### **ACCOUNTING FOR EXPECTATIONS ABOUT THE FUTURE**

# A DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL (DDCCM) OF CAR OWNERSHIP, USAGE AND FUEL TYPE

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# **OUTLINE**

- Introduction
- Background and data
- The dynamic discrete-continuous choice modeling framework
- Illustration of model application
- Conclusion and future works





### INTRODUCTION

#### Aim of the research:

- Model dynamics of car transactions, usage and choice of fuel type in the Swedish car fleet
- Motivations
  - Governmental policies to reduce carbon emissions / car usage:
    - Stockholm congestion tax
    - Independence of fossil fuels
  - Technology changes:
    - Increase of alternative-fuel vehicles
  - Economical features:
    - Financial crisis
    - Fuel price changes
  - Car ownership and usage vary importantly over time.
  - Model needed to analyze and predict impact of policies on ownership





### Register data of Swedish population and car fleet:

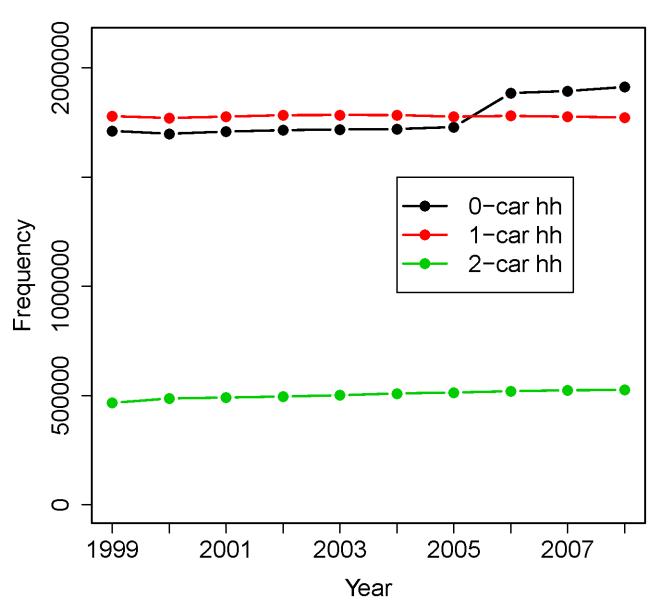
- Data from 1998 to 2008
- All individuals
  - **Individual information**: socio-economic information on car holder (age, gender, income, home/work location, employment status/sector, etc.)
  - Household information: composition (families with children and married couples)
- All vehicles
  - Privately-owned cars, cars from privately-owned company and company cars
  - Vehicle characteristics (make, model, fuel consumption, fuel type, age)
  - Annual mileage from odometer readings
  - Car bought new or second-hand





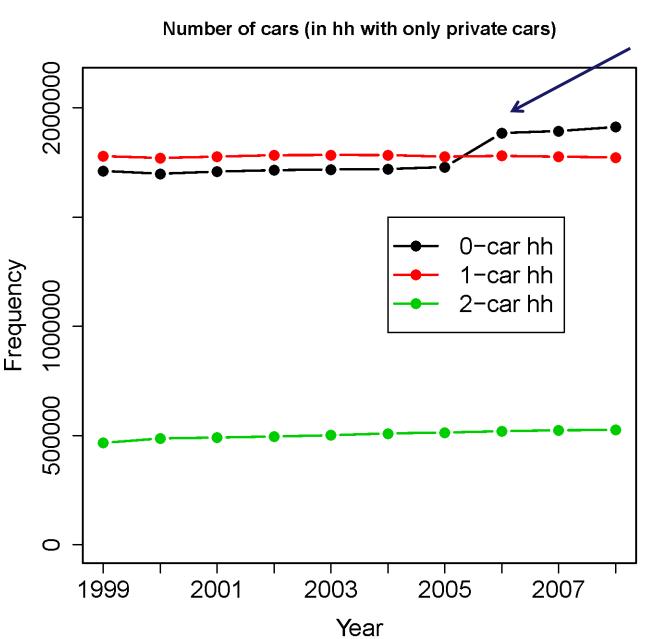
**OWNERSHIP** 

Number of cars (in hh with only private cars)









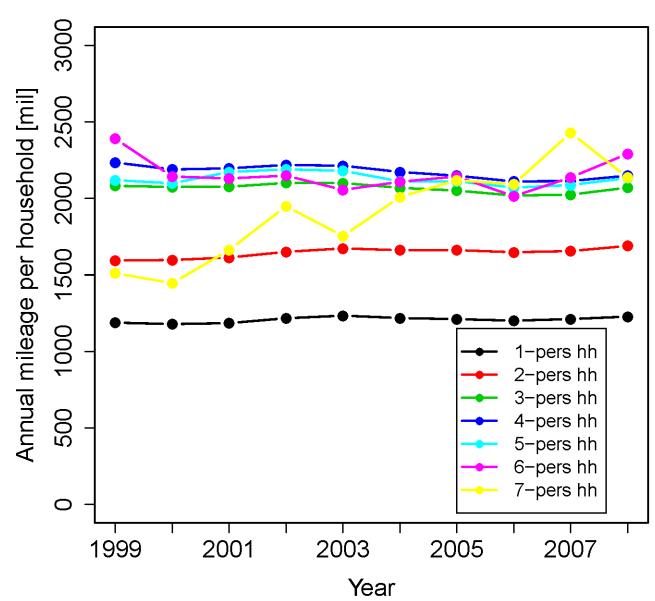






**USAGE** 

#### Household annual mileage per household size







#### LITERATURE

- Car ownership models in transportation literature:
  - Discrete choice models (DCM) widely used, but mostly **static models**.
    - Main drawback: do not account for forward-looking behavior
    - Important aspect to account for since car is a durable good
  - Econometric literature: dynamic programming (DP) models + DCM
  - Recently, dynamic discrete choice models (DDCM) starting to be applied in transportation field (Cirillo and Xu, 2011; Schiraldi, 2011)
- Joint models of car ownership and usage:
  - Duration models and regression techniques for car holding duration and usage (De Jong, 1996)
  - Vehicle type, usage and replacement decisions using dynamic programming, discrete-continuous, mixed logit (Schjerning, 2008, and Munk-Nielsen, 2012)
  - Discrete-continuous model of vehicle choice and usage based on register data (Gillingham, 2012)
- Wide literature on car ownership and usage models





#### RESEARCH ISSUES

- Car are durable goods 

   Need to account for forward-looking behavior of agents
- Difficulty of modeling a discrete-continuous choice when jointly modeling car ownership and usage
- Many models focus on individual decisions, but choices regarding car ownership and usage made at household level





#### RESEARCH ISSUES

- Car are durable goods 

  Need to account for forward-looking behavior of agents
- Difficulty of modeling a discrete-continuous choice when jointly modeling car ownership and usage
- Many models focus on individual decisions, but choices regarding car ownership and usage made at household level

### **Proposed methodology:**

 Attempt to address these issues by applying dynamic discrete-continuous choice model (DDCCM)



Large register data of all individuals and cars in Sweden



#### MAIN FEATURES

- In the area of dynamic choice modeling
  - Choices modeled at household level
  - Up to two cars allowed
- Constant elasticity of substitution (CES) utility to model annual driving distance for 2-car households
- Several choices modeled simultaneously





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>12</sup>

#### THE CHOICE VARIABLE IN DETAILS

### **Objective**

Model simultaneously car ownership, usage and fuel type.

In details: model simultaneous choice of

Transaction type 
$$\times$$
  $\begin{bmatrix} Annual & Private/company & Fuel type - car c & car c$ 





# cars

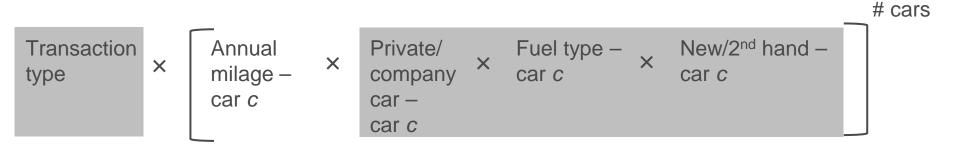
# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>13</sup>

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Discrete variables





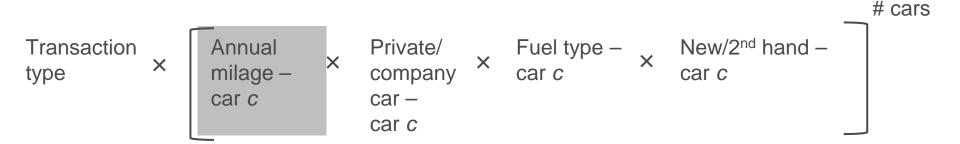
# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>14</sup>

#### THE CHOICE VARIABLE IN DETAILS

### **Objective**

Model simultaneously car ownership, usage and fuel type.

In details: model simultaneous choice of



**Continuous variables** 





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>15</sup>

**ASSUMPTIONS** 

- 1. Choice at household level: up to 2 cars in household
- 2. Strategic choice of:
  - Transaction
  - Type(s) of ownership (company vs private car)
  - Fuel type(s)
  - Car state(s) (new vs 2<sup>nd</sup>-hand)
  - Account for forward-looking behavior of households
- 3. Myopic choice of:
  - Annual mileage(s)
- 4. Choice of mileage conditional on choice of discrete variables





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>16</sup>

**ASSUMPTIONS** 

### Myopic choice (static case)

$$P(\text{action}) = \frac{\exp\{\text{instantaneous utility}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities}\}}$$

### Strategic choice (dynamic case)

$$P(\text{action}) = \frac{\exp\{\text{instantaneous utility} + \text{expected discounted utility of future choices}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities} + \text{expected discounted utilities of future choices}\}}$$





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>17</sup>

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Embeds a choice model into a dynamic programming framework





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>18</sup>

#### **DEFINITION OF THE COMPONENTS**

### Components of the DDCCM:

- Agent
- Time step
- State space
- Action space
- Transition rule
- Instantaneous utility function

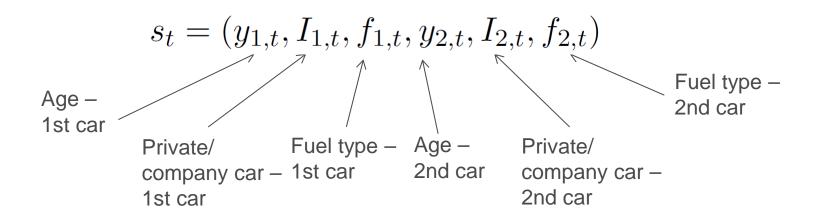




# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>19</sup>

#### **DEFINITION OF THE COMPONENTS**

- Agent: household
- Time step *t*: year
- State space S



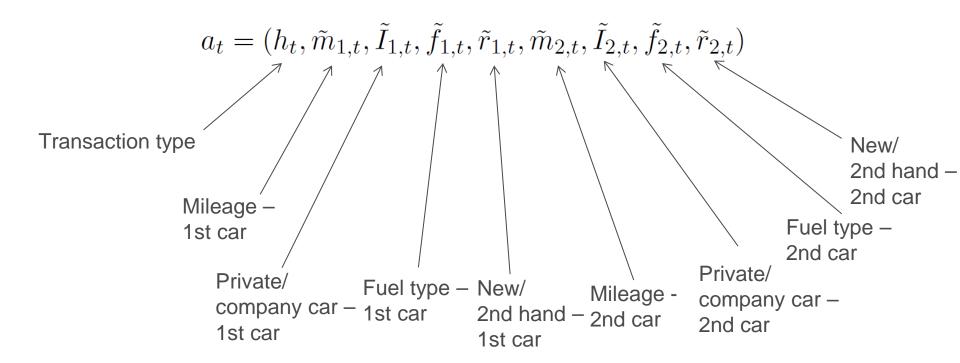




# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>20</sup>

#### **DEFINITION OF THE COMPONENTS**

Action space A





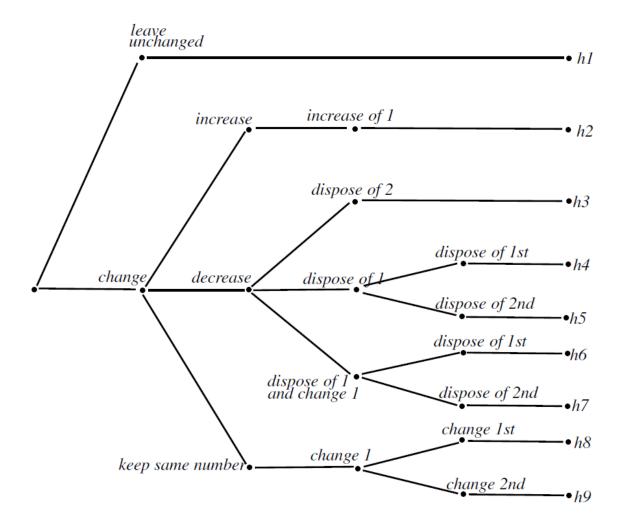


# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>21</sup>

#### **DEFINITION OF THE COMPONENTS**

Action space A

Transaction types







# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>22</sup>

#### **DEFINITION OF THE COMPONENTS**

• Transition rule: deterministic rule: each state  $s_{t+1}$  can be inferred exactly once  $s_t$  and  $a_t$  are known.





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>23</sup>

#### **DEFINITION OF THE COMPONENTS**

- Transition rule: deterministic rule: each state  $s_{t+1}$  can be inferred exactly once  $s_t$  and  $a_t$  are known.
- Instantaneous utility function:

$$u(s_t, a_t^C, a_t^D, x_t, \theta) = v(s_t, a_t^C, a_t^D, x_t, \theta) + \mathcal{E}_D(a_t^D)$$
Deterministic term Random term for discrete choices

Assume additive deterministic utility for simplicity (see also Munk-Nielsen, 2012):

$$v(s_t, a_t^C, a_t^D, x_t, \theta) = v_t^D(s_t, a_t^D, x_t, \theta) + v_t^C(s_t, a_t^D, a_t^C, x_t, \theta)$$
Utility for discrete actions
Utility for continuous actions





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>24</sup>

#### **DEFINITION OF THE COMPONENTS**

- Instantaneous utility function
  - Utility for continuous actions:
     Constant elasticity of substitution (CES) utility function (e.g. Zabalza, 1983):

$$v_t^C(s_t, a_t^D, a_t^C, x_t, \theta) = (m_{1,t}^{-\rho} + \alpha \cdot m_{2,t}^{-\rho})^{-1/\rho}$$

- Captures substitution patterns between the choice of both annual driving distances
- $\rho$  = elasticity of substitution
- $\alpha$  = share parameter
- Formulation could be extended to introduce randomness in α.





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>25</sup>

SOLVING THE DYNAMIC PROGRAMMING PROBLEM

1. Finding the optimal value(s) of annual mileage conditional on the discrete choices

2. Solving the Bellman equation to find expected utility of future choices (value function)





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>26</sup>

#### SOLVING THE DYNAMIC PROGRAMMING PROBLEM

### Finding the optimal value(s) of mileage

• Maximization of the continuous utility:  $\max_{m_{1,t},m_{2,t}} v_t^C$  s.t.  $p_{1,t}m_{1,t} + p_{2,t}m_{2,t} = \mathrm{Inc}_t$ 

• Find analytical solutions:  $m_{1,t}^*$  and  $m_{2,t}^*$ 

$$m_{2,t}^* = \frac{\operatorname{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))} \alpha^{(-1/(\rho+1))}}$$

$$m_{1,t}^* = \frac{\operatorname{Inc}_t}{p_{1,t}} - \frac{p_{2,t}}{p_{1,t}} m_{2,t}^*$$

• Optimal continuous utility  $v_t^{C*}(s_t, a_t^D, a_t^{C*}, x_t, \theta)$ 





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>27</sup>

#### SOLVING THE DYNAMIC PROGRAMMING PROBLEM

### Solving the Bellman equation

- Logsum formula used in the completely discrete case (DDCM)
   (Aguirregabiria and Mira, 2010; Cirillo and Xu, 2011)
- Logsum can be applied here given the key assumptions:
  - Choice of mileage(s) is conditional on discrete actions
  - Choice of mileage(s) is myopic

$$\bar{V}(s_t, x_t, \theta) = \log \sum_{a_t^D} \left\{ \exp\{v_t^D(s_t, a_t^D, x_t, \theta) + v_t^{C*}(s_t, a_t^D, a_t^{C*}, x_t, \theta)\} + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1} | s_t, a_t) \right\}$$

• Iterate on Bellman equation to find integrated value function  $\overline{V}$ 





# DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL<sup>28</sup>

MODEL ESTIMATION

Parameters obtained by maximizing likelihood:

$$\mathcal{L} = \prod_{n=1}^{N} \prod_{t=1}^{T_n} P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta)$$

- Optimization algorithm is Rust's nested fixed point algorithm (NFXP) (Rust, 1987):
  - Outer optimization algorithm: search algorithm to obtain parameters maximizing likelihood
  - Inner value iteration algorithm: solves the DP problem for each parameter trial
- Plan to investigate variants of NFXP to speed up computational time (e.g. swapped algorithm from Aguirregabiria and Mira, 2002)





### **Assumptions for the example:**

Deterministic utility function

$$v_t^D(s_t, a_t^D, x_t, \theta) = C(s_t) + \tau(a_t^D) + \beta_{Age}(a_t^D, s_t) \cdot \max(Age1_t, Age2_t)$$

Constant for households with at least one car

Transaction costs

Transaction-dependant parameters relative to age of oldest car

Chose arbitrary values for parameters





			$eta_{ m Age}$		τ
Transaction name	Case	0 car	1 car	2 cars	all households
$h_1$ : leave unchanged		0	-1	-1	0
$h_2$ : increase 1		0	0	-	-3
$h_3$ : dispose 2		-	-	1	0
h <sub>4</sub> : dispose 1st	1st car is oldest	-	1.5	1.5	0
	2nd car is oldest	-	-	0	0
h <sub>5</sub> : dispose 2nd	1st car is oldest	-	-	0	0
	2nd car is oldest	-	-	1.5	0
$h_6$ : dispose 1st and change 2nd		-	-	0	-4
$h_7$ : dispose 2nd and change 1st		-	-	0	-4
h <sub>8</sub> : change 1st	1st car is oldest	-	1.5	1.5	-4
	2nd car is oldest	-	_	0	-4
h <sub>9</sub> : change 2nd	1st car is oldest	-	_	0	-4
	2nd car is oldest	_	_	1.5	-4





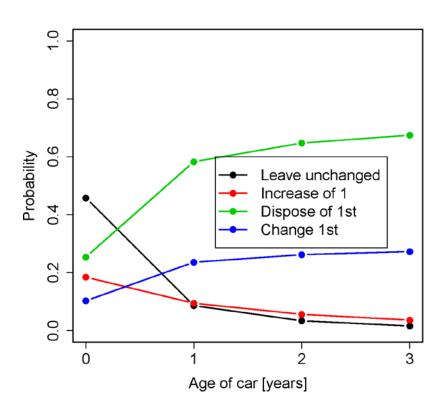
### **Assumptions for the example:**

- Visualize choice probabilities for one observation:
  - 1-car household
  - Annual income = 530'000 SEK (≈ 58300 €; 84800 AUD)
  - 8% expenses on fuel

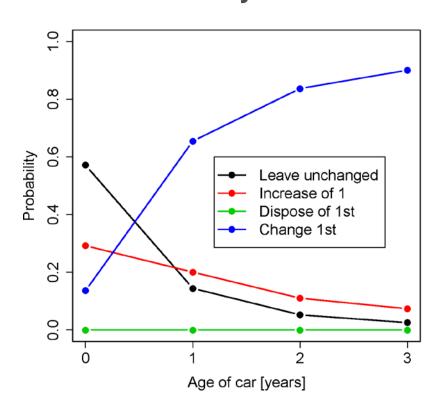




### Static



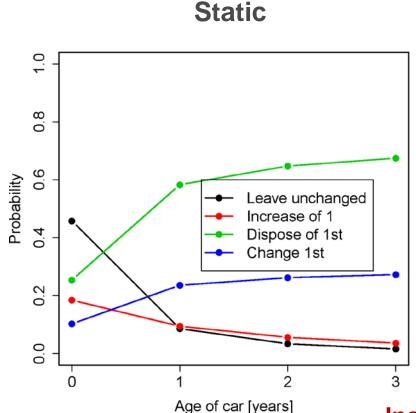
# FROM MODEL APPLICATION **Dynamic**

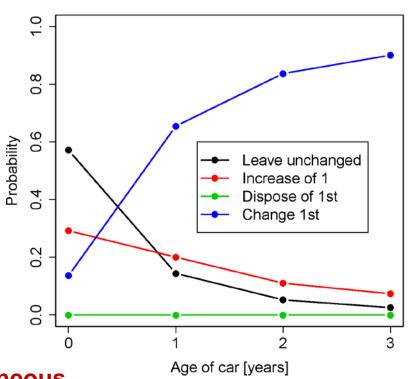






# FROM MODEL APPLICATION **Dynamic**





Instantaneous

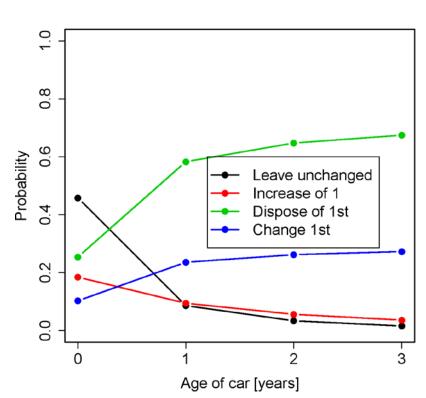
$$P(a_{n,t}^{D}|s_{n,t},x_{n,t},\theta) = \frac{v_{n,t}^{D} + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V}f}{\sum_{a_{n,t}^{\tilde{D}}} \left\{ v_{n,t}^{D} + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V}f \right\}}$$

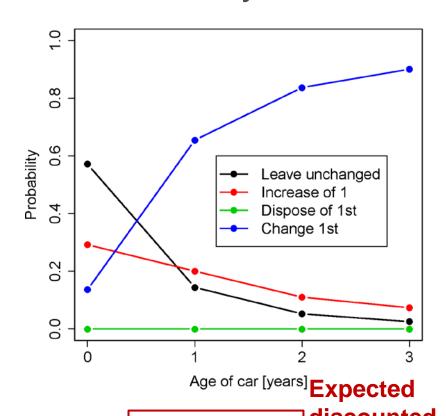




# FROM MODEL APPLICATION **Dynamic**

#### **Static**





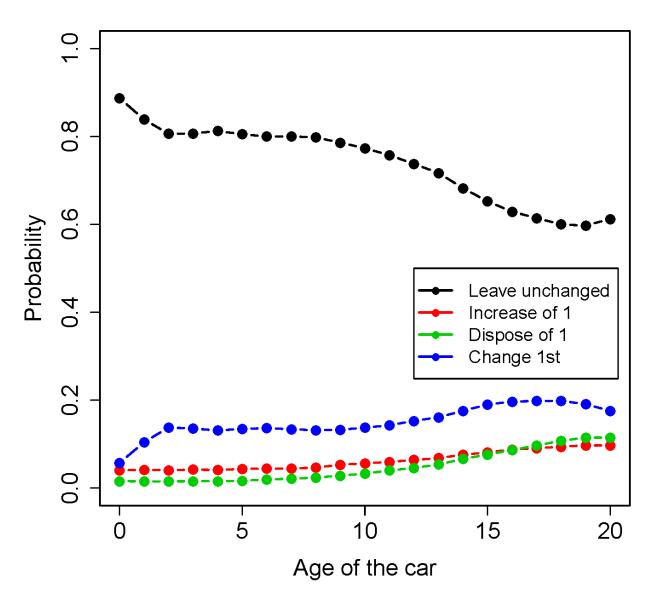
$$P(a_{n,t}^D|s_{n,t},x_{n,t},\theta) = \frac{v_{n,t}^D + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V}f}{\sum_{a_{n,t}^D} \left\{ v_{n,t}^D + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V}f \right\}} \frac{\text{discounted utility}}{\sum_{a_{n,t}^D} \left\{ v_{n,t}^D + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V}f \right\}}$$





#### FROM REGISTER DATA

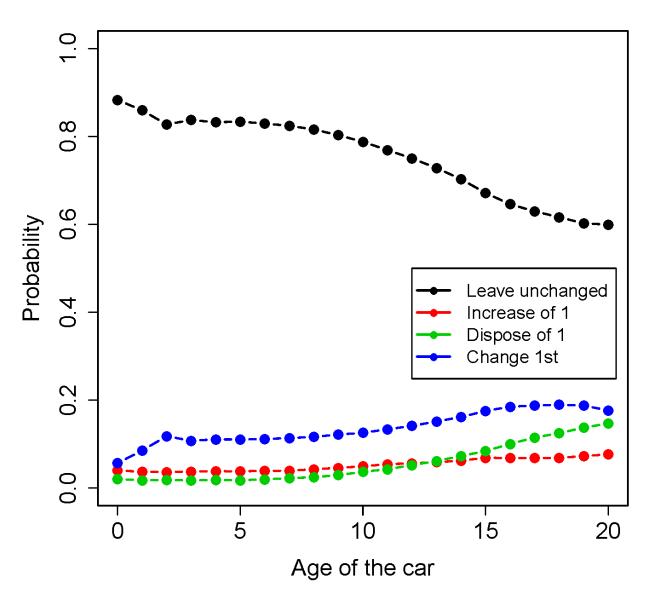
Transitions 1999-2000: 1-car households







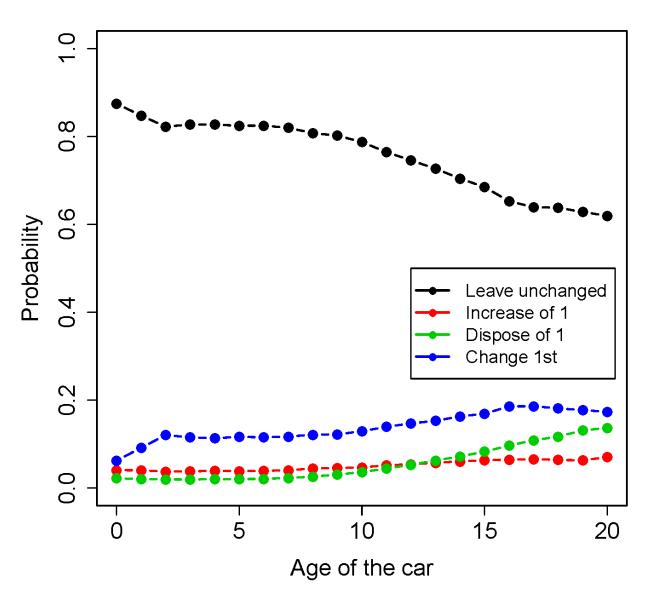
Transitions 2000-2001: 1-car households







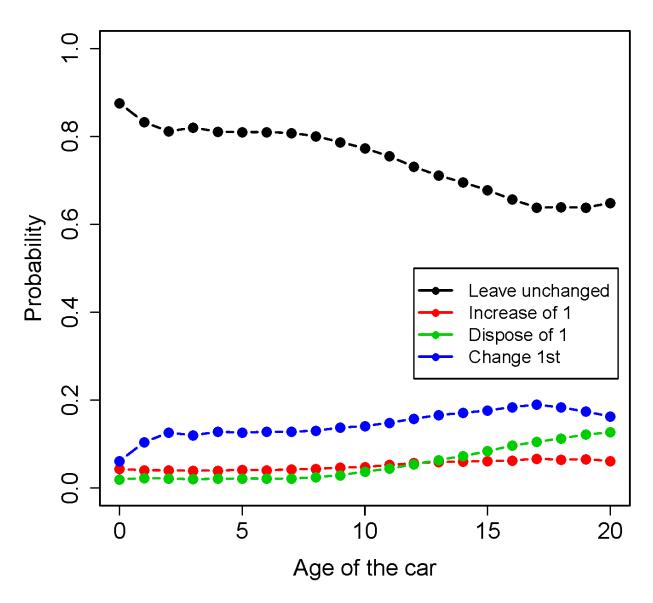
Transitions 2001-2002: 1-car households







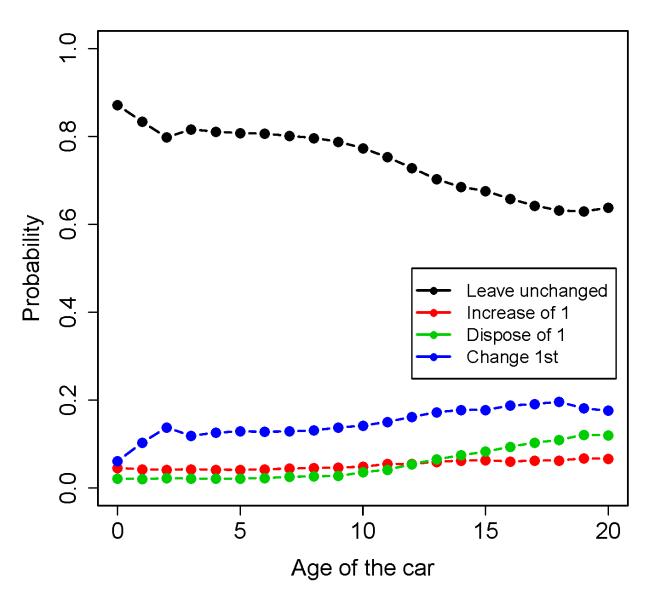
Transitions 2002-2003: 1-car households







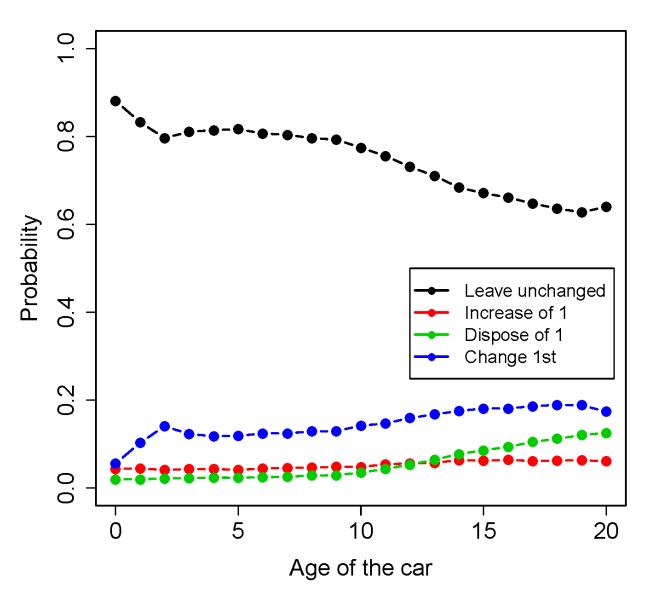
Transitions 2003-2004: 1-car households







Transitions 2004-2005: 1-car households

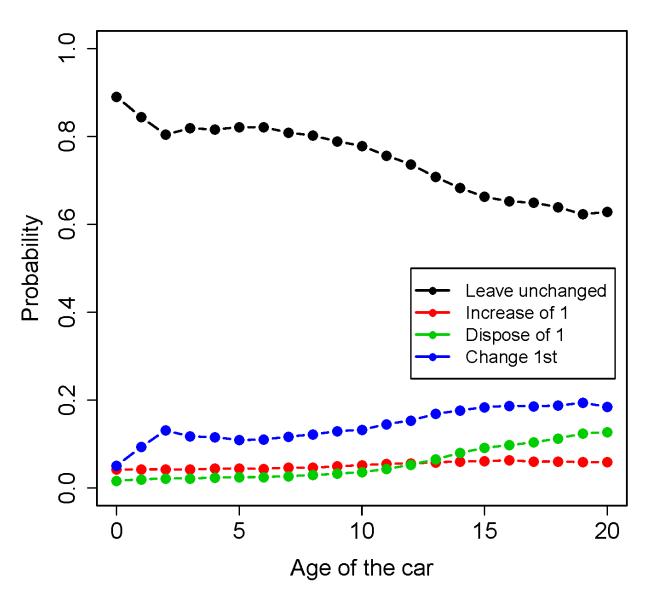






#### FROM REGISTER DATA

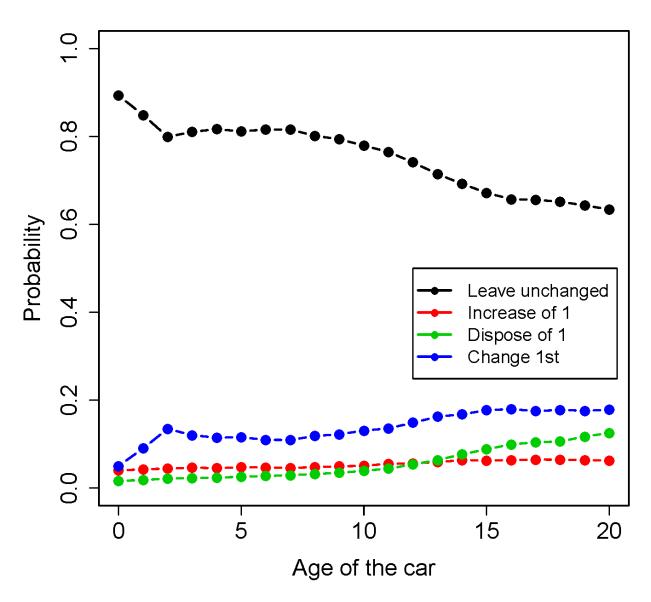
Transitions 2005-2006: 1-car households







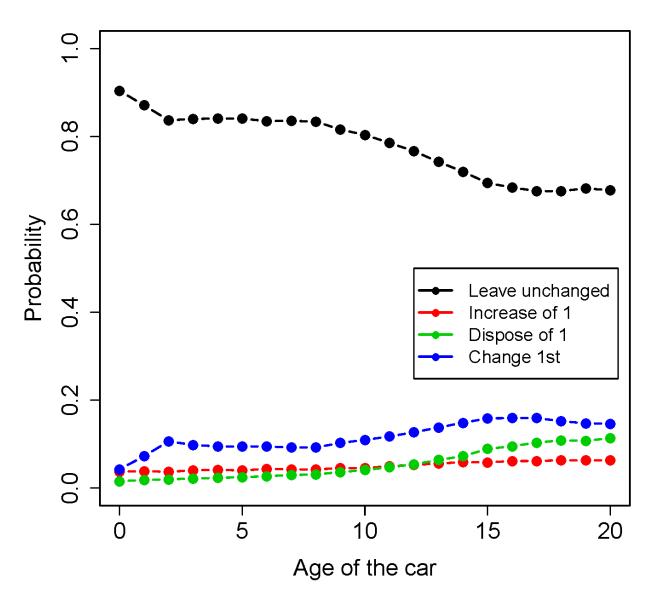
Transitions 2006-2007: 1-car households







Transitions 2007-2008: 1-car households







### **CONCLUSION AND FUTURE WORKS**

#### **Conclusion:**

- Methodology to model choice of car ownership and usage dynamically
- Example of application shows feasibility of approach

### **Next steps:**

- Model estimation on small sample of synthetic data
- Model estimation on register data
- Scenario testing:
  - Validation of policy measures taken during the years available in the data
  - Test policy measures that are planned to be applied in future years





# Thanks!



