A DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL OF CAR OWNERSHIP AND USAGE

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• Introduction

• Background and data

• The dynamic discrete-continuous choice modeling framework
  • Assumptions
  • Definition of the components
  • Solving the dynamic programming problem
  • Model estimation

• Illustration of model application

• Conclusion and future works
Aim of the research project:

- Model dynamics of car transactions, usage and choice of fuel type in the Swedish car fleet

- Motivations
  - Governmental policies:
    - Goals of reducing carbon emissions
  - Technology changes:
    - Increase of alternative-fuel vehicles
    - Changes in the supply
  - Company cars: represent important share of new car sales
Current literature on car ownership and usage modeling:

• **Car ownership models in transportation literature:**
  • Mostly **static models:**
    • Main drawback: do not account for forward-looking behavior
    • Important aspect to account for since car is a durable good
  • **Econometric literature: dynamic programming (DP) models + discrete choice models (DCM)**
  • Recently, **dynamic discrete choice models (DDCM)** starting to be applied in transportation field (Cirillo and Xu, 2011; Schiraldi, 2011)

• **Joint models of car ownership and usage:**
  • Early references: e.g. **duration models** and regression techniques for car holding duration and usage (De Jong, 1996)
  • **Dynamic programming mixed logit (DPMXL)** (Schjerning, 2007) used to model car ownership, type of car and usage (Munk-Nielsen, 2012)
  • **Discrete-continuous model** of vehicle choice and usage based on register data (Gillingham, 2012)
Research issues:

• Car are durable goods $\implies$ Need to account for forward-looking behavior of individuals

• Difficulty of modeling a discrete-continuous choice

• Many models focus on individual decisions, but choices regarding car ownership and usage made at household level
Research issues:

- Car are durable goods $\Rightarrow$ Need to account for **forward-looking behavior** of individuals
- Difficulty of modeling a **discrete-continuous choice**
- Many models focus on individual decisions, but choices regarding car ownership and usage made at **household level**

Proposed methodology:

- Attempt to address these issues by applying **dynamic discrete-continuous choice model (DDCCM)**
- Large **register data** of all **individuals** and **cars** in Sweden
BACKGROUND AND DATA

Register data of Swedish population and car fleet:

- Data from 1998 to 2008

- All individuals
  - **Individual information**: socio-economic information on car holder (age, gender, income, home/work location, employment status/sector, etc.)
  - **Household information**: composition (families with children and married couples)

- All vehicles
  - Privately-owned cars, cars from privately-owned company and **company** cars
  - Vehicle **characteristics** (make, model, fuel consumption, fuel type, age)
  - **Annual mileage** from odometer readings
  - Car bought **new or second-hand**
Aim of the project:

• Model simultaneously car ownership, usage and fuel type.

In details: model simultaneous choice of

\[
\text{Transaction type} \times \left[ \text{Annual milage} - \text{car } c \right] \times \left[ \text{Private/ company car} - \text{car } c \right] \times \left[ \text{Fuel type} - \text{car } c \right] \times \left[ \text{New/2}^{\text{nd}} \text{ hand} - \text{car } c \right] \times # \text{ cars}
\]
Aim of the project:

- Model simultaneously car ownership, usage and fuel type.

In details: model simultaneous choice of:

- Transaction type
- Annual milage – car $c$
- Private/company car – car $c$
- Fuel type – car $c$
- New/2$^{nd}$ hand – car $c$
- # cars

Discrete variables
Aim of the project:

- Model simultaneously car ownership, usage and fuel type.

In details: model simultaneous choice of

Transaction type \( \times \) Annual milage – car \( c \) \( \times \) Private/ company car – car \( c \) \( \times \) Fuel type – car \( c \) \( \times \) New/2\(^{nd}\) hand – car \( c \) \( \times \) # cars

Continuous variables
Motivations for discrete-continuous vs discrete model

- **Mileage** variable(s) are **continuous**: lose information by discretizing it.

- In a discrete-continuous approach:

  If choice of mileage **conditional** on the discrete choice

  Reduction of size of the discrete action space
• Decisions at household level: up to 2 cars in household

• **Strategic choice** of:
  • Transaction
  • Type(s) of ownership (company vs private car)
  • Fuel type(s)
  • Car state(s) (new vs 2\textsuperscript{nd}-hand)

  \(\Rightarrow\) Account for forward-looking behavior of households

• **Myopic choice** of:
  • Annual mileage(s)

• **Choice of mileage conditional** on choice of discrete variables
• **Agent**: household

• **Time step** \( t \): year

• **State space** \( S \)

\[
s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})
\]

\[
|S| = (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 + (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)
\]

\[
+ 1.
\]
1. **Agent**: household

2. **Time step** $t$: year

3. **State space** $S$

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

- **Age** – 1st car
- **Private/company car** – 1st car
- **Fuel type** – 1st car
- **Age** – 2nd car
- **Private/company car** – 2nd car
- **Fuel type** – 2nd car

$$|S| = \left( |Y| \times (|I_C| - 2) \times (|F| - 1) + 1 \right)^2$$

2-car households

$$+ \left( |Y| \times (|I_C| - 2) \times (|F| - 1) + 1 \right)$$

$$+ 1.$$
Agent: household

Time step $t$: year

State space $S$

\[ s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t}) \]

\[ |S| = (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 \]

+ 1-car households

\[ + (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1) \]

+ 1.
Agent: household

Time step $t$: year

State space $S$

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

$$|S| = (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2$$

$$+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)$$

$$+ 1.$$ 0-car households
• Agent: household

• Time step $t$: year

• State space $S$

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

$$|S| = (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2$$

$$+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)$$

$$+ 1 = 1407 \Rightarrow \text{relatively small state space}$$
• Action space $A$

$$a_t = (h_t, \tilde{m}_{1,t}, \tilde{I}_{1,t}, \tilde{f}_{1,t}, \tilde{r}_{1,t}, \tilde{m}_{2,t}, \tilde{I}_{2,t}, \tilde{f}_{2,t}, \tilde{r}_{2,t})$$
• Action space $A$

Transaction type: details

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL
DEFINITION OF THE COMPONENTS
### Action space $A$

<table>
<thead>
<tr>
<th>Transaction name</th>
<th>0 car</th>
<th>1 car</th>
<th>2 cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$: leave unchanged</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$h_2$: increase 1</td>
<td>18</td>
<td>18</td>
<td>-</td>
</tr>
<tr>
<td>$h_3$: dispose 2</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$h_4$: dispose 1st</td>
<td>-</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$h_5$: dispose 2nd</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$h_6$: dispose 1st and change 2nd</td>
<td>-</td>
<td>-</td>
<td>18</td>
</tr>
<tr>
<td>$h_7$: dispose 2nd and change 1st</td>
<td>-</td>
<td>-</td>
<td>18</td>
</tr>
<tr>
<td>$h_8$: change 1st</td>
<td>-</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>$h_9$: change 2nd</td>
<td>-</td>
<td>-</td>
<td>18</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>19</strong></td>
<td><strong>38</strong></td>
<td><strong>76</strong></td>
</tr>
</tbody>
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Sum: 19 38 76

Size of the **discrete part** of the action space assuming:
- 3 levels company car
- 3 levels fuel type
- 2 levels new/2nd hand

Maximum size of action space $<<$ for DDCM
• **Transition rule**: deterministic rule: each state $s_{t+1}$ can be inferred exactly once $s_t$ and $a_t$ are known.

**Example:**

If $s_t = [2,1,2,0,0,0]$ and $a_t = [1,12'000,0,0,0,0,3,0,0]$ then $s_{t+1} = [3,1,2,0,3,0]$. 
• Instantaneous utility function:

\[ u(s_t, a_t^C, a_t^D, x_t, \theta) = v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta) + \varepsilon_D(a_t^D) \]
• Instantaneous utility function:

\[ u(s_t, a_t^C, a_t^D, x_t, \theta) = v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C, \theta)) + \varepsilon_D(a_t^D) \]

- Deterministic term
- Random term
• Instantaneous utility function:

\[ u(s_t, a^C_t, a^D_t, x_t, \theta) = v(s_t, a^C_t, a^D_t, x_t, \varepsilon_C(a^C_t), \theta) + \varepsilon_D(a^D_t) \]

Assume additive **deterministic utility** for simplicity (see also Munk-Nielsen, 2012):

\[ v(s_t, a^C_t, a^D_t, x_t, \varepsilon_C(a^C_t), \theta) = v^D_t(s_t, a^D_t, x_t, \theta) + v^C_t(s_t, a^D_t, a^C_t, x_t, \varepsilon_C(a^C_t), \theta) \]
Instantaneous utility function:

\[ u(s_t, a_t^C, a_t^D, x_t, \theta) = v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta) + \varepsilon_D(a_t^D) \]

**Deterministic term**

**Random term**

Assume additive **deterministic utility** for simplicity (see also Munk-Nielsen, 2012):

\[ v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta) = v_t^D(s_t, a_t^D, x_t, \theta) + v_t^C(s_t, a_t^D, a_t^C, x_t, \varepsilon_C(a_t^C), \theta) \]

**Utility for discrete actions**

**Utility for continuous actions**
Instantaneous utility function:

\[ u(s_t, a_t^C, a_t^D, x_t, \theta) = v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta) + \varepsilon_D(a_t^D) \]

Define:

- Deterministic term: \[ v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta) \]
- Random term: \[ \varepsilon_D(a_t^D) \]

Assume additive deterministic utility for simplicity (see also Munk-Nielsen, 2012):

\[ v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta) = v_t^D(s_t, a_t^D, x_t, \theta) + v_t^C(s_t, a_t^D, a_t^C, x_t, \varepsilon_C(a_t^C), \theta) \]

- Utility for discrete actions: \[ v_t^D(s_t, a_t^D, x_t, \theta) \]
- Utility for continuous actions: \[ v_t^C(s_t, a_t^D, a_t^C, x_t, \varepsilon_C(a_t^C), \theta) \]
Instantaneous utility function

Utility for continuous actions: **Constant elasticity of substitution (CES) utility function:**

- Captures substitution patterns between the choice of both annual driving distances
- $\rho$ is elasticity of substitution

$$v_t^C(s_t, a_t^D, a_t^C, x_t, \varepsilon_C(a_t^C), \theta) = (m_1 t^{-\rho} + \alpha \cdot m_2 t^{-\rho})^{-1/\rho}$$

- Randomness introduced in $\alpha := \exp\{\gamma x_t - \varepsilon_C(a_t^C)\}$

- $x_t$ contains price of fuel, car consumption and other socio-economic characteristics
1. Finding the optimal value(s) of annual mileage conditional on the discrete choices

2. Solving the Bellman equation
Finding the optimal value(s) of mileage

• Maximization of the continuous utility:
  \[
  \max_{m_{1,t},m_{2,t}} v_t^C
  \]
  \[\text{s.t. } p_{1,t}m_{1,t} + p_{2,t}m_{2,t} = \text{Inc}_t\]

• Find analytical solutions: \(m_{1,t}^*, \text{ and } m_{2,t}^*\)

\[
m_{2,t}^* = \frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))} \alpha^{(-1/(\rho+1))}}
\]

\[
m_{1,t}^* = \frac{\text{Inc}_t}{p_{1,t}} - \frac{p_{2,t}m_{2,t}^*}{p_{1,t}}
\]

• Optimal continuous utility

\[
v_t^{C*}(s_t, d_t^D, d_t^{C*}, x_t, \theta)
\]
Solving the Bellman equation

• **Logsum** formula used in the completely discrete case (DDCM) (Aguirregabiria and Mira, 2010; Cirillo and Xu, 2011)

• Logsum can be applied here given the **key assumptions**:  
  • Choice of mileage(s) is conditional on discrete actions  
  • Choice of mileage(s) is myopic

\[
\bar{V}(s_t, x_t, \theta) = \log \sum_{a_t^D} \left\{ \exp\{ v_t^D(s_t, a_t^D, x_t, \theta) + v_t^C(s_t, a_t^D, a_t^C, x_t, \theta) \} + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1}|s_t, a_t) \right\}
\]

• Iterate on **Bellman equation** to find integrated value function \( \bar{V} \)
• Parameters obtained by maximizing likelihood:

\[ L = \prod_{n=1}^{N} \prod_{t=1}^{T_n} P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta) \]

• Optimization algorithm is Rust’s nested fixed point algorithm (NFXP) (Rust, 1987):

  • **Outer optimization algorithm**: search algorithm to obtain parameters maximizing likelihood

  • **Inner value iteration algorithm**: solves the DP problem for each parameter trial

• Plan to investigate variants of NFXP to speed up computational time
Assumptions for the example:

- **Size state space = 651**
  - Max age = 3
  - Company car levels = 3
  - Number of fuel types = 3

- **Size action space = max 745**
  - Number of transaction types = 9
  - Number of state levels (new/old) = 2

- **Utility function contains:**
  - Transaction cost $\tau$
  - Transaction-dependent parameters for age of oldest car

$$v_t^D(s_t, a_t^D, x_t, \theta) = \tau(a_t^D) + \beta_{\text{Age}}(a_t^D, s_t) \cdot \max(\text{Age 1}_t, \text{Age 2}_t)$$

- **Parameters of DP problem:**
  - Discount factor $\beta = 0.7$
  - Stopping criterion $\varepsilon = 0.01$
Program:

- Code in C++
- 2 minutes on 20-core server

Graph:

- $V$ vs age of oldest of cars in household fleet
ILLUSTRATION OF MODEL APPLICATION

\[ P(a_{n,t}^D | x_{n,t}, s_{n,t}, \theta) = \frac{v_{n,t}^D + v_{n,t}^{C^*} + \beta \sum_{s_{n,t+1} \in S} \tilde{V} f}{\sum_{a_{n,t}^D} \left\{ v_{n,t}^D + v_{n,t}^{C^*} + \beta \sum_{s_{n,t+1} \in S} \tilde{V} f \right\}} \]
ILLUSTRATION OF MODEL APPLICATION

Instantaneous utility

\[
P(\alpha_{n,t}^D | s_{n,t}, x_{n,t}, \theta) = \frac{v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \tilde{V} f}{\sum_{\alpha_{n,t}} \left( v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \tilde{V} f \right)}
\]
ILLUSTRATION OF MODEL APPLICATION

\[ P(\alpha_{n,t}^D | s_{n,t}, x_{n,t}, \theta) = \frac{v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \tilde{V} f}{\sum_{\alpha_{n,t}^D} \left( v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \tilde{V} f \right)} \]
Conclusion:

• Methodology to model choice of car ownership and usage dynamically
• Example of application shows feasibility of approach

Next steps:

• Exploratory analysis to specify instantaneous utility
• Model estimation on small sample of synthetic data
• Model estimation on register data
• Scenario testing:
  • Validation of policy measures taken during the years available in the data
  • Test policy measures that are planned to be applied in future years
Thanks!