td>
<td>10</td>
<td>300</td>
<td>1.43% 14.18%</td>
</tr>
<tr>
<td>3</td>
<td>69,901</td>
<td>931</td>
<td>22</td>
<td>274</td>
<td></td>
<td>5</td>
<td>82,311</td>
<td>1145</td>
<td>16</td>
<td>1148</td>
<td>0.30%</td>
</tr>
<tr>
<td>4</td>
<td>904,054</td>
<td>1448</td>
<td>10</td>
<td>1148</td>
<td>0.30%</td>
<td>6</td>
<td>135,656</td>
<td>1814</td>
<td>32</td>
<td>498</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>115,983</td>
<td>2236</td>
<td>26</td>
<td>691</td>
<td></td>
<td>7</td>
<td>854,902</td>
<td>1270</td>
<td>10</td>
<td>1016</td>
<td>0.43% 5.83%</td>
</tr>
<tr>
<td>6</td>
<td>1016</td>
<td>135,656</td>
<td>1814</td>
<td>32</td>
<td>498</td>
<td>0.30%</td>
<td>8</td>
<td>137,428</td>
<td>1517</td>
<td>34</td>
<td>391</td>
</tr>
<tr>
<td>7</td>
<td>93,347</td>
<td>1144</td>
<td>20</td>
<td>387</td>
<td>3.36%</td>
<td>9</td>
<td>27,076</td>
<td>448</td>
<td>10</td>
<td>207</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>49,448</td>
<td>1050</td>
<td>12</td>
<td>370</td>
<td></td>
<td>10</td>
<td>52,369</td>
<td>599</td>
<td>10</td>
<td>267</td>
<td>1.45% 16.69%</td>
</tr>
<tr>
<td>9</td>
<td>26,486</td>
<td>504</td>
<td>6</td>
<td>185</td>
<td></td>
<td>11</td>
<td>27,076</td>
<td>448</td>
<td>10</td>
<td>207</td>
<td></td>
</tr>
</tbody>
</table>

**Results**
Heuristic method

- We are limited in terms of the computational time
- A heuristic based on two simplified versions of the model:
  - $FAM^{LS}$: price-inelastic schedule planning model $\Rightarrow$ MILP
    - Explores new fleet assignment solutions based on a local search
    - Price sampling
    - Variable neighborhood search
  - $REV^{LS}$: Revenue management with fixed capacity $\Rightarrow$ NLP
    - Optimizes the revenue for the explored fleet assignment solution
Heuristic method

Require: $\bar{x}_0, \bar{y}_0, \bar{d}_0, \bar{p}_0, \bar{t}_0, \bar{b}_0, \bar{\pi}_0, z^*, z_{opt}, k_{max}, \varepsilon, n_{min}, n_{max}$

$k := 0, n_{fixed} := n_{min}$

repeat

- $\bar{p}_k :=$ Price sampling
- $\{\bar{d}_k, \bar{b}_k\} :=$ Demand model($\bar{p}_k$)
- $\{\bar{x}_k, \bar{y}_k, \bar{\pi}_k, \bar{t}_k\} :=$ solve $z_{FAMLS}(\bar{d}_k, \bar{b}_k, n_{fixed})$
- $\{\bar{p}_k, \bar{d}_k, \bar{b}_k, \bar{\pi}_k, \bar{t}_k\} :=$ solve $z_{REVLS}(\bar{x}_k, \bar{y}_k)$

if improvement($z_{REVLS}$) then

- Update $z^*$

  Intensification: $n_{fixed} := n_{fixed} + 1$ when $n_{fixed} < n_{max}$

else

  Diversification: $n_{fixed} := n_{fixed} - 1$ when $n_{fixed} > n_{min}$

end if

$k := k + 1$

until $||z_{opt} - z^*||^2 \leq \varepsilon$ or $k \geq k_{max}$
Performance of the heuristic

The omitted instances are the ones where the sequential approach has the same solution as the integrated model.

<table>
<thead>
<tr>
<th>Flights</th>
<th>Best solution reported by Bonmin</th>
<th>Sequential approach</th>
<th>Heuristic results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Profit Time (sec)</td>
<td>Profit % dev.</td>
<td>Time (sec) Time red.</td>
</tr>
<tr>
<td></td>
<td>(sec)</td>
<td></td>
<td>(sec)</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>37,335 27</td>
<td>35,372 5.26%</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>70,904 2,479</td>
<td>69,901 1.41%</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>906,791 12,964</td>
<td>904,054 0.30%</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>858,544 7,343</td>
<td>854,902 0.42%</td>
</tr>
<tr>
<td>10</td>
<td>39</td>
<td>138,575 37,177</td>
<td>137,428 0.83%</td>
</tr>
<tr>
<td>11</td>
<td>23</td>
<td>96,486 17,142</td>
<td>93,347 3.25%</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>53,128 141</td>
<td>52,369 1.43%</td>
</tr>
<tr>
<td>16</td>
<td>77</td>
<td>194,598 42,360</td>
<td>208,561 -7.18%</td>
</tr>
<tr>
<td>17</td>
<td>61</td>
<td>227,364 22,174</td>
<td>226,615 0.33%</td>
</tr>
<tr>
<td>18</td>
<td>48</td>
<td>153,789 4,387</td>
<td>163,114 -6.06%</td>
</tr>
</tbody>
</table>

max 43200 max 3600
Conclusions and future work

- Heuristic
  - Inclusion of larger instances
- Further solution methods for the resulting mixed integer nonlinear problem
  - Convex approximation of the nonlinearity
  - Decomposition methods $\Rightarrow$ FAM and REV models
  - Subgradient optimization
Thank you for your attention!
Discrete choice analysis

- Finite and discrete set of alternatives
  - Choice of transportation mode: car, bus, etc.
  - Choice of brand: Leonidas, Lindt, Suchard, Toblerone, etc.
  - Choice of flight: GVA-NCE 10:00, GVA-NCE 06:30, etc.
- Individual \( n \) associates a utility to alternative \( i \)
- Represented by a random function

\[
U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}
\]
Individual $n$ chooses alternative $i$ if $U_{in} \geq U_{jn}$, for all $j$.

Utility is random, so we have a probabilistic model

$$P_n(i|C_n) = \Pr(U_{in} \geq U_{jn}) = \Pr(V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn})$$

Concrete models require
- specification of $V_{in}$
- assumptions about $\varepsilon_{in}$
- estimation of the parameters from data