SIMULATION OF URBAN ENERGY FLOW: A GRAPH THEORY INSPIRED APPROACH

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ABSTRACT

Following the increased consciousness regarding global warming and other environmental issues, there is a growing urge in cities for a more rational energy supply and use in buildings. In parallel, the range of possible actions has broaden, from building refurbishment and construction options, through local energy production (solar panels, heat pumps) to the larger scale re-use of waste heat, green electricity production or trading.

This evolution has increased the complexity of urban energy management, and more information regarding the energy efficiency of the energy demand and supply is required to evaluate possible improvements. Crude models fail to include all aspects of this complexity, while specialised simulation tools make global assessments and comparisons difficult. There is thus an opportunity for research to provide tools dealing with the increased complexity of urban energy flow to study the numerous options available to decrease the primary energy use of buildings and the related greenhouse gas emissions.

Focusing on the scale of a few hundred buildings, we model in this paper the disaggregated energy flow, from the resources through networks and energy conversion system towards the energy use in buildings for distinct services, as an oriented graph. Combining the limited available data with existing simulation tools and energy consumption data, a new graph theory inspired simulation method is formulated. The resulting detailed picture of energy flow can give access to a large amount of information but, most importantly, will provide a unified tool to compare energy efficiency scenarios regarding a broad range of technological aspects of energy demand and supply.

Keywords: disaggregated urban energy flow simulation, graph theory, factor graph, belief propagation, energy demand and supply, energy use measurements

INTRODUCTION

Graphical models is a framework used in various fields to represent and study probability distributions. Developed independently in statistical physics, coding theory and artificial intelligence, message passing algorithm are used to perform inference on such models and calculate marginal distributions [1].

Let Ψ be a probability density function of N variables $\mathbf{x} = \{x_1, x_2, ..., x_N\}$ that can be written as a product of M factors: $\Psi(\mathbf{x}) = \prod_{a=1}^{M} \psi_a(\mathbf{x}_{\partial_a})$. Each factor ψ_a depends on a subset of variables; the set of the corresponding variable indexes is noted ∂_a and the set of variables is noted $\mathbf{x}_{\partial_a}$. Factor graphs [1, 2] represent such functions as graphs composed of **variable nodes** associated with the variables x_i and **factor or function nodes** associated with functions ψ_a. There is an edge between the variable node i and the factor node a if and only if the function ψ_a depends on the variable x_i, i.e. iff $i \in \partial_a$.

Message-passing algorithms have been developed to compute the marginal or max-marginal distributions of the variables. This approach was proven to be very efficient in particular on tree graphs with discrete variables [1]. The message-passing algorithms operate by passing messages along the edges of the graph; the messages are probability distributions over the variable nodes' domains. Variable nodes inform neighbouring factor nodes of their probability distribution according to the other factor nodes they are connected to ($\nu_{i\to a}^{(t+1)}(x_i)$). In turn, each factor node send neighbouring variable nodes the distribution it would assign to them according to the other connected variable nodes ($\hat{\nu}_{a\to i}^{(t)}(x_i)$).

Considering in particular the **max-product** algorithm, the purpose of which is to find the configuration $\mathbf{x}$ maximising the probability $\Psi(\mathbf{x})$, the message update rules are given by the following equations:

$$\hat{\nu}_{a\to i}^{(t)}(x_i) \propto \max_{\mathbf{x}_{\partial_a \setminus i}} \left\{ \psi_a(\mathbf{x}_{\partial_a}) \prod_{j\in\partial_a\setminus i} \nu_{j\to a}^{(t)}(x_j) \right\} \tag{1}$$

$$\nu_{i\to a}^{(t+1)}(x_i) \propto \prod_{b\in\partial_i\setminus a} \hat{\nu}_{b\to i}^{(t)}(x_i) \tag{2}$$

(note that the message distributions need to be normalised). On a tree-graphical model, these updates converge to the correct max-marginals after at most $t^\star$ iterations, where $t^\star$ is the *diameter* of the graph, i.e. the maximum distance between two variable nodes [1].

While the numerical implementation of this algorithm is straightforward for binary or discrete variables, continuous variables prove to be more challenging. Loeliger [3] discusses options to treat continuous variables; of these the **max-product particle belief propagation** [4] consists in using a list of P samples to discretise the continuous domain. An optimisation can be performed iteratively using the max-product algorithm on the particle sample, and then choosing a more refined discretisation based on the results on the previous sample.

In order to fully define the algorithm, one must fix the number of particles to be used for each variable and their initial value. The initial messages $\nu_{j\to a}^{(0)}(x_j)$ of the variable nodes are set to a uniform distribution, although other initial conditions could be used. An update rule to define the new set of particles based on the results is also necessary, as well as a convergence criterion.

This paper presents an application of the factor graph formalism to the simulation of urban energy flow. Modelling the energy flow as a graph, the necessary simulation is formalised as an optimisation problem that can be solved using the max-product particle belief propagation algorithm.

METHODOLOGY

Modelling the disaggregated urban energy flow, from primary energy to energy services in buildings, naturally leads to an oriented graph representation, with *energy nodes* representing any grouping, transformation or distribution of energy, and edges corresponding to energy flows between nodes. In this conceptual graph, a distribution network is a node which collects the production of any number of energy conversion system (ECS) (or other networks) and distributes this energy, minus losses, amongst other nodes. ECS nodes are

similar, while the energy use of a building is represented by several "sink" energy service nodes. We consider distinctly the space heating, electricity and domestic hot water (DHW) services.

The N variables of interest $\mathbf{x} = \{x_1, x_2, ..., x_N\}$ are the quantities of energy passing through each edge in the chosen unit of time (we consider here annual energy flow, although the formalism is strictly identical for any unit of time). The energy nodes of the conceptual graph can be seen as constraints or information about these variables.

Provided enough information about the structure of the disaggregated energy system is available to build this conceptual graph, the available figures regarding the intensity of the energy flow are diverse:

- Energy losses at network and ECS nodes and energy conservation rules.
- Possible monitored energy flow for any edge (usually for billing purpose).
- Simulated values of energy demand per service.
- Estimated provision mode of each building's services (for instance, which fraction of a building's DHW service is provided by each ECS node).

The first two items correspond to hard constraints on the energy flow through the graph. However, the usually low availability of monitored data is insufficient to guarantee a determined problem. Using building simulation software or other methods to estimate the energy demands of building provide a consistent groundwork of data. Still, the system might be underdetermined where several edges provide the same service for instance. If the simulated demand of space heating for a building is known, and the model shows this service is provided both by a boiler and a heat pump, the system will only be determined once it is specified which part of the demand is provided by each ECS, or once a method (and enough data) to determine this information is defined.

Carefully building the model and setting rules about accepted structures can thus lead to a fully determined problem, which actually becomes overdetermined if monitored data is available together with simulated energy demands. Calculating the most coherent picture of the energy flow over the graph thus corresponds to an optimisation problem, satisfying (when possible) all the constraints regarding energy conservation and measurements, while approaching the simulated or estimated values as much as possible where degrees of freedom remain.

Optimisation problem

In order to formally define this optimisation problem, a cost or energy function E_a must be assigned to each node. The cost of a node is minimal when its constraints are satisfied and increases when the variables stray from the expected values. The variables for which measurements are available can either be fixed, or kept as variable to ensure that a solution might be found even in overdetermined cases (such a solution can be treated later and has the advantage to evidence the conflicting variables). The objective of the optimisation problem is then to find the configuration that minimises the total cost function; a robust method which allows the number of variables to become quite large (a few thousand to simulate a few hundred buildings zone) is preferred.

In a statistical physics analogy, the total cost function can be seen as an energy function $E(\mathbf{x})$. Finding the minimum energy configuration is then equivalent to finding the most probable configuration according to a (fixed low temperature) Bolzmann distribution $\mu_\beta(x) \propto \mathrm{e}^{-E(\mathbf{x})}$. This distribution has two properties supporting the use of factor graph:

- $E(\mathbf{x})$ being a sum of partial cost functions, $\Psi(\mathbf{x})$ can easily be factorized, each factor corresponding to a node: $\Psi(\mathbf{x}) \propto e^{-\sum_a E_a(\mathbf{x}_{\partial_a})} = \prod_a \psi_a(\mathbf{x}_{\partial_a})$, with $\psi_a(\mathbf{x}_{\partial_a}) = e^{-E_a(\mathbf{x}_{\partial_a})}$.
- Most factors concern only a small number of variables.

CASE STUDY

The methodology discussed above was applied to a test case representing a seven-storey apartment building. The conceptual model of the energy flow to supply electricity, space heating and domestic hot water in the building is represented in Fig. 1. The ECS are

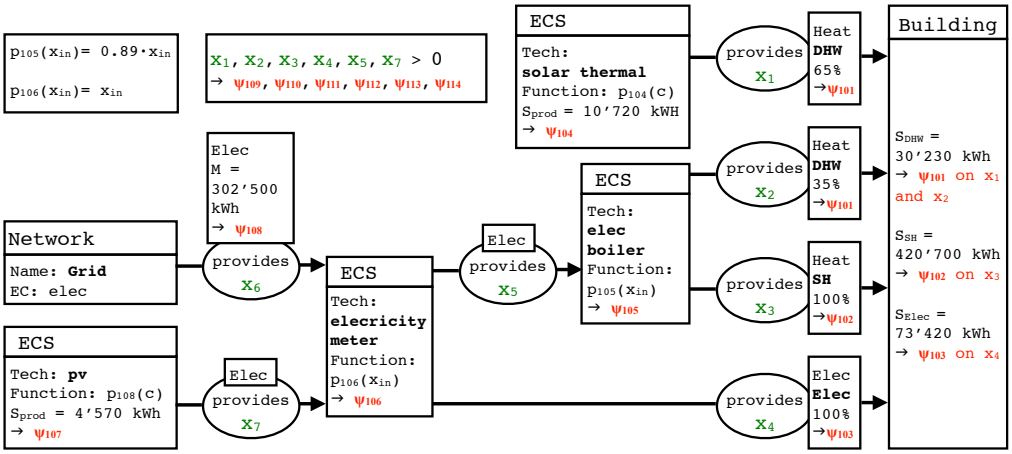

Figure 1: Example of conceptual graph representation of the energy flow concerning the provision of a building's energy services.

modelled through the p_a functions, estimating their energy production x_{out} based either on their consumption x_{in} or on climatic data c. The energy demands have been simulated by external means, as well as the solar panel production; the simulated values are noted S. Finally, the purchased electricity is known (M value). The sign of the energy flow variables x_i is fixed through the orientation of the graph from resources nodes to energy service sink nodes (building).

Each constraint or information is assigned a weight function $E_a(\mathbf{x}_{\partial_a})$ and a probability distribution $\psi_a(\mathbf{x}_{\partial_a}) \propto e^{-E_a(\mathbf{x}_{\partial_a})}$, resulting in the full probability distribution $\Psi(\mathbf{x})$. The corresponding factor graph is shown in Fig. 2, with the same x_i variable nodes and the ψ_a representing the factor nodes. The information available at the energy nodes of the

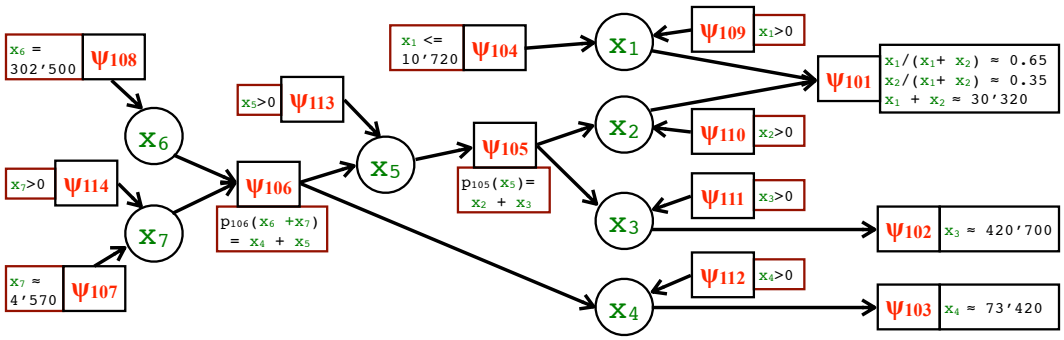

Figure 2: The factor graph corresponding to the test case energy flow.

conceptual model is embodied in the factor nodes ψ_{101} to ψ_{107}. These are completed by measurement data (ψ_{108}) and positive flow constraints (ψ_{109}-ψ_{114}) factor nodes.

The translation of the node properties as cost functions offers a large range of possibilities; the functions must however be chosen carefully in order to convey the intended behaviour. The selected cost functions $E_a(\mathbf{x}_{\partial_a})$ are detailed in Table 1.

Factor node type	Cost (or energy) function	Definitions
Positive flow	$E_{positive,a} = 100 \cdot \max\{0, -\mathbf{x}_{\partial_a}\}$	∂_a contains only one index i
Measurement	$E_{meas,a} = 1000\,\lvert m_{\partial_a} - \mathbf{x}_{\partial_a}\rvert$	∂_a contains only one index i, m_{∂_a} is the measured value of x_{∂_a}
Usual energy conversion systems	$E_{ECS,a}(\mathbf{x}_{\partial_a}) = \left\lvert p_a\left(\sum_{i\in\partial_a}\frac{1+o_{i,a}}{2}x_i\right) - \sum_{i\in\partial_a}\frac{1-o_{i,a}}{2}x_i\right\rvert$	$p_a(x_{in}) = x_{out}$ is the function linking the input and output energy flows of the ECS a, and $o_{i,a} :=$ {1 if orientation $i \to a$, -1 otherwise}
Solar energy production systems	$E_{SEPS,a} = \left\lvert S_a - \sum_{i\in\partial_a} x_i\right\rvert$	S_a is the simulated production of the energy system a
Building's service	$E_{service,a}(\mathbf{x}_{\partial_a}) = 10^{-3}\cdot\max\{T_a, S_a\}\cdot \left[\left(\max\{\frac{T_a}{S_a}, \frac{S_a}{T_a}\} - 1\right) + \left(\sum_{j\in\partial_a}\left(f_j - \frac{x_j}{T_a}\right)^2\right)^2\right]$	S_a is the simulated demand for the service, f_j the fraction of the service the edge (j,a) is expected to provide, and $T_a = \sum_{i\in\partial_a} x_i$

Table 1: Cost functions associated with the different kinds of node. These functions where adapted to avoid unwanted behaviour with zero or (forbidden) negative values.

RESULTS

First tests where performed with a sampling of $P = 3$ or $P = 5$ particles per variable, evenly spread on the domain chosen for this test case to $D_i^{(0)} = \{-100'000, 500'000\}$ for all variables (based on the available simulated and measured values). The parameter $s^{(u)}$ represents the size of the domain $D_i^{(u)}$. Once the belief propagation algorithm has converged, the most probable particle $x_i^{(u)}$ is chosen for each variable based on its max-marginal (given by Eq. 2 summing on *all* neighbouring factor nodes b). To force convergence, the search domain is decreased according to the rule $s^{(u+1)} = k \cdot s^{(u)}$, with k chosen at 0.7 for these tests, and the new particles are evenly distributed on the new domains $D_i^{(u)} = \{x_i^{(u)} - \frac{1}{2}s^{(u+1)}, x_i^{(u)} + \frac{1}{2}s^{(u+1)}\}$. These steps are repeated until $s^{(u)} \leq 0.1$.

The algorithm was found to be stable only if the cost functions are altered to match the granularity of the chosen discretisation, i.e. to ignore any variations in the values smaller than $\frac{s^{(u)}}{P}$: $E^{(u)}(\mathbf{x}) = \min\left\{E(\mathbf{x}') \mid x_i - x_i' \leq \frac{s^{(u)}}{P}\ \forall\, i\right\}$. As the types of cost function are in limited number, this can be performed easily. Using those adapted weight functions, the algorithm converges to the $\mathbf{x}$ values pictured in Fig. 3.

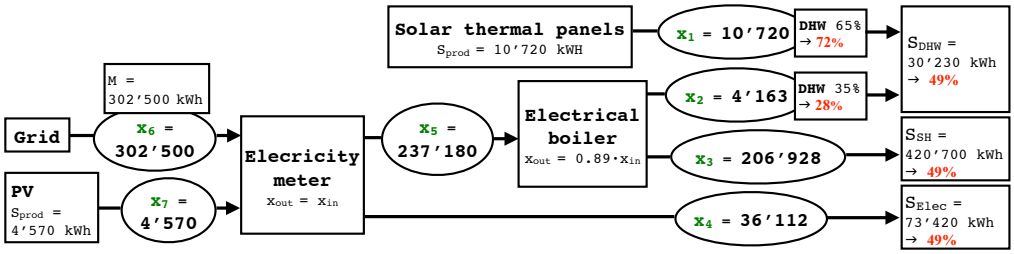

Figure 3: Simulation results: optimal energy flow according to available data.

The solution found fulfills the capital constraints regarding monitored data, energy conservation and simulated energy production. The second point of interest is the repartition

of the energy amongst the services. In this test case, the total supply is known and lower than the total simulated energy demand. As intended, this is reported evenly on all services, which are provided only 49% of the simulated demands. The indications about the production of DHW are intended to be influential only when other constraints are satisfied, and thus do not weight much in this example.

Despite the limited number and rough choice of particles discussed above, the algorithm converges correctly to the values expected given the available data. This is partly due to the deliberately "nice" nature of the energy functions used and their lowered sensibility. The convergence remains stable with small variations of k; smaller k can be used when the number of particle P is increased, however the computational cost increases faster with P than with k.

CONCLUSION

A conceptual model of urban energy flow considered as a graph has been developed. In order to simulate the disaggregated energy flow, the transformation in an optimisation problem through the building up of a factor graph is proposed. Using this formalism, the max-product particle belief propagation algorithm has been successfully applied to simulate the energy flow related to a test-case building. Based on a few other tests, this resolution method is expected to behave correctly in most situations, the main problems originating rather in the number of variables.

Still, in order to refine and possibly accelerate the simulation process, several aspects of the method can be further investigated. The robustness and efficiency of the algorithm in particular could be increase by exploring possible improvements:

- The domain of the variables does not need to be fixed beforehand if the parameter $s^{(u)}$ is not decreased after each iteration, but only once some convergence criterion has been met.
- The optimal combination of the parameters P and k requires more investigation.
- The properties of the cost functions have a dominating role: other possibilities and relative weighting need to be tested.

Nevertheless, the first concluding results encourage studies be carried out at a larger scale and on real cases, in order to assess the robustness and efficiency of the suggested algorithm.

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