Integration of explicit supply-demand interactions in airline schedule planning and fleet assignment

Bilge Atasoy, Matteo Salani, Michel Bierlaire

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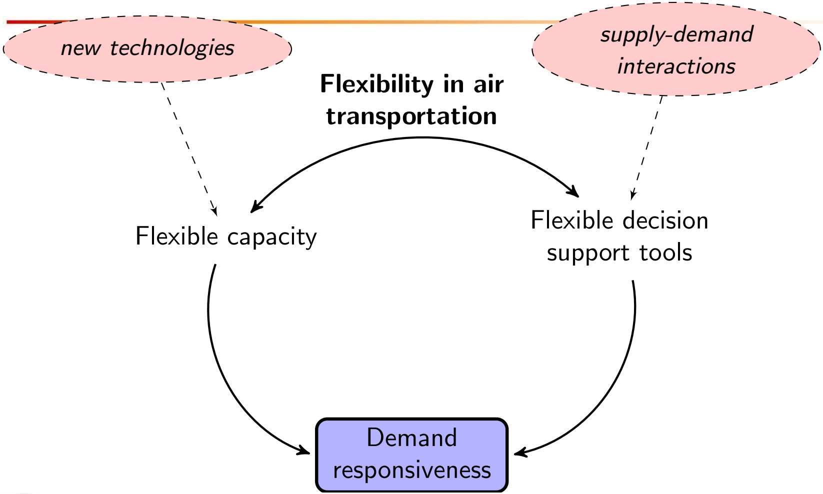
Motivation


new technologies
supply-demand interactions
Flexibility in air transportation
Flexible capacity
Flexible decision support tools
Demand responsiveness

Motivation

new technologies

supply-demand interactions

Flexibility in air transportation

Flexible decision support tools

Demand responsiveness

Today's talk

- Advance supply-demand interactions
  - Sensitivity analysis
- Solution methods for the integrated model
  - A local search heuristic
  - Log transformation of the logit model

Itinerary choice model

- Market segments, s, defined by the class and each OD pair
- Itinerary choice among the set of alternatives, I_s, for each segment s
- For each itinerary $i \in I_s$ the utility is defined by:

$$V_i = ASC_i + \beta_p \cdot \ln(p_i) + \beta_{time} \cdot time_i + \beta_{morning} \cdot morning_i$$

$$V_i = V_i(p_i, z_i, \beta)$$

  - ASC_i : alternative specific constant
  - p is the **only policy variable** and included as log
  - p and $time$ are interacted with non-stop/stop
  - $morning$ is 1 if the itinerary is a morning itinerary
- *No-revenue* represented by the subset $I_s^{'} \in I_s$ for segment s.

Estimation

- **Revealed preferences (RP) data:** Booking data from a major European airline
  - Lack of variability
  - Price inelastic demand
- RP data is combined with a **stated preferences (SP) data**
- Time, cost and morning parameters are **fixed** to be the same for the two datasets.
- A **scale** parameter is introduced for SP to capture the differences in variance.

Integrated airline scheduling, fleeting and pricing

Decision variables:

- $x_{k,f}$: binary, assignment of aircraft k to flight f
- $\pi^h_{k,f}$: allocated seats for class h on flight f aircraft k
- p_i: price of itinerary i
- d_i: demand of itinerary i
- $t_{i,j}$: spilled passengers from itinerary i to j

Integrated model - Scheduling & fleeting

$$Max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s')}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I_s')} t_{j,i} b_{j,i})p_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f} x_{k,f} : \text{ revenue - cost} \quad (1)$$

$$s.t. \sum_{k\in K} x_{k,f} = 1 : \text{ mandatory flights} \qquad \forall f \in F^M \quad (2)$$

$$\sum_{k\in K} x_{k,f} \leq 1 : \text{ optional flights} \qquad \forall f \in F^O \quad (3)$$

$$y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f} : \text{ flow conservation} \qquad \forall [k,a,t] \in N \quad (4)$$

$$\sum_{a\in A} y_{k,a,minE_a^-} + \sum_{f\in CT} x_{k,f} \leq R_k : \text{ fleet availability} \qquad \forall k \in K \quad (5)$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+} : \text{ cyclic schedule} \qquad \forall k \in K, a \in A \quad (6)$$

$$\sum_{h\in H} \pi^h_{k,f} = Q_k x_{k,f} : \text{ seat capacity} \qquad \forall f \in F, k \in K \quad (7)$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \quad (8)$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \quad (9)$$

- *Itinerary-based fleet assignment & Spill and recapture*
- Lohatepanont and Barnhart 2004

Integrated model - Revenue management - Pricing

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s')}\delta_{i,f}(d_i-\sum_{j\in I_s}t_{i,j}+\sum_{j\in(I_s\setminus I_s')}t_{j,i}b_{j,i})\leq\sum_{k\in K}\pi^h_{k,f}: \textit{ demand-capacity} \qquad \forall h\in H, f\in F \qquad (10)$$

$$\sum_{j\in I_s}t_{i,j}\leq d_i: \textit{ total spill} \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s') \qquad (11)$$

$$\tilde{d}_i=D_s\frac{\exp(V_i(p_i,z_i,\beta))}{\sum_{j\in I_s}\exp(V_j(p_j,z_j,\beta))}: \textit{ logit demand} \qquad \forall h\in H, s\in S^h, i\in I_s \qquad (12)$$

$$b_{i,j}=\frac{\exp(V_j(p_j,z_j,\beta))}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k(p_k,z_k,\beta))}: \textit{ recapture ratio} \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s'), j\in I_s \qquad (13)$$

$$d_i\leq\tilde{d}_i: \textit{ realized demand} \qquad \forall h\in H, s\in S^h, i\in I_s \qquad (14)$$

$$LB_i\leq p_i\leq UB_i: \textit{ bounds on price} \qquad \forall h\in H, s\in S^h, i\in I_s \qquad (15)$$

$$t_{i,j}, b_{i,j}\geq 0 \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s'), j\in I_s \qquad (16)$$

$$\pi^h_{k,f}\geq 0 \qquad \forall h\in H, k\in K, f\in F \qquad (17)$$

- Schön (2008): integration of pricing

Heuristic method

- Mixed Integer Non-convex Problem
- We devised a heuristic procedure based on two subproblems:

- FAM^{LS}: price-inelastic schedule planning model $\Rightarrow$ MILP
  - Prices fixed
  - Optimizes the schedule design and fleet assignment
- REV^{LS}: Revenue management with fixed capacity $\Rightarrow$ NLP
  - Schedule design and fleet assignment fixed
  - Optimizes the revenue

- Local search based on spill information

Data

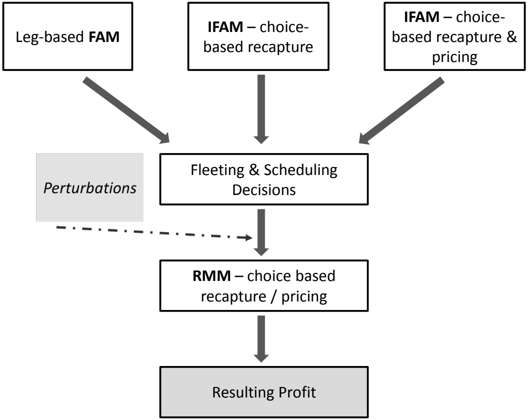
no	airports	flights	flights per route	demand per flight	fleet composition	
1	3	10	1.67	51.90	2	50-37
2	3	11	2.75	83.10	2	117-50
3	3	12	2.00	113.80	2	164-100
4	3	12	2.00	113.80	6	164-146-128-124-107-100
5	3	26	4.33	56.10	3	100-50-37
6	3	19	3.17	96.70	3	164-117-72
7	3	19	3.17	96.70	5	124-107-100-85-72
8	3	12	3.00	193.40	3	293-195-164
9	3	33	8.25	71.90	3	117-70-37
10	3	32	5.33	100.50	3	164-117-85
11	3	32	5.33	100.50	5	128-124-107-100-85
12	2	11	5.50	173.70	3	293-164-127
13	4	39	4.88	64.50	4	117-85-50-37
14	4	23	3.83	86.10	4	117-85-70-50
15	4	19	3.17	101.40	4	134-117-100-85
16	4	19	3.17	101.40	5	128-124-107-100-85
17	4	15	1.88	58.10	5	117-85-70-50-37
18	4	14	2.33	87.60	5	134-117-85-70-50
19	4	13	2.60	100.10	5	164-134-117-100-85
20	3	33	8.25	71.90	4	85-70-50-35
21	3	46	7.67	86.85	5	128-124-107-100-85
22	7	48	2.29	101.94	4	124-107-100-85
23	3	61	15.25	69.15	4	117-85-50-37
24	8	77	2.08	67.84	4	117-85-50-37
25	8	97	3.46	90.84	5	164-117-100-85-50

Data instances are derived from ROADEF 2009 dataset.

Computational results

	BONMIN Integrated model		Sequential approach (SA)			Local search heuristic *Average over 5 replications*			
	Profit	Time *(sec)*	Profit	% deviation from BONMIN	Time (sec)	Profit	%deviation from BONMIN	%impr. over SA	Time *(sec)*
1	15,091	2	15,091	0.00%	1	15,091	0.00%	0.00%	1
2	37,335	22	35,372	-5.26%	1	37,335	0.00%	5.55%	13
3	50,149	62	50,149	0.00%	1	50,149	0.00%	0.00%	1
4	46,037	2,807	43,990	-4.45%	1	46,037	0.00%	4.65%	3
5	70,904	1,580	69,901	-1.41%	1	70,679	-0.32%	1.11%	6
6	82,311	1,351	82,311	0.00%	1	82,311	0.00%	0.00%	1
7	87,212	32,400	84,186	-3.47%	1	87,212	0.00%	3.59%	60
8	779,819	8,137	779,819	0.00%	1	779,819	0.00%	0.00%	1
9	135,656	666	135,656	0.00%	2	135,656	0.00%	0.00%	2
10	107,927	482	107,927	0.00%	1	107,927	0.00%	0.00%	1
11	85,820	31,705	85,535	-0.33%	2	85,820	0.00%	0.33%	88
12	858,544	5,598	854,902	-0.42%	1	858,544	0.00%	0.43%	1
13	112,881	32,713	109,906	-2.64%	1	112,881	0.00%	2.71%	151
14	85,808	10,643	82,440	-3.93%	1	85,808	0.00%	4.09%	9
15	49,448	33	49,448	0.00%	1	49,448	0.00%	0.00%	1
16	38,205	240	37,100	-2.89%	1	38,205	0.00%	2.98%	1
17	27,076	35	27,076	0.00%	1	27,076	0.00%	0.00%	1
18	45,070	78	44,339	-1.62%	1	45,070	0.00%	1.65%	1
19	26,486	13	26,486	0.00%	1	26,486	0.00%	0.00%	1
20	146,773	30 846	146,464	-0.21%	1	147,506	0.50%	0.71%	406
21	194,987	4,963	210,134	7.77%	10	214,251	9.88%	1.96%	1,499
22	152,126	68,864	158,978	4.50%	2	159,258	4.69%	0.18%	39
23	227,643	40,862	226,615	-0.45%	12	227,284	-0.16%	0.30%	1,283
24	153,384	59,708	154,301	0.60%	4	158,099	3.07%	2.46%	2,314
25	313,943	82,780	331,920	5.73%	13	332,744	5.99%	0.25%	1,451

Sensitivity Analysis

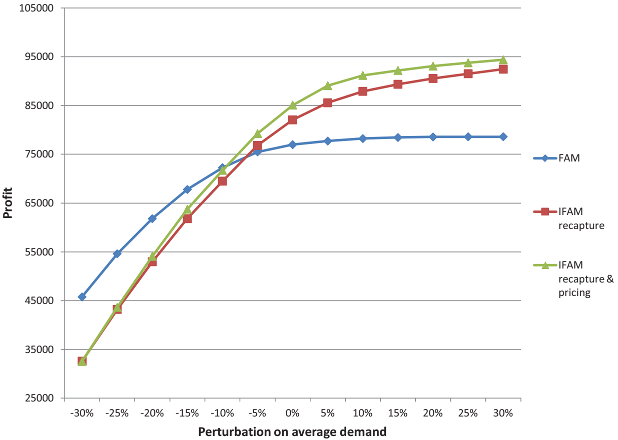


Sensitivity to demand fluctuations

- Total market segment demand is assumed to be known
- Fluctuations in reality

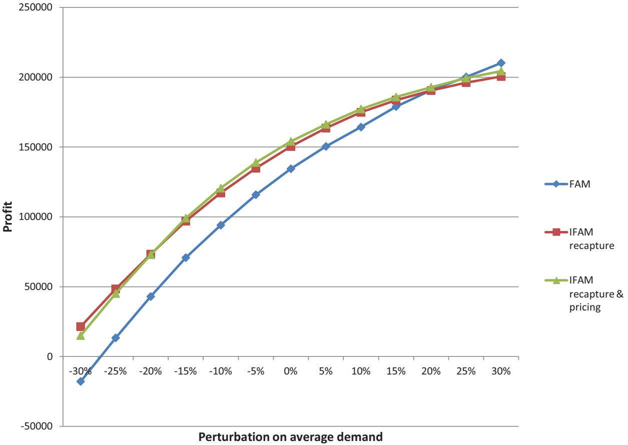
- Average demand is perturbed in a range [-30%, +30%]
- For each average demand 500 simulations with Poisson

Sensitivity to demand fluctuations



23 flights 4 aircraft types

Sensitivity to demand fluctuations



77 flights 4 aircraft types - heuristic solution

Transformation of the logit model

$$ms_i = \frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)}, \qquad V_i = \beta \ln(p_i) + c_i$$

A new variable υ_s and log transformation:

$$\upsilon_s = \frac{1}{\sum_{j \in I_s} \exp(V_j)}$$

$$\mathrm{ms}_i = \upsilon_s \exp(\beta \ln(p_i) + c_i)$$

$$\mathrm{ms}_i^{'} = \upsilon_s^{'} + \beta p_i^{'} + c_i,$$

Reformulated concave revenue subproblem

$$\max \sum_{s \in S} \sum_{i \in (I_s \setminus I_s')} (\ln(D_s) + \mathrm{ms}_i' + p_i') - \mathrm{penalty}(\mathrm{ms}_i - \exp(\mathrm{ms}_i'))^2 \tag{18}$$

$$\sum_{i \in I_s} \mathrm{ms}_i = 1 \qquad \forall s \in S \tag{19}$$

$$\mathrm{ms}_i' = \upsilon_s' + \beta p_i' + c_i \qquad \forall s \in S, i \in I_s \tag{20}$$

$$\sum_{s \in S} D_s \sum_{i \in (I_s \setminus I_s')} \delta_{i,f} \mathrm{ms}_i \leq \sum_{k \in K} Q_k X_{k,f} \qquad \forall f \in F \tag{21}$$

$$p_i' \leq \ln(UB_i) \qquad \forall s \in S, i \in (I_s \setminus I_s') \tag{22}$$

$$\mathrm{ms}_i' \in \Re \qquad \forall s \in S, i \in I_s \tag{23}$$

$$\mathrm{ms}_i \geq 0 \qquad \forall s \in S, i \in I_s \tag{24}$$

Reformulated concave revenue subproblem

$$\max \sum_{s\in S} \sum_{i\in(I_s\setminus I'_s)} (\ln(D_s) + \mathrm{ms}'_i + p'_i)$$

$$- \mathrm{penalty}(\mathrm{ms}_i - \exp(\mathrm{ms}'_i))^2 \quad (18)$$

$$\sum_{i\in I_s} \mathrm{ms}_i = 1 \qquad \forall s \in S \quad (19)$$

$$\mathrm{ms}'_i \leq \upsilon'_s + \beta p'_i + c_i \qquad \forall s \in S, i \in I_s \quad (20)$$

$$\sum_{s\in S} D_s \sum_{i\in(I_s\setminus I'_s)} \delta_{i,f}\mathrm{ms}_i \leq \sum_{k\in K} Q_k X_{k,f} \qquad \forall f \in F \quad (21)$$

$$p'_i \leq \ln(UB_i) \qquad \forall s \in S, i \in (I_s \setminus I'_s) \quad (22)$$

$$\mathrm{ms}'_i \in \Re \qquad \forall s \in S, i \in I_s \quad (23)$$

$$\mathrm{ms}_i \geq 0 \qquad \forall s \in S, i \in I_s \quad (24)$$

Conclusions

- The integrated model has promising results
- ... which motivates the effort in devising solution methodologies
- Logarithmic transformation provides a concave formulation of the revenue problem
- ... is expected to facilitate efficient solution methodologies

Ongoing work

- Logarithmic transformation provides a concave formulation of the revenue problem
- ... which is flexible for the integration of advanced demand models
- $\sum_{i \in I_s} (\ln(D_s) + \mathrm{ms}_i^{'} + p_i^{'}) \Rightarrow \sum_{i \in I_s} \sum_{n \in N} p_i^{'} + \mathrm{Prob}_{i,n}^{'}$
- Generalized framework for the integration of endogenous demand models

Ongoing work

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- Generalized framework for the integration of endogenous demand models

Thank you for your attention!

Itinerary choice model

- Market share and demand for itinerary i in market segment s:

$$\mathrm{ms}_i = \frac{\exp(V_i(p_i, z_i, \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j, \beta))} \quad \Rightarrow \quad d_i = D_s ms_i$$

  - D_s is the total expected demand for market segment s.

- **Spill and recapture effects**: Capacity shortage ⇒ passengers may be recaptured by other itineraries (instead of their desired itineraries)
- **Recapture ratio** is given by:

$$b_{i,j} = \frac{\exp(V_j(p_j, z_j, \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k, \beta))}$$

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- **Value of time (VOT):**

$$VOT_i = \frac{\partial V_i / \partial time_i}{\partial V_i / \partial cost_i} = \frac{\beta_{time} \cdot cost_i}{\beta_{cost}}$$

For the same OD pair...

- VOT for economy, non-stop: 8 €/hour
- VOT for economy, one-stop: 19.8, 11, 9.2 €/hour
- VOT for business, non-stop: 21.7 €/hour

Improvement due to the local search

	Sequential approach (SA)	Random neighborhood		Neighborhood based on spill		% Improvement	
	Profit	Profit	Time(sec)	Profit	Time(sec)	Quality of the solution	Reduction in time
2	35,372	37,335	116	37,335	13	-	89.10%
4	43,990	44,302	27	46,037	3	3.92%	88.88%
5	69,901	*No imp. over SA*		70,679	6	1.11%	-
7	84,186	85,335	1,649	87,212	60	2.20%	96.36%
8	904,054	906,791	209	906,791	2	-	99.04%
11	93,920	*No imp. over SA*		94,203	10	0.30%	-
12	854,902	*No imp. over SA*		858,545	1	0.43%	-
13	137,428	*No imp. over SA*		138,575	173	0.83%	-
14	93,347	96,365	943	96,486	89	0.13%	90.56%
16	37,100	38,205	6	38,205	1	-	80.65%
18	52,369	53,128	334	53,128	1	-	99.80%
20	146,464	*No imp. over SA*		147,506	380	0.71%	-
21	217,169	*No imp. over SA*		219,136	1,395	0.91%	-
22	163,114	*No imp. over SA*		163,393	126	0.17%	-
23	226,615	*No imp. over SA*		227,284	1,283	0.30%	-
24	208,561	*No imp. over SA*		210,395	791	0.88%	-
25	469,136	*No imp. over SA*		470,494	1,117	0.29%	-

A small example

- 2 airports CDG-MRS
- 4 flights - all are mandatory
- 2 aircraft types: 37-50 seats

We start with an initial FAM solution:

	AC1	AC2
F1	X	
F2	X	
F3	X	
F4	X	

A small example - GBD iterations

Iteration 1		
	Sub	Master
	12522.8	16923.4
	LB	**UB**
	12522.8	16923.4
	AC1	AC2
F1		X
F2		X
F3		X
F4		X

⟹

Iteration 2		
	Sub	Master
	10734.4	14822.8
	LB	**UB**
	12522.8	14822.8
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	

Iteration 3		
	Sub	Master
	12696.8	14822.8
	LB	**UB**
	12696.8	14822.8
	AC1	AC2
F1	X	
F2		X
F3		X
F4	X	

⟹

Iteration 4		
	Sub	Master
	12474.4	12696.8
	LB	**UB**
	12696.8	**12696.8**
	AC1	AC2
F1		X
F2		X
F3	X	
F4	X	