
Berth Allocation Problem in Real Time

Nitish Umang* Michel Bierlaire*

* TRANSP-OR, Ecole Polytechnique Fédérale de Lausanne

10th Joint Operations Research Days
June 28-29, Neuchatel, Switzerland

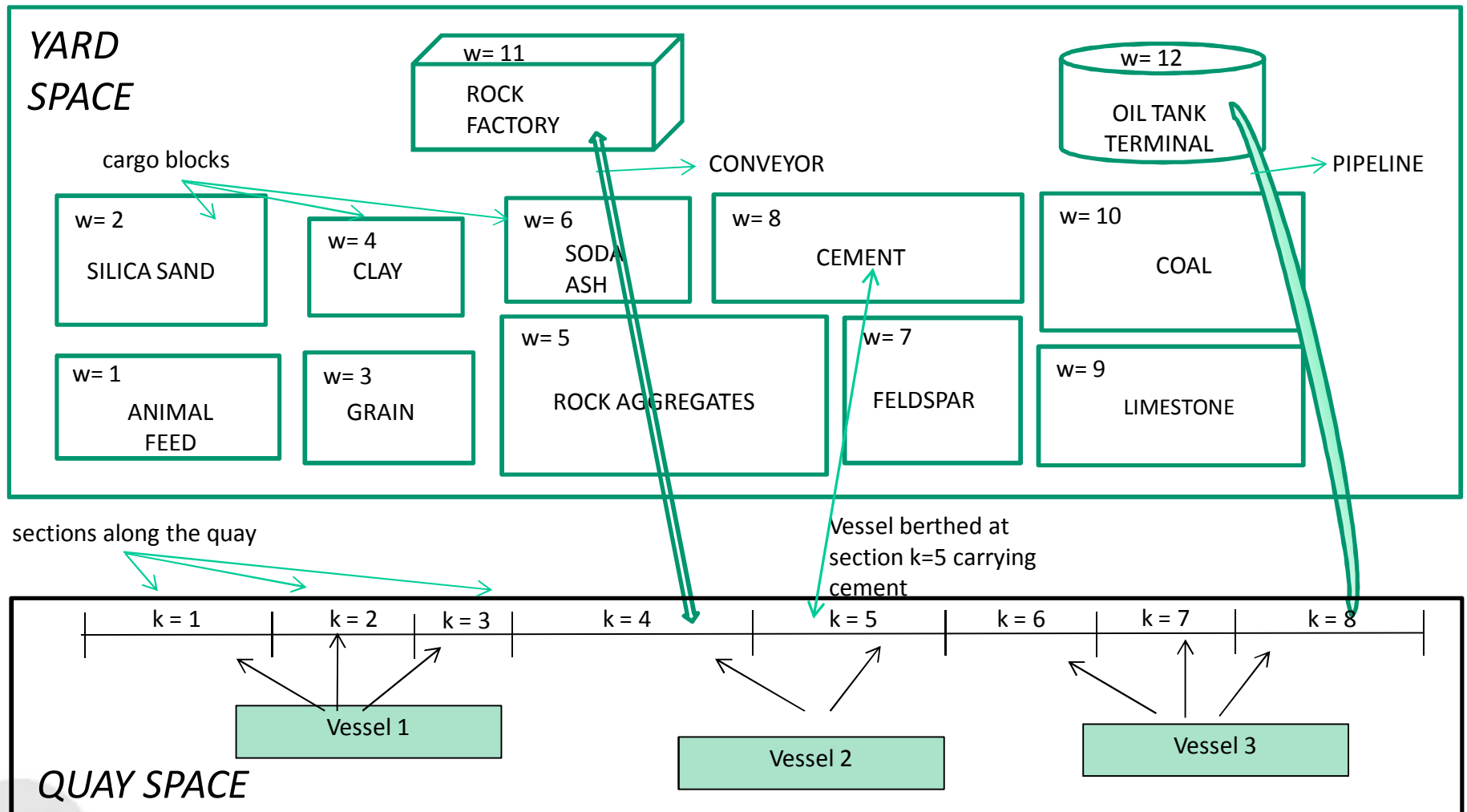
Contents

- Motivation
- Research Objectives
- Deterministic Berth Allocation Problem
- Real Time Recovery in BAP
- Preliminary Results
- Conclusions and Future Work

Motivation

- International shipping tonnage in solid bulk and liquid bulk trade has registered an increase by 52% and 48% respectively. The total volume of dry bulk cargoes loaded in 2008 stood at 5.4 billion tons, accounting for 66.3 per cent of total world goods loaded (UNCTAD, 2009)
- Bulk port terminals have received significantly less attention than container terminals in the field of large scale optimization
- High level of uncertainty in bulk port operations due to weather conditions, mechanical problems etc.
 - Disrupt the normal functioning of the port
 - Require quick real time action.
- Very few studies address the problem of real time recovery in port operations, while the problem has not been studied at all in context of bulk ports.
- Our research problem derives from the realistic requirements at the SAQR port, Ras Al Khaimah, UAE

Schematic Diagram of a Bulk Terminal



Research Objectives

- Study the crucial problems of
 - **Berth Allocation** : scheduling and assignment of vessels to sections along the quay
 - **Yard Assignment** : assignment of vessels and cargo types to specific locations on the yard
- **Large Scale Integrated Planning**: Integration of the berth allocation and yard assignment for better coordination between berthing and yard activities
- Develop **real time** and **robust optimization algorithms** to account for uncertainties in arrival times and handling times of vessels, and other unforeseen disruptions and delays in operations.
- The focus of this talk is solving the berth allocation problem in real time for a given baseline schedule.

Research Objectives

- Develop real time algorithms for disruption recovery in berth allocation problem (BAP)
- For a given baseline berthing schedule, minimize the total realized costs of the updated schedule as actual arrival data is revealed. The total realized costs include
 - The total service cost of all vessels berthing at the port which is the sum total of the handling times and berthing delays of all vessels berthing in the planning horizon.
 - Inconsistent cost of rescheduling over space and time to account for the cost of re-allocating human labor, handling equipment and availability of cargo.

Literature Review

- Very scarce studies on real time and robust algorithms in container terminals . To the best of our knowledge, no literature on bulk ports.
- Comprehensive literature surveys on BAP in container terminals can be found in Steenken et al. (2004), Stahlobock and Voss (2007), Bierwirth and Meisel (2010).
- OR literature related to BAP under uncertainty in container terminals
 - **Pro-active Robustness**
 - Stochastic programming approach used by Zhen et al. (2011), Han et al. (2010)
 - Define surrogate problems to define the stochastic nature of the problem: Moorthy and Teo (2006), Zhen and Chang (2012), Xu et al. (2012) and Hendriks et al. (2010)
 - **Reactive approach or disruption management**
 - Zeng et al.(2012) and Du et al. (2010) propose reactive strategies to minimize the impact of disruptions.

Baseline Schedule

- Any feasible berthing assignment and schedule of vessels along the quay respecting the spatial and temporal constraints on the individual vessels
- Best case: Optimal solution of the deterministic berth allocation problem (without accounting for any uncertainty in arrival information)

Deterministic BAP: Problem Definition

- **Find**

- Optimal assignment and schedule of vessels along the quay (without accounting for any uncertainty in arrival information)

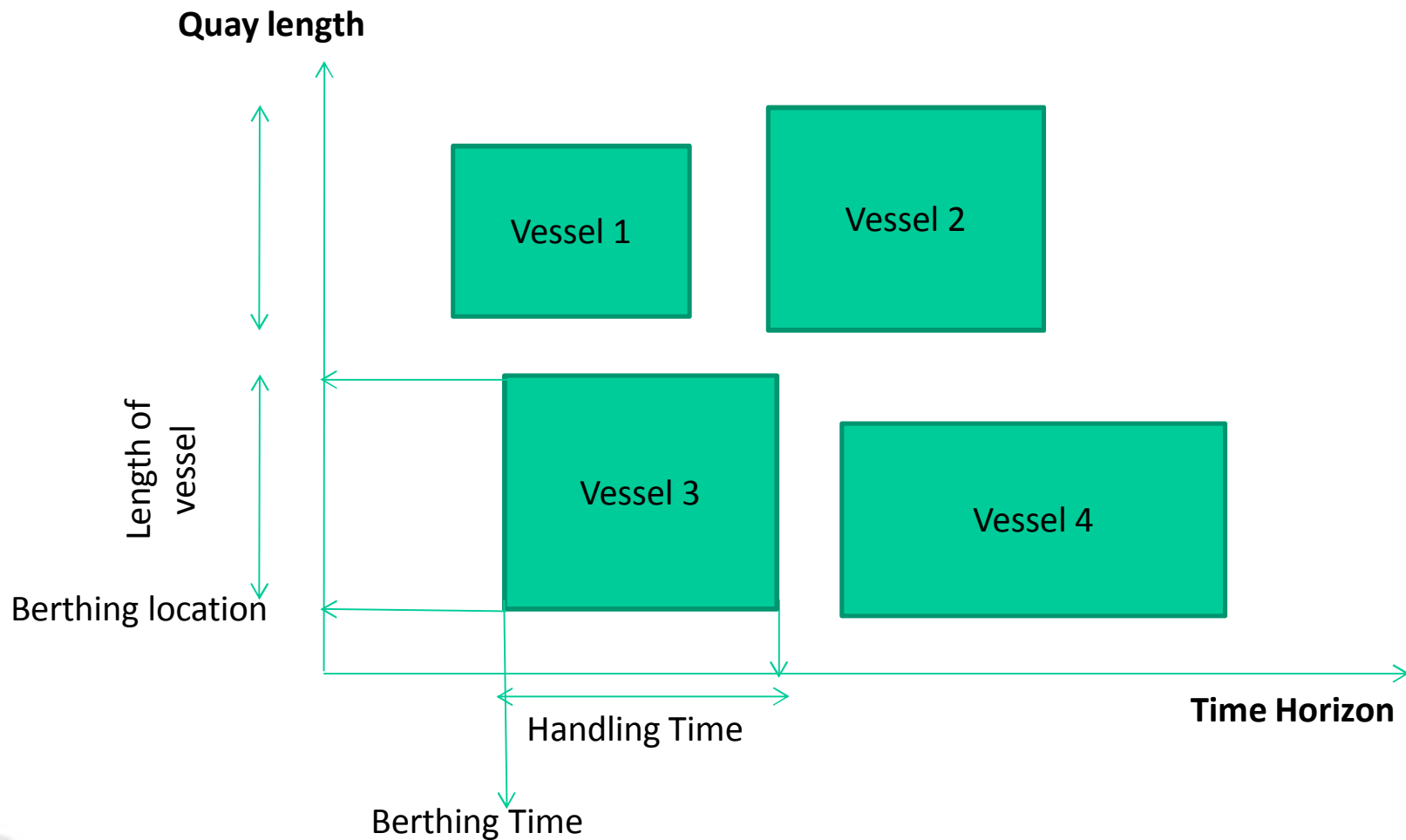
- **Given**

- Expected arrival times of vessels
- Handling times dependent on
 - **Cargo type** on the vessel (the relative location of the vessel along the quay with respect to the cargo location on the yard)
 - Number of cranes operating on the vessel

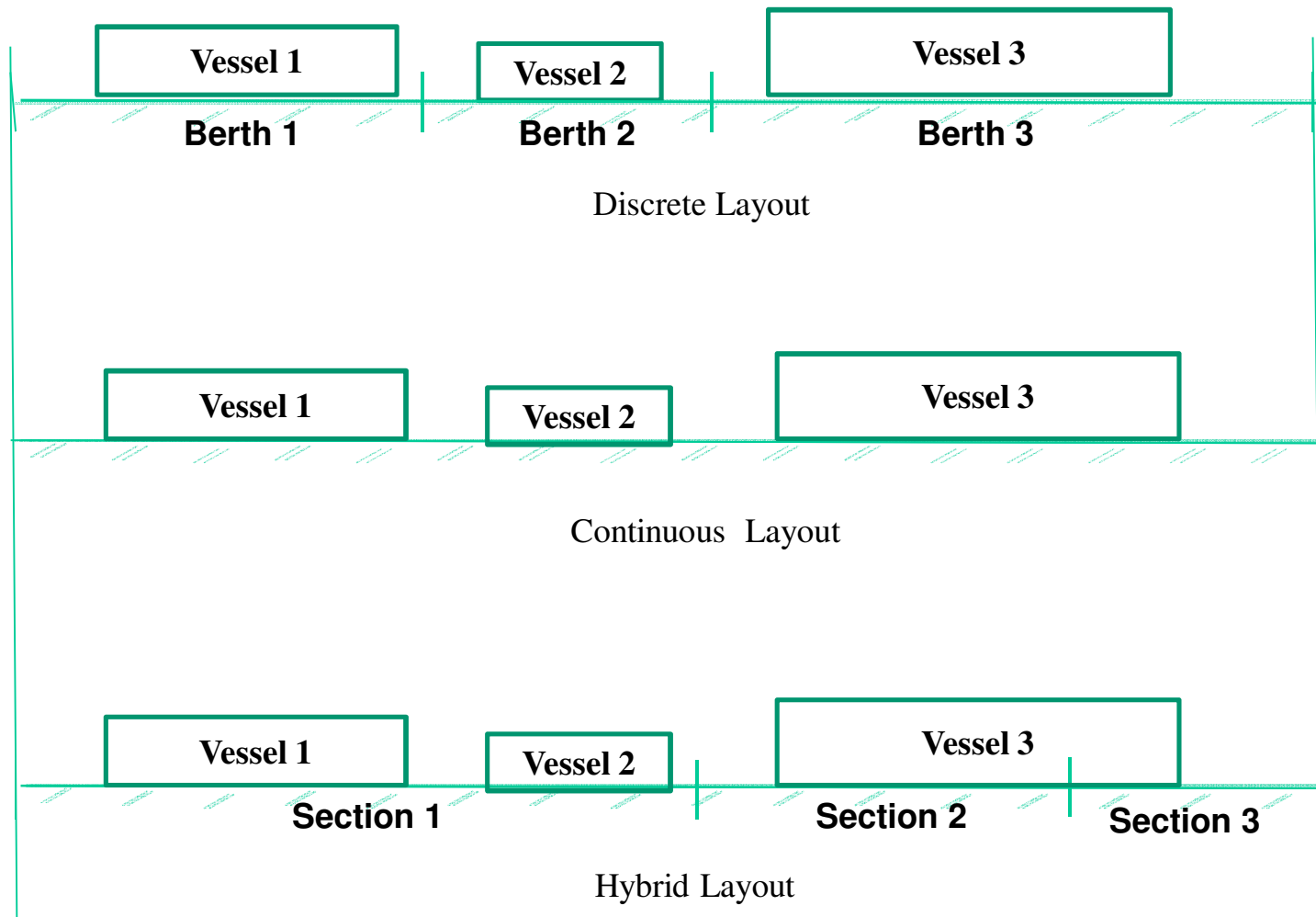
- **Objective**

- Minimize total service times (waiting time + handling time) of vessels berthing at the port

BAP Solution



Discretization



MILP Model

Objective Function

$$\min \sum_{i \in N} (m_i - A_i + c_i)$$

Decision variables:

m_i starting time of handling of vessel $i \in N$;

c_i total handling time of vessel $i \in N$;

MILP Model

Dynamic vessel arrival constraints

$$m_i - A_i \geq 0 \quad \forall i \in N,$$

Non overlapping constraints

$$\sum_{k \in M} (b_k s_k^j) + B(1 - y_{ij}) \geq \sum_{k \in M} (b_k s_k^i) + L_i \quad \forall i, j \in N, i \neq j,$$

$$m_j + B(1 - z_{ij}) \geq m_i + c_i \quad \forall i, j \in N, i \neq j,$$

$$y_{ij} + y_{ji} + z_{ij} + z_{ji} \geq 1 \quad \forall i, j \in N, i \neq j,$$

MILP Model

Section covering constraints

$$\sum_{k \in M} s_k^i = 1 \quad \forall i \in N,$$

$$\sum_{k \in M} (b_k s_i^k) + L_i \leq L \quad \forall i \in N,$$

$$\sum_{l \in M} (d_{ilk} s_l^i) = x_{ik} \quad \forall i \in N, \forall k \in M,$$

Draft Restrictions

$$(d_k - D_i) x_{ik} \geq 0 \quad \forall i \in N, \forall k \in M,$$

MILP Model

Determination of Handling Times

- Given an input vector of unit handling times for each combination of cargo type and section along the quay
- Specialized facilities (conveyors, pipelines etc.) also modeled as cargo types
- All sections occupied by the vessel are operated simultaneously

$$c_i \geq h_k^w p_{ilk} Q_i s_l^i \quad \forall i \in N, \forall k \in M, \forall l \in M, \forall w \in W_i$$

Q_i quantity of cargo to be loaded on or discharged from vessel i

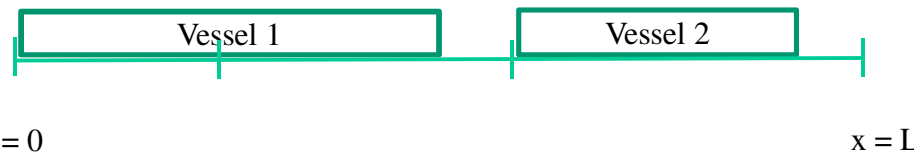
h_k^w handling time for unit quantity of cargo $w \in W$ and vessel berthed at section $k \in M$;

p_{ilk} fraction of cargo handled at section $k \in M$ when vessel i is berthed at starting section $l \in M$

GSPP Model

- Used in context of container terminals by Christensen and Holst (2008)
- Generate set P of columns, where each column $p \in P$ represents a feasible assignment of a single vessel in both space and time
- Generate two matrices
 - Matrix $A = (A_{ip})$; equal to 1 if vessel $i \in N$ is the assigned vessel in the feasible assignment represented by column $p \in P$
 - Matrix $B = (b_p^{st})$; equal to 1 if section $s \in M$ is occupied at time $t \in H$ in column $p \in P$

GSPP Formulation: A simple example



- $|N| = 2, |M| = 3, |H| = 3$
- Vessel 1 cannot occupy section 3 owing to spatial constraints (does not have conveyor facility), vessel 2 arrives at time $t = 1$
- Constraint matrix P has 4 feasible assignments:

Vessel 1	1	1	0	0
Vessel 2	0	0	1	1
Section 1 , Time 1	1	0	0	0
Section 1, Time 2	1	1	1	0
Section 1, Time 3	0	1	1	0
Section 2, Time 1	1	0	0	0
Section 2, Time 2	1	1	1	1
Section 2, Time 3	0	1	1	1
Section 3, Time 1	0	0	0	0
Section 3, Time 2	0	0	0	1
Section 3, Time 3	0	0	0	1

GSPP Model Formulation

Objective Function:

$$\min \sum_{p \in P} (d_p \lambda_p + h_p \lambda_p)$$

Constraints:

$$\sum_{p \in P} (A_{ip} \lambda_p) = 1 \quad \forall i \in N$$

$$\sum_{p \in P} (b_p^{st} \lambda_p) \leq 1 \quad \forall s \in M, \forall t \in H$$

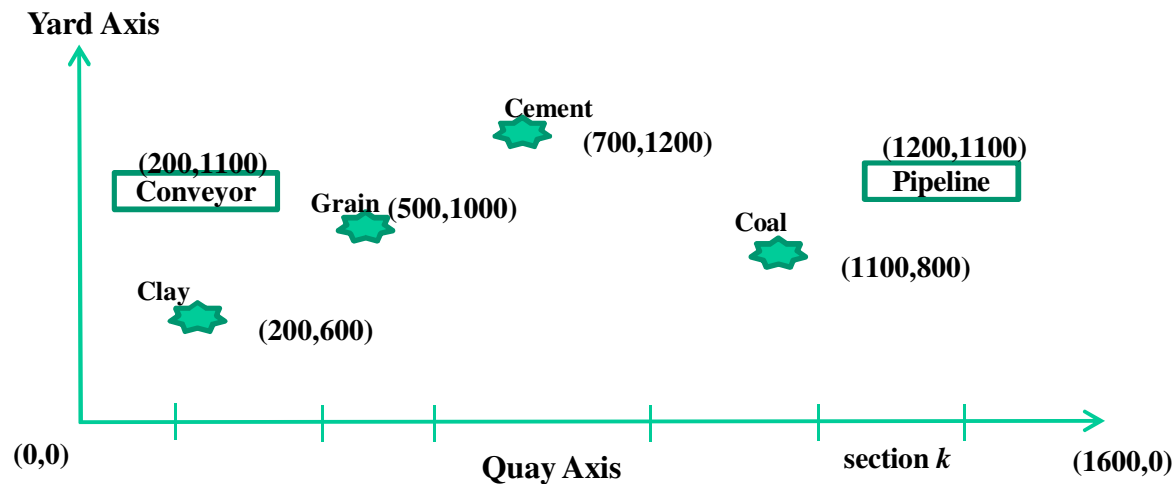
d_p : delay in service associated with assignment $p \in P$

h_p : handling time associated with assignment $p \in P$

λ_p : binary parameter, equal to 1 if assignment $p \in P$ is part of the optimal solution

Generation of Instances

- Instances based on data from SAQR port with quay length of 1600 meters and vessel lengths in the range 80-260 meters.
- Generate 6 instances sizes with $|N| = 10, 25$ and 40 vessels, and $|M| = 10$ and 30 sections, with 9 instances for each instance size.
- Handling times generated for 6 cargo types.

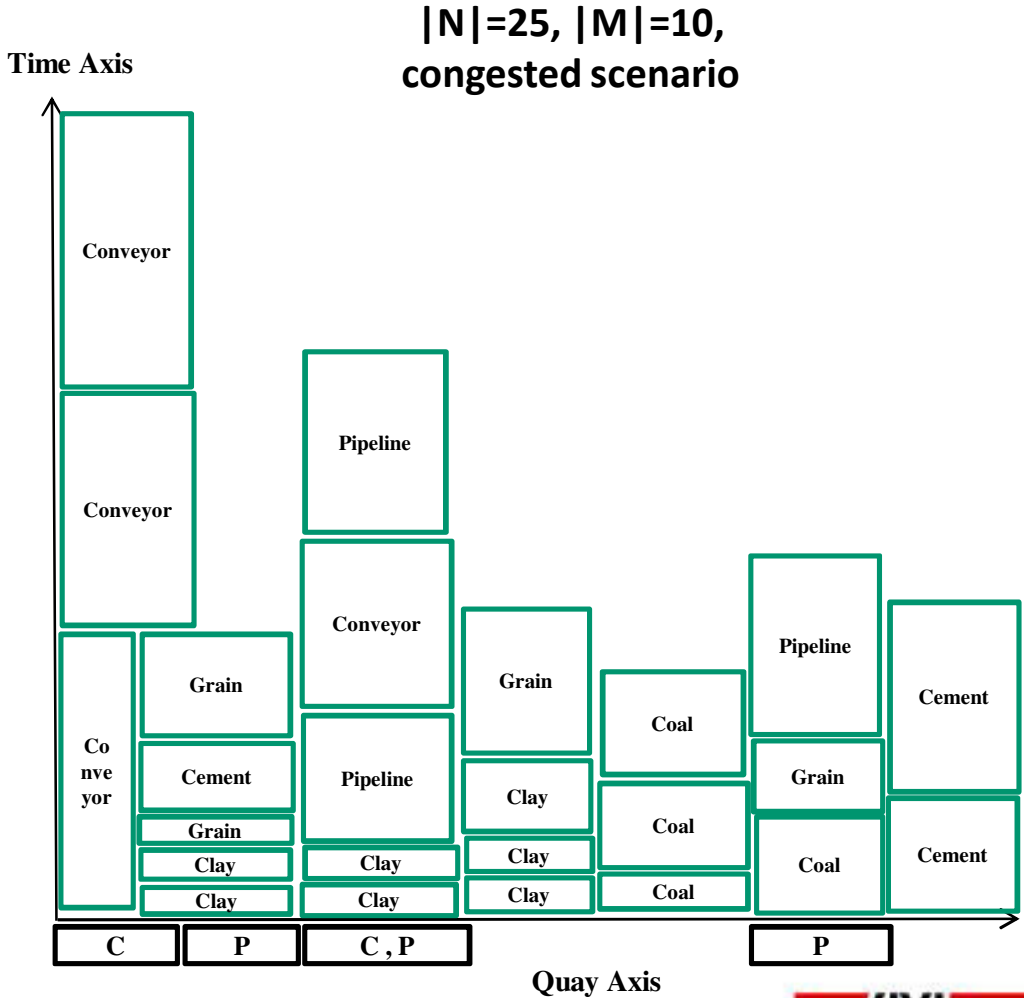
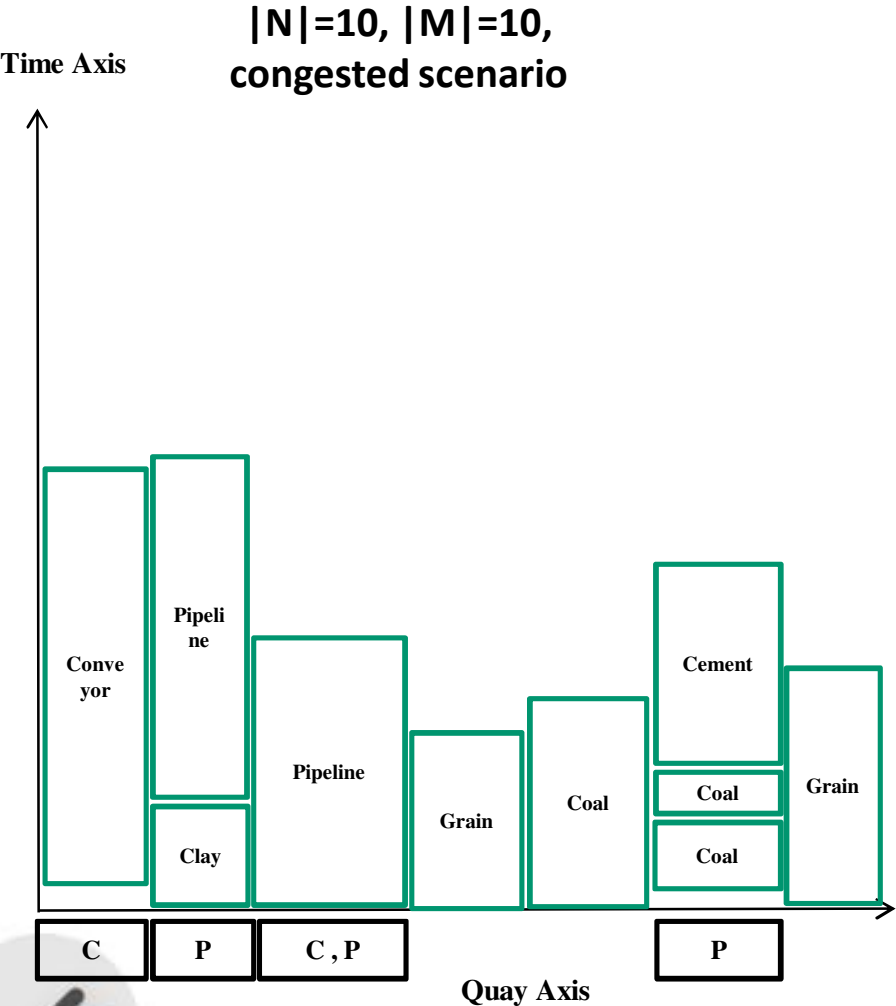


- Drafts of all vessels D_i are less than the minimum draft along the quay.

Computational Results

- Instances based on data from SAQR port
- All tests were run on an Intel Core i7 (2.80 GHz) processor and used a 32-bit version of CPLEX 12.2.
- Results inspired by port data show that the problem is complex !
- MILP formulation fails to produce optimal results for even small instances with $|N|=10$ vessels within CPLEX time limit of 2 hours.
- The performance of the GSPP model is quite remarkable!
 - Can solve instances up to $|N| = 40$ vessels
 - Limitations: For larger instances, or longer horizon H solver runs out of memory (use dynamic column generation!)
- Alternate heuristic approach based on squeaky wheel optimization (SWO) performs reasonably well for not so large instances. Optimality gap is less than 10% (with respect to exact solution obtained from GSPP approach) averaged over all tested instances.

Results Analysis



Real Time Recovery in Berth Allocation Problem

Problem Definition: Real time recovery in BAP

- **Objective:** For a given baseline berthing schedule, minimize the total realized costs including the total actual service costs and total cost of rescheduling in space and time

$$\min Z = \sum_{i \in N_u} (m_i - A_i + h_i) + \sum_{i \in N_u} (c_1 |b_i(k') - b_i(k)| + c_2 \mu_i |e'_i - e_i|)$$

N_u : set of unassigned vessels

c_1 : cost coefficient of shifting berthing location

$b_i(k')$: actual berthing location of vessel i

$b_i(k)$: estimated berthing location of vessel i

c_2 : cost coefficient of departure delay

μ_i : service priority assigned to vessel i

e'_i : actual departure time of vessel i

e_i : estimated departure time of vessel i

Problem Definition: Real time recovery in BAP

- Solving the BAP as arrival delay information is released in real time
 - For each vessel $i \in N$, we are given an expected arrival time A_i which is known in advance.
 - The expected arrival time of a given vessel may be updated $|P|$ times during the planning horizon of length $|H|$ at time instants $t_{i1}, t_{i2}, \dots, t_{iP}$ such that

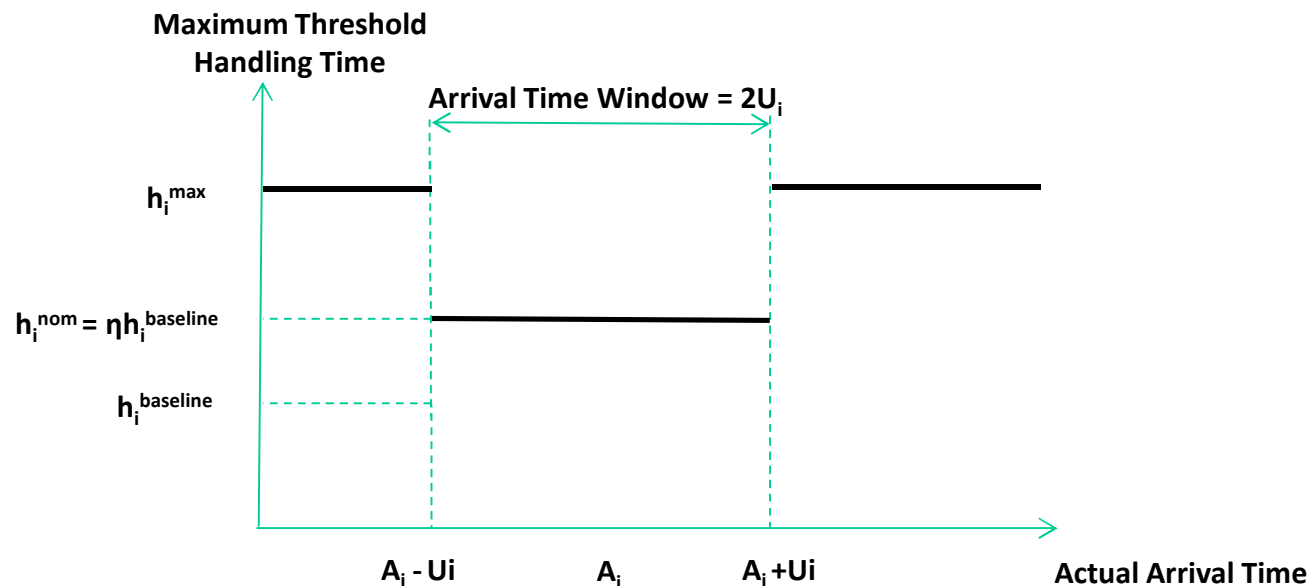
$$0 \leq t_{i1} < t_{i2} < t_{i3} \dots t_{i(P-1)} < t_{iP} < a_i$$

where a_i is the actual arrival time of the vessel, and the corresponding arrival time update at time instant t_{iP} is A_{iP} for all $i \in N$.

Problem Definition: Real time recovery in BAP

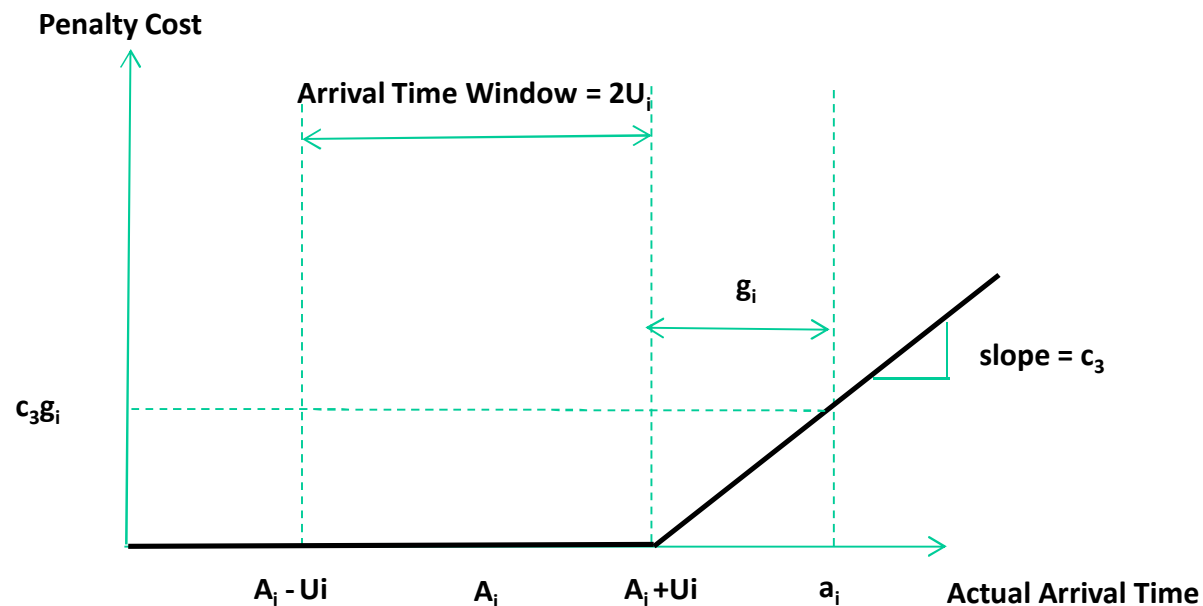
To maximize revenues earned by the port while guaranteeing a minimum level of service, we propose that the bulk terminal managers adopt and implement certain strategic measures

- **Handling Time Restrictions:** Impose an upper bound on the maximum handling time of a vessel $i \in N$ if it arrives within a pre-defined arrival time window $[A_i - U_i, A_i + U_i]$



Problem Definition: Real time recovery in BAP

- **Penalty Cost on late arriving vessels:** Impose a penalty fees on vessels arriving beyond the right end of the arrival window, $A_i + U_i$



Solution Algorithms

- **Optimization based recovery algorithm**

- re-optimize the berthing schedule of all unassigned vessels using set-partitioning approach every time the arrival time of any vessel is updated and it deviates from its expected value.
- the berthing assignment of a vessel determined after its arrival update is frozen and unchangeable

- **Heuristic based recovery algorithm**

- If a vessel has arrived and current time in the planning horizon is greater than or equal to the estimated berthing time of the vessel (as per baseline schedule), assign it to the section(s) at which the total realized cost of all unassigned vessels at that instant is minimized
- Assumption : All other unassigned vessels are assigned to the estimated berthing sections as per the baseline schedule

$$\min Z = \sum_{i \in N_u} (m_i - A_i + h_i) + \sum_{i \in N_u} (c_1 |b_i(k') - b_i(k)| + c_2 \mu_i |e'_i - e_i|)$$

Optimization based Recovery Algorithm

Require: Baseline schedule of set N of vessels, set M of sections

Initialize set N_u of unassigned vessels to N

Initialize boolean array arrivalUpdated of size $N = \text{false}$ *forall* $i \in N$

Initialize counter = 0

while $|N_u| > 0$ and counter $\leq |H|$ **do**

Initialize boolean shouldOptimize = false

for $i = 1$ to N **do**

if arrivalUpdated[i] = false and counter $\geq a_i - \tau$ and $a_i \neq A_i$ **then**

Set arrivalUpdated[i] = true

Set $A_i = a_i$

Set shouldOptimize = true

end if

end for

if shouldOptimize **then**

Re-optimize *forall* $i \in N_u$

end if

for $i = 1$ to N_u **do**

if counter = latest updated start time m'_i **then**

Assign vessel i to latest updated location $b_i(k')$

Set N_u to $N_u - \{i\}$

end if

end for

counter++

end while

Heuristic based Recovery Algorithm

Require: Baseline schedule of set N of vessels, set M of sections

Initialize set N_u of unassigned vessels to N

Initialize boolean array arrivalUpdated of size $N = \text{false}$ *forall* $i \in N$

Initialize counter = 0

while $|N_u| > 0$ and counter $\leq |H|$ **do**

for *berthing Schedule: b* **do**

if $b.\text{hasArrived}$ AND $\neg b.\text{isAssigned}$ **then**

 Set boolean foundSection = false

for $k = 1$ to M **do**

if $\text{isStartSectionAvailable}(b.\text{vessel}, k)$ **then**

 foundSection = true;

 break;

end if

end for

if foundSection AND counter $\geq b.\text{estimatedBerthingTime}$ **then**

 Scan the entire quay and assign the vessel to the set of sections with minimum total cost *forall* $i \in N_u$

end if

end if

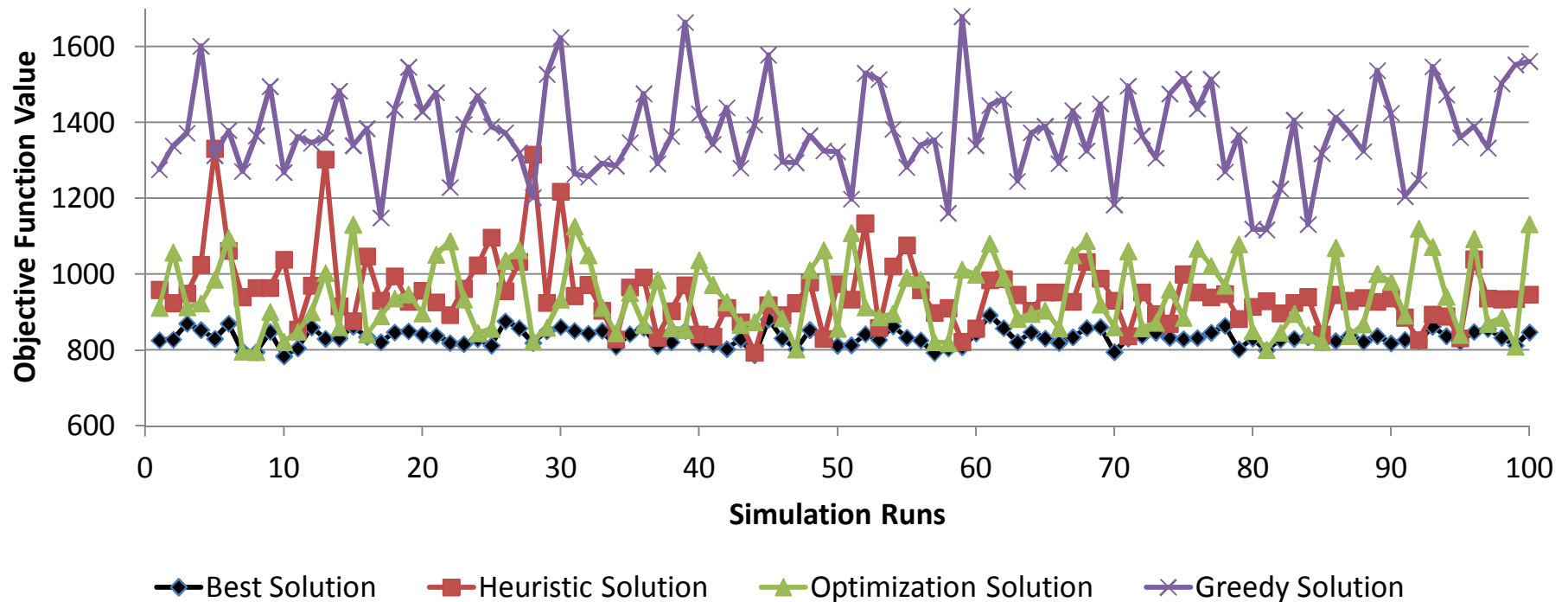
end for

 counter++

end while

Preliminary Results

- $|N|=25$ vessels, $|M|=10$ sections, $c_1 = 1.0$, $c_2 = 0.002$, $U_i = 8$ hours, $\tau = 5$ hours, $\eta = 1.2$, $D_v = 5$

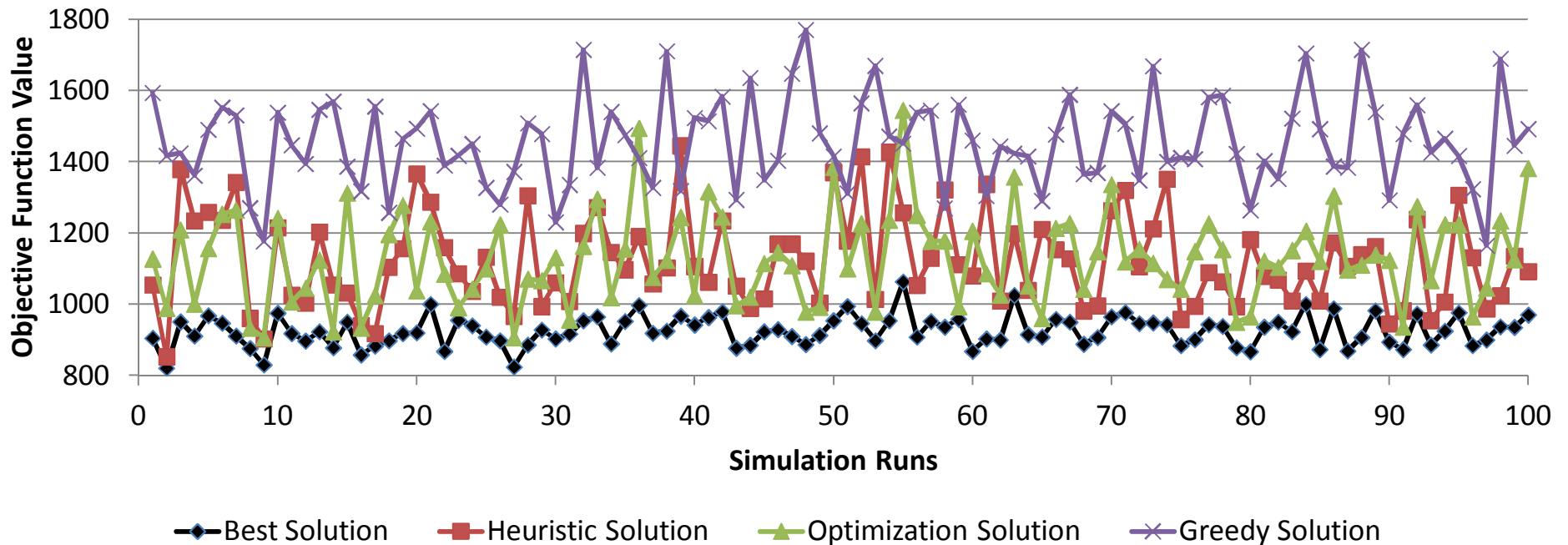


- Mean Gap with respect to the best solution

Greedy Approach	Optimization based Approach	Heuristic based Approach
65.04 %	12.59 %	14.01 %

Preliminary Results

- $|N|=25$ vessels, $|M|=10$ sections, $c_1 = 1.0$, $c_2 = 0.002$, $U_i = 8$ hours, $\tau = 5$ hours, $\eta = 1.2$, $D_v = 15$



- Mean Gap with respect to the best solution

Greedy Approach	Optimization based Approach	Heuristic based Approach
57.46 %	21.85 %	21.31 %

Summary of Results

- Modeled and solved the dynamic, hybrid berth allocation problem in bulk ports
- Addressed the problem of recovering a baseline berthing schedule in bulk ports in real time as actual arrival data is revealed.
- Discussed strategies that the port can adopt and implement to maximize their revenues while ensuring a desired level of service
- Developed solution algorithms to solve the BAP in real time in bulk ports with the objective to minimize the total realized costs of the updated schedule.
- Conducted simple numerical experiments to validate the efficiency of the algorithms..

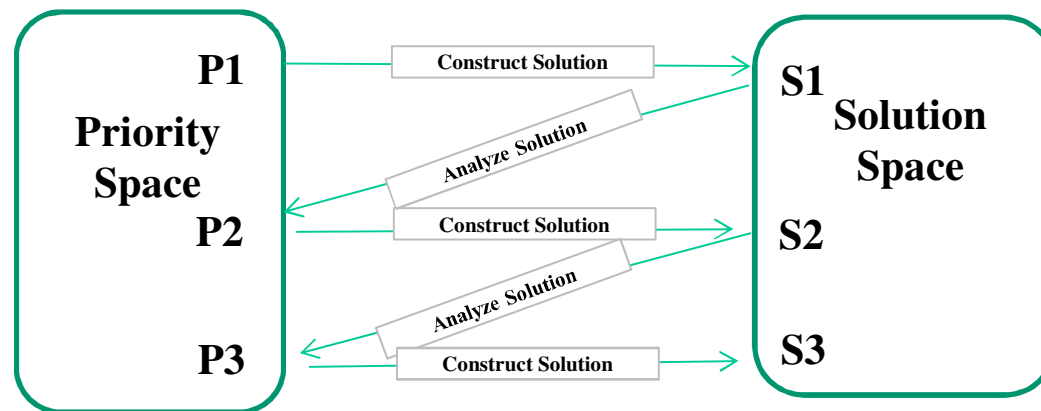
Ongoing and Future Work

- More extensive numerical analysis to study the impact of
 - parameter values related to rescheduling of vessels including cost of shifting the vessel along the quay and cost of departure delay of a vessel
 - bounds on the maximum handling times for vessels arriving within the prescribed arrival window.
 - penalty cost function dependent on the late arriving vessels for arrival delay beyond the prescribed arrival window of the vessel
- Develop a robust formulation of the berth allocation problem in bulk ports with a certain degree of anticipation of variability in information.

Thank you!

SWO Heuristic Approach

- Introduced by Clements (1997), typically successful in problems where it is possible to quantify the contribution of each single problem element to the overall solution quality
- Construct/ Analyze/ Prioritize: Solution generated at each successive iteration is constructed and analyzed, results of analysis used to generate a new priority order



- Moves in search space are motivated by the weak performing elements and not the overall objective function value

SWO Heuristic Approach

- **Construction heuristic:** Returns a feasible berthing assignment for given priority order of vessels
- **Initial Solution:** First-Cum-First-Served ordering based on arrival times of vessels
- **Algorithm:** In each successive iteration, a new priority order is constructed based on the service quality measure of each berthing vessel in the previous solution
 - Service time of the vessel in the solution found in the last iteration
 - Deviation of service time of vessel from the minimum service time possible for that vessel (zero delay + minimum handling time)
 - Sum of service times of the vessel in all iterations completed so far!
- If a priority order is already evaluated, introduce randomization by swapping two or more vessels, until we obtained a priority order that has not been evaluated so far
- Algorithm terminates after a preset number of iterations and best solution is selected as the final solution

Results and Analysis

$|N| = 10$ vessels, and $|M| = 10$ sections

Instance	MILP			GSPP		FCFS		SWO		
	OFV	Gap	Time	OFV	Time (H=150, h=1)	OFV	RE	OFV	RE	Time
A1	230.21	0.01%	67.67	231.21	5.94	262.09	13.36%	230.48	-0.32%	15.81
A2	237.35	0.01%	15.31	238.49	5.54	250.44	5.01%	239.08	0.25%	16.66
A3	223.99	0.01%	9.58	226.61	5.96	280.04	23.58%	225.33	-0.57%	16.97
A4	227.12	0.01%	10.31	227.22	5.68	240.91	6.03%	228.00	0.35%	16.60
A5	234.20	0.01%	5.60	234.22	5.43	251.09	7.20%	234.47	0.11%	16.30
A6	233.12	0.01%	11.06	234.06	6.85	262.61	12.20%	233.12	-0.40%	16.90
A7	203.23	0.00%	0.56	203.23	4.99	208.44	2.57%	203.38	0.07%	15.95
A8	218.87	0.00%	0.56	219.99	5.29	220.90	0.41%	218.87	-0.51%	16.72
A9	198.03	0.00%	0.60	199.89	5.16	214.17	7.14%	198.03	-0.93%	17.53
Mean							8.61%		-0.22%	

$|N| = 10$ vessels, and $|M| = 30$ sections

Instance	MILP			GSPP		FCFS		SWO		
	OFV	Gap	Time	OFV	Time (H=150, h=1)	OFV	RE	OFV	RE	Time
B1	188.39	0.01%	15.80	189.73	94.552	216.56	14.14%	192.81	1.62%	49.95
B2	178.07	0.01%	15.78	179.10	86.08	186.41	4.08%	178.08	-0.57%	48.42
B3	200.16	0.01%	1094.61	202.33	101.93	230.14	13.74%	216.14	6.82%	49.79
B4	182.57	0.01%	3.04	184.27	89.58	224.00	21.56%	182.80	-0.80%	47.73
B5	178.48	0.01%	10.97	179.23	85.01	185.60	3.55%	179.01	-0.12%	48.37
B6	199.82	0.01%	87.78	201.17	96.19	240.09	19.35%	223.37	11.04%	48.83
B7	173.02	0.01%	1.30	173.02	86.00	175.72	1.56%	175.30	1.32%	48.61
B8	162.51	0.00%	1.57	162.81	81.67	169.20	3.92%	166.20	2.08%	50.29
B9	175.29	0.00%	1.39	175.81	95.74	192.41	9.44%	191.26	8.79%	50.27
Mean							10.15%		3.35%	

$|N| = 25$ vessels, and $|M| = 10$ sections

Instance	MILP			GSPP		FCFS		SWO		
	OFV	Gap	Time	OFV	Time (H=150, h=1)	OFV	RE	OFV	RE	Time
C1	812.32	33.08%	-	819.22	14.09	976.49	19.20%	869.31	6.11%	22.29
C2	783.45	30.27%	-	781.72	11.70	924.35	18.25%	825.92	5.65%	22.58
C3	903.51	33.39%	-	900.43	20.19	1107.59	23.01%	929.32	3.21%	22.24
C4	795.71	23.18%	-	791.18	15.26	877.90	10.96%	852.03	7.69%	23.28
C5	751.19	27.47%	-	747.88	10.41	846.26	13.15%	774.17	3.51%	22.16
C6	874.53	24.50%	-	863.86	19.38	979.26	13.36%	898.44	4.00%	23.19
C7	735.13	19.72%	-	741.16	15.91	840.85	13.45%	806.23	8.78%	22.54
C8	689.37	22.26%	-	699.14	11.23	761.48	8.92%	735.46	5.20%	22.32
C9	800.00	22.76%	-	793.24	12.82	936.55	18.07%	872.76	10.03%	23.56
Mean							15.37%		6.02%	

$|N| = 25$ vessels, and $|M| = 30$ sections

Instance	MILP			GSPP		FCFS		SWO		
	OFV	Gap	Time	OFV	Time (H=150, h=1)	OFV	RE	OFV	RE	Time
D1	690.79	23.14%	-	670.42	219.04	857.99	27.98%	785.91	17.23%	105.36
D2	617.31	34.23%	-	591.06	185.44	723.13	22.34%	667.20	12.88%	96.03
D3	809.55	30.75%	-	784.94	387.40	984.28	25.40%	866.82	10.43%	105.66
D4	657.48	20.62%	-	636.19	220.40	778.50	22.37%	728.60	14.53%	100.95
D5	560.65	25.96%	-	556.37	172.09	622.14	11.82%	614.72	10.49%	91.35
D6	754.87	21.97%	-	739.44	253.67	909.53	23.00%	836.23	13.09%	102.34
D7	581.54	8.36%	-	590.24	194.80	731.55	23.94%	706.82	19.75%	100.82
D8	510.80	16.20%	-	506.30	167.09	565.87	11.77%	565.87	11.77%	111.46
D9	704.76	19.18%	-	677.97	200.63	848.97	25.22%	778.67	14.85%	99.87
Mean							21.54%		13.89%	

$|N| = 40$ vessels, and $|M| = 10$ sections

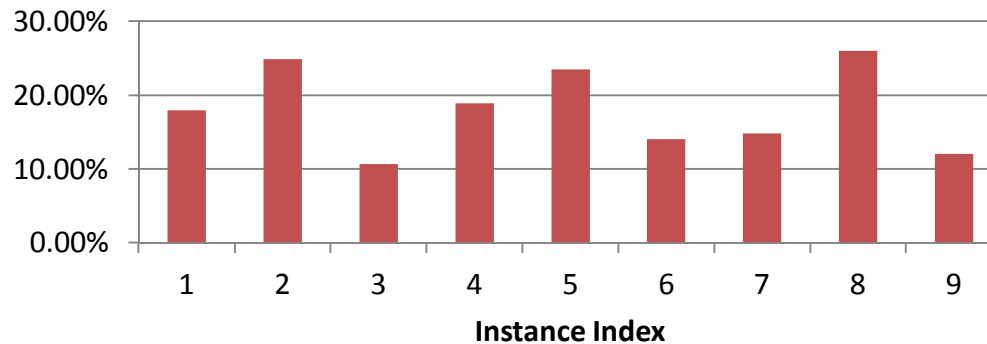
Instance	MILP			GSPP		FCFS		SWO		
	OFV	Gap	Time	OFV	Time (H=150, h=1)	OFV	RE	OFV	RE	Time
E1	1243.64	63.77%	-	1140.60	41.73	1536.78	34.73%	1289.88	13.09%	28.24
E2	1193.05	59.69%	-	1138.16	18.80	1571.07	38.04%	1273.09	11.86%	28.44
E3	1341.65	67.35%	-	1249.06	139.47	1878.78	50.42%	1416.54	13.41%	31.50
E4	1113.36	59.53%	-	1051.50	30.87	1408.95	33.99%	1137.20	8.15%	30.11
E5	1105.34	56.98%	-	1063.85	19.06	1447.39	36.05%	1202.50	13.03%	32.06
E6	1361.62	68.15%	-	1160.05	167.58	1903.39	64.08%	1330.64	14.71%	34.01
E7	1011.20	55.47%	-	946.35	26.04	1291.11	36.43%	1148.14	21.32%	31.69
E8	1013.41	53.02%	-	953.24	20.03	1183.57	24.16%	1094.15	14.78%	32.01
E9	1181.97	64.95%	-	1071.46	94.88	1500.71	40.06%	1296.79	21.03%	35.64
Mean							39.77%		14.60%	

$|N| = 40$ vessels, and $|M| = 30$ sections

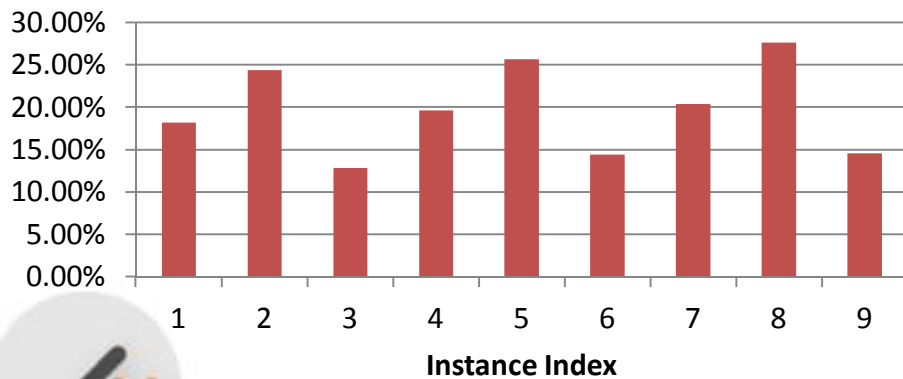
Instance	MILP			GSPP		FCFS		SWO		
	OFV	Gap	Time	OFV	Time (H=150, h=2)	OFV	RE	OFV	RE	Time
F1	1193.42	70.56%	-	920.73	506.02	1278.04	38.81%	1092.44	18.65%	169.53
F2	913.59	62.66%	-	863.43	127.91	1168.60	35.34%	911.47	5.56%	159.28
F3	+	+	+	1089.48	932.56	1784.98	63.84%	1411.24	29.53%	173.89
F4	902.74	59.15%	-	856.41	3341.62	1143.59	33.53%	1035.72	20.94%	160.113
F5	881.37	61.20%	-	786.27	137.91	973.94	23.87%	857.47	9.06%	163.41
F6	1121.14	66.39%	-	1015.53	2281.78	1628.76	60.39%	1286.73	26.71%	265.29
F7	922.04	62.05%	-	777.06	829.88	932.34	19.98%	932.34	19.98%	166.52
F8	728.48	52.93%	-	679.58	131.81	774.12	13.91%	745.00	9.63%	160.49
F9	934.35	58.59%	-	920.29	1767.57	1458.45	58.48%	1214.66	31.99%	171.97
Mean							38.68%		19.12%	

Results Analysis

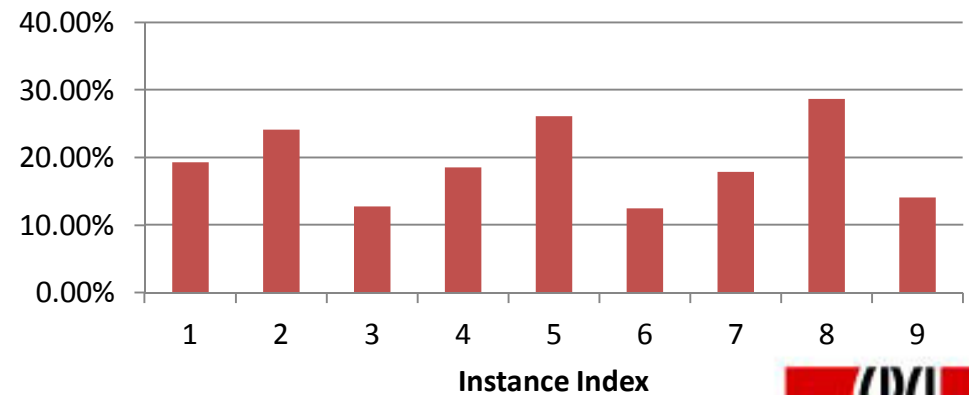
Percentage gap in optimal service times between 10x10 and 10x30



Percentage gap in optimal service times between 25x10 and 25x30



Percentage gap in optimal service times between 40x10 and 40x30



Disruption: Arrival Delay Scenario

Vessel	EAT	AAT
0	3	-1
1	3	-3
2	3	1
3	2	4
4	2	-2
5	2	4
6	2	4
7	1	0
8	4	12
9	2	1
10	2	4
11	0	4
12	1	-2
13	0	-2
14	1	-8
15	4	-4
16	4	-2
17	5	1
18	1	2
19	0	9
20	3	9
21	3	5
22	5	1
23	0	-2
24	1	10

Problem Definition: Real time recovery in BAP

- **Key Assumptions**

- **Vessel Priorities:** In practice, if a vessel with higher priority arrives late, it may still be given preference over a vessel with low service priority.
- **Release of information:** Each incoming vessel updates its exact arrival time a certain fixed time period τ before its actual arrival time, and once updated it does not change again.
- **Future vessel arrivals:** At any time instant t , the arrival time of an unassigned vessel $i \in N_v$ that is not updated is assumed equal to the expected arrival time A_i if current time t is less than $A_i - \tau$, or otherwise assumed equal to $t + \tau$. (The handling time restrictions are imposed accordingly.)