A DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL OF CAR OWNERSHIP, USAGE AND FUEL TYPE

Aurélie Glerum EPFL
Emma Frejinger Université de Montréal
Anders Karlström KTH
Muriel Beser Hugosson KTH
Michel Bierlaire EPFL

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• Introduction
• Background and data
• Dynamic discrete-continuous choice model
• Illustrative example
• Estimation on synthetic data
• Conclusion and future works
Aim of the research:

- Model dynamics of car transactions, usage and choice of fuel type in the Swedish car fleet

- Motivations
  - Governmental policies to reduce carbon emissions / car usage:
    - Stockholm congestion tax
    - Independence of fossil fuels
  - Technology changes:
    - Increase of alternative-fuel vehicles
  - Economical features:
    - Financial crisis
    - Fuel price changes

Car ownership and usage vary importantly over time.
Model needed to analyze and predict impact of policies on ownership.
Register data of Swedish population and car fleet:

- Data from 1998 to 2008

- All individuals
  - **Individual information**: socio-economic information on car holder (age, gender, income, home/work location, employment status/sector, etc.)
  - **Household information**: composition (families with children and married couples)

- All vehicles
  - Privately-owned cars, cars from privately-owned company and company cars
  - Vehicle **characteristics** (make, model, fuel consumption, fuel type, age)
  - **Annual mileage** from odometer readings
  - Car bought **new or second-hand**
BACKGROUND AND DATA

Number of cars (in hh with only private cars)

- 0-car hh
- 1-car hh
- 2-car hh


Frequency: 0, 500000, 1000000, 2000000, 2500000

OWNERSHIP
Number of cars (in hh with only private cars)

- 0-car hh
- 1-car hh
- 2-car hh

Year:
- 1999
- 2001
- 2003
- 2005
- 2007

Frequency:
- 0
- 5000000
- 10000000
- 15000000
- 20000000

Congestion tax
Stockholm
BACKGROUND AND DATA

Household annual mileage per household size

- **Annual mileage per household [mil]**
  - 0
  - 500
  - 1000
  - 1500
  - 2000
  - 2500
  - 3000

- **Year**
  - 1999
  - 2001
  - 2003
  - 2005
  - 2007

Legend:
- 1-pers hh
- 2-pers hh
- 3-pers hh
- 4-pers hh
- 5-pers hh
- 6-pers hh
- 7-pers hh
Car ownership models in transportation literature:
- Discrete choice models (DCM) widely used, but mostly static models.
  - Main drawback: do not account for forward-looking behavior
  - Important aspect to account for since car is a durable good
- Econometric literature: dynamic programming (DP) + DCM
  - Recently, dynamic discrete choice models (DDCM) starting to be applied in transportation field (Cirillo and Xu, 2011; Schiraldi, 2011)

Joint models of car ownership and usage:
- Duration models and regression techniques for car holding duration and usage (De Jong, 1996)
- Vehicle type, usage and replacement decisions using dynamic programming, discrete-continuous, mixed logit (Schjerning, 2008, and Munk-Nielsen, 2012)
- Discrete-continuous model of vehicle choice and usage based on register data: includes expectation of fuel prices & car future resale price (Gillingham, 2012)
Car are durable goods \(\implies\) Need to account for forward-looking behavior of agents

- Difficulty of modeling a discrete-continuous choice when jointly modeling car ownership and usage
- Many models focus on individual decisions, but choices regarding car ownership and usage made at household level
• Car are durable goods \( \rightarrow \) Need to account for forward-looking behavior of agents

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• Many models focus on individual decisions, but choices regarding car ownership and usage made at household level

Proposed methodology:

• Attempt to address these issues by applying dynamic discrete-continuous choice model (DDCCM)

Large register data of all individuals and cars in Sweden
In the area of **dynamic choice modeling**
- Choices modeled at **household level**
- **Up to two cars** allowed (only 4% households with > 2 cars in 2007)

- **Constant elasticity of substitution (CES) utility** to model annual driving distance for 2-car households

- Several **choices modeled simultaneously**
Objective

Model simultaneously car ownership, usage and fuel type.

In details: model simultaneous choice of

Transaction type \( \times \) [Annual milage – car \( c \)] \( \times \) [Private/company – car \( c \)] \( \times \) [Fuel type – car \( c \)] \( \times \) [New/2\(^{nd}\) hand – car \( c \)] \( \times \) # cars
Objective

Model simultaneously car ownership, usage and fuel type.
In details: model simultaneous choice of

Discrete variables
Objective

Model simultaneously car ownership, usage and fuel type.
In details: model simultaneous choice of

- Transaction type
- Annual milage – car \( c \)
- Private/ company car – car \( c \)
- Fuel type – car \( c \)
- New/2\(^{nd}\) hand – car \( c \)

Continuous variables

# cars
1. **Choice at household level**: up to 2 cars in household

2. **Strategic choice** of:
   - Transaction
   - Type(s) of ownership (company vs private car)
   - Fuel type(s)
   - Car state(s) (new vs 2\textsuperscript{nd}-hand)

   \(\rightarrow\) Account for forward-looking behavior of households

3. **Myopic choice** of:
   - Annual mileage(s)

4. **Choice of mileage conditional** on choice of discrete variables
Myopic choice (static case)

\[ P(\text{action}) = \frac{\exp\{\text{instantaneous utility}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities}\}} \]

Strategic choice (dynamic case)

\[ P(\text{action}) = \frac{\exp\{\text{instantaneous utility} + \text{expected discounted utility of future choices}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities} + \text{expected discounted utilities of future choices}\}} \]
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Strategic choice (dynamic case)

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Embeds a choice model into a dynamic programming framework
Components of the DDCCM:

- Agent
- Time step
- State space
- Action space
- Transition rule
- Instantaneous utility function
• **Agent**: household

• **Time step** $t$: year

• **State space** $S$

\[
s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})
\]

- Age – 1st car
- Private/company car – 1st car
- Fuel type – 1st car
- Age – 2nd car
- Private/company car – 2nd car
- Fuel type – 2nd car
DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Action space $A$

$$a_t = (h_t, \tilde{m}_{1,t}, \tilde{I}_{1,t}, \tilde{f}_{1,t}, \tilde{r}_{1,t}, \tilde{m}_{2,t}, \tilde{I}_{2,t}, \tilde{f}_{2,t}, \tilde{r}_{2,t})$$
• Action space \( A \)

Transaction types

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL
DEFINITION OF THE COMPONENTS
**Transition rule:** deterministic rule: each state $s_{t+1}$ can be inferred exactly once $s_t$ and $a_t$ are known.
Transition rule: deterministic rule: each state $s_{t+1}$ can be inferred exactly once $s_t$ and $a_t$ are known.

Instantaneous utility function:

$$u(s_t, a_t^C, a_t^D, x_t, \theta) = v(s_t, a_t^C, a_t^D, x_t, \theta) + \varepsilon_D(a_t^D)$$

- **Deterministic term**
- **Random term for discrete choices**

Assume additive **deterministic utility** for simplicity (see also Munk-Nielsen, 2012):

$$v(s_t, a_t^C, a_t^D, x_t, \theta) = v_t^D(s_t, a_t^D, x_t, \theta) + v_t^C(s_t, a_t^D, a_t^C, x_t, \theta)$$

- **Utility for discrete actions**
- **Utility for continuous actions**
• Instantaneous utility function

• Utility for continuous actions: Constant elasticity of substitution (CES) utility function (e.g. Zabalza, 1983):

\[ v_t^C(s_t, a_t^D, a_t^C, x_t, \theta) = (m_{1,t}^{-\rho} + \alpha \cdot m_{2,t}^{-\rho})^{-1/\rho} \]

- Captures substitution patterns between the choice of both annual driving distances
- \( \rho = \) elasticity of substitution
- \( \alpha = \) share parameter

• Formulation could be extended to introduce randomness in \( \alpha \).
• Parameters obtained by maximizing likelihood:

\[ \mathcal{L} = \prod_{n=1}^{N} \prod_{t=1}^{T_n} P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta) \]

• Optimization algorithm: Rust’s nested fixed point algorithm (NFXP) (Rust, 1987):
  
  • **Outer optimization algorithm**: search algorithm to obtain parameters maximizing likelihood
  
  • **Inner value iteration algorithm**: solves the dynamic programming problem for each parameter trial
  
• Plan to investigate variants of NFXP to speed up computational time (e.g. swapped algorithm from Aguirregabiria and Mira, 2002)
Outer algorithm:

- Standard estimation procedure (as for static discrete choice models)
- Here: BHHH algorithm

Inner algorithm:

Two steps
1. Finding the optimal value(s) of annual mileage conditional on the discrete choices
2. Finding the expected discounted utility of future choices (= value function)
1. Finding the optimal value(s) of mileage

- Maximization of the continuous utility:
  \[
  \max_{m_{1,t}, m_{2,t}} v_t^C
  \]
  \[\text{s.t. } p_{1,t} m_{1,t} + p_{2,t} m_{2,t} = \text{Inc}_t\]

- Find analytical solutions: \( m_{1,t}^* \) and \( m_{2,t}^* \)

\[
m_{2,t}^* = \frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/((\rho+1))} + p_{1,t}^{(\rho/(1+\rho))} \alpha^{(-1/(\rho+1))}}
\]

\[
m_{1,t}^* = \frac{\text{Inc}_t}{p_{1,t}} - \frac{p_{2,t}}{p_{1,t}} m_{2,t}^*
\]

- Optimal continuous utility
  \[
v_t^C(s_t, a_t^D, a_t^{C*}, x_t, \theta)
\]
2. Finding the expected discounted utility of future choices
(= value function)

- **Logsum** formula used in the completely discrete case (DDCM)
  (Aguirregabiria and Mira, 2010; Cirillo and Xu, 2011)

- Logsum can be applied here given the **key assumptions**: 
  - Choice of mileage(s) is conditional on discrete actions
  - Choice of mileage(s) is myopic

\[
\bar{V}(s_t, x_t, \theta) = \log \sum_{a_t^D} \left\{ \exp\{v_t^D(s_t, a_t^D, x_t, \theta) + v_t^C(s_t, a_t^D, a_t^C, x_t, \theta)\} + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1}|s_t, a_t) \right\}
\]

- Iterate on **Bellman equation** to find integrated value function \( \bar{V} \)
ILLUSTRATIVE EXAMPLE

Assumptions for the example:

• Deterministic utility function

\[ v^D_t(s_t, a^D_t, x_t, \theta) = C(s_t) + \tau(a^D_t) + \beta_{\text{Age}}(a^D_t, s_t) \cdot \max(Age1_t, Age2_t) \]

- Constant for households with at least one car
- Transaction costs
- Transaction-dependant parameters relative to age of oldest car

• Chose arbitrary values for parameters
## ILLUSTRATIVE EXAMPLE

<table>
<thead>
<tr>
<th>Transaction name</th>
<th>Case</th>
<th>$\beta_{Age}$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0 car</td>
<td>1 car</td>
</tr>
<tr>
<td>$h_1$: leave unchanged</td>
<td></td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$h_2$: increase 1</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$h_3$: dispose 2</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_4$: dispose 1st</td>
<td>1st car is oldest</td>
<td>-</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>2nd car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_5$: dispose 2nd</td>
<td>1st car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2nd car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_6$: dispose 1st and change 2nd</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_7$: dispose 2nd and change 1st</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_8$: change 1st</td>
<td>1st car is oldest</td>
<td>-</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>2nd car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_9$: change 2nd</td>
<td>1st car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2nd car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

$C = 5$, for 1- or 2-car households
Assumptions for the example:

- Visualize choice probabilities for one observation:
  - 1-car household
  - Annual income = 530’000 SEK (≈ 60’800 €)
  - 8% expenses on fuel
ILLUSTRATIVE EXAMPLE

Static

Dynamic

Age of car [years]

Probability

- Leave unchanged
- Increase of 1
- Dispose of 1st
- Change 1st

Age of car [years]
ILLUSTRATIVE EXAMPLE

Static

Dynamic

Instantaneous utility

\[
P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta) = \frac{v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \tilde{V}_f}{\sum_{a_{n,t}} \left( v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \tilde{V}_f \right)}
\]
ILLUSTRATIVE EXAMPLE

Static

Dynamic

\[ P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta) = \frac{v_{n,t}^D + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \tilde{V}f}{\sum_{a_{n,t}^D} \left( v_{n,t}^D + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \tilde{V}f \right)} \]
Approach to validate the model framework

• Generate 100 observations based on distributions of variables in the Swedish register data

• Generate choice (for each observation) based on postulated parameters

• Estimation of model

• Approach validated once postulated parameters are retrieved
Conclusion:
• Methodology to model choice of car ownership and usage dynamically
• Example of application shows feasibility of approach
• Currently validating the approach (using synthetic data)

Next steps:
• Model estimation on register data
• Scenario testing:
  • Validation of policy measures taken during the years available in the data
  • Test policy measures that are planned to be applied in future years
Thanks!