Integration of explicit supply-demand interactions in airline schedule planning and fleet assignment

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Abstract

In airline scheduling problems, integrated decision methodologies have aroused intense interest in the last decade. These integrated approaches allow for more efficient solutions due to the simultaneous decision making process. The studied integrated models mostly consist of planning phase subproblems. However in order to be demand responsive, demand related information should also be included in planning models. For that matter supply-demand interactions are now being introduced in airline decision problems. This is often carried out with simplified demand models that are exogenous to the supply model. In this paper we present an integrated airline scheduling, fleeting and pricing model which includes an explicit demand model formulation. The integrated model combines supply and demand related decisions. On the supply side, we have the decisions on the subset of flights to be flown and the aircraft types to be assigned to each flight. On the demand side, we have the pricing decision and spill and recapture effects which are facilitated through an itinerary choice model. This itinerary choice model is estimated based on real data which is a combination of RP and SP datasets. In this paper a sensitivity analysis is presented for the added-value of the integrated supply-demand interactions. Furthermore a reformulation of the model is proposed to reduce the complexity.

Keywords

Airline fleet assignment, airline schedule planning, supply-demand interactions, itinerary choice models, spill and recapture, mixed integer nonlinear problems

1 Introduction

In airline fleet assignment literature the integration of supply-demand interactions attract an increased interest in the last decades. These interactions are either directly integrated into the model or external demand models/simulators are developed that provide better inputs to the planning process. When there is a direct representation of supply-demand interactions in the planning problems it is assumed that the airlines have a control on the revenue side. On the other hand, researchers who believe that airlines can not have such a control prefer to keep the revenue related decisions external to the model. This enables to keep the stochastic nature of the demand and use advance demand modeling techniques.

In this section we provide a brief review of airline fleet assignment literature where supply-demand interactions are considered through network effects. Since our focus is on how supply-demand interactions are treated we do not provide a comprehensive review on fleet assignment models (FAM). We refer to Sherali *et al.* (2006) for a recent review on FAM literature. In Table 1 we categorize different conventions in airline fleet assignment literature regards to supply-demand interactions compared to the basic FAM.

Table 1: FAM literature with network effects

Basic FAM	**FAM**	**IFAM**
fixed price fixed demand	externally estimated revenue network effects	fixed price network effects
e.g. Abara (1989) Hane *et al.* (1995)	Jacobs *et al.* (2008) Dumas *et al.* (2009)	Barnhart *et al.* (2002) Lohatepanont and Barnhart (2004) Wang *et al.* (2012) Atasoy *et al.* (2013b)

As mentioned earlier keeping the revenue estimations external to the FAM enables to use stochastic revenue models. With that motivation Jacobs *et al.* (2008) present a leg-based FAM where network effects are estimated with a passenger mix model. The passenger mix model is a nonlinear network flow model which estimates the total revenue given the capacity. Simplified network effects are included directly in FAM in order to keep a linear formulation. Dumas *et al.* (2009) present a framework where a passenger flow model and a leg-based FAM are iteratively solved. Their aim is to keep the stochastic nature of the demand and reflect the time dimension in the booking process. They assume that demand distributions of itineraries and recapture ratios are known.

Direct integration of supply-demand interactions lead to itinerary-based fleet assignment (IFAM) since the information on the demand is at the itinerary level (Barnhart *et al.*, 2002). With an itinerary-based setting Lohatepanont and Barnhart (2004) present an integrated schedule design and fleet assignment model with network effects. The considered effects are demand correction for the market demand in case of flight cancellations and recapture effects. Recapture ratios are estimated based on the Quality Service Index (QSI) and introduced as fixed inputs to the model. Sherali *et al.* (2010) also present an integrated schedule design and fleet assignment model where they work with itinerary-based demands for multiple fare classes. They optimize the allocation of seats for each fare class as we do in our integrated model. However they do not include network effects in the model.

The advances in demand modeling enables to better understand the underlying travel behavior. Discrete choice methodology is widely used in the context of air transportation for itinerary choice estimation. We refer to Garrow (2010) where the motivation for the usage of discrete choice methodology in air travel demand is presented together with several case studies. Various specifications are provided such as logit and nested logit models.

Wang *et al.* (2012) use utility models similar to discrete choice modeling in order to represent the spill and recapture effects. They present the idea with a basic passenger mix model. Extensions to the model are proposed with fleet assignment and schedule design decisions as well as market and departure time selections. Atasoy *et al.* (2013b) estimate the recapture ratios based on a logit model. The logit model is estimated based on a mixed revealed preferences and stated preferences dataset. The recapture ratios are explained by the price, departure time, number of stops and trip length. They are provided as inputs to the schedule design and fleet assignment model.

The explicit representation of demand models in optimization frameworks is a recent trend in order to improve the supply-demand interactions. Talluri and van Ryzin (2004) introduce a revenue management model based on a discrete choice methodology. They decide on the subset of fare products to offer at each point in time according to the discrete choice model. They consider single-leg, multiple-fare-class products.Schön (2008) presents an integrated schedule design, fleet assignment and pricing model. She provides different specifications of the demand model as logit and nested logit where the only explanatory variable is the price. Spill and recapture effects are not considered. The concavity of the model is achieved through an inversion of the demand model (see Appendix D) which is a limiting factor in case of the existence of other policy variables in addition to the price. Atasoy *et al.* (2012) propose an integrated scheduling, fleeting and pricing model where an itinerary choice model is estimated based on a real dataset. They present several tests in order to understand the behavior of the integrated model. The explicit representation of the demand model results with a non-convex mixed integer nonlinear problem (MINLP). Therefore a heuristic method is proposed for the solution of the problem (Atasoy *et al.*, 2013a).

In this paper we present a sensitivity analysis in order to understand the added-value of supply-demand interactions at different levels (see section 4). Leg-based FAM (e.g. Hane *et al.*, 1995) is the reference model and the two IFAMs presented by Atasoy *et al.* (2013b) and Atasoy *et al.*

(2012) are tested compared to the leg-based FAM. Furthermore we present a reformulation of the itinerary choice model which provides a concave revenue sub-problem (see section 5). This reformulation is expected to facilitate the optimization framework with iterative solutions of the schedule planning and revenue problems.

2 The itinerary choice model

In order to understand the air travel demand an itinerary choice model is developed. Origin-destination (OD) pairs are considered as market segments. Economy and business classes are treated separately. A market segment therefore consists of all the available itineraries for an OD pair and a cabin class. For example, all the economy itineraries offered from Geneva to Boston constitute a market segment. The notation is provided in Table 2.

Table 2: Notation for the demand model

H	set of cabin classes - indexed by h
S^h	set of market segments in class h - indexed by s
I_s	set of itineraries in segment s - indexed by i
$I_s^{'}$	set of competing/no-revenue itineraries in segment s
D_s	unconstrained demand of segment s
V_i	deterministic utility of itinerary i
ms_i	market share of itinerary i
d_i	demand of itinerary i
p_i	price of itinerary i
c_i	constant term for itinerary i that represents several attributes
β	parameters of the demand model

The itinerary choice for the market segments is considered to be independent from each other. The choice set for each market segment consists of all the available itineraries serving the market. These itineraries may vary in terms of the offered level of service, price, trip length and departure time of day. A logit model is developed in order to estimate the choice probability of each itinerary in each segment based on the listed variables. The estimation of the parameters are performed using maximum likelihood estimation over a mixed RP/SP dataset. The details for the demand model and the estimation procedure is provided in Atasoy and Bierlaire (2012).

As mentioned earlier the demand model is integrated into the airline schedule planning model. The only policy variable which could be controlled by the optimization model is the price (p_i). The other variables (trip length, departure time of day, number of stops) improve the estimation of choice probabilities and can be represented by a constant (c_i) in the optimization model. The utility is represented by:

$$V_i = \beta \ln(p_i) + c_i \qquad \forall h \in H, s \in S^h, i \in I_s \tag{1}$$

Depending on the utility, the choice probability/market share and the demand associated with itinerary i in segment s is given by:

$$\mathrm{ms}_i = \frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)} = \frac{\exp(\beta \ln(p_i) + c_i)}{\sum_{j \in I_s} \exp(\beta \ln(p_j) + c_j)} \tag{2}$$

$$d_i = D_s \mathrm{ms}_i \qquad \forall h \in H, s \in S^h, i \in I_s \tag{3}$$

3 Integrated schedule planning models

In order to reveal the impact of integrated supply-demand interactions we carry out a comparative analysis for different FAMs. The first model is a basic leg-based FAM with schedule design decisions. The second is an IFAM with choice-based recapture and the third model is an IFAM with choice-based recapture and pricing. In this section we present these three models.

3.1 Leg-based FAM with schedule design

The considered leg-based FAM is based on the work of Hane *et al.* (1995). The notation for the parameters and decision variables is provided in Table 3 and Table 4 respectively.

$$z^*_{\textbf{FAM}} = \min \sum_{k \in K} \sum_{f \in F} C_{k,f} x_{k,f} + \text{spill costs} \tag{4}$$

$$\sum_{k \in K} x_{k,f} = 1 \qquad \forall f \in F^M \tag{5}$$

$$\sum_{k \in K} x_{k,f} \leq 1 \qquad \forall f \in F^O \tag{6}$$

$$y_{k,a,t^-} + \sum_{f \in \mathrm{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in \mathrm{Out}(k,a,t)} x_{k,f} \qquad \forall [k,a,t] \in N \tag{7}$$

$$\sum_{a \in A} y_{k,a,\mathrm{minE}_a^-} + \sum_{f \in CT} x_{k,f} \leq R_k \qquad \forall k \in K \tag{8}$$

$$y_{k,a,\mathrm{minE}_a^-} = y_{k,a,\mathrm{maxE}_a^+} \qquad \forall k \in K, a \in A \tag{9}$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \tag{10}$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \tag{11}$$

$$\tag{12}$$

The objective (4) is to minimize the operating costs and the spill costs. Since the leg-based formulation does not carry information regarding demand, assumptions are made for the estimation of spill costs. First assumption is the *full fare allocation* where each flight in the itinerary is assigned the full fare of the itinerary. The spill estimation is performed in a de-

Table 3: Notation

Set	Definition
F	the set of flight legs indexed by f
F_M	the set of mandatory flight legs
F_O	the set of optional flight legs
CT	the set of flights flying at count time
A	the set of airports indexed by a
K	the set of fleet types indexed by k
T	the set of time of the events in the network indexed by t
$N(k, a, t)$	the set of the nodes in the time-line network for fleet type k, airport a and time t
In(k, a, t)	set of inbound flight legs for node (k,a,t)
Out(k, a, t)	set of outbound flight legs for node (k,a,t)
Parameter	**Definition**
$C_{k,f}$	operating cost for flight f when operated by fleet type k
R_k	available number of planes for type k
Q_k	the capacity of fleet type k in number of seats
$min\mathrm{E}_a^-$	the time just before the first event at airport a
$max\mathrm{E}_a^+$	the time just after the last event at airport a

Table 4: Variables

Variable	Definition
$x_{k,f}$	binary variable, 1 if fleet type k is assigned to flight f, 0 otherwise
y_{k,a,t^-}	the number of type k planes at airport a just before time t
y_{k,a,t^+}	the number of type k planes at airport a just after time t

terministic way where the itineraries are listed in order of decreasing fare and the demand is assigned respecting this order. These assumptions on the fare allocation and spill estimation are explained by Kniker (1998). A similar approach is also considered by Lohatepanont (2002).

In addition to the fleet assignment decisions, the schedule design feature with the sets of mandatory and optional flights is integrated (5)-(6). Th remaining constraints are classical FAM constraints: flow of aircraft should be conserved (7), the number of available aircraft for each type should be respected (8) and a cyclic schedule is maintained (9).

3.2 IFAM with choice-based recapture

The considered IFAM is presented by Atasoy *et al.* (2013b). As proposed by Barnhart *et al.* (2002) an itinerary-based setting is adopted. The additional notation for the IFAM model is presented in Table 5.

Table 5: Additional notation for IFAM

Parameter	Definition
$\delta_{i,f}$	1 if itinerary i uses flight leg f, 0 otherwise
$b_{i,j}$	recapture ratio for the passengers spilled from itinerary i and redirected to itinerary j
Decision variable	**Definition**
$t_{i,j}$	redirected passengers from itinerary i to itinerary j
$\pi^h_{k,f}$	assigned seats for flight f in a type k plane for cabin class h

$$z^*_{\textbf{IFAM}} = \max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} t_{j,i}b_{j,i})p_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f} \tag{13}$$

$$s.t. \sum_{k\in K} x_{k,f} = 1 \qquad \forall f\in F^M \tag{14}$$

$$\sum_{k\in K} x_{k,f} \leq 1 \qquad \forall f\in F^O \tag{15}$$

$$y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f} \qquad \forall [k,a,t]\in N \tag{16}$$

$$\sum_{a\in A} y_{k,a,t_n} + \sum_{f\in CT} x_{k,f} \leq R_k \qquad \forall k\in K \tag{17}$$

$$y_{k,a,minE^-_a} = y_{k,a,maxE^+_a} \qquad \forall k\in K, a\in A \tag{18}$$

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)} \delta_{i,f}d_i - \sum_{j\in I_s}\delta_{i,f}t_{i,j} + \sum_{j\in(I_s\setminus I'_s)}\delta_{i,f}t_{j,i}b_{j,i} \leq \sum_{k\in K}\pi^h_{k,f} \qquad \forall h\in H, f\in F \tag{19}$$

$$\sum_{h\in H}\pi^h_{k,f} \leq Q_k x_{k,f} \qquad \forall f\in F, k\in K \tag{20}$$

$$\sum_{j\in I_s} t_{i,j} \leq d_i \qquad \forall h\in H, s\in S^h, i\in I_s \tag{21}$$

$$x_{k,f}\in\{0,1\} \qquad \forall k\in K, f\in F \tag{22}$$

$$y_{k,a,t}\geq 0 \qquad \forall [k,a,t]\in N \tag{23}$$

$$\pi^h_{k,f}\geq 0 \qquad \forall h\in H, k\in K, f\in F \tag{24}$$

$$d_i \leq \tilde{d}_i \qquad \forall i\in I \tag{25}$$

$$t_{i,j}\geq 0 \qquad \forall i\in I, j\in I \tag{26}$$

This model uses the given price values in the dataset. It assumes that the recapture ratios, $b_{i,j}$ are given by a logit formula. With the given itinerary attributes in the dataset, recapture ratios are calculated and taken as an input. The model optimizes the revenue minus operating costs (13). The revenue functions explicitly includes the spill and recapture effects. In addition to the schedule planning decisions, seat allocation is optimizes to each class of passengers. Constraints (19) maintain the balance between demand and allocated seats. Constraints (20) ensure that actual capacity of aircraft is respected. The number of spilled passengers from an itinerary cannot be more than the expected demand of that itinerary which is given by constraints (21).

The presented IFAM is similar to what Lohatepanont and Barnhart (2004) proposes. However they work on demand correction terms which is not considered by Atasoy *et al.* (2013b). Furthermore Lohatepanont and Barnhart (2004) introduces decision variables on the cancellation of the itineraries which is related to the cancellation of the associated flight. Therefore, in Atasoy *et al.* (2013b) the demand of the itineraries can only be reduced through spill which underestimates the potential revenue. This phenomenon is also valid for the model presented in the next section.

3.3 IFAM with choice-based recapture and pricing

The final IFAM integrates pricing decisions into the framework (Atasoy *et al.*, 2012). The pricing decision is also determined by the itinerary choice model. Therefore the representation of supply-demand interactions becomes explicit in the modeling framework. The price, p_i, the demand d_i and the recapture ratios, $b_{i,j}$, are now all decision variables in contrast to the previously presented models.

The demand is given by the itinerary choice model as in (36). This demand variable, $\tilde{d}_i$, serves as an upper bound on the actual demand variable, d_i, as given by (41). It is important to note that $\tilde{d}_i$ is the expected demand for each itinerary without any capacity limit. Therefore for the decision on $\tilde{d}_i$, the attributes of all the itineraries play a role. In case where all flights are mandatory this is similar to the model by Wang *et al.* (2012). However in case of canceled flights this provides more conservative results compared to the model by Wang *et al.* (2012) since the expected demand is affected by the attributes of all flights. This way of modeling has such an effect which was addressed by Lohatepanont and Barnhart (2004) with demand correction terms. The relation for the recapture ratios are presented in constraints (37). When a number of passengers is spilled from itinerary i, each remaining itinerary j gets $b_{i,j}$ portion of the spilled passengers.

$$
\begin{aligned}
z^*_{\textbf{IFAM-P}} = \max & \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} t_{j,i}b_{j,i})p_i \\
& - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f} && (27)\\
s.t. & \sum_{k\in K} x_{k,f} = 1 && \forall f\in F^M \quad (28)\\
& \sum_{k\in K} x_{k,f} \le 1 && \forall f\in F^O \quad (29)\\
& y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f} && \forall [k,a,t]\in N \quad (30)\\
& \sum_{a\in A} y_{k,a,t_n} + \sum_{f\in CT} x_{k,f} \le R_k && \forall k\in K \quad (31)\\
& y_{k,a,minE^-_a} = y_{k,a,maxE^+_a} && \forall k\in K, a\in A \quad (32)\\
& \sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)} \delta_{i,f}d_i - \sum_{j\in I_s}\delta_{i,f}t_{i,j} + \sum_{j\in(I_s\setminus I'_s)}\delta_{i,f}t_{j,i}b_{j,i} \\
& \quad \le \sum_{k\in K}\pi^h_{k,f} && \forall h\in H, f\in F \quad (33)\\
& \sum_{h\in H}\pi^h_{k,f} \le Q_k x_{k,f} && \forall f\in F, k\in K \quad (34)\\
& \sum_{j\in I_s} t_{i,j} \le d_i && \forall h\in H, s\in S^h, i\in I_s \quad (35)\\
& \tilde{d}_i = D_s\frac{\exp(V_i)}{\sum_{j\in I_s}\exp(V_j)} && \forall h\in H, s\in S^h, i\in I_s \quad (36)\\
& b_{i,j} = \frac{\exp(V_j)}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k)} && \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s), j\in I_s \quad (37)\\
& x_{k,f}\in\{0,1\} && \forall k\in K, f\in F \quad (38)\\
& y_{k,a,t}\ge 0 && \forall [k,a,t]\in N \quad (39)\\
& \pi^h_{k,f}\ge 0 && \forall h\in H, k\in K, f\in F \quad (40)\\
& d_i \le \tilde{d}_i && \forall i\in I \quad (41)\\
& 0\le p_i\le UB_i && \forall i\in I \quad (42)\\
& t_{i,j}\ge 0 && \forall i\in I, j\in I \quad (43)\\
& b_{i,j}\ge 0 && \forall i\in I, j\in I \quad (44)
\end{aligned}
$$

4 Sensitivity analysis

In this section we perform sensitivity analysis in order to address the key assumptions of the choice-based IFAM models. The addressed assumptions are related to the unconstrained demand, price parameter in the logit model and the assumptions regarding the competitors' offers.

We perform a comparative analysis between the leg-based FAM, IFAM with choice-based recapture and IFAM with choice-based recapture and pricing that are presented in section 3.

The sensitivity analysis for each of the assumptions starts with the solution of different FAM/IFAM models. With the resulting fleeting and scheduling decisions, a passenger allocation sim-

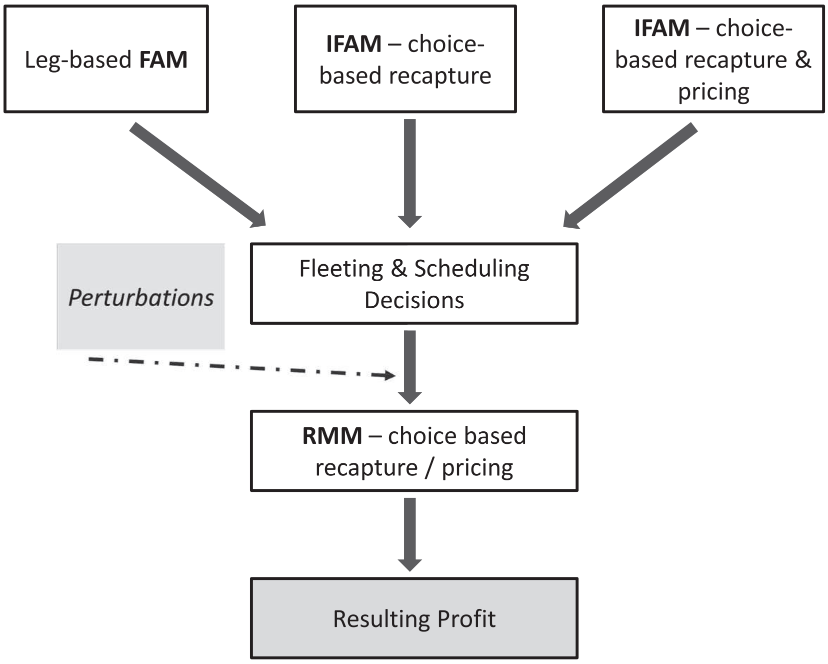

Figure 1: Framework for the sensitivity analysis

ulator is employed to determine the quality of the planning decisions given by the models. The passenger allocation simulator is considered as a revenue management model (RMM) which consists of all the demand/revenue related equations of the IFAM. In order to have a fair comparison for the models we have utilized two versions of the RMM. The first one (see Appendix A) uses the price values given in the dataset and therefore does not have a pricing decision. The second RMM (see Appendix B) on the other hand has the pricing decision. Both of the RMMs use the capacity given by the FAM/IFAMs therefore the fleet assignment variables are fixed and represented by $X_{k,f}$.

When solving the RMMs, perturbations are introduced and the profit obtained by the decision of different FAM/IFAM models are compared. This methodology for sensitivity analysis is inspired by the work of Lohatepanont (2002). It is illustrated by Figure 1.

The tests for the sensitivity analysis are performed with the two data instances presented in Table 6.

4.1 Demand uncertainty

In the presented IFAMs we assume that the total demand for each market segment is known. The demand share of each itinerary in a market segment is proportional to this total demand. This is a strong assumption given the daily and seasonal fluctuations of the demand. In this section we address this assumption and test the added value of IFAMs compared to FAM with simulated values of demand.

Table 6: The data instances used for the sensitivity analysis

Experiment 14	
Airports	4
Flights	23
Itineraries	35
	All economy
	6 one-stop - 29 non-stop
Total demand	1918 pax.
Available fleet	4 aircraft types
	117-85-70-50 seats
Experiment 24	
Airports	8
Flights	77
Itineraries	167
	19 business - 90 economy
	19 one-stop - 90 non-stop
Total demand	5051 pax.
Available fleet	4 aircraft types
	117-85-50-37 seats

We randomly draw 500 realizations of the unconstrained itinerary demand from a Poisson distribution with a mean equal to the average demand value provided in the dataset. Furthermore, we introduce perturbations on the average demand value in a range of -30%-+30%. Therefore for each perturbation on the average demand value we have 500 simulations. In order to decide on a meaningful range for the demand fluctuations we checked passenger flow statistics from The Airline Origin and Destination Survey (DB1B). For example for the OD pair JFK-FLL the total demand varies at most by around 15% from quarter to quarter in a year (data range 2005-2010). Since the daily fluctuation would be higher than the quarterly fluctuation we kept the range wider.

First of all the added value of IFAM with choice-based recapture is tested compared to leg-based FAM using the experiments 14 and 24. The considered RMM for passenger allocation is a model with choice-based recapture but without pricing (see Appendix A).

In experiment 14, supply demand interactions embedded in IFAM results with a higher capacity allocation since spilled passengers can be accommodated by other itineraries and the cost of larger capacity aircraft can be compensated. However leg-based FAM decides to assign less capacity since it cannot make use of recapture effects. Therefore, as seen in Figure 2, when the demand is over-estimated FAM performs better in the range -30% to -5%. However starting with -5% IFAM performs better. When the demand is under-estimated the improvement due to IFAM can be clearly observed. After a level of underestimation (around 15%) leg-based FAM cannot change the revenue since the allocated capacity is not enough to accommodate all the demand.

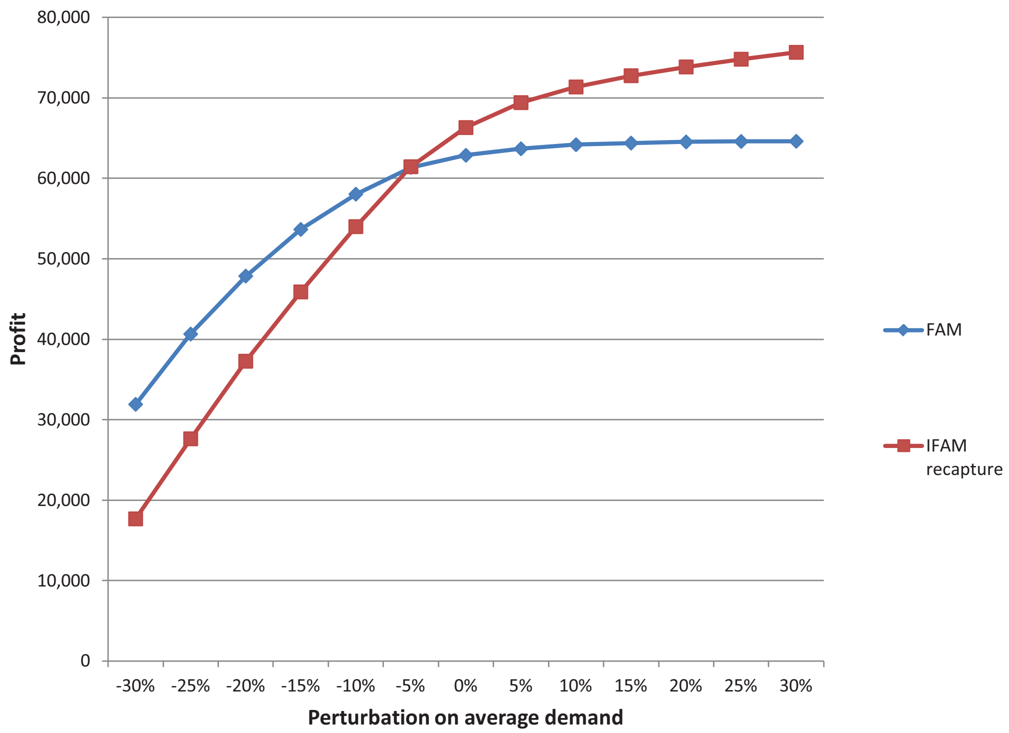

Figure 2: Sensitivity to demand fluctuations - Experiment 14

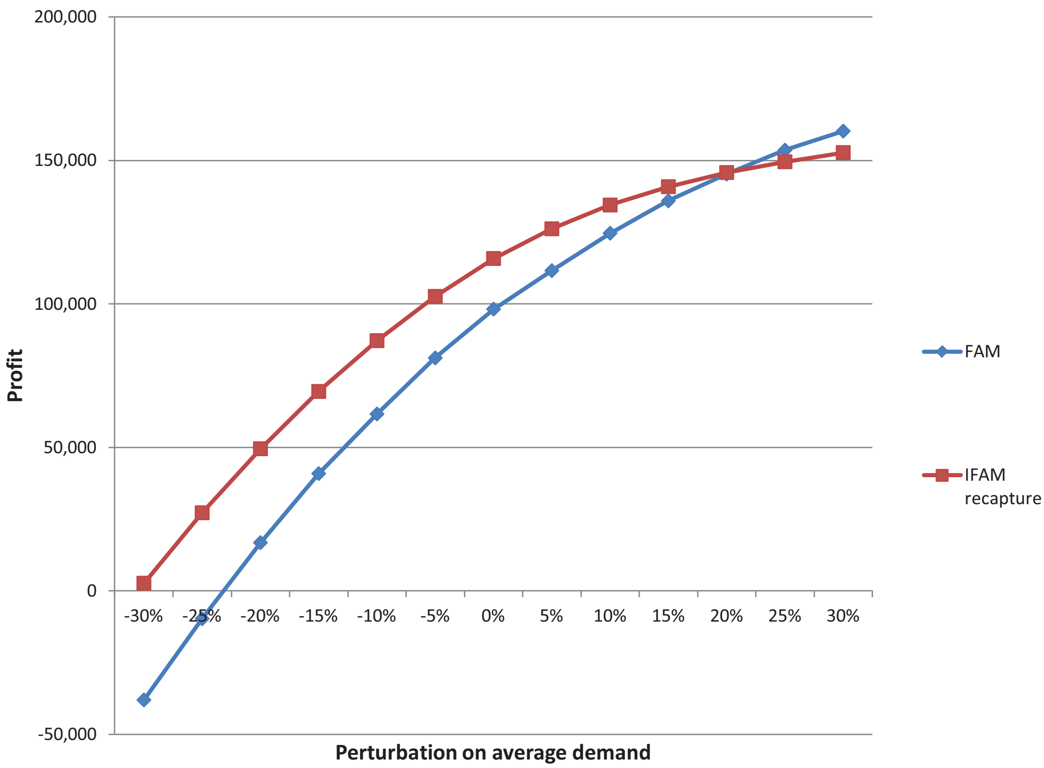

Figure 3: Sensitivity to demand fluctuations - Experiment 24

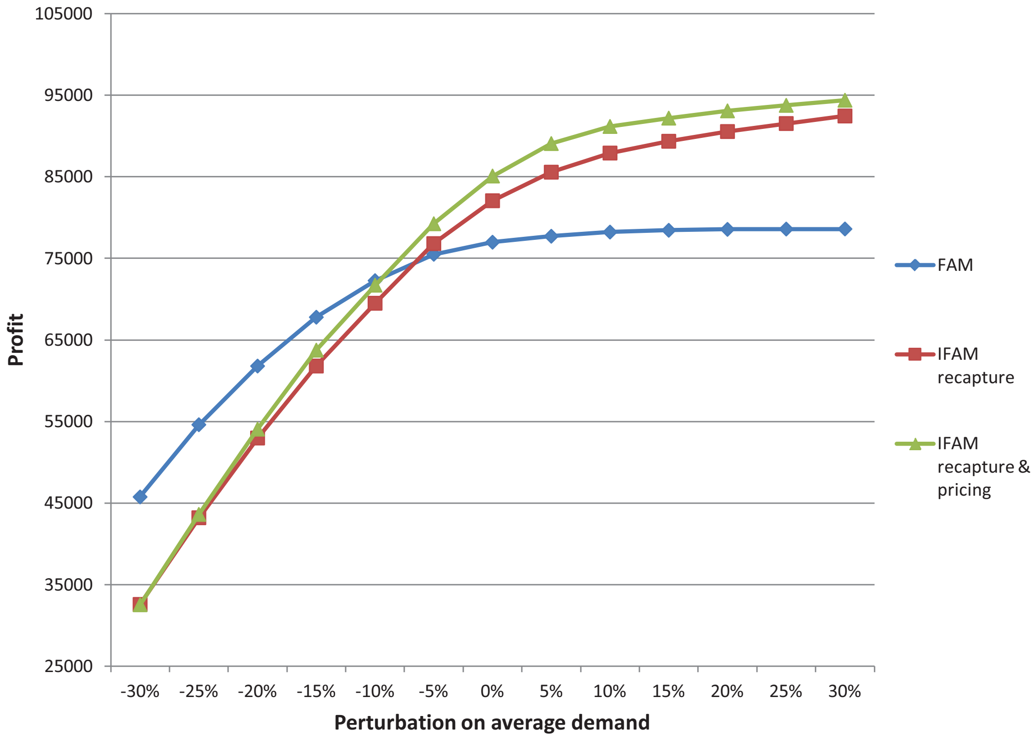

Figure 4: Sensitivity to demand fluctuations - Pricing - Experiment 14

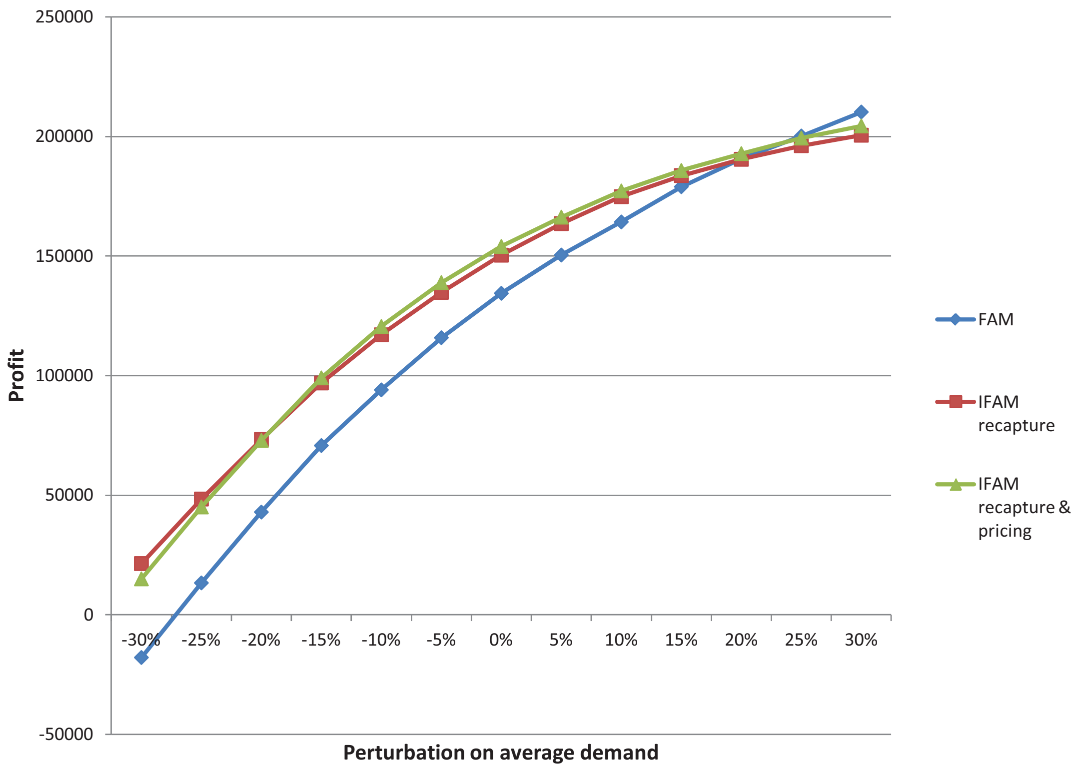

Figure 5: Sensitivity to demand fluctuations - Pricing - Experiment 24

Experiment 24 includes business class passengers which have higher average demand compared to economy passengers. The existence of business passengers increases the revenue that could be obtained through recapture and therefore IFAM decides to allocate less capacity compared to leg-based FAM. As seen in Figure 3, IFAM performs better up to the perturbation level +20%. When the demand is under-estimated by more than 20% FAM performs better. The higher capacity allocation of FAM helps to recover the unexpected increase in the demand.

In order to be able to test the sensitivity of IFAM with choice-based recapture and pricing we also performed the same analysis using the RMM with pricing (see Appendix B). In Figures 4 and 5 the results for leg-based FAM, IFAM with choice-based recapture and IFAM with choice-based recapture and pricing are presented. It is observed that the pricing decision increases the robustness of IFAM. For experiment 14, IFAM is outperformed by FAM up to -5% fluctuation. However with IFAM pricing this is pulled back to -10%. A similar phenomenon is observed with experiment 24. IFAM is outperformed by FAM when demand is under-estimated by more than 20%. With IFAM pricing this is shifted towards 25%. This supports the fact that when the planning model has more flexibility and information from the revenue side, the perturbations can be absorbed better.

It is important to note that experiment 14 is a smaller size instance compared to experiment 24 and its solution is provided by BONMIN solver with a 0% duality gap. However for the experiment 24 we work with the heuristic solution. Therefore the improvement due to pricing is less evident for experiment 24 compared to experiment 14.

The analysis performed for the demand uncertainty shows that the performance of IFAMs are not affected by slight changes on the average demand. The sensitivity analysis performed by Lohatepanont (2002) concludes that the improvement due to IFAM is not clear even with a perturbation of 2-3% on average demand. In our case, the improvement provided by the IFAMs is more evident. It can be concluded that a flexible planning model provides robust solutions since it can deal with more significant perturbations.

4.2 Price parameter of the itinerary choice model

The parameters of the demand model are estimated based on a mixed RP/SP dataset as explained in section 2. Since the only policy variable is the price among the set of explanatory variables, its coefficient has a direct impact on the market shares. Depending on the data set used, the estimated value of the parameter will be different. Therefore in this section we perform a sensitivity analysis with perturbations on the price parameter in a range from -50% to +50%. This range of perturbations modifies the elasticity of passengers is perturbed considerably.

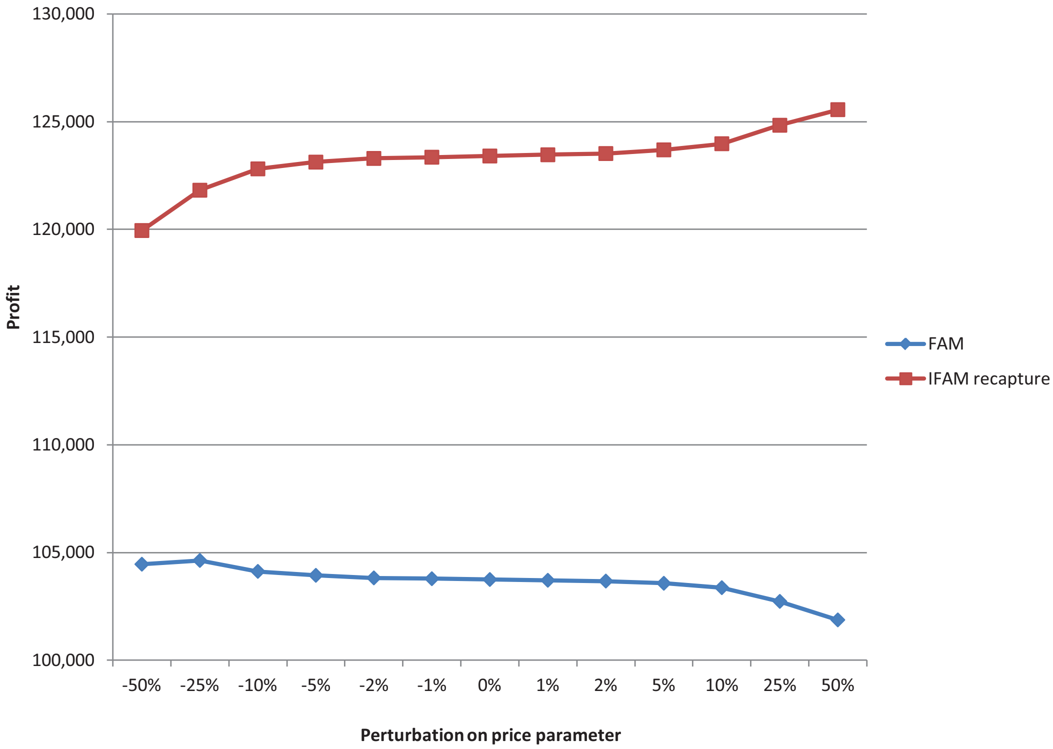

Figure 6: Sensitivity to price parameter - Experiment 24

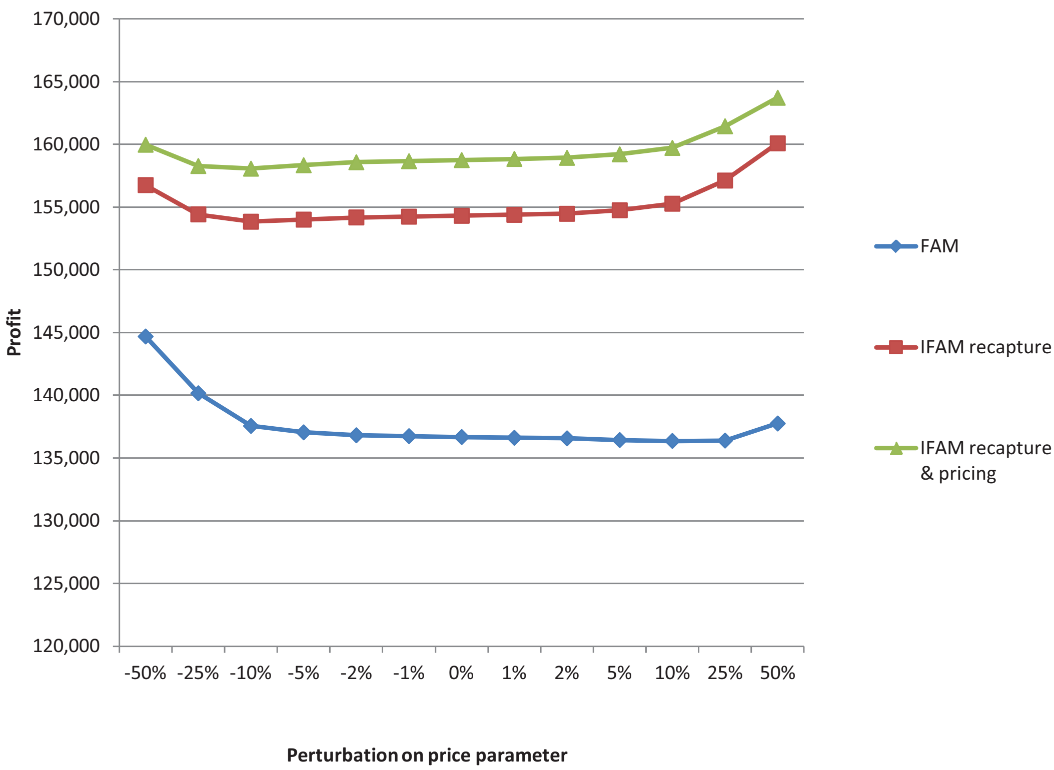

Figure 7: Sensitivity to price parameter - Pricing - Experiment 24

We perform the analysis on the price parameter with experiment 24. In Figure 6 we compare the behavior of FAM and IFAM with choice-based recapture. The considered RMM has choice-based recapture but does not have pricing decision (see Appendix A). It is observed that IFAM is outperforming FAM for all the cases which suggests that the added-value of IFAM is systematic and robust to perturbations on the price parameter.

In Figure 7 we compare the behavior of FAM, IFAM with choice-based recapture, and IFAM with both recapture and pricing. In this case RMM has a decision on pricing as well (see Appendix B). It is observed that IFAM with recapture provides a better fleeting solution compared to the two other models. It can be concluded that the added-value of IFAM with pricing is robust to the perturbations on price sensitivity. Another important observation is that the profit is higher when passengers are highly sensitive to price and when they are almost insensitive to price. In the former case RMM is able to attract more passengers with a slight price reduction. In the latter case it is able to increase the prices without decreasing the demand significantly. Both phenomenon result with an increase in the profit.

This section concludes that the improvement provided by the supply-demand interactions through the itinerary choice model is not sensitive to the price parameter.

4.3 Competitors' price

As mentioned in section 2 we include no-revenue options in the choice set which represent the alternatives offered by the competitors. The price of these alternatives are assumed to be known and even if there is a pricing decision they are kept fixed. Since the market share of the itineraries depends on the offer by the competitors, in this section we perform analysis on the price of the competitor' itineraries. The prices are perturbation in a range from -50% to +50%. The analysis is done with experiment 24.

In Figure 8 we compare the behavior of FAM and IFAM with choice-based recapture. The RMM is solved with choice-based recapture but without pricing (see Appendix A). It is observed that the price offered by the competitors does not have a significant impact on the added-value of IFAM. Only when the prices are under-estimated by around 50% FAM performs better than IFAM. As mentioned in section 4.1 leg-based FAM allocates more capacity compared to IFAM in experiment 24. Therefore with such a significant under-estimation of competitors' price, the market shares of the airline increase and FAM performs better with a higher capacity at hand.

In Figure 9 we compare the behavior of FAM, IFAM with choice-based recapture, and IFAM with both recapture and pricing. Consequently, considered RMM has a decision on pricing as well (see Appendix B). It is observed that IFAM with pricing outperforms both of the models in

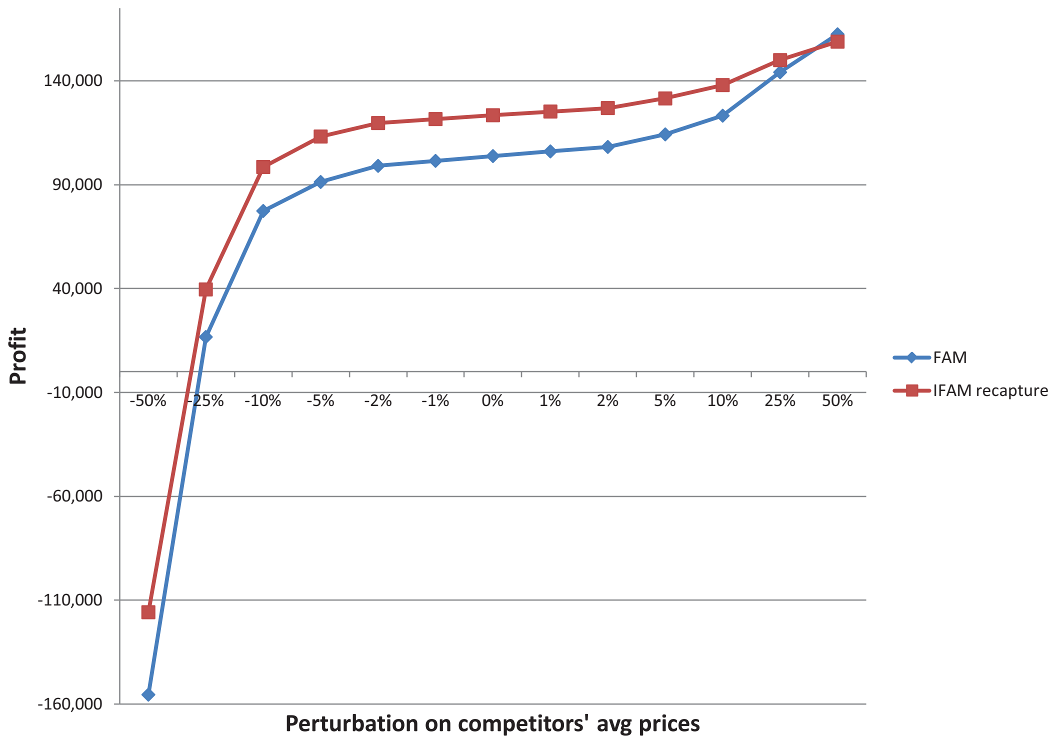

Figure 8: Sensitivity to competitors' price - Experiment 24

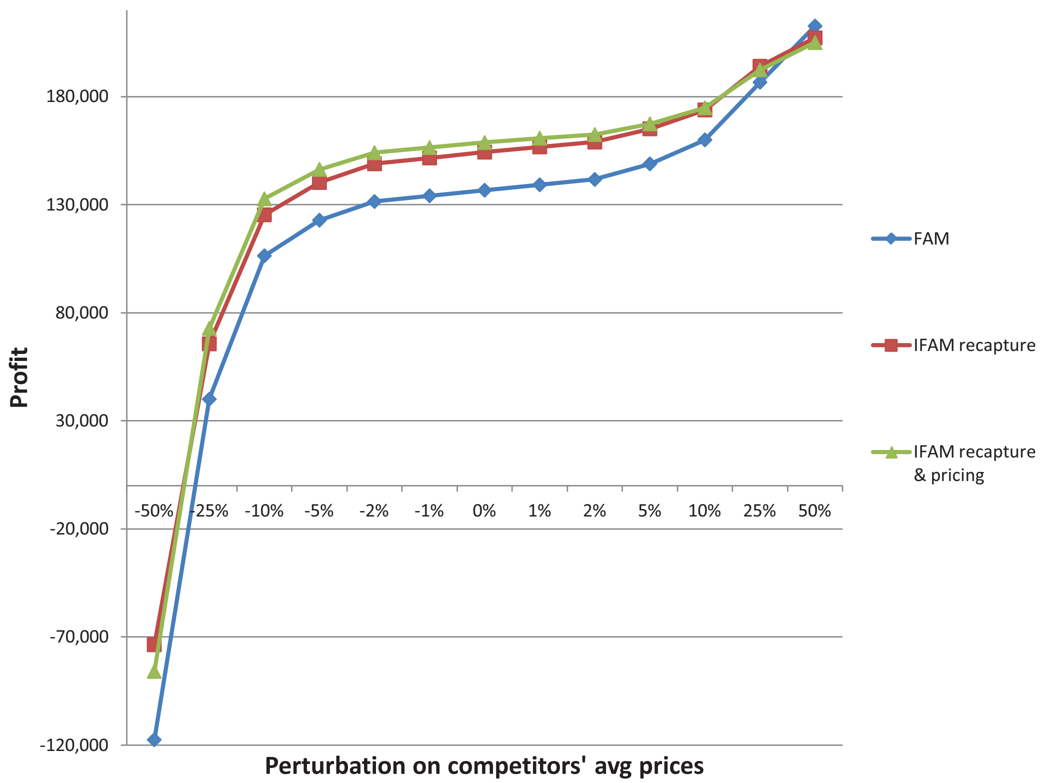

Figure 9: Sensitivity to competitors' price - Pricing - Experiment 24

the range from -25% to +25%. However when the price of competitors change more than 25% IFAM with pricing is outperformed by IFAM. The reasoning behind is that pricing decision is taken given the competitors' prices and the planning decisions are directly affected by those prices. All in all this only occurs with very high perturbations and even in those cases the improvement compared to leg-based FAM can be clearly observed.

The analysis shows that integrating supply-demand interactions with a choice model enables to react to market conditions which cannot be achieved through a leg-based FAM setting. IFAM gives this reaction by updating the recapture effects and the decisions on spill. IFAM with pricing also reacts with the changes on the prices in addition to the spill and recapture effects.

5 Reformulation of the model

In this section we present a logarithmic transformation of the itinerary choice model in order to reduce the complexity of the integrated schedule planning model presented by Atasoy *et al.* (2012).

We first present how the logarithmic transformation is done in section 5.1. Then we present the relative market shares idea proposed by Wang *et al.* (2012) in section 5.2. In section 5.3 we provide the RMM sub-problem as a result of the reformulation. The logarithmic transformation necessitates the information on the selected flights since 0 market share cannot be treated in the logarithmic space. Therefore the new formulation is appropriate for bi-level programming framework where the schedule planning decisions and revenue related decisions are separated. In section 5.4 we present a Benders' Decomposition framework for the iterative solutions of IFAM and RMM. Finally we discuss the flexibility of the reformulated problem for the extensions of the demand model in section 5.5.

5.1 Log transformation of the logit model

The integration of the presented logit model into an optimization framework brings complexity issues. In addition to the nonlinearity, depending on the formulation it may result with a non-convex problem. The integrated scheduling, fleeting and pricing model presented in Atasoy *et al.* (2012) is mixed integer nonlinear problem where the convexity is not guaranteed. The logit model is explicitly integrated in the model representing the decisions on pricing and spill effects.

In order to develop a framework where the non-convexity can be avoided, we propose a logarithmic transformation of the problem.

The denominator in equation 2 is same for all the itineraries in the segment. Similar to Schön (2008) a new variable υ is defined as follows :

$$\upsilon_s = \frac{1}{\sum_{j \in I_s} \exp(\beta \ln(p_j) + c_j)} \qquad \forall h \in H, s \in S^h \tag{45}$$

Therefore the market share of each itinerary in the segment can be written as in equation 46.

$$\text{ms}_i = \upsilon_s \cdot \exp(\beta \ln(p_i) + c_i) \qquad \forall h \in H, s \in S^h, i \in I_s \tag{46}$$

Since we do not use the full logit formula we need to make sure that the market shares sum up to 1. Therefore we need the following relation for each market segment:

$$\sum_{i \in I_s} \text{ms}_i = 1 \qquad \forall h \in H, s \in S^h \tag{47}$$

The logarithm of equation 46 is given by:

$$\ln(\text{ms}_i) = \ln(\upsilon_s) + \beta \ln(p_i) + c_i \qquad \forall h \in H, s \in S^h, i \in I_s \tag{48}$$

We can denote $\ln(\text{ms}_i)$ by $\text{ms}_i^{'}$, $\ln(p_i)$ by $p_i^{'}$, an similarly $\ln(\upsilon_s)$ by $\upsilon_s^{'}$. Therefore we can write the following linear relation:

$$\text{ms}_i^{'} = \upsilon_s^{'} + \beta p_i^{'} + c_i \qquad \forall h \in H, s \in S^h, i \in I_s \tag{49}$$

5.2 Relative market shares and spill effects

Atasoy *et al.* (2012) present a model where the logit function is explicitly integrated in the model. The pricing decision is given by the full logit formula. Similarly the recapture ratios are also modeled by the logit function where the actually desired itinerary is removed from the choice set. Wang *et al.* (2012) represent the spill and recapture effects with a neater formulation. They keep the market shares proportional to the utility of the itineraries however they do not limit it to the exact market share given by the logit. It serves as an upper bound but it is not necessarily realized. This allows the spill and recapture of the passengers. With this formulation there is no decision variable on the spilled number of passengers and no parameter on the recapture ratios. Therefore no control is available on the spill. Furthermore, they do not have any pricing decision in the formulation.

Combined with our logarithmic transformation the market share formulation can be re-written as follows:

$$
\begin{aligned}
&\mathrm{ms}_i \leq \mathrm{ms}_j \frac{\exp(V_i)}{\exp(V_j)} \\
&\mathrm{ms}_i^{'} \leq \mathrm{ms}_j^{'} + V_i - V_j \\
&\mathrm{ms}_i^{'} \leq \mathrm{ms}_j^{'} + \beta p_i^{'} + c_i - \beta p_j^{'} - c_j \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}), j \in I_s^{'},
\end{aligned} \tag{50}
$$

where itineraries i and j are in the same market segment s and itinerary j represents the competing itineraries available in the market. The expected utility of j is constant since there is no pricing decision over the competing itineraries. This relation ensures that the market share of an itinerary is proportional to its relative expected utility. In our setting there is only a single competing itinerary in a market segment. If there are several the market share and the expected utility should be replaced by their sum over all the competing itineraries.

With this formulation there is no need for the variables for the number of spilled passengers and the recapture ratios. Moreover the defined variable υ_s is also redundant since relative market shares are considered rather than the full logit representation.

5.3 Sub-problem - RMM

RMM maximizes the revenue with the decisions on the market share, pricing and seat allocation. The decisions on the operated flights and the fleet assignment are inputs to the model. Fleet assignments are represented by $X_{k,f}$ parameters. The transformed model can be given as follows:

$$
z^*_{\text{RMM}} = \max \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s')} (\ln(D_s) + \mathrm{ms}_i^{'} + p_i^{'}) - \mathrm{M}(\mathrm{ms}_i - \exp(\mathrm{ms}_i^{'}))^2 \tag{51}
$$

$$
\sum_{i \in I_s} \mathrm{ms}_i = 1 \qquad \forall h \in H, s \in S^h \tag{52}
$$

$$
\mathrm{ms}_i^{'} \leq \mathrm{ms}_j^{'} + \beta p_i^{'} + c_i - \beta p_j^{'} - c_j \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}), j \in I_s^{'} \tag{53}
$$

$$
\sum_{s \in S^h} D_s \sum_{i \in (I_s \setminus I_s')} \delta_{i,f} \mathrm{ms}_i \leq \sum_{k \in K} \pi_{k,f}^h \qquad \forall h \in H, f \in F \tag{54}
$$

$$
\sum_{h \in H} \pi_{k,f}^h \leq Q_k X_{k,f} \qquad \forall f \in F, k \in K \tag{55}
$$

$$
\pi_{k,f}^h \geq 0 \qquad \forall h \in H, k \in K, f \in F \tag{56}
$$

$$
\ln(LB_i) \leq p_i^{'} \leq \ln(UB_i) \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}) \tag{57}
$$

$$
\mathrm{ms}_i^{'} \in \Re \qquad \forall h \in H, s \in S^h, i \in I_s \tag{58}
$$

$$
\mathrm{ms}_i \geq 0 \qquad \forall h \in H, s \in S^h, i \in I_s \tag{59}
$$

Objective is to maximize the revenue (51).$\exp\left(\ln\left(D_s\right) + \mathrm{ms}_i^{'} + p_i^{'}\right)$ is mathematically equivalent to $D_s \cdot \mathrm{ms}_i \cdot p_i$. Furthermore we can remove the $\exp\left(\right)$ and have the presented objective function. There is a penalty term, M, which penalizes the deviation from the real market share when represented with the logarithmic transformation. We need this penalty because the relation between the market share and its logarithm could not be given by an equality constraint such as $\exp\left(\mathrm{ms}_i^{'}\right) = \mathrm{ms}_i$ in order to avoid non-convexity. Constraints (53) ensure the relation between the expected utility of the itineraries and their market shares in a market segment as explained in section 5.2. Constraints (54) ensure that the allocated number economy/business seats for a flight should satisfy the realized economy/business demand. Constraints (55) maintain that the actual capacity of the aircraft is respected.

The presented RMM has linear constraints and continuous variables. The only nonlinearity is in the objective function. It can be shown that the objective function is a concave function (seeAppendix C). Therefore the problem is a concave nonlinear problem (NLP) given the decisions on the schedule design and fleet assignment. This problem can be embedded in an iterative framework in the context of bi-level modeling where the fleet assignment model (IFAM) is solved and the $x_{k,f}$ variables are transferred to RMM.

5.4 Generalized Benders' Decomposition framework

In this section we provide the Generalized Benders' Decomposition framework for the reformulated problem based on the Mixed Integer Nonlinear Programming chapter of Li and Sun (2006).

The idea is an iterative solution of the RMM sub-problem and the master problem which is the IFAM in our case. The sub-problem optimizes the price, market share and the seat allocation to each class of passengers given the fleet assignment decisions. This solution provides Benders' cuts to the master problem through constraints (55). Let's consider a simplified version of the problem with all flights being mandatory, all itineraries being non-stop and economy. In this case we have one itinerary per flight and no decision on seat allocation. With such a simplification the master problem can be written as follows:

$$\max \quad \alpha \tag{60}$$

$$\text{s.t. } \alpha \leq \sum_{s \in S} D_s \sum_{i \in (I_s \setminus I'_s)} \exp\left(P_i^{c'} + MS_i^{c'}\right) - \sum_{\substack{k \in K \\ f \in F}} C_{k,f} X_{k,f}^c$$

$$+ \sum_{\substack{k \in K \\ f \in F}} (Q_k \lambda_f^c - C_{k,f})[x_{k,f} - X_{k,f}^c] \qquad \forall c \in \text{CUTS} \tag{61}$$

$$\sum_{k \in K} x_{k,f} = 1 \qquad \forall f \in F \tag{62}$$

$$y_{k,a,t^-} + \sum_{f \in \text{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f \in \text{Out}(k,a,t)} x_{k,f} \qquad \forall [k,a,t] \in N \tag{63}$$

$$\sum_{a \in A} y_{k,a,\text{minE}_a^-} + \sum_{f \in CT} x_{k,f} \leq R_k \qquad \forall k \in K \tag{64}$$

$$y_{k,a,\text{minE}_a^-} = y_{k,a,\text{maxE}_a^+} \qquad \forall k \in K, a \in A \tag{65}$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \tag{66}$$

$$y_{k,a,t} \geq 0 \qquad \forall [k,a,t] \in N \tag{67}$$

The Lagrangian multipliers associated with constraints (55) is represented by λ_f. Constraints (61) represent the main idea of the framework. The information on the potential revenue change by a modification on the fleet assignment is carried with λ_f^c's at each iteration c. The multipliers should be obtained through the optimality conditions of the RMM sub-problem.

5.5 The generalization of the itinerary choice model

The presented integrated schedule planning model with pricing includes an itinerary choice model where the only policy variable is the price. Furthermore the considered explanatory variables of the logit model are only itinerary attributes, in other words there is no socio-economic characteristics related to passengers which enables to have an aggregate level model. In this section we discuss the flexibility of the reformulated model for the extensions of the demand model.

5.5.1 Additional policy variables

When there are more additional variables they will appear as an additional term in the market share constraint (53). Let's say this additional variable is the departure time of the itinerary, dt_i. The new market share relation would be as follows:

$$\text{ms}_i^{'} \leq \text{ms}_j^{'} + \beta p_i^{'} + \beta_{dt} dt_i + c_i - \beta p_j^{'} - \beta_{dt} dt_i - c_j \tag{68}$$

The complexity of the constraint does not change and since the time does not appear directly in the objective function the structure of the problem is conserved. When the decision on the departure time is introduced the schedule design part of the model will be updated. However this will not change the revenue problem as already mentioned.

5.5.2 Socio-economic characteristics

In this section we investigate the case with socio-economic characteristics in the presence of individual data availability. The utility for each individual n and itinerary i with such individual level characteristics can be represented by:

$$V_{i,n} = \beta \ln(p_i) + \beta_{i,n} z_n + c_i \qquad \forall n \in N, h \in H, s \in S^h, i \in I_s, \tag{69}$$

where z_n is a socio-economic variable for individual n, and $\beta_{i,n}$ is the corresponding alternative specific parameter. Note that one of the β_{i_n}'s should be fixed to 0 for identification purposes.

In the previous cases we presented the logit model with market shares since the demand model is aggregate. When we have individual level characteristics we need the choice probability for each individual n. And the demand for an itinerary i is the sum of the choice probabilities over individuals as presented below:

$$d_i = \sum_{n \in N} \text{Prob}_{i,n} \qquad \forall h \in H, s \in S^h, i \in I_s, \tag{70}$$

where $\text{Prob}_{i,n}$ is the choice probability for alternative i for individual n. The choice probability is represented by:

$$Prob_{i,n} = \upsilon_{s,n} \cdot \exp(\beta \ln(p_i) + \beta_{i,n} z_n + c_i) \qquad \forall n \in N, h \in H, s \in S^h, i \in I_s, \tag{71}$$

where a new variable $\upsilon_{s,n}$ is defined for each market segment s and individual n similar to (45) in section 5.1:

$$\upsilon_{s,n} = \frac{1}{\sum_{j \in I_s} \exp(V_{j,n})} \qquad \forall h \in H, s \in S^h. \tag{72}$$

The market share relation in RMM (53) needs to be replaced by the choice probability relation as given by:

$$\text{Prob}'_{i,n} \leq \text{Prob}'_{j,n} + \beta p'_i + \beta_{i,n} z_n + c_i - \beta p'_j - \beta_{j,n} z_n - c_j, \tag{73}$$

where $\text{Prob}^{'}_{i,n}$ represents $\ln(\text{Prob}_{i,n})$. Similarly the constraints (52) should be replaced by:

$$\sum_{i \in I_s} \text{Prob}_{i,n} = 1 \qquad \forall n \in N, h \in H, s \in S^h. \tag{74}$$

The constraints on the demand-capacity balance (54) should also be modified accordingly:

$$\sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} \sum_{n \in N} \delta_{i,f} \text{Prob}_{i,n} \leq \sum_{k \in K} \pi^h_{k,f} \qquad \forall h \in H, f \in F. \tag{75}$$

Finally the objective function of the RMM should be adapted. Total revenue is represented by:

$$\sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} p_i \sum_{n \in N} \text{Prob}_{i,n}, \tag{76}$$

which can be reformulated using the logarithmic transformation as given in equation(77). $\exp()$ is removed since for the maximization problem it does not bring any change. In this case penalty should be imposed on the deviation of $\exp(\text{Prob}^{'}_{i,n})$ from $\text{Prob}_{i,n}$ similar to (51).

$$\sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} \sum_{n \in N} p^{'}_i + \text{Prob}^{'}_{i,n}. \tag{77}$$

Therefore the presented framework is valid when there are socio-economic characteristics in the demand model.

6 Conclusions and Future Research

In this paper we present the added value of different level of supply-demand interactions through a sensitivity analysis. It is concluded that choice-based supply-demand interactions are not sensitive to slight changes on demand model parameters. This shows the robustness of the choice-based framework.

The integration of choice-based pricing into the IFAM brings nonlinearities that cannot be characterized as convexity/concavity. Therefore in this paper we propose a logarithmic transformation of the logit model which enables us to have a concave RMM subproblem. With this reformulation, the IFAM and RMM can be considered in an iterative framework where the duals of RMM provide information to the IFAM.

Acknowledgments

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A RMM with choice-based recapture

$$z^*_{\mathbf{RMM}} = \max \sum_{h\in H} \sum_{s\in S^h} \sum_{i\in(I_s\setminus I'_s)} (d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} t_{j,i} b_{j,i}) p_i \tag{78}$$

$$\begin{aligned} s.t. \sum_{s\in S^h} \sum_{i\in(I_s\setminus I'_s)} \delta_{i,f} d_i - \sum_{j\in I_s} \delta_{i,f} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} \delta_{i,f} t_{j,i} b_{j,i} & \\ \leq \sum_{k\in K} \pi^h_{k,f} & \qquad \forall h\in H, f\in F \qquad (79) \\ \sum_{h\in H} \pi^h_{k,f} \leq Q_k X_{k,f} & \qquad \forall f\in F, k\in K \qquad (80) \\ \sum_{j\in I_s} t_{i,j} \leq d_i & \qquad \forall h\in H, s\in S^h, i\in I_s \qquad (81) \\ \pi^h_{k,f} \geq 0 & \qquad \forall h\in H, k\in K, f\in F \qquad (82) \\ d_i \leq \tilde{d}_i & \qquad \forall i\in I \qquad (83) \\ t_{i,j} \geq 0 & \qquad \forall i\in I, j\in I \qquad (84) \end{aligned}$$

Note that this RMM does not have the pricing decision and therefore price (p_i) and recapture ratios ($b_{i,j}$) are input parameters to the model.

B RMM with choice-based recapture and pricing

$$z^*_{\textbf{RMM-P}} = \max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I_s^{'})} t_{j,i}b_{j,i})p_i \tag{85}$$

$$s.t. \sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})} \delta_{i,f}d_i - \sum_{j\in I_s}\delta_{i,f}t_{i,j} + \sum_{j\in(I_s\setminus I_s^{'})}\delta_{i,f}t_{j,i}b_{j,i}$$

$$\leq \sum_{k\in K}\pi^h_{k,f} \qquad \forall h\in H, f\in F \tag{86}$$

$$\sum_{h\in H}\pi^h_{k,f} \leq Q_k X_{k,f} \qquad \forall f\in F, k\in K \tag{87}$$

$$\sum_{j\in I_s} t_{i,j} \leq d_i \qquad \forall h\in H, s\in S^h, i\in I_s \tag{88}$$

$$\tilde{d}_i = D_s\frac{\exp(V_i)}{\sum_{j\in I_s}\exp(V_j)} \qquad \forall h\in H, s\in S^h, i\in I_s \tag{89}$$

$$b_{i,j} = \frac{\exp(V_j)}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k)} \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}), j\in I_s \tag{90}$$

$$\pi^h_{k,f}\geq 0 \qquad \forall h\in H, k\in K, f\in F \tag{91}$$

$$d_i \leq \tilde{d}_i \qquad \forall i\in I \tag{92}$$

$$0\leq p_i \leq UB_i \qquad \forall i\in I \tag{93}$$

$$t_{i,j}\geq 0 \qquad \forall i\in I, j\in I \tag{94}$$

$$b_{i,j}\geq 0 \qquad \forall i\in I, j\in I \tag{95}$$

C Concavity of the reformulated RMM sub-problem

The objective function (51) can be evaluated in two parts. First part is a linear term which is other words both convex and concave. The Hessian matrix for the second part with respect to the variables ms_i and $\text{ms}_i^{'}$ is provided as follows:

$$H = \begin{pmatrix} \frac{\partial^2 z_i}{\partial \text{ms}_i^2} & \frac{\partial^2 z_i}{\partial \text{ms}_i \partial \text{ms}_i^{'}} \\ \frac{\partial^2 z_i}{\partial \text{ms}_i^{'} \partial \text{ms}_i} & \frac{\partial^2 z_i}{\partial \text{ms}_i^{'2}} \end{pmatrix}.$$

In order to have concavity we need to have $\frac{\partial^2 z_i}{\partial \text{ms}_i^2}$ and $\frac{\partial^2 z_i}{\partial \text{ms}_i^{'2}} \leq 0$. Furthermore we need to have the determinant nonnegative, $\frac{\partial^2 z_i}{\partial \text{ms}_i^2}\frac{\partial^2 z_i}{\partial \text{ms}_i^{'2}}$ - $\frac{\partial^2 z_i}{\partial \text{ms}_i \partial \text{ms}_i^{'}}\frac{\partial^2 z_i}{\partial \text{ms}_i^{'} \partial \text{ms}_i} \geq 0$.

When we re-write the Hessian with the appropriate partial derivatives we obtain:

$$H = \begin{pmatrix} -2M & 2M\exp(\text{ms}_i^{'}) \\ 2M\exp(\text{ms}_i^{'}) & 2M\exp(\text{ms}_i^{'})(\text{ms}_i - 2\exp(\text{ms}_i^{'})) \end{pmatrix},$$

where $-2M$ is clearly negative and off-diagonal entries are positive. The last term of the Hessian is $2M\exp(\text{ms}_i^{'})(\text{ms}_i - 2\exp(\text{ms}_i^{'}))$.

We can check the first derivative with respect to ms_i which is given by:

$$\frac{\partial z_i}{\partial \text{ms}_i} = -2M(\text{ms}_i - \exp(\text{ms}_i^{'})). \quad (96)$$

This first derivative is only zero when $\text{ms}_i = \exp(\text{ms}_i^{'})$. Similarly, the derivative with respect to $\text{ms}_i^{'}$ is given by:

$$\frac{\partial z_i}{\partial \text{ms}_i^{'}} = 2M \exp(\text{ms}_i^{'})(\text{ms}_i - \exp(\text{ms}_i^{'})), \quad (97)$$

which is also zero only when $\text{ms}_i = \exp(\text{ms}_i^{'})$. Therefore we have:

$$\frac{\partial^2 z_i}{\partial \text{ms}_i^{'2}} = 2M \exp(\text{ms}_i^{'})(\text{ms}_i - 2\exp(\text{ms}_i^{'})) \quad (98)$$

$$= -2M \exp(\text{ms}_i^{'})^2 < 0. \quad (99)$$

With a similar analysis the determinant of the Hessian matrix $(\frac{\partial^2 z_i}{\partial \text{ms}_i^2} \cdot \frac{\partial^2 z_i}{\partial \text{ms}_i^{'2}} - \frac{\partial^2 z_i}{\partial \text{ms}_i \partial \text{ms}_i^{'}} \cdot \frac{\partial^2 z_i}{\partial \text{ms}_i^{'} \partial \text{ms}_i})$ is computed as zero. This shows the concavity (not strictly) of the penalty term.

The objective function is therefore the sum of a linear term and a concave function which completes the proof of the concavity.

D Inverse demand function proposed by Schön, 2008

As explained in section 5.1 market share, ms_i, is given by:

$$\text{ms}_i = \upsilon_s \exp(\beta p_i + c_i), \quad (100)$$

which is similar to (46). If we have the inverse function we can write the price as a function of the choice probability:

$$p_i = \frac{1}{\beta}(\ln(\frac{\text{ms}_i}{\upsilon_s}) - c_i) \quad (101)$$

The revenue for each itinerary, R_i, is given by $\text{ms}_i p_i D_s$ which can be written as:

$$R_i = \frac{1}{\beta} D_s \text{ms}_i (\ln(\frac{\text{ms}_i}{\upsilon_s}) - c_i) \quad (102)$$

The Hessian for R_i is therefore given by:

$$H = \begin{pmatrix} \frac{\partial^2 R_i}{\partial \text{ms}_i^2} = D_s \frac{1}{\beta} \frac{1}{\text{ms}_i} & \frac{\partial^2 R_i}{\partial \text{ms}_i \partial p_i} = -D_s \frac{1}{\beta} \frac{1}{\upsilon_s} \\ \frac{\partial^2 R_i}{\partial p_i \partial \text{ms}_i} = -D_s \frac{1}{\beta} \frac{1}{\upsilon_s} & \frac{\partial^2 R_i}{\partial p_i^2} = D_s \text{ms}_i \frac{1}{\beta} \frac{1}{\upsilon_s^2} \end{pmatrix}$$

where $\text{ms}_i, D_s, \upsilon_s$ are ≥ 0 by definition. $\beta \leq 0$ since it gives the effect of price on the utility. Therefore $\frac{\partial^2 R_i}{\partial \text{ms}_i^2}$ and $\frac{\partial^2 R_i}{\partial p_i^2} \leq 0$ and $\frac{\partial^2 R_i}{\partial \text{ms}_i \partial p_i}$, $\frac{\partial^2 R_i}{\partial p_i \partial \text{ms}_i} \geq 0$. $\frac{\partial^2 R_i}{\partial \text{ms}_i^2} \times \frac{\partial^2 R_i}{\partial p_i^2} - \frac{\partial^2 R_i}{\partial \text{ms}_i \partial p_i} \times \frac{\partial^2 R_i}{\partial p_i \partial \text{ms}_i} = 0$, which shows the concavity (not strictly concave) of the revenue function.

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