Demand Based Timetabling of Passenger Railway Service

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Railway Planning

STRATEGIC - several years

TACTICAL - >= 1 year

OPERATIONAL - < 1 year

Demand → Line Planning → Lines → Train Timetabling → Actual Timetables → Train Platforming → Platform Assignments

→ Actual Timetables → Rolling Stock Planning → Train Assignments

→ Actual Timetables → Crew Planning → Crew Assignments
Line Planning Problem

Railway Infrastructure

Passenger Demand

Potential Lines

Model

Min Cost

Max Direct Pass.

Trade-Off

Supply and Demand

SURPLUS

Shortage

Equilibrium

Price

Quantity

Supply

Demand
Train Timetabling Problem – Non-Cyclic
Train Timetabling Problem – Cyclic
Arising Issues

Figure: Outside peak hour

Figure: Inside peak hour

Figure: Train station in China
Do We Keep Traditions?

TRADITION

Doing Stupid Things Since 1876 Is No Reason To Continue Doing Stupid Things.
Railway Planning Improved

**STRATEGIC** - several years
- Demand

**TACTICAL** - >= 1 year
- Line Planning
- Ideal Train Timetabling
- Ideal Timetables
- Train Timetabling

**OPERATIONAL** - < 1 year
- Actual Timetables
- Train Platforming
- Rolling Stock Planning
- Crew Planning

TOC
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8 / 35
Agenda

1. Motivation
2. Ideal Train Timetabling Problem
3. Conclusions
4. Future Work
1 Motivation

2 Ideal Train Timetabling Problem
   - Assumptions
   - Inputs
   - Decision Variables
   - Objective
   - Constraints
   - Cyclicality
   - Connections

3 Conclusions

4 Future Work
Assumptions I

User Cost

Ideal Time

Time
Assumptions II
**Inputs**

\[ t \in T \quad \text{– set of time steps} \]
\[ l \in L \quad \text{– set of lines} \]
\[ f \quad \text{– fraction by which it is better to be early} \]
\[ d_t^l \quad \text{– demand captured along the line } l, \text{ when scheduling} \]
\[ \text{a train at time } t \]
\[ d_t^{ll'} \quad \text{– connection demand captured along the line } l \text{ and } l', \]
\[ \text{when scheduling a train at time } t \text{ on the line } l \]
\[ n^l \quad \text{– number of trains available for line } l \]
\[ h_{l}^{l'} \quad \text{– relative headway to reach a connection point of lines} \]
\[ l \text{ and } l' \text{ from the first station on line } l \text{ and } l' \]
\[ c^l \quad \text{– size of the cycle on line } l \]
\[ s \quad \text{– preferred start of the planning horizon} \]
\[ M \in \mathbb{M} \quad \text{– set of sufficiently large numbers} \]
Primary Decision(s)

\[
x_t^l = \begin{cases} 
1 & \text{if a train on line } l \text{ is scheduled at time } t, \\
0 & \text{otherwise.}
\end{cases}
\]
Secondary Decisions I

- $y_{tb}^{l} \in \mathbb{R}^{+}$ – cost of the passengers wanting to travel at time $t$ on the line $l$, when taking a closest train at $t$ or before
- $y_{ta}^{l} \in \mathbb{R}^{+}$ – cost of the passengers wanting to travel at time $t$ on the line $l$, when taking a closest train after $t$
- $y_{t}^{l} \in \mathbb{R}^{+}$ – cost of the passengers wanting to travel at time $t$ on the line $l$
Secondary Decisions II

\[ z_t^l = \begin{cases} 
1 & \text{if passengers wanting to travel at time } t \\
& \text{on the line } l \text{ take the closest train after the time } t, \\
0 & \text{otherwise.} 
\end{cases} \]
Objective

$$\min \sum_{l \in L} \sum_{t \in T} y^l_t \cdot d^l_t$$
Constraints I

\[ y_{lb}^{t} \geq \frac{(t - t')}{f} \cdot \left( x_{t'}^{l} - \sum_{t''=t'+1}^{t} x_{t''}^{l} \right) \quad \forall l \in L, \forall t, \forall t' \in T : t \geq t', \]

\[ y_{la}^{t} \geq (t' - t) \cdot \left( x_{t'}^{l} - \sum_{t''=t+1}^{t'-1} x_{t''}^{l} \right) \quad \forall l \in L, \forall t, \forall t' \in T : t < t', \]
\[ y_{lb}^I \geq M_1 \cdot \left( 1 - \sum_{t' = s}^{t} x^l_{i'} \right) \quad \forall l \in L, \forall t \in T, \]

\[ y_{la}^I \geq M_1 \cdot \left( 1 - \sum_{t' = t}^{T} x^l_{i'} \right) \quad \forall l \in L, \forall t \in T, \]
Constraints III

\[ y_t^l \geq y_t^{lb} - z_t^l \cdot M_2 \quad \forall l \in L, \forall t \in T, \]
\[ y_t^l \geq y_t^{la} - (1 - z_t^l) \cdot M_2 \quad \forall l \in L, \forall t \in T, \]
\[ M_2 > M_1 \]
Constraints IV

$$\sum_{t \in T} x^l_t \leq n^l \quad \forall l \in L,$$
Motivation

Ideal Train Timetabling Problem
- Assumptions
- Inputs
- Decision Variables
- Objective
- Constraints
- Cyclicity
- Connections

Conclusions

Future Work
Introducing Cyclicity

\[ x^l_{t+c^l} = x^l_t \]

\[ \forall l \in L, \forall t \in T : t + c^l \leq T : t \geq s, \]

\[ \min(t+c^l, T) \]

\[ \sum_{t'=t+1} x^l_{t'} \leq (1 - x^l_t) \cdot M_3 \]

\[ \forall l \in L, \forall t \in T : t \geq s, \]
Introducing Cyclicity

\[ x_{t+c'}^l = x_t^l \]

\[ \min(t + c', T) \sum_{t'=t+1} x_t' \leq (1 - x_t^l) \cdot M_3 \]

\[ \forall l \in L, \forall t \in T : t + c' \leq T : t \geq s, \]

\[ \forall l \in L, \forall t \in T : t \geq s, \]
1 Motivation

2 Ideal Train Timetabling Problem
   - Assumptions
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   - Cyclicity
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Extra Decisions I

- \( y_t^{ll'} \in \mathbb{R}^+ \) – cost of the passengers wanting to travel at time \( t \) on the line \( l \), when taking a closest train at \( t \) or before and connecting to line \( l' \)
- \( y_t^{ll''} \in \mathbb{R}^+ \) – cost of the passengers wanting to travel at time \( t \) on the line \( l \), when taking a closest train after \( t \) and connecting to line \( l' \)
- \( y_t^{ll'''} \in \mathbb{R}^+ \) – cost of the passengers wanting to travel at time \( t \) on the line \( l \) and connecting to line \( l' \)
Extra Decisions II

\[ z_{t}^{ll'} = \begin{cases} 
1 & \text{if passengers wanting to travel at time } t \text{ on the line } l \text{ take the closest train after the time } t \text{ and connecting to line } l', \\
0 & \text{otherwise.} 
\end{cases} \]
Objective

\[
\min \sum_{l \in L} \sum_{t \in T} y^l_t \cdot d^l_t + \sum_{l \in L} \sum_{l' \in L} \sum_{t \in T} y_{l'l}^t \cdot d_{l'l}^t
\]
Extra Constraints I

\[ y_{t''}^b \geq (t - t') / f \cdot \left( x_{t'}^{l'} - \sum_{t'''=t'+1}^{t} x_{t'''}^{l''} \right) + \left( t'' - (t' + h_i') \right) \cdot \left( x_{t''}^{l''} - \sum_{t'''=t'+h_i'+1}^{t''-1} x_{t'''}^{l''} \right) - M_4 \cdot \left( 1 - x_{t'}^{l'} + \sum_{t'''=t'+1}^{t} x_{t'''}^{l''} \right) \]

\( \forall l, \forall l' \in L : l \neq l' \),

\( \forall t, \forall t', \forall t'' \in T : t \geq t' \) and \( t' + h_i' < t'' \),

\[ y_{t''}^a \geq (t' - t) \cdot \left( x_{t'}^{l'} - \sum_{t'''=t+1}^{t'-1} x_{t'''}^{l''} \right) + \left( t'' - (t' + h_i') \right) \cdot \left( x_{t''}^{l''} - \sum_{t'''=t'+h_i'+1}^{t''-1} x_{t'''}^{l''} \right) - M_4 \cdot \left( 1 - x_{t'}^{l'} + \sum_{t'''=t+1}^{t'-1} x_{t'''}^{l''} \right) \]

\( \forall l, \forall l' \in L : l \neq l' \),

\( \forall t, \forall t', \forall t'' \in T : t < t' \) and \( t' + h_i' < t'' \),
Extra Constraints II

User Cost

Ideal Time $t$

Regular Time Step

Arrival to Line $l'$ at time $t'+h$

Departure

$yt_{lb}$

$yt_{la}$

Time $t''$
Extra Constraints III

\[ y_{t}^{ll'} \geq y_{t}^{ll'}^{b} - z_{t}^{ll'} \cdot M_{2} \quad \forall l, \forall l' \in L: l \neq l', \forall t \in T, \]

\[ y_{t}^{ll'} \geq y_{t}^{ll'}^{a} - (1 - z_{t}^{ll'}) \cdot M_{2} \quad \forall l, \forall l' \in L: l \neq l', \forall t \in T, \]

Constraints to add

- Beginning and the end of horizon, when no connections are possible
1 Motivation

2 Ideal Train Timetabling Problem

3 Conclusions

4 Future Work
Conclusions

- New planning phase, based on the demand
- User cost rather than demand to capture (no need for discrete choice model)
- Can handle both non- and cyclic timetables
- Connections are demand imposed

...AND IN CONCLUSION I RESTATE MY MAIN POINTS IN A SUPER WORDY FASHION TO SQUEEZE ANOTHER PARAGRAPH OUT OF THIS PAPER.
1 Motivation

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Future Work

- Methodology design (cyclic is tighter than the non-)
- Actually solving the problem
- Analysis of the general results
- Analysis of the connections
Thank you for your attention.