Models and algorithms for integrated airline schedule planning and revenue management

Bilge Atasoy* **Matteo Salani+**
Michel Bierlaire*
*Transport and Mobility Laboratory (TRANSP-OR), ENAC,
Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland
+Dalle Molle Institute for Artificial Intelligence (IDSIA), Switzerland
Email: bilge.kucuk@epfl.ch

1 Motivation

In airline scheduling problems, it is well accepted that there is a potential for obtaining superior solutions by integrating the sub-problems of the process. However revenue management and schedule planning models are mostly treated independently. Revenue management models usually work with a fixed capacity which is provided by the schedule planning phase (Talluri and van Ryzin, 2004). Schedule planning models consider the demand and the price as inputs based on forecasts (Schön, 2008). In this study, we develop an integrated schedule planning model with explicit supply-demand interactions. These interactions are represented by a logit model and they enable to integrate revenue management decisions early in the planning phase. This integration results with superior planning decisions and expected to provide valuable information to the actual revenue management process.

2 Integrated model

The presented integrated model simultaneously decides on the *schedule design*, *fleet assignment* and *pricing*. The schedule design and fleet assignment decisions are modeled similar to the work of Lohatepanont and Barnhart (2004). The contribution of the model to the fleet assignment literature is the explicit integration of supply-demand interactions via an itinerary choice model. The integration of pricing decision is inspired by Schön (2008). The originality of our work is the estimation based on a real data[1]. The demand model is developed separately for economy and business classes which facilitates the decision on the capacity allocated to each class.

[1]A mixed RP/SP data set, the details can be found in Atasoy and Bierlaire (2012)

$$\max \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} \mathrm{ms}_i p_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f} \tag{1}$$

$$\text{s.t.} \sum_{k\in K} x_{k,f} = 1 \qquad \forall f \in F^M \tag{2}$$

$$\sum_{k\in K} x_{k,f} \le 1 \qquad \forall f \in F^O \tag{3}$$

$$y_{k,a,t^-} + \sum_{f\in \mathrm{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in \mathrm{Out}(k,a,t)} x_{k,f} \qquad \forall [k,a,t] \in N \tag{4}$$

$$\sum_{a\in A} y_{k,a,\mathrm{minE}_a^-} + \sum_{f\in CT} x_{k,f} \le R_k \qquad \forall k \in K \tag{5}$$

$$y_{k,a,\mathrm{minE}_a^-} = y_{k,a,\mathrm{maxE}_a^+} \qquad \forall k \in K, a \in A \tag{6}$$

$$\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} \delta_{i,f}\mathrm{ms}_i \le \sum_{k\in K} \pi^h_{k,f} \qquad \forall h \in H, f \in F \tag{7}$$

$$\sum_{h\in H} \pi^h_{k,f} \le Q_k x_{k,f} \qquad \forall f \in F, k \in K \tag{8}$$

$$\sum_{i\in I_s} \mathrm{ms}_i = 1 \qquad \forall h \in H, s \in S^h \tag{9}$$

$$\mathrm{ms}_i \le \upsilon_s \exp\left(V_i(p_i, z_i; \beta)\right) \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I'_s) \tag{10}$$

$$\mathrm{ms}_j = \upsilon_s \exp\left(V_j(p_j, z_j; \beta)\right) \qquad \forall h \in H, s \in S^h, j \in I'_s \tag{11}$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \tag{12}$$

$$y_{k,a,t} \ge 0 \qquad \forall [k,a,t] \in N \tag{13}$$

$$\pi^h_{k,f} \ge 0 \qquad \forall h \in H, k \in K, f \in F \tag{14}$$

$$\mathrm{LB}_i \le p_i \le \mathrm{UB}_i \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I'_s) \tag{15}$$

$$\mathrm{ms}_i \ge 0 \qquad \forall h \in H, s \in S^h, i \in I_s \tag{16}$$

$$\upsilon_s \ge 0 \qquad \forall h \in H, s \in S^h \tag{17}$$

The objective (1) is to maximize the profit, which is the revenue minus the operating cost. The main decision variable for the schedule planning is $x_{k,f}$ which is 1 if aircraft k is assigned to flight f, 0 otherwise. Constraints (2) and (3) constitute the schedule design decisions where *mandatory flight*s needs to be served and *optional flights* can be canceled. Constraints (4) maintain the conservation of flow and constraints (5) respect the availability of fleet. The network has a daily cyclic schedule which is provided by the constraints (6). Constraints (7) represent the coverage of the realized demand where $\pi^h_{k,f}$ is the decision variable for the allocated number of class h seats on aircraft k for flight f. Constraints (8) constitute the key component in the integration such that the allocated seats by the revenue management cannot exceed the actual capacity given by the fleet assignment. The subsequent constraints are related to the demand. The market share, ms_i, is given by a logit formula (10) with the associated the utilities, V_i. The pricing decision, p_i, is also embedded in the same set of constraints. Competitive itineraries are represented by the set I'_s and the airline does not have control on these itineraries, i.e. their prices are fixed and provide a reference market price (11). The constraints (10)-(11) represent a reformulation of the full logit model. It is equivalent to the logit function due to the constraints (9) which maintain that market shares sum up to 1. The spill and recapture effects are also facilitated with the logit model due to the $\leq$ relation in (10). The resulting model is a non-convex MINLP where non-convexity is due to the revenue in the objective function and the constraints (10)-(11).

As a further reformulation of the logit model, a logarithmic transformation is proposed:

$$\mathrm{ms}_i^{'} \leq \upsilon_s^{'} + V_i \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}), \tag{18}$$

$$\mathrm{ms}_j^{'} = \upsilon_s^{'} + V_j \qquad \forall h \in H, s \in S^h, j \in I_s^{'}, \tag{19}$$

where $\mathrm{ms}^{'}$ represents $\ln(\mathrm{ms})$ and $\upsilon_s^{'}$ represents $\ln(\upsilon_s)$.

3 A local search heuristic for the integrated model

The heuristic model is based on the two sub-problems of the model: a fleet assignment model where the price and demand are inputs to the model, and a revenue management model where the capacity is an input. The fleet assignment problem is a mixed integer linear problem. With the proposed logarithmic transformation the revenue sub-problem can be represented with a concave objective function and linear constraints. The two subproblems are solved in an iterative process where local search techniques are employed to explore good feasible solutions.

Local search is composed of *price sampling* and *variable neighborhood search (VNS)*. Price sampling is based on the spilled number of passengers so that if there is a high spill the price is decreased and vice versa. VNS is carried out by fixing a subset of fleet assignment solutions. The probability of a flight to be fixed to its aircraft in the previous iteration is based on the spilled passengers as well. The number of fixed assignments is increased to intensify the search when a better solution is found and diversification is applied otherwise.

4 Results

Data instances are generated using a dataset from a major European airline[2]. The integrated model is solved with BONMIN (Bonami et al., 2008) which cannot guarantee optimality for non-convex problems. However the first aim was to quantify the added value of the integrated approach. The integrated approach is compared with a *sequential approach (SA)* which is the current practice of airlines. In SA, the fleet assignment problem is solved to optimality with the given price. Subsequently the revenue is maximized with the given capacity by the fleet assignment. It is observed that, in more than half of the instances the integrated model results with a higher profit compared to SA (Atasoy et al., 2013).

The performance of the heuristic is evaluated in comparison to the best feasible solution provided by BONMIN and to the solution of SA. In fact, the heuristic starts with the given demand-price values in the data and the first iteration is equivalent to SA. Heuristic has a

[2] `http://challenge.roadef.org/2009/en`

Table 1: Performance of the heuristic

	Integrated model by BONMIN		**Sequential approach (SA)**		**Heuristic results** *Avg. over 5 replications*				
	Profit	Time (sec)	Profit	%dev.	Profit	%dev.	%imp. over SA	Time (sec)	%time reduction
20	155,772	1,429	154,322	-0.93%	155,772	0.00%	0.94%	990	30.7%
21	303,726	*86,400*	303,469	-0.08%	307,021	1.08%	1.17%	3,824	95.6%
22	161,197	*86,400*	163,324	1.32%	163,767	1.59%	0.27%	235	99.7%
23	284,269	*86,400*	278,942	-1.87%	282,226	-0.72%	1.18%	1,480	98.3%
24	155,457	*86,400*	150,844	-2.97%	156,761	0.84%	3.92%	2,189	97.5%
25	409,496	*86,400*	394,716	-3.61%	401,019	-2.07%	1.60%	4,864	94.4%

time limit of 2 hours on the other hand BONMIN has 24 hours. In Table 1 we present the results for 6 instances, which consist of 33, 46, 48, 61, 77, and 97 flights respectively. The heuristic is able to improve the solution by SA thanks to local search mechanisms for all the instances. It provides superior feasible solutions compared to BONMIN in 4 of the instances in significantly less computational time.

References

B. Atasoy and M. Bierlaire. An air itinerary choice model based on a mixed RP/SP dataset. Technical Report TRANSP-OR 120426, Transport and Mobility Laboratory, Ecole Polytechnique Fédérale de Lausanne, 2012.

B. Atasoy, M. Salani, and M. Bierlaire. An integrated airline scheduling, fleeting and pricing model for a monopolized market. Forthcoming in Computer-Aided Civil and Infrastructure Engineering, 2013.

P. Bonami, L. T. Biegler, A. R. Conn, G. Cornuéjols, I. E. Grossmann, C. D. Laird, J. Lee, A. Lodi, F. Margot, N. Sawaya, and A. Wächter. An algorithmic framework for convex mixed integer nonlinear programs. *Discrete Optimization*, 5(2):186 – 204, 2008.

M. Lohatepanont and C. Barnhart. Airline schedule planning: Integrated models and algorithms for the schedule design and fleet assignment. *Transportation Science*, 38:19–32, 2004.

C. Schön. Integrated airline schedule design, fleet assignment and strategic pricing. In *Multikonferenz Wirtscaftsinformatik (MKWI)*, February 2008.

K. T. Talluri and G. J. van Ryzin. *The Theory and Practice of Revenue Management*. Kluwer Academic Publishers, Boston, first edition, 2004.