Integrated supply-demand models for the optimization of flexible transportation systems

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The only real wisdom is
knowing you know nothing.
— Socrates

To my parents . . .
Sevgili anne ve babama . . .

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Abstract

This thesis investigates methodologies for improving the demand responsiveness of transportation systems through flexibility. The methodologies propose advances both in demand and supply models having a focus on supply-demand interactions. The demand side enables to understand the underlying travel behavior and is important to identify the most important aspects of flexibility that needs to be offered with new transportation alternatives. Supply models that integrate supply-demand interactions lead to more efficient and flexible decision support tools with integrated decision problems. Furthermore the supply models enable to understand the impact of flexibility on transportation operations with appropriate representation of flexibility aspects. The main study area of the thesis is air transportation however we believe that the methodological contributions of the thesis are not limited to any mode and have the potential to provide improvements in various systems.

In the context of demand modeling, advanced demand models are studied. In the first place, hybrid choice models are developed in the context of a mode choice study motivated by a rich data set. Attitudes and perceptions of individuals are integrated in choice modeling framework and an enhanced understanding of preferences is obtained. Secondly, an air itinerary choice model is developed based on a real dataset. A mixed revealed preferences (RP) and stated preferences (SP) dataset is used for the estimation of the demand model. A demand model is obtained with reasonable demand elasticities due to the existence of the SP data.

Advances in demand models can be exploited early in the planning phase when deciding on the capacity. For this matter an integrated airline scheduling, fleeting and pricing model is studied with explicit supply-demand interactions represented by the air itinerary choice model. The integrated model simultaneously decides on schedule design, fleet assignment, pricing, spill, and seat allocation to each class. Several scenarios are analyzed in order to understand the added-value of the integrated model. It is observed that the simultaneous decisions on capacity and revenue bring flexibility in decision making and provide higher profitability compared to state-of-the art models. The main reference model is called the sequential approach that solves the planning and revenue problems sequentially representing the current practice of airlines.

The explicit integration of the demand model brings nonlinearities which cannot be characterized as convexity/concavity. For the solution of the model a heuristic method is implemented which iteratively solves two sub-problems of the integrated model. The first sub-problem is an integrated schedule planning model with fixed prices and the second sub-problem is a revenue management problem with fixed capacity. The heuristic is found to provide better quality feasible solutions, in considerably reduced computational time, compared to the mixed integer nonlinear solver BONMIN. Local search techniques are embedded in the heuristic method which enable to obtain better feasible solutions compared to the sequential approach in reasonable computational time even for instances that are similar to real flight networks.

In order to reduce the complexity of the problem a logarithmic transformation of the logit model is proposed. The transformation results with a stronger formulation of the revenue problem. Price is the only explanatory variable of the logit model that is defined as a decision variable of the optimization model. However the methodology is flexible for other specifications. The reformulation of the model is again a mixed integer non-convex problem however as illustrated with examples and the airline case study, the model can be handled easier. In order to obtain valid bounds on the revenue a piecewise linear approximation is proposed for the non-convexities in the model.

In the last part of the thesis, we focus on analyzing the impact of flexibility by a new design of aircraft called Clip-Air. The main property of Clip-Air is the flexible capacity due to the decoupling of the wing and the capsules (cabin). One, two, or three capsules can be attached under the wing and the configuration of Clip-Air can be adapted to the demand volume. Clip-Air is the main motivation for the contributions of the thesis in the context of supply modeling. The developed integrated models are therefore used in order to carry out a comparative analysis between Clip-Air and standard aircraft. It is found that Clip-Air utilizes the available capacity more efficiently and carries more passengers with less allocated capacity for several scenarios. A sensitivity analysis is performed for different realizations of cost figures. In a nutshell it is observed that the solutions are improved as the level of flexibility is increased, in other words as we move from standard systems to flexible alternatives and from classical planning models to integrated models with explicit representation of demand.

Keywords: flexible transportation systems, airline fleet assignment, integrated planning, revenue management, nonlinear programming, local search heuristic, mode choice, air itinerary choice

Résumé

Cette thèse a pour but le développement de méthodologies visant à améliorer la réactivité des systèmes de transport face aux fluctuations de la demande, en permettant plus de flexibilité. Les méthodologies ont des contributions dans la modélisation de la demande, d'une part, et dans la modélisation de l'offre, d'une autre part, et se focalisent en particulier sur les interactions entre l'offre et la demande. La modélisation de la demande permet de comprendre le comportement sous-jacent des gens en matière de déplacements. De plus, elle est importante pour identifier les aspects de la flexibilité qui doivent être comblés à l'aide de nouvelles alternatives de transport. Les modèles d'offre intégrant les interactions entre l'offre et la demande produisent des outils de décision plus efficaces et flexibles. De plus, les modèles d'offre permettent de comprendre l'impact de la flexibilité sur les opérations de transport avec une représentation appropriée des aspects de flexibilité. Le principal domaine d'étude de cette thèse est le transport aérien. Cependant, nous pensons que les contributions méthodologiques de cette thèse ne sont pas limitées à un seul mode et ont la possibilité d'améliorer divers systèmes.

Dans le contexte de la modélisation de la demande, nous étudions des modèles de demande avancés. Premièrement, des modèles de choix hybrides sont élaborés dans le contexte d'une étude de choix modal motivée par un jeu de données riche. Les attitudes et les perceptions des individus sont intégrés dans un modèle de choix et mènent à une compréhension approfondie des préférences. Deuxièmement, un modèle de choix d'itinéraire aérien est développé sur la base d'un jeu de données réel. Un jeu de données combinant des préférences révélées et déclarées est utilisé pour l'estimation d'un modèle de demande. Un modèle de demande avec des élasticités raisonnables est obtenu grâce à l'existence des données de préférences déclarées.

Les développements dans la modélisation de la demande peuvent être exploités tôt dans la phase de planification lorsque les décisions sont prises quant à la capacité de transport. Dans ce but, un modèle intégré de planification des horaires, de la flotte et de la tarification dans un contexte de déplacements aériens est étudié. Les interactions entre l'offre et la demande sont représentées de manière explicite par le modèle de choix d'itinéraire aérien. Le modèle intégré permet de déterminer simultanément l'horaire, l'affectation de la flotte, la tarification, le surplus de passagers, et l'attribution des sièges à chaque classe. Plusieurs scénarios sont analysés afin de comprendre la valeur ajoutée du modèle intégré. Il peut être observé que les décisions simultanées concernant la capacité et le revenu sont davantage flexibles en termes de prise de décision et génèrent des profits plus élevés par rapport aux modèles actuels. Le

modèle de référence principal consiste en une approche séquentielle qui résout les problèmes de planification et de revenu de manière séquentielle. Celle-ci est couramment utilisée par les companies aériennes.
L'intégration explicite du modèle de demande comporte des non-linéarités qui ne peuvent être considérées ni comme convexes ni comme concaves. Afin de résoudre ce problème, une méthode heuristique est implémentée et résout de manière itérative deux sous-problèmes du modèle intégré. Le premier sous-problème est un modèle intégré de planification des horaires et de la flotte avec des prix fixes. Le second sous-problème est un problème d'optimisation du revenu avec capacité fixe. Nous montrons que l'heuristique permet d'obtenir des solutions admissibles de meilleure qualité que celles obtenues à l'aide du solveur BONMIN, en un temps de calcul considérablement réduit. Des techniques de recherche locale sont intégrées à la méthode heuristique et permettent d'obtenir de meilleures solutions admissibles par rapport à l'approche séquentielle, dans un temps de calcul raisonnable même pour des instances représentant un vrai réseau aérien.
Afin de réduire la complexité du problème, nous proposons une transformation logarithmique du modèle logit. Cette transformation résulte en une formulation plus solide du problème de revenu. Le prix est la seule variable explicative du modèle logit qui est une variable de décision du modèle d'optimisation. Cependant, la méthodologie peut être adaptée à d'autres spécifications. La reformulation du modèle est à nouveau un problème de programmation mixte en nombres entiers et non-convexe, mais le modèle peut être résolu plus facilement, comme le montrent des exemples et l'étude de cas aérien. Pour obtenir des bornes de revenu valides, une approximation linéaire par morceaux est proposée pour les parties non-convexes du modéle.
Dans la dernière partie de la thèse, nous nous concentrons sur l'analyse de l'impact de la flexibilité par une nouvelle conception d'avion appelée Clip-Air. L'avantage principal de Clip-Air est sa capacité flexible due au découplage des ailes et des capsules (cabine). Une, deux ou trois capsules peuvent être attachées sous l'aile et la configuration de Clip-Air peut être adaptée au volume de demande. Clip-Air est la motivation principale aux contributions de cette thèse dans le contexte de la modélisation de l'offre. Les modèles développés sont donc utilisés afin d'effectuer une analyse comparative entre Clip-Air et les avions standards. Nous montrons que Clip-Air utilise la capacité à disposition de manière plus efficace et peut transporter davantage de passagers avec une capacité attribuée plus faible pour plusieurs scénarios. Une analyse de sensibilité est effectuée pour différents coûts. En résumé, nous observons que les solutions sont améliorées lorsque le niveau de flexibilité augmente, c'est-à-dire lorsque les systèmes standards sont remplacés par des alternatives flexibles, et lorsque les modèles de planification classiques deviennent des modèles intégrés avec une représentation explicite de la demande.
Mots clés : systèmes de transport flexibles, affectation de la flotte aérienne, planification intégrée, gestion du revenu, programmation non-linéaire, heuristique de recherche locale, choix modal, choix d'itinéraire aérien

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1 Introduction

1.1 Context and motivation

Flexible transportation systems concept is a key interest for today's world with increasing mobility needs of individuals. The increasing amount of transportation activities and the complexity of travel patterns bring the need for more efficient use of current transportation systems. However in order to achieve substantial improvements new alternatives should be offered with increased flexibility. Therefore, it is an increasing interest to provide flexible alternatives in the context of different transportation modes.

In order to introduce flexibility, actions should be taken in both demand and supply modeling. An advanced understanding of travel behavior helps to understand which notions of flexibility are the most critical. Therefore advanced demand models are needed which take into account the characteristics of individuals in a disaggregate way. From the supply side, new models and methodologies should be developed in order to provide the planning of flexible transportation systems. Furthermore, building flexible decision support tools can be achieved through integrated models where several decisions are taken simultaneously. These integrated models can be extended with an explicit representation of demand in the optimization problems in order to provide an extended flexibility in responding to market demand.

In demand modeling, recent studies focus on disaggregate demand models where the behavior of individuals can be better analyzed. Discrete choice methodology attracts an increasing interest in several contexts. The choice of individuals is explained with various explanatory variables including attitudes and perceptions of individuals. Hybrid choice models can tackle the modeling of attitudes and perceptions for an enhanced forecasting power. Furthermore different data sources can be combined in order to take advantage of different datasets with different characteristics.

In the context of supply modeling, recent models have two main directions. Firstly, several decision problems are integrated in order to obtain superior planning decisions. This is challenging both in terms of modeling and solution methodologies. Secondly, the demand aspect is being introduced in planning problems through supply-demand interactions. These interactions extend the classical planning problems, where the demand is a fixed input, by introducing the reaction of demand to changing planning decisions. This direction is also challenging in terms of various aspects. An appropriate demand model estimation, an integrated planning model with supply-demand interactions and the design of solution methodologies.

In the context of this thesis, advanced demand models are developed motivated by a rich dataset in the context of a mode-choice study. This dataset is obtained from a data collection campaign for a collaborative project between PostBus and the Ecole Polytechnique Fédérale de Lausanne (EPFL). PostBus is the public transport branch of the Swiss postal service, which typically serves in low-density areas of Switzerland. The data contains detailed information on the actual trips of individuals, their socio-economic characteristics, their attitudes and perceptions.

The contributions of the thesis to the supply modeling are motivated by an innovative aircraft, Clip-Air, that is being designed at EPFL. It is an aircraft with flexible capacity. The main property of this new design is the decoupling of the wing and the capsules which changes the concept of fleet. In order to evaluate the advantages of Clip-Air, airline fleet assignment is considered and the methodological contributions of the thesis are built around fleet assignment models.

This thesis is motivated by the above listed research directions that bring together various flexibility notions through advanced demand models, integration of explicit demand models in supply models and the new design of aircraft. The specific contributions in each of the given fields will be provided in the next section.

1.2 Thesis contributions

The contributions of the thesis can be categorized as *advance demand models*, *integrated supply models* and *innovative application*.

Advanced demand models

- We propose hybrid choice models in the context of mode choice taking into account the perceptions and attitudes of individuals.

  Two types of hybrid choice models are developed. The first is a latent variable model where the attitudes of individuals are considered as continuous variables. The second model is a latent class model, where two classes of people, are identified with psychometric indicators and socio-economic characteristics. The use of psychometric indicators for the identification of the classes under a simultaneous estimation of class membership and choice models is one of the initial attempts in the literature. The heterogeneity in individuals' behavior is clearly observed thanks to latent segmentation of the population. Both of the models show the applicability of hybrid choice models using psychometric indicators. Since the heterogeneity is explained through structural equation models the model can be applied in different contexts without the need for attitudinal variables. Indeed we provide forecasting results for both of the models. Furthermore demand indicators such as demand elasticities, market shares, and value of time are also provided.

- We propose an air itinerary choice model based on a mixed dataset consisting of revealed and stated preferences data.

  In the context of air itinerary choice, obtaining a price-elastic demand model using booking data is difficult due to lack of information regarding unobserved choices. Therefore the presented methodology of combining the observed choices with a stated preferences data is a solution for understanding the price elasticity of air travelers.

  As mentioned before, the supply modeling part of this thesis is carried out on air transportation. In order to model supply-demand interactions in the context of airline optimization the proposed demand is a valuable input.

Integrated supply models

- We propose an integrated airline scheduling, fleeting and pricing model where a demand model is explicitly represented.

  The estimated air itinerary choice model is integrated into an airline schedule design and fleet assignment model. The novelty of the model is that the supply-demand interactions are explicitly integrated. The pricing decision and spill and recapture effects are based on the air itinerary choice model. The resulting model optimizes the decisions of scheduling, fleeting, pricing, and seat allocation. The proposed model is shown to provide superior schedule planning decisions compared to classical itinerary-based fleet assignment models for small-medium size instances.

- We propose a local search heuristic for the integrated model.

  The integrated airline scheduling, fleeting and pricing model is a mixed integer nonlinear programming problem where convexity is not guaranteed. For large instances a solution methodology is needed in order to deal with the complexity.

  A local search heuristic is developed which iterates with two sub-problems of the integrated model and visits improved solutions with the mechanisms of price sampling and variable fixing. The heuristic is shown to provide good quality feasible solutions for realistic size instances up to around 300 flights in the network. It is also used to solve the reformulations of the model and the extended model for the case of Clip-Air. Therefore the heuristic method is shown to be a potential solution method for integrated schedule planning models with explicit representation of demand.

- We propose a reformulation of the integrated model and perform sensitivity analysis for the two formulations with perturbed demand model parameters.

  The integrated model is reformulated with a new representation of the logit model. There is no need for the definition of spill variables in this formulation. The local search heuristic is adapted for the reformulation. The resulting profit is higher compared to the previous formulation since the pricing and spill is fully-market driven without the control of the airline.

  Secondly, a sensitivity analysis performed for the integrated model. This sensitivity analysis is important to see the added-value of the integration of supply-demand interactions and understand how robust it is. It is found out that the model is not sensitive to changes in the demand model parameters except high perturbations.

- We propose a logarithmic transformation of the logit model which leads to a stronger formulation.

  The logarithmic transformation of the logit model enables to have linear representation of the demand/market share variable in the model. The transformation is flexible in terms of the specification of the utility function. The added-value of the approach is shown with illustrative examples with aggregate and disaggregate demand models and a case-study of airline revenue management. The model cannot be shown as a convex programming problem and therefore can only provide feasible solutions. In order to obtain a valid bound on the revenue a piecewise linear approximation is proposed which is an ongoing work.

Innovative application

- We extend the itinerary-based airline schedule design and fleet assignment model to the case of Clip-Air.

  The novelty of the model for Clip-Air is that it handles the two-level fleet assignment: wing to flights and capsules to the wing. A comparative analysis is performed between Clip-Air and standard aircraft. Several tests are performed in order to show the advantages and disadvantages of Clip-Air. An enhanced performance is obtained for well-connected networks with a high flight density. In majority of the instances Clip-Air carries more passengers with less allocated capacity. A sensitivity analysis is performed for the cost figures and it is found out that the results are robust to the changes in the costs. The integrated scheduling, fleeting and pricing model is also tested with Clip-Air. It is shown that the potential of Clip-Air compared to standard aircraft is even higher with the integrated approach.

1.3 Thesis outline

The thesis is organized in three main parts.

Part I focuses on the estimation and application of demand models.

Chapter 2 provides the mode-choice study where hybrid choice models are developed using a rich data set with individuals' attitudes and perceptions. A latent variable and a latent class model is developed; demand indicators such as demand elasticities, market shares, value of time are provided; and model validation is presented.

This chapter is published and presented as:

Atasoy, B., Glerum, A., and Bierlaire, M. (2013).Attitudes towards mode choice in Switzerland. disP - The Planning Review, 49 (12): 101 - 117.

Atasoy, B., Glerum, A., and Bierlaire, M. (2011). Mode choice with attitudinal latent class: a Swiss case-study. Proceedings of the Second International Choice Modeling Conference (ICMC) July 4-6, 2011.

Chapter 3 presents the air itinerary choice model based on a mixed RP/SP dataset. The estimation procedure, results and demand indicators are presented.

This chapter is based on:

Atasoy, B., and Bierlaire, M. (2012). An air itinerary choice model based on a mixed RP/SP dataset. Technical report TRANSP-OR 120426. Transport and Mobility Laboratory, ENAC, EPFL.

Part II focuses on airline schedule planning models integrated with the air itinerary choice models, their analysis and solution methodologies.

Chapter 4 presents the integrated airline scheduling, fleeting and pricing model. The demand model presented in Chapter 3 represents the pricing decision and spill effects. Tests with small to medium size instances are presented in order to understand the added-value of the integrated model.

This chapter is published and presented as:

Atasoy, B., Salani, M., and Bierlaire, M. (2013). An integrated airline scheduling, fleeting and pricing model for a monopolized market. Computer-aided Civil and Infrastructure Engineering. doi: 10.1111/mice.12032 (article first published online: July 19, 2013).

Atasoy, B., Salani, M., and Bierlaire, M. (2011). Integrated schedule planning with supply-demand interactions for a new generation of aircrafts . Proceedings of the International Conference on Operations Research (OR) August 30 - September 2, 2011.

Atasoy, B., Salani, M., and Bierlaire, M. Integrated airline schedule planning with supply-demand interactions, IFORS, presented on July 14, 2011, Melbourne, Australia

Chapter 5 presents a heuristic method for the solution of the integrated model presented in Chapter 4. The results of the heuristic are presented in comparison to the BONMIN solver and the sequential approach.

This chapter is published and presented as:

Atasoy, B., Salani, M., and Bierlaire, M. (2013). A local search heuristic for a mixed integer nonlinear integrated airline schedule planning problem. Technical report TRANSP-OR 130402. Transport and Mobility Laboratory, ENAC, EPFL (Submitted to EJOR - First set of reviews are incorporated).

Atasoy, B., Salani, M., and Bierlaire, M. An integrated fleet assignment model with supply-demand interactions, 25th European Conference on Operational Research (EURO), presented on July 09, 2012, Vilnius, Lithunia

Atasoy, B., Bierlaire, M., and Salani, M. An integrated schedule planning and revenue management model, LATSIS Symposium: 1st European Symposium on Quantitative Methods in Transportation Systems, presented on September 07, 2012, Lausanne, Switzerland

Chapter 6 presents a reformulation of the integrated model presented in Chapter 4. Results are obtained with an adapted version of the heuristic given in Chapter 5. Secondly, a sensitivity analysis is provided for the two formulations of the integrated model where the demand model parameters are perturbed and the robustness of solutions is analyzed.

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Atasoy, B., Salani, M., and Bierlaire, M. (2013). Integration of explicit supply-demand interactions in airline schedule planning and fleet assignment. Proceedings of the Swiss Transportation Research Conference (STRC) 24-26 April, 2013.

Chapter 7 presents a log transformation of the logit model which results with a stronger formulation of the revenue sub-problem. Even though the revenue sub-problem is still a non-convex problem, it is shown to perform better compared to the original formulations of the logit with illustrative examples and the airline revenue management model that is used in chapter 6.

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Part III provides the application of the developed airline schedule planning models and methodologies in the context of the innovative aircraft, Clip-Air.

Chapter 8 provides an extension of the airline schedule design and fleet assignment model for Clip-Air. A comparative analysis is performed between Clip-Air and standard aircraft.

This chapter is published as:

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Finally, **Chapter 9** concludes the thesis together with future research directions.

Appendix:
A.1 presents supplementary results for the air itinerary choice model given in chapter 3. **A.2** provides the formulation of IFAM sub-problems used in the heuristic algorithm in chapters 5 and 6 and also in the sensitivity analysis in chapter 6. Similarly, **A.3** provides the RMM sub-problems for different formulations of the model that are used in the heuristic algorithm (chapters 5 and 6) and for the sensitivity analysis (chapter 6). In **A.5**, we extend the integrated model given in chapter 6 to the case of Clip-Air. **A.6** gives the concavity of the inverse-demand function approach used in the literature in order to obtain a convex formulation for revenue maximization models. **A.7** provides the integrated model with the logarithmic transformation that is proposed in chapter 7. In **A.8**, we provide the framework for Lagrangian relaxation procedure with subgradient optimization for the solution of the integrated model given in A.7. Moreover, for the same integrated model, A Generalized Benders' Decomposition framework is sketched in **A.9**.

Advanced demand models Part I

Part I constitutes the demand modeling part of the thesis and combines two chapters on advance demand models. Chapter 2 presents hybrid choice models with integrated perception and attitudes of individuals in the context of mode choice. These models enable to better understand the demand behavior of individuals through latent variables/classes. Chapter 3 proposes a logit model for air itinerary choice based on a mixed data set. This chapter provides an important input for the continuing chapters with integrated supply and demand models.

2 Attitudes towards mode choice in Switzerland

We integrate latent attitudes of the individuals into a transport mode choice model through latent variable and latent class models. Psychometric indicators are used to measure these attitudes. The aim of the inclusion of attitudes is to better understand the underlying choice preferences of travelers and therefore increase the forecasting power of the choice model. We first present an integrated choice and latent variable model, where we include attitudes towards public transport and environmental issues, explaining the utility of public transport. Secondly, we present an integrated choice and latent class model, where we identify two segments of individuals having different sensitivities to the attributes of the alternatives, resulting from their individual characteristics. The calibration of these types of advanced models on our sample has demonstrated the importance of attitudinal variables in the characterization of heterogeneity of mode preferences within the population.

2.1 Introduction

Transport mode choice behavior of the individuals is explained by socio-economic characteristics and attributes of the mode. However these are not the only variables that explain heterogeneity in the mode preferences. It has been well accepted that attitudes and perceptions play an important role in the decision-making process McFadden (1986). Attitudes and perceptions cannot be directly observed from the data and hence considered *latent variables*.

Structural equation models (SEM) provide a powerful methodology to translate attitudes and other latent variables into a statistical model Bollen (1989). SEM has been widely applied in social sciences Bielby and Hauser (1977). An early example of such application is the evaluation of the effect of an individual's occupational aspiration, as a latent variable, on his best friend's Duncan et al. (1968). Later, the development of Linear Structural Relation (LISREL) model Joreskog et al. (1979) contributed to a wider use of SEM in social sciences. One of the major difficulties in SEM is the collection of adequate measurements for the latent variable, since it cannot be observed directly from the data. Research in this context has been concentrating on the measurement of attitudes via psychometrics (Likert, 1932, Bearden and

Netemeyer, 1999, Schüessler and Axhausen, 2011), and more recently by generating data from words Kaufmann et al. (2010).

In transport research, attitudinal variables are studied to explain the travel behavior of individuals through structural equation models. Golob (2003) provides a detailed literature review on numerous applications of SEM in transport. Scheiner and Holz-Rau (2007) analyze the interrelation between socioeconomic characteristics, lifestyle, residential choice and travel behavior of the individuals. Structural equations are developed by using data from a survey in Cologne, Germany. They have found out that lifestyle preferences play a key role in the residential choice of individuals, which in turn has an important impact on the travel mode choice. Similarly, Van Acker et al. (2010) study how residential and travel attitudes affect the decision of residential location and travel behavior with data from an Internet survey in the region of Flanders, Belgium. It is shown that car ownership is significantly affected by the residential attitudes. Furthermore, Van Acker et al. (2011) extend the model by including interrelations between residential and travel mode choices for leisure trips. They point out that the strength of interrelations depends on the mode as well as the activity performed. They also come up with different lifestyle characteristics that result in different decisions on travel mode. By comparing the models with and without lifestyle characteristics, they conclude that there is an improvement in terms of the explained variance in mode choice, with the inclusion of these subjective variables.

The structural equation models of attitudinal variables are integrated into choice models, in order to make use of simultaneous estimation of choice and attitudinal variables. These integrated models are called hybrid choice models, which are introduced by Ben-Akiva et al. (1999), Walker and Ben-Akiva (2002) and Ben-Akiva et al. (2002). They provide a general framework where attitudinal variables are considered as latent variables. These variables are introduced in the choice context through latent variable models and latent classes.

In integrated choice and latent variable models, the attitudinal variables are included as explanatory variables of the choice. Vredin Johansson et al. (2006) analyze the effect of the latent variables of environmental preferences, safety, comfort, convenience and flexibility on the mode choice using a sample of Swedish commuters. They provide insights for policy-makers so as to improve the transport systems through the use of the attitudinal variables. Espino et al. (2006) study the mode choice behavior for suburban trips by including the latent variable of comfort. Abou-Zeid et al. (2010) explain the variability in individuals' willingness to pay, with individuals' attitudes toward travel, through a latent variable model. They introduce a *car-loving* attitude and show that the individuals who dislike public transport are more sensitive to the time and cost changes of public transport compared to others.

Latent class models are used to identify different classes of individuals by making use of the attitudinal variables Collins and Lanza (2004). Different classes may have different taste parameters, choice sets, and decision protocols. Ben-Akiva and Boccara (1995) study the mode choice behavior of commuters and allow different choice sets for different segments of the population. Gopinath (1995) presents latent class models for mode choice behavior and shows that different segments of population have different decision protocols for the choice process as well as different sensitivities for time and cost. Hosoda (1995) works on the mode choice models for shopping trips where both latent variables and latent classes are included in the framework. It is shown that without a proper modeling of heterogeneity in the sample, there can be significant bias in the parameter estimates, even for travel time and travel cost. Therefore attitudinal variables are proposed to be included through appropriate hybrid choice models. More recently, Walker and Li (2007) study lifestyle preferences with a data from Portland, Oregon. They identify different latent classes of individuals that have different residential location choices, resulting from their lifestyle preferences.

In this chapter we present models that integrate attitudes into choice context through latent variables and latent classes. These latent variables and classes are identified with psychometric indicators that are related to the attitudes of individuals in the context of transport modes. With the presented models, we show that the attitudinal variables have significant impacts on the transport mode preferences. The models show two different methodologies to integrate attitudinal variables in a mode choice context. In the first model heterogeneity in the sample is captured through latent attitudinal variables and in the second model through a latent segmentation of the population. We show that the models are operational in the sense that they can be used in order to predict the market shares for different transport modes; to compute elasticities of demand and willingness to pay for individuals. Moreover, in the area of behavior modeling, the presented models are advanced behavioral models compared to classical models and the resulting complexity brings in a better understanding of the travel behavior.

For the preliminary analysis regarding the same research we refer to Atasoy et al. (2010) and Atasoy et al. (2011) where latent variables or classes are used to better explain the travel behavior.

The rest of the chapter is organized as follows: section 2.2 summarizes the data collection campaign. Section 2.3 provides the model specification and estimation results regarding the integrated choice and latent variable model and the integrated choice and latent class model. In section 2.4 we present the validation of the model and the analysis of demand indicators including market shares, demand elasticities and values of time (VOT). Finally we conclude and discuss the future directions of our research in section 2.5.

2.2 Data Collection

A comprehensive data collection campaign is carried out between 2009 and 2010 within the framework of a collaborative project between PostBus and the Ecole Polytechnique Fédérale de Lausanne (EPFL) on travel mode choice. PostBus is the public transport branch of the Swiss postal service, which typically serves in low-density areas of Switzerland.

The first step of the data collection campaign was a qualitative survey conducted by the Urban Sociology Laboratory (LASUR) of the Ecole Polytechnique Fédérale de Lausanne (EPFL). It consisted of interviews of 20 individuals in the Swiss canton of Vaud, the purpose of which was to obtain information on their mobility habits and residential choice. In addition to the interviews, all trips of the respondents were recorded using GPS devices. A complete description of the qualitative survey is reported in Doyen (2005). The qualitative survey provided important insights about the individuals' opinions on transport modes. These outcomes were used in the construction of a revealed preferences (RP) survey.

The second step consists of the RP survey, which is the data source used for the models presented in this chapter. Data on the mobility of inhabitants of suburban areas of Switzerland was collected. Questionnaires were sent to households in 57 towns/villages, which were selected in order to be representative of the PostBus network. For small villages, all the households were included in the sample. For larger towns the sample included all the households in the center and a portion of the surrounding neighborhoods. In total 28'193 respondents received a questionnaire and in return 1763 valid questionnaires (6.25%) were collected. Respondents were asked to report information about all trips performed during one day, including origins, destinations, travel durations, costs, chosen modes and activities at destination. In addition, data about the respondents' opinions on topics related to environment, mobility, residential choice or lifestyle were collected, as well as information about their mobility habits, perceptions of various transport modes, household composition and socio-economic situation. Part of the survey that was dedicated to collect information on opinions, included a series of 54 statements. The respondents had to rate their level of agreement on a five-point Likert scale (Likert, 1932) ranging from a total disagreement (response of 1) to a total agreement (response of 5). These statements, refered as *psychometric indicators*, were designed on the basis of examples in the existing literature (see Kitamura et al., 1997, Redmond, 2000, Ory and Mokhtarian, 2005, and Vredin Johansson et al., 2006) and using the outcomes of the qualitative survey mentioned above.

Examples of the sentences related to the environmental concern of respondents in the revealed preference survey are reported below:

- I am concerned about global warming.
- We should increase the price of gasoline to reduce congestion and air pollution.
- We must act and take decisions to limit emissions of greenhouse gases.
- We need more public transport services, even if taxes are set up to pay for the additional costs.

In this chapter, we present discrete choice models which aim at identifying the factors driving individuals' mode choices over the reported sequences of trips departing from their home and returning to that same place. For instance, a sequence of trips could include a first trip

Table 2.1: Proportions of socio-demographic categories

Category		Sample	Population
Education			
	University	14.2%	6.2%
	Vocational university	16.2%	10.6%
	Certificate of Vocational Training and Education	61.0%	50.9%
	Compulsory school	7.6%	27.6%
	No school diploma	1.0%	4.7%
Age			
	16-19 years	2.3%	8.2%
	20-39 years	21.2%	33.4%
	40-64 years	55.9%	41.6%
	65-79 years	18.7%	12.7%
	80 years and above	1.8%	4.1%
Gender			
	Male	53.0%	49.0%
	Female	47.0%	51.0%

from home to work, a second trip from work to leisure, and a last trip from leisure to home. For each of these sets of trips, the main mode was identified. Therefore, the data we used for estimating the models presented in this chapter consists of 2265 sequences of trips reported by 1763 respondents.

It is to be noted that due to the inaccuracy of the travel durations and costs reported by the respondents for each of their trips, the times and costs used in the models presented in this chapter were imputed using the websites of the Swiss railways (SBB) `http://www.cff.ch` and of ViaMichelin `http://fr.viamichelin.ch`. To be able to use these websites, for each trip, we entered the origin and destination information which was reported by the respondents. The same websites were used to infer the times and costs for the non-chosen alternatives.

In this sample, some socio-demographic categories were oversampled, i.e. individuals with a high education level, male respondents or individuals aged between 40 and 79 years. The proportions of individuals in each category in the sample and in the population of the regions considered in the survey are reported in Table 2.1. For the percentages of each socio-demographic category in the population, we report the data of the Federal Census of 2000.

In section 2.4.1, we are presenting the aggregate indicators of demand including, market shares, elasticities and value of time. These indicators must be computed by weighting each observation of the survey according to the representation of its age category, gender and education level in the regions considered in the survey, in order to evaluate the real demand for private motorized modes, public and soft transport modes in these regions. The weights are calculated by applying the *iterative proportional fitting (IPF)* algorithm.

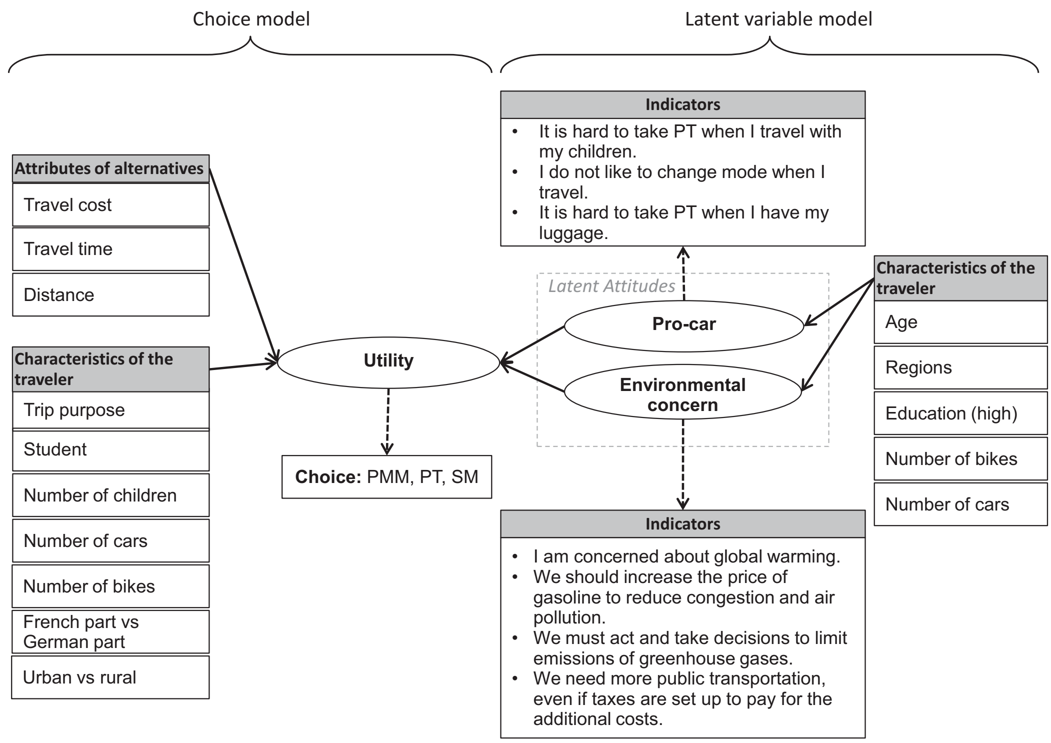


Figure 2.1: Continuous model framework

2.3 Model Specification and Estimation Results

The two models are represented by Figures 2.1 and 2.2. Observed variables such as explanatory variables, psychometric indicators, and choices are represented by rectangular boxes and latent variables such as utilities, attitudinal variables, and classes are represented by ovals. Structural equations are represented by straight arrows while measurement equations are represented by dashed arrows.

The model pictured in Figure 2.1 is called the *continuous model*, since latent attitudes are integrated as continuous explanatory variables in the choice model. It consists of two components: a latent variable model and a discrete choice model.

The model in Figure 2.2 is called the *discrete model*, since two separate choice models are specified for the two latent classes. These classes are identified by attitudinal indicators. The integrated model is composed of a latent class model and two class-specific choice models.

As a base reference we estimate a logit model, which has the same specification as the choice models included in the continuous and discrete models. In sections 2.3.3 and 2.4 we use this base model as a reference to evaluate the added value of latent variables and classes.

It is important to note that for the construction of structural equations for latent variables as well as the identification of latent classes, we have performed a factor analysis as an exploratory

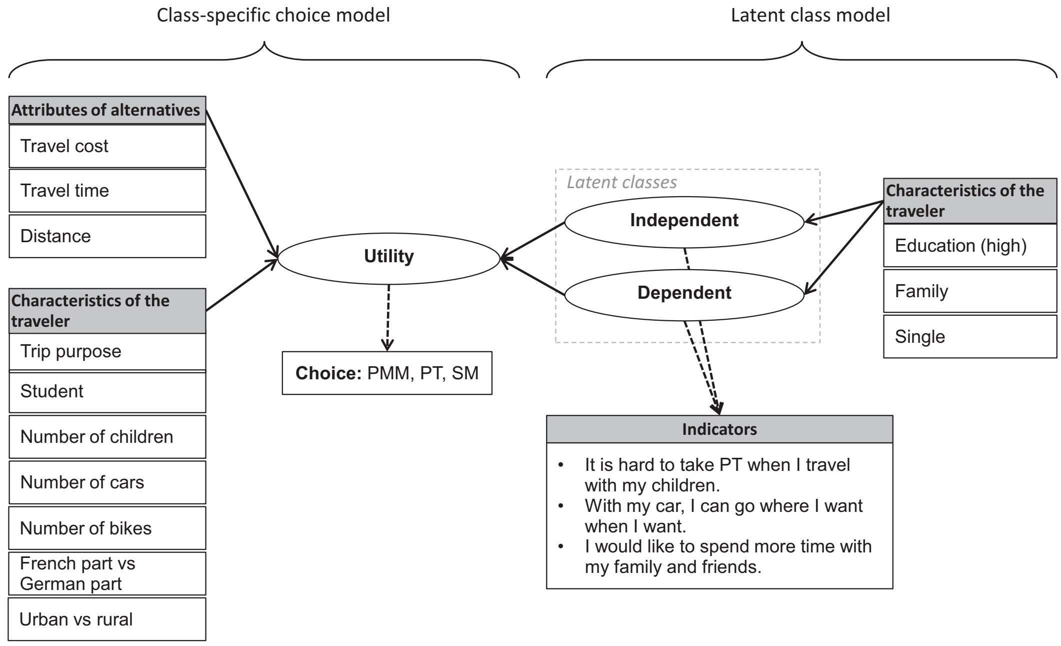


Figure 2.2: Discrete model framework

step with the relevant variables.

2.3.1 Continuous model

Psychometric indicators are studied using factor analysis techniques to identify the most important ones that explain the choice behavior. In Table 2.2 we present results for the first three factors with indicators having a factor loading higher than 0.2 (in absolute sense), which is used as the cut-off value. When we analyze the results, we observe that the first factor corresponds to a negative attitude towards public transport, being positively correlated with the indicators that are related to the inconvenience of public transport. When we do a similar analysis for factor 2 and 3, we observe that the second one is related to the environmental attitude and the third one represents the public transport awareness.

From these results we selected the first and second factors, and named them as *pro-car* and *environmental concern* respectively. For *pro-car* we included the indicators 8, 9 and 10 and for *environmental concern* we worked with 1, 2, 4 and 5 which were found to improve the model.

Structural equations for latent attitudes

In the latent variable model, the structural equations for the attitudes were built as specified in Table 2.3. The *pro-car* attitude is represented by A_{car} and the *environmental concern* is represented by A_{env}. The explanatory variables can be listed as follows:

Table 2.2: Factor analysis results for indicators

Indicators	Factor 1	Factor 2	Factor 3
1- We should increase the price of gasoline to reduce congestion and air pollution.	-0.375	0.453	
2- We need more public transport, even if it means higher taxes.		0.410	
3- Environmentalism harms the small businesses.	0.237		
4- I am concerned about global warming.		0.674	
5- We must act and make decisions to reduce emissions of greenhouse gases.		0.675	
6- I'm not comfortable when I travel with people I do not know well.	0.342		
7- Taking the bus helps to make the city more comfortable and welcoming.		0.311	
8- It's hard to take public transport when I travel with my children.	0.448		
9- It's hard to take public transport when I travel with bags or luggage.	0.587		
10- I don't like to change transport modes when I travel.	0.493		
11- If I use public transport instead of my car, I have to cancel some activities.	0.563		
12- The bus schedule is sometimes hard to understand.	0.398		
13- I know well which bus or train I must take, regardless of where I'm going.			0.709
14- I know the bus schedule by heart.			0.515
15- I use the Internet for schedules and departure times of buses or trains.			0.308
16- I have used public transport all my life.	-0.240		0.370
17- I know some of the drivers of the buses I take.			0.279

Table 2.3: Specification table of the structural equations of the continuous model

Attitudes	A_{car}	A_{env}
$\overline{A_{car}}$	1	-
$\overline{A_{env}}$	-	1
θ_{Ncars}	N_{cars}	-
θ_{educ}	$-Educ$	$Educ$
θ_{Nbikes}	-	N_{bikes}
θ_{age}	-	$Age \cdot (Age > 45)$
θ_{Valais}	$Valais$	-
θ_{Bern}	$Bern$	-
$\theta_{Basel-Zurich}$	$Basel - Zurich$	-
θ_{East}	$EastSwitzerland$	-
$\theta_{Graubünden}$	$Graubünden$	-

- $\overline{A_{car}}$ and $\overline{A_{env}}$ are the constants for the corresponding attitudes,
- N_{cars} represents the number of cars in the household,
- a set of dummy variables ($Valais$, $Bern$, $Basel - Zurich$, $EastSwitzerland$, $Graubünden$) represent the regions that are in the German speaking part except $Valais$ where both French and German are spoken,
- $Educ$ is a dummy variable which is 1 for respondents who have a university degree,
- N_{bikes} is the number of bikes in the household,
- $Age \cdot (Age > 45)$ is a piecewise linear variable which is 0 for the individuals under age 45. Therefore individuals under the age of 45 constitute a reference value and the parameter is estimated for the remaining population.

Let us remark that the parameter for $Educ$ variable is kept the same for the two attitudes, but introduced with a minus sign for *pro-car*. Indeed, considering separate parameters for both equations did not give significantly different results.

Measurement equations for latent attitudes

As mentioned previously, for the attitude *pro-car*, indicators 8, 9 and 10 were used and for *environmental concern*, indicators 1, 2, 4 and 5 were included in the model. Therefore, measurement equations were built with the corresponding indicators of the attitudes as given in equation (2.1).

$$I_k = \alpha_k + \lambda_k A + \upsilon_k \qquad \forall k, \tag{2.1}$$

where α_k and λ_k are parameters to be estimated. A denotes the latent attitudes. I_k represents the psychometric indicators. The error term υ_k is normally distributed with mean 0 and standard deviation σ_{υ_k}.

Structural equations for utilities

Mode choice is assumed to be between the alternatives of *private motorized modes (PMM)*, which include car as a user and passenger, motorbike and taxi, *public transport (PT)*, which consists of bus, train and car postal, and *soft modes (SM)*, that represents walking and bike. Utilities of the alternatives are defined with explanatory variables of modal attributes, individual characteristics and latent attitudes represented by Table 2.4.

The explanatory variables used in the utilities are listed as follows:

- TT_{PMM} and TT_{PT} represent the travel time,
- C_{PMM} and C_{PT} are the travel costs,
- N_{cars} is the number of cars in the household,
- $N_{children}$ is the number of children under age 15 in the household,
- $French$ is a dummy variable being 1 for the respondents in the French speaking part,
- $WorkTrip$ is a dummy variable being 1 for the work related chain of trips,
- $Urban$ is a dummy variable representing the urban regions,
- $Student$ is a dummy variable for the respondents who are either a student or a trainee,
- D_{SM} is the total distance traveled.

Measurement equations for utility

Utilities of the alternatives are measured with the observed choices of the respondents as given in equation (2.2), where C_n is the choice set of individual n.

$$y_{in} = \begin{cases} 1 & \text{if } U_{in} \geq U_{jn}, \forall j \in C_n, \\ 0 & \text{otherwise.} \end{cases} \quad . \tag{2.2}$$

Having defined the structural and measurement models for the latent attitudes and utilities, the likelihood of a given observation is built. It is given by the joint probability of observing choice and indicators of the latent attitudes.

Table 2.4: Specification table of the utilities

	Continuous model			**Discrete model**					**Base model**		
				Class independent			**Class dependent**				
Utilities	V_{PMM}	V_{PT}	V_{SM}	V_{PMM}	V_{PT}	V_{SM}	V_{PMM}	V_{PT}	V_{PMM}	V_{PT}	V_{SM}
ASC_{PMM}	1	-	-	-	-	-	-	-	1	-	-
ASC^1_{PMM}	-	-	-	1	-	-	-	-	-	-	-
ASC^2_{PMM}	-	-	-	-	-	-	1	-	-	-	-
ASC_{SM}	-	-	1	-	-	-	-	-	-	-	-
ASC^1_{SM}	-	-	-	-	-	1	-	-	-	-	-
β_{cost}	C_{PMM}	C_{PT}	-	-	-	-	-	-	C_{PMM}	C_{PT}	-
β^1_{cost}	-	-	-	C_{PMM}	C_{PT}	-	-	-	-	-	-
β^2_{cost}	-	-	-	-	-	-	C_{PMM}	C_{PT}	-	-	-
$\beta_{TT_{PMM}}$	TT_{PMM}	-	-	-	-	-	-	-	TT_{PMM}	-	-
$\beta^1_{TT_{PMM}}$	-	-	-	TT_{PMM}	-	-	-	-	-	-	-
$\beta^2_{TT_{PMM}}$	-	-	-	-	-	-	TT_{PMM}	-	-	-	-
$\beta_{TT_{PT}}$	-	TT_{PT}	-	-	-	-	-	-	-	TT_{PT}	-
$\beta^1_{TT_{PT}}$	-	-	-	-	TT_{PT}	-	-	-	-	-	-
$\beta^2_{TT_{PT}}$	-	-	-	-	-	-	-	TT_{PT}	-	-	-
$\beta_{distance}$	-	-	D_{SM}	-	-	-	-	-	-	-	D_{SM}
$\beta^1_{distance}$	-	-	-	-	-	D_{SM}	-	-	-	-	-
$\beta_{N_{cars}}$	N_{cars}	-	-	N_{cars}	-	-	N_{cars}	-	N_{cars}	-	-
$\beta_{N_{children}}$	$N_{children}$	-	-	-	-	-	-	-	$N_{children}$	-	-
$\beta^1_{N_{children}}$	-	-	-	$N_{children}$	-	-	-	-	-	-	-
$\beta^2_{N_{children}}$	-	-	-	-	-	-	$N_{children}$	-	-	-	-
$\beta_{language}$	$French$	-	-	$French$	-	-	$French$	-	$French$	-	-
β_{work}	$WorkTrip$	-	-	-	-	-	-	-	$WorkTrip$	-	-
β^1_{work}	-	-	-	$WorkTrip$	-	-	-	-	-	-	-
β^2_{work}	-	-	-	-	-	-	$WorkTrip$	-	-	-	-
β_{urban}	-	$Urban$	-	-	$Urban$	-	-	$Urban$	-	$Urban$	-
$\beta_{student}$	-	$Student$	-	-	$Student$	-	-	$Student$	-	$Student$	-
$\beta_{N_{bikes}}$	-	-	N_{bikes}	-	-	-	-	-	-	-	N_{bikes}
$\beta^1_{N_{bikes}}$	-	-	-	-	-	N_{bikes}	-	-	-	-	-
$\beta_{A_{car}}$	-	A_{car}	-	-	-	-	-	-	-	-	-
$\beta_{A_{env}}$	-	A_{env}	-	-	-	-	-	-	-	-	-

Table 2.5: Results of factor analysis

	Factor 1	**Factor 2**
Choice PT		**0.250**
Socio-economic information		
$N_{children}$	**0.517**	
Student/trainee	0.117	**0.770**
N_{cars}	**0.203**	
HighIncome	**0.252**	
Education		-0.123
Age ≥ 60	**-0.375**	
Family status		
Couple without children	**-0.606**	
Couple with children	**0.927**	**-0.368**
Living with parents	0.159	**0.956**
Single	**-0.371**	
Single parent	-0.170	
Roommate	-0.142	
Psychometric Indicators		
PT children		
Flexibility car		-0.130
Family oriented	0.135	

2.3.2 Discrete model

In this section a discrete attitude model is presented where a latent segmentation of the individuals is simultaneously performed with the choice model. With the latent segmentation our aim is to identify the classes of travelers who have different sensitivities to changes in the attributes of the mode alternatives. We decided to work with two latent classes with different demand elasticities.

To start with a reasonable model, a factor analysis is performed as an exploratory analysis with socio-economic characteristics, psychometric indicators and the choice variable. This analysis provides information on the two segments of the individuals with respect to their characteristics and travel behavior.

The indicators included in the presented factor analysis are:

- **PT children:** *It is hard to take public transport when I travel with my children.*
- **Flexibility car:** *With my car, I can go where I want when I want.*
- **Family oriented:** *I would like to spend more time with my family and friends.*

It is observed that family attributes of individuals play an important role in the segmentation, together with their income level and age category. The factor loadings with an absolute value higher than 0.1 can be seen in Table 2.5, where the ones with an absolute value higher than 0.2 are presented in bold. Looking at the results, the two classes are defined as follows:

Table 2.6: Specification table of the structural equations of the discrete model

Latent class	$V_{independent}$	$V_{dependent}$
ASC_{ind}	1	-
γ_{family}	$Family$	-
γ_{income}	$HighIncome$	-
γ_{single}	-	$Single$

- **Class 1 - Independent**: Middle-aged individuals that live with their family and children, are typically active in the professional life, and have high income.

- **Class 2 - Dependent**: Young individuals who are mostly students and old people. This class of individuals are typically singles or couples without children.

The idea behind the naming of the classes is that the second group of individuals are either very young and students/trainees, which makes them economically dependent, or they are old, which limits their physical activities. We note that the factor loading for the indicator *PT children* is not strong. However, this indicator is included in the model since it is observed that it has a significant role in the segmentation as explained in section 2.3.3.

Structural equations for latent classes

With the help of the exploratory analysis the structural equations for the class membership model are built as in Table 2.6, where:

- *Family* is equal to 1 if the individual is living with his/her children, i.e. couples with children and single parent,

- *High Income* is 1 if household income is high,

- *Single* is 1 if the person lives either alone or with parents.

Although there were other characteristics suggested by the factor analysis, these are the ones who are estimated with success in the integrated model.

Measurement equations for the indicators

The class membership model is strengthened with the inclusion of the measurement model of psychometric indicators that are mentioned in the beginning of this section.

The probability of an individual n in latent class s giving a particular response r to an indicator k, $P(I_{nk} = r|s)$ for $r = 1,...,5$, which is called item-response probability, is defined as a parameter to be estimated from the model and measured with the psychometric indicators.

Structural equations for the utilities

For the two latent classes, *independent* and *dependent*, a specific mode choice model is developed. For the class *independent* we have all three alternatives available. However, the individuals belonging to the class *dependent* do not have the soft mode alternative. The reason is that a low proportion (< 5%) of individuals in the dataset chose soft mode as their main mode. Therefore the second class, which includes the old people as well, did not allow the inclusion of soft mode. It is hence assumed that individuals belonging to class *dependent* do not consider soft mode as an alternative.

The specification of the utilities is displayed in Table 2.4 and is similar to the specification of the continuous model. The superscripts 1 and 2 are used to specify the latent class that the parameters are defined for. Superscript 1 specifies the latent class *independent* and superscript 2 is for the class *dependent*. Time and cost parameters are specific to each class to capture taste heterogeneity. Explanatory variables of $N_{children}$ and $WorkTrip$ are also defined specific to each class since the characteristics of classes significantly differ in terms of family attributes and professional life.

The specification of the measurement equations of the utilities for the discrete model are the same as the continuous model.

2.3.3 Estimation results

The maximum likelihood method is used for model estimation where the likelihood function is defined over the joint probability of observing the choice and the indicators of the latent components. The estimation is done by using the software package BIOGEME which allows for the estimation of advanced behavioral modeling as explained in Bierlaire and Fetiarison (2009). Estimation results are presented in Table 2.7 for the continuous model, the discrete model and the base model. The log-likelihood values and goodness of fit results are reported in Table 2.8 for the three models. The log-likelihood values for the continuous and the discrete models are calculated for only the choice probabilities to be comparable with the base model. It can be noticed that the discrete model has the best fit compared to the continuous and base models.

When we look at the utility parameters regarding the modal attributes of time, cost and distance, it is seen that they have the expected signs such that they affect the utility negatively. For the base model and the continuous model, the values of the estimates are close to each other. On the other hand, since latent class model allows the segmentation of the population, we have different sensitivities for the two classes. Individuals in the class *dependent* are more sensitive to the changes in travel cost and time, as expected. The differences in the time and cost sensitivities are also observed by looking at the demand elasticities and willingness to pay values, which will be discussed in section 2.4. Since we do not have the soft mode alternative for the second class, the distance parameter only appears in the utility of the first class. It results in a lower absolute value, compared to the other models, since *active* individuals are less sensitive to changes in distances.

The parameters for the other explanatory variables also have the expected signs and some further observations are presented below:

- The number of children in the household positively affects the utility of private motorized modes, since it brings the need for more flexible forms of transport. When we compare the continuous model with the base model, we observe that the value of the parameter becomes higher with the inclusion of the attitudes regarding the children. When we look at the latent class model, the effect is stronger for the individuals in class *independent* who are typically living with their children. On the other hand, the parameter is not significant for the class *dependent*, which prevents to make any conclusion, since the children related issues are not applicable to this class. Although it is not statistically significant, it is decided to be included for the purpose of presenting the different behavior of the latent segments.

- Individuals performing work related trips have a lower utility for private motorized modes which is expected due to the nature of these trips, being more frequent and almost identical from one day to the next. The latent class model allows to capture the fact that individuals in the class *dependent* do not behave in the same way since they are not active in professional life, being either students or retired people.

- The *pro-car* attitude decreases the utility of public transport and the effect increases with the number of cars in the household. On the other hand, individuals with high education and living in the German speaking part of Switzerland have a lower level of same attitude, which increases the utility of public transport.

- The *environmental concern* increases the utility of public transport so that the individuals who are sensitive to environmental issues use public transport more. This effect is more evident for the individuals with high level of education and increases with age and the number of bikes in the household.

- The integration of attitudes into the choice models enables us to see the effect of variables on the utilities as well as on the attitudes. In the continuous model, we have a variable N_{cars} both in the structural equation of *pro-car* and the utility of private motorized modes. Both parameters support that the utility of public transport decreases with the number of cars in the household. Similarly, N_{bikes} appears both in the structural equation of *environmental concern* and the utility of soft mode.

- Analyzing the results of the discrete model, individuals who are living with their children and have high income have higher probability to belong to class *independent*. On the other hand, single individuals have higher probability to belong to class *dependent*. This shows that our assumptions based on the factor analysis is supported by the model.

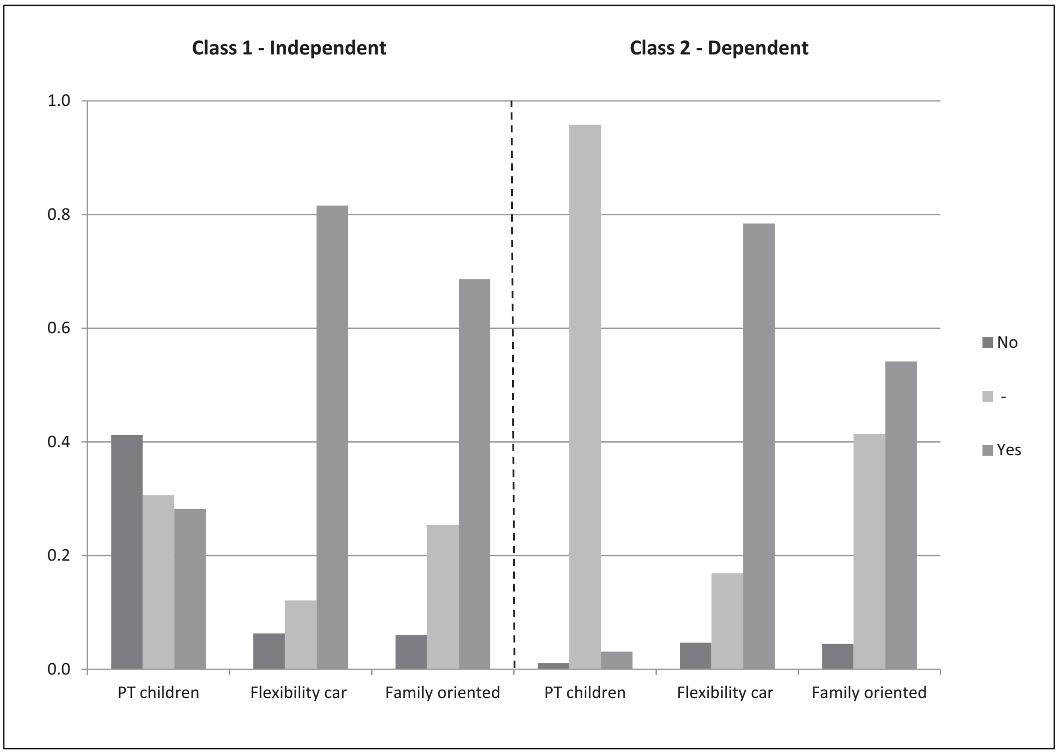


Figure 2.3: Estimated item-response probabilities

For the measurement equations of the discrete model regarding the psychometric indicators, we provide the estimated item-response probabilities in Figure 2.3. We group the probabilities of responding 1 and 2 under the name of *No* and 4 and 5 under the name of *Yes*, and represent the probability of responding 3 as *Neutral (-)*. It is seen that individuals in class *dependent* have very high probability to give a neutral response to the the first indicator which is related to the difficulty of using public transport when traveling with children. This is parallel to our assumptions for defining the two classes as explained in section 2.3.2. The second indicator is related to the flexibility of car and for the two classes, we do not have very different response probabilities, but the probability to agree with the statement is higher for class *independent*. The last indicator is related to the desire to spend time with family and friends and the probability to give a higher value of response is higher for class *independent*, who are living with their family and having their social network. Including these class-specific item-response probabilities in the model strengthens the class membership model by considering the attitudes of individuals related to their travel behavior.

2.4 Model Application

The estimation results of the models presented in section 2.3 enabled us to uncover the variables explaining individuals' mode choices as well as characterizing population segments with different mobility behaviors. We will now explain how these results can be used to

Table 2.7: Estimation results

	Continuous model		**Discrete model**		**Base model**	
Parameter	Estimate	t-test	Estimate	t-test	Estimate	t-test
Utilities						
ASC_{PMM}	-0.599	-0.810*	-	-	-0.413	-2.39
ASC^1_{PMM}	-	-	-0.945	-3.63	-	-
ASC^2_{PMM}	-	-	-0.936	-3.21	-	-
ASC_{SM}	-0.772	-0.930*	-	-	-0.470	-1.27*
ASC^1_{SM}	-	-	0.512	1.31*	-	-
β_{cost}	-0.0559	-5.11	-	-	-0.0592	-5.61
β^1_{cost}	-	-	-0.027	-2.74	-	-
β^2_{cost}	-	-	-0.302	-3.68	-	-
$\beta_{TT_{PMM}}$	-0.0294	-4.79	-	-	-0.0299	-4.96
$\beta^1_{TT_{PMM}}$	-	-	-0.0161	-2.59	-	-
$\beta^2_{TT_{PMM}}$	-	-	-0.111	-5.71	-	-
$\beta_{TT_{PT}}$	-0.0119	-4.40	-	-	-0.0121	-4.55
$\beta^1_{TT_{PT}}$	-	-	-0.00692	-2.5	-	-
$\beta^2_{TT_{PT}}$	-	-	-0.0445	-4.96	-	-
$\beta_{distance}$	-0.224	-4.25	-	-	-0.227	-4.28
$\beta^1_{distance}$	-	-	-0.199	-3.69	-	-
$\beta_{N_{cars}}$	0.970	9.88	1.23	9.8	1.00	10.3
$\beta_{N_{children}}$	0.215	3.23	-	-	0.154	2.37
$\beta^1_{N_{children}}$	-	-	0.404	4.64	-	-
$\beta^2_{N_{children}}$	-	-	-1.03	-1.19*	-	-
$\beta_{language}$	1.06	6.59	1.20	6.78	1.09	6.89
β_{work}	-0.583	-4.94	-	-	-0.582	-5.01
β^1_{work}	-	-	-0.785	-4.83	-	-
β^2_{work}	-	-	-0.130	-0.410*	-	-
β_{urban}	0.283	2.25	0.390	2.81	0.286	2.33
$\beta_{student}$	3.26	9.62	3.70	7.46	3.21	9.33
$\beta_{N_{bikes}}$	0.385	6.85	-	-	0.347	6.34
$\beta^1_{N_{bikes}}$	-	-	0.205	3.46	-	-
$\beta_{A_{car}}$	-0.574	-3.51	-	-	-	-
$\beta_{A_{env}}$	0.393	2.98	-	-	-	-
Attitudes						
$\overline{A_{car}}$	3.02	45.11	-	-	-	-
$\overline{A_{env}}$	3.23	66.49	-	-	-	-
θ_{Ncars}	0.104	4.37	-	-	-	-
θ_{educ}	0.235	6.92	-	-	-	-
θ_{Nbikes}	0.0845	7.42	-	-	-	-
θ_{age}	0.00445	2.22	-	-	-	-
θ_{Valais}	-0.223	-2.8	-	-	-	-
θ_{Bern}	-0.361	-4.74	-	-	-	-
$\theta_{Basel-Zurich}$	-0.256	-4.11	-	-	-	-
θ_{East}	-0.228	-3.21	-	-	-	-
$\theta_{Graubünden}$	-0.303	-3.37	-	-	-	-
Latent class						
ASC_{ind}	-	-	-0.629	-2.64	-	-
γ_{family}	-	-	3.92	3.8	-	-
γ_{income}	-	-	0.46	1.93	-	-
γ_{single}	-	-	0.704	3.51	-	-

(* Statistical significance < 90%)

Table 2.8: Statistics

	Continuous model	Discrete model	Base model
Log-likelihood	-1069.8	-1032.5	-1067.4
ρ^2	0.489	0.507	0.490

Table 2.9: Market shares

Model		PMM	PT	SM
Base model		62.31%	32.09%	5.60%
Continuous model		63.11%	31.20%	5.69%
Discrete model	Class 1	54.91%	36.13%	8.96%
	Class 2	65.73%	34.27%	-
	Overall	62.70%	32.35%	4.94%

quantify the demand by defining several indicators. Moreover an analysis of the validity of the models will be provided.

2.4.1 Demand indicators

In this section, we present several aggregate indicators which reveal the demand of individuals for the different transport modes considered in this study. These indicators consist of *market shares*, *elasticities* and *values of time*.

Let us note that for the discrete model, the demand indicators were computed using the individual class membership probabilities. When these probabilities are weighted according to the representation in the population, their aggregate values are 54.5% for the *independent* class and 45.5% for the *dependent* class.

The first demand indicators that we are interested in are the market shares of each transport mode. Table 2.9 reports the market shares predicted by the logit model, the continuous model and the discrete model, for each mode. The latter do not vary much across models and are the highest for private motorized modes, ranging from 62.31% to 63.11%, the second highest for public transport, ranging from 31.20% to 32.35%, and the lowest for soft mode, ranging from 4.94% to 5.69%.

The market shares predicted by the continuous model differ from class 1, consisting of *independent* individuals, to class 2, representing *dependent* ones, since it cannot predict the choice for soft mode of individuals in class 2.

In order to evaluate the variations in the market shares caused by the increase or decrease of time and cost parameters, the second indicator we report in this chapter are demand elasticities. The aggregate elasticities for the base model and the continuous model are computed using formula (2.3), to assess the effect on demand of changes in a variable $x \in$

$\{C_{PMM}, TT_{PMM}, C_{PT}, TT_{PT}\}$ representing travel costs and times in private motorized modes and public transport, respectively.

$$E_x^i = \frac{\sum_{n=1}^N w_n P_n(i) E_{x_n}^i}{\sum_{n=1}^N w_n P_n(i)}, \tag{2.3}$$

where w_n is the sample weight described in section 2.2 for individual n, $P_n(i)$ is the probability that individual n chooses alternative i and $E_{x_n}^i$ is the elasticity of the demand of person n for variations in individual quantity x_n. The complete formula of this disaggregate elasticity is the following:

$$E_{x_n}^i = \frac{\partial P_n(i)}{\partial x_n} \frac{x_n}{P_n(i)}.$$

For the discrete model, the formula differs slightly since we need to include the membership probabilities to the classes of *independent* and *dependent* individuals. It is given as follows:

$$E_x^i = \frac{\sum_{n=1}^N w_n (P_n(i|Class1) \cdot P_n(Class1) \cdot E_{x_n}^{i,Class1} + P_n(i|Class2) \cdot P_n(Class2) \cdot E_{x_n}^{i,Class2})}{\sum_{n=1}^N w_n (P_n(i|Class1) \cdot P_n(Class1) + P_n(i|Class2) \cdot P_n(Class2))},$$

where $P_n(Class1)$ and $P_n(Class2)$ are the class membership probabilities for classes *independent* and *dependent*, respectively, for an individual n, $P_n(i|Class1)$ and $P_n(i|Class2)$ are the probabilities that n chooses alternative i given that he belongs to class *independent*, respectively class *dependent*, and $E_{x_n}^{i,Class1}$ and $E_{x_n}^{i,Class2}$ are disaggregate elasticities of the demand of person n for variations in individual quantity x_n, given that n belongs to class *independent*, respectively class *dependent*. Precisely, $E_{x_n}^{i,Class1}$ and $E_{x_n}^{i,Class2}$ are given by the following formulas:

$$E_{x_n}^{i,Class1} = \frac{\partial P_n(i|Class1)}{\partial x_n} \frac{x_n}{P_n(i|Class1)}$$

$$E_{x_n}^{i,Class2} = \frac{\partial P_n(i|Class2)}{\partial x_n} \frac{x_n}{P_n(i|Class2)}$$

Table 2.10 reports the aggregate demand elasticities for each of the three models. Let us first note that the latter are lower than 1 in absolute value, implying that demand is not very elastic with respect to changes in time and cost Arnold (2008). No obvious differences in the elasticities can be noticed between the base model and the continuous model. The elasticities for the discrete model are slightly higher.

The cost elasticities for private motorized modes are the lowest ($|\cdot| \leq 0.086$), implying that an increase of 1% in the travel costs for such modes, e.g. caused by an increase of the gasoline price, would result in a decrease in their market shares of less than 0.086%. For public transport, the cost elasticities are higher ($0.2 < |\cdot| < 0.3$), showing that an increase of 1% of travel fares results in a decrease of the market share of public transport slightly higher than 0.2%.

Time elasticities are higher than cost elasticities for all three models and this demonstrates that

Table 2.10: Demand elasticities

Model		PMM		PT	
		Cost elas.	Time elas.	Cost elas.	Time elas.
Base model		−0.064	−0.247	−0.216	−0.471
Continuous model		−0.058	−0.234	−0.202	−0.465
Discrete model	Class 1	−0.037	−0.165	−0.104	−0.275
	Class 2	−0.145	−0.425	−0.441	−0.879
	Overall	−0.086	−0.282	−0.263	−0.580

individuals are more sensitive to changes in travel durations than to changes in travel costs. Similar to cost elasticities, time elasticities computed for private motorized modes and public transport differ: the time elasticities for private motorized modes ($0.234 < |\cdot| < 0.282$) are lower than the ones for public transport ($0.465 < |\cdot| < 0.580$), meaning that private motorized mode users are less sensitive to changes in their travel durations than users of public transport.

For the discrete model, differences occur in the sensitivity to variations in the travel costs and times. For individuals in the *dependent* class, i.e. class 2, an increase in the travel costs of 1% would result in a larger decrease in their probability to choose their current transport mode than for individuals in the *independent* class, i.e. class 1. This is consistent with the fact that individuals in class *independent* have larger incomes than individuals in class *dependent* (see Table 2.6 for the characterization of the classes). The same effect can be noticed for changes in travel times, i.e. individuals in class *dependent* are more sensitive to variations in travel durations than individuals in class *independent*.

The third demand indicator we investigate is the value of time. It expresses the willingness to pay of individuals to gain a travel duration of one hour. Table 2.11 reports the values of time for private motorized modes and public transport, predicted by all three models. It can be noticed that for both types of modes, the values of time do not differ much across models: for private motorized modes, the value of time is close to 30 CHF per hour and for public transport, it is slightly above 12 CHF per hour. These values are comparable with those reported in a study on the value of time in Switzerland Axhausen et al. (2008). Precisely, that paper reports a value of time for public transport of 14.10 CHF per hour, which is close to our results, and a value of time for car travels of 20.98 CHF per hour, which is slightly lower than the values of time we obtained for the three models. Nevertheless, a similar trend appears between the study on the value of time and our research, which demonstrates that individuals are ready to spend more in order to gain time in private motorized modes than in public transport.

Let us also note that the values of time are different in the two classes of the discrete model. For both private motorized modes and public transport, they are higher for *independent* individuals. This can be explained by the fact that most of the individuals in this class are active workers for whom gaining an hour in travel is very important, contrary to part of the individuals in the *dependent* class who are students or retired persons. Let us remark that Axhausen et al. (2008) report a value of time of 27.66 CHF/hour for business travels in car,

Table 2.11: Value of time

Model		PMM	PT
Base model		30.30 CHF/hour	12.26 CHF/hour
Continuous model		31.54 CHF/hour	12.81 CHF/hour
Discrete model	Class 1	35.78 CHF/hour	15.38 CHF/hour
	Class 2	22.05 CHF/hour	8.84 CHF/hour
	Overall	29.53 CHF/hour	12.40 CHF/hour

Table 2.12: Percentages of choice probabilities higher than 0.5 and 0.9

Threshold	Base model	Continuous model	Discrete model
0.5	72.87%	73.67%	75.00%
0.9	25.80%	25.53%	27.93%

which is close to the value of time obtained for individuals in class *independent*.

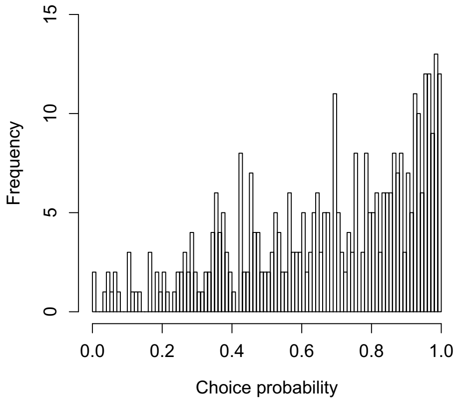
In order to assess if the continuous model and discrete model presented in section 2.3 could be applied on other potential data sets, we perform a validation. As only one data set is available, that is, the one on which we calibrated the models, it is split into two parts. First we select randomly 80% of its observations and estimate the model on the latter and second we apply the model on the remaining 20% of the observations.

Histograms of the choice probabilities predicting the choice of the individuals in the 20% of the observations are shown in Figure 2.4 for the base model, the continuous model and the discrete model.

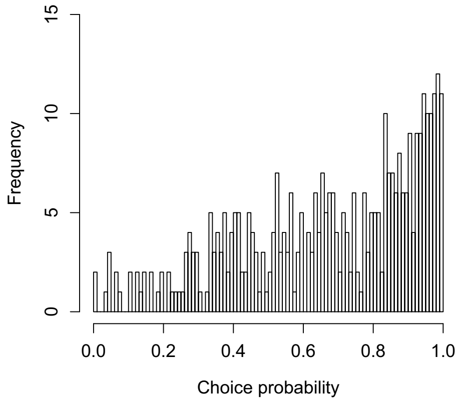
We observe that choice probabilities are well predicted by all three models, but best by the discrete model. As a confirmation of this result, Table 2.12 shows the percentages of choice probabilities higher than 0.5 and 0.9 for each model. For all three models, the percentage of choice probabilities above 0.5 and 0.9 are quite large, i.e. between 72% and 75% and between 25% and 28%, respectively. We notice that for the discrete model, the percentages of choice probabilities above 0.5 (75.00%) and above 0.9 (27.93%) are higher than for the two other models, which shows that the characterization of the two latent classes of *independent* and *dependent* individuals within the choice model results in a better prediction power.

2.5 Conclusions and future research directions

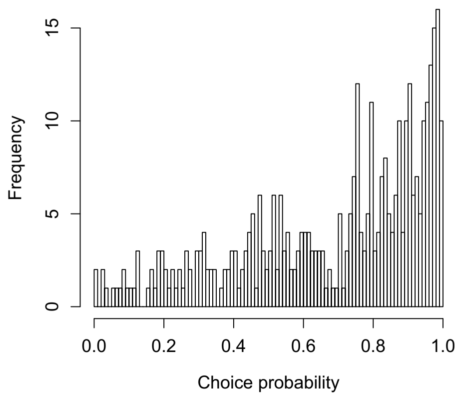
In this chapter we presented two models that aim at characterizing better mode choice behavior by using attitudinal indicators. In the first model, we integrated latent attitudes regarding public transport dislike and care for environment within a choice model. Moreover, in the second model, we could observe and capture heterogeneity in mode preferences for two different segments of the population via an integrated choice and latent class model.



(a) Base model



(b) Continuous model



(c) Discrete model

Figure 2.4: Histograms of the choice probabilities

In order to analyze the demand for the different mode choices, several indicators are computed, i.e. market shares, elasticities and values of time. The indicators obtained for integrated choice and latent class model showed evidence of differences in the sensitivities to variations in the travel fares and durations between individuals of the two segments. Such model also demonstrated a higher prediction power over a simple logit model.

The presented results are obtained with the estimation of the models using 2,265 observations that are collected with the RP survey. The presented methodologies can simply be repeated in the existence of richer datasets. In such a case, there is a potential of discovering further interactions between individuals' attitudes and their mode choice behavior.

In the presented models the heterogeneity in the sample is explained through structural equation models for attitudinal variables. Therefore, provided that the necessary variables are available, the models can be applied for other samples. This is an added value of the presented models compared to mixtures of models which incorporate heterogeneity within the population through random distributions.

Regarding the specification of the integrated choice and latent variable model, further research could consist of the inclusion of more attitudinal variables as well as a better characterization of their indicators. The integrated choice and latent class model could include additional classes. Finally, a combination of both models could be considered in order to have a comprehensive framework of the complexity and heterogeneity lying in the population.

3 An air itinerary choice model based on a mixed RP/SP dataset

In this chapter, we present an itinerary choice model based on a mixed RP/SP dataset. The aim of the combination of the two datasets is to exploit the variability of the SP data for the estimation of the RP model parameters. As a result a price elastic demand model is obtained which will be integrated in an airline schedule planning framework. This integration will enable us to explicitly model the supply-demand interactions which is critical for airlines for superior schedule planning decisions.

3.1 Introduction and related literature

Demand forecasting models of airlines are critical in a profitable planning of the network and schedule. In the last decade discrete choice methodology has been introduced in the context of demand analysis of airlines (Garrow, 2010). It has been shown by Coldren et al. (2003) that discrete choice modeling leads to superior forecasts compared to a widely used Quality Service Index (QSI).

In air transportation context an *itinerary* is defined as a product between an origin and destination pair which can be composed of several flight legs. Since the information on the demand is available on the itinerary level, choice models are developed for the itineraries. In the literature, random utility models have been used to model the choice of itinerary depending on various attributes. We refer to the work of Garrow (2010) for a comprehensive review of different specifications of discrete choice models for air travel demand. Coldren et al. (2003) propose logit models and Coldren and Koppelman (2005) extend the previous work with the introduction of GEV and nested logit models. Gramming et al. (2005) propose a probit model where there is a large set of alternatives in the context of non-IIA problems. Koppelman et al. (2008) model the time of day preferences under a logit setting in order to analyze the effect of schedule delay. Carrier (2008) and Wen and Lai (2010) propose some advanced demand modeling in which customer segmentation is modeled as a latent class.

In this study we develop an itinerary choice model based on a real dataset. The dataset is a mixed revealed preferences (RP) and stated preferences (SP) dataset. Combining different sources of data is a common practice in demand modeling literature in order to make use of the advantages of different characteristics of the sources (Ben-Akiva and Morikawa, 1990; Ben-Akiva et al., 1994; Louviere et al., 1999). The considered RP data is a booking data from a major European airline and the SP data is based on an Internet survey in US. The contribution of this study is the price elasticity of the resulting demand model which is lacking in the models based on booking data. This demand model is aimed to be integrated with a schedule planning model. This integration provides simultaneous decisions on the schedule plan and revenue management.

The rest of the chapter is organized as follows. Section 3.2 presents the itinerary choice model we develop. Section 3.3 presents the mixed RP/SP data used for the estimation. The methodology for the joint estimation of RP and SP models is presented in section 3.4. We provide the estimation results in section 3.5 together with the indicators of demand including willingness to pay and elasticities. Finally, we conclude in section 3.6.

3.2 Itinerary choice model

We develop an itinerary choice model for the choice of alternative itineraries in the same market segments. The market segments , $s \in S^h$, are defined by the origin and destination (OD) pairs and they are differentiated for each cabin class h. Considered classes are economy and business classes and therefore we have two segments for each OD pair. The choice situation is defined for each segment s with a choice set of all the alternative itineraries in the segment represented by I_s. The index i for each alternative itinerary in segment I_s carries the information of the cabin class of the itinerary due to the definition of the segments. Therefore we do not use the index h for the itineraries. As an example, consider a market segment of economy passengers between Geneva and Washington. The alternatives for this segment includes all the available economy itineraries which can be non-stop or connecting itineraries with different departure times. Finally, in order to better represent the reality, we include *no-revenue options* ($I_s^{'} \subset I_s$), which represent the itineraries offered by competitive airlines.

The utility of each alternative itinerary i, including the no-revenue options, is represented by V_i and the specification is provided in Table 3.1. The alternative specific constants, ASC_i, are included for each itinerary in each segment except one of them which is normalized to 0 for identification purposes. Other parameters are represented by β for each of the explanatory variables. Since we have different models for economy and business classes all the parameters and variables are specified accordingly. Superscripts E and B indicate the economy and business classes respectively. Moreover, the superscripts NS and S indicate the non-stop and one-stop itineraries respectively. The explanatory variables are described as follows:

- $price_i$ is the price of itinerary i in €, which is normalized by 100 for scaling purposes,
- $time_i$ is the elapsed time for itinerary i in hours,

Table 3.1: Specification table of the utilities

	Parameters	Explanatory variables
constants	ASC_i^E	$1 \times \text{economy}_i$
	ASC_i^B	$1 \times \text{business}_i$
price	$\beta_{\text{price}}^{E,NS}$	$\ln(\text{price}_i/100) \times \text{non-stop}_i \times \text{economy}_i$
	$\beta_{\text{price}}^{B,NS}$	$\ln(\text{price}_i/100) \times \text{non-stop}_i \times \text{business}_i$
	$\beta_{\text{price}}^{E,S}$	$\ln(\text{price}_i/100) \times \text{stop}_i \times \text{economy}_i$
	$\beta_{\text{price}}^{B,S}$	$\ln(\text{price}_i/100) \times \text{stop}_i \times \text{business}_i$
time	$\beta_{\text{time}}^{E,NS}$	$\text{time}_i \times \text{non-stop}_i \times \text{economy}_i$
	$\beta_{\text{time}}^{B,NS}$	$\text{time}_i \times \text{non-stop}_i \times \text{business}_i$
	$\beta_{\text{time}}^{E,S}$	$\text{time}_i \times \text{stop}_i \times \text{economy}_i$
	$\beta_{\text{time}}^{B,S}$	$\text{time}_i \times \text{stop}_i \times \text{business}_i$
time-of-day	$\beta_{\text{morning}}^{E}$	$\text{morning}_i \times \text{economy}_i$
	$\beta_{\text{morning}}^{B}$	$\text{morning}_i \times \text{business}_i$

- *non-stop*$_i$ is a dummy variable which is 1 if itinerary i is a non-stop itinerary, 0 otherwise,
- *stop*$_i$ is a dummy variable which is 1 if itinerary i is a one-stop itinerary, 0 otherwise,
- *economy*$_i$ is a dummy variable which is 1 if itinerary i is an economy itinerary, 0 otherwise,
- *business*$_i$ is a dummy variable which is 1 if itinerary i is a business itinerary, 0 otherwise,
- *morning*$_i$ is a dummy variable which is 1 if itinerary i is a morning itinerary departing between 07:00-11:00, 0 otherwise. The time slot is inspired by the studies in literature that show that the individuals have higher utility for the departures in this slot Garrow (2010).

As seen in Table 3.1 all the parameters are interacted with the *economy* and *business* dummies in order to be able to have two different models for the two classes. In addition to the interaction with the cabin class, the time and price variables are interacted with the number of stops, i.e. the dummies of *non-stop* and *stop* since there are strong correlations between the number of stops and both the time and price of the itinerary. Furthermore, the price variable is included as a log formulation since it improved the model significantly. The idea behind is that, the effect of the increase in price is not linear for different levels of the price.

The choice model is formulated as a logit model. It gives the choice probability for each itinerary i in segment s as represented by equation 3.1.

$$P^s(i) = \frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)} \qquad \forall h \in H, s \in S^h, i \in I_s \tag{3.1}$$

3.3 Data

For the estimation of the demand model we use an RP data provided in the context of ROADEF Challenge 2009[1]. This is a booking data from a major European airline which provides the set of airports, flights, aircraft and itineraries. The information provided for the itineraries includes the corresponding flight legs therefore we can deduce the information on the departure and arrival time of itinerary, the trip length and the number of stops. Additionally, we have information on the demand and average price (€) for each cabin class. Three main sets of instances are provided in the context of the challenge. The first set includes 608 flights and 1,943 itineraries, the second has 1,422 flights and 11,214 itineraries; and the last one consists of 2,178 flights and 28,308 itineraries. For the estimation of the demand model a subset of the whole dataset is used as explained in section 3.5. The same dataset is used for the integrated models presented in Part II of the thesis. Several data instances are generated as presented in Table 5.1 in Chapter 5.

The RP data does not include any information concerning the competitive airlines. Therefore the no-revenue options are not considered in the estimation process. However for applying the model we assume that these itineraries have the same type of utility functions as presented in Table 3.1 and their attributes are assigned according to the other available itineraries in the market offered by competitive airlines.

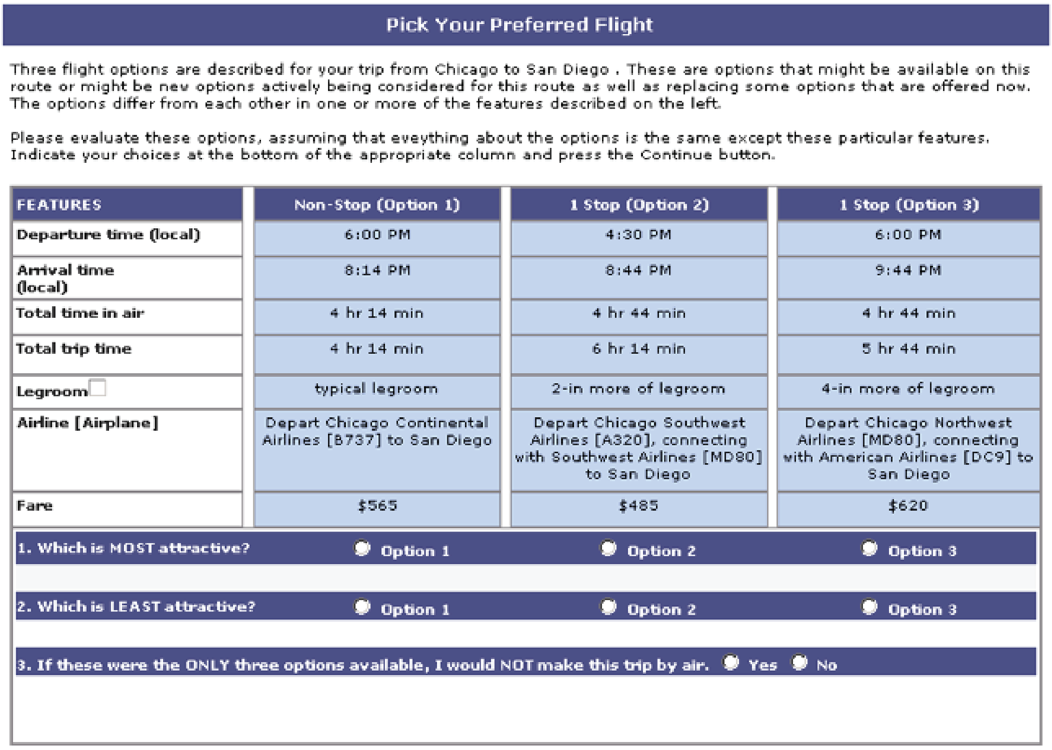
As it is common with RP data, the lack of variability in some attributes precludes a statistically significant estimation of key parameters of the choice models. Many models in literature, which are based on airline booking data, have insignificant price parameters (Garrow, 2010). Therefore, in this study, the RP data is combined with SP data, where the variability is obtained by design.

The SP data, which is used to overcome the inelastic nature of RP data, is based on an Internet choice survey collected in 2004 in the US. Let us note that, the combined dataset therefore contains both European and US data. The Internet survey was organized to understand the sensitivity of air passengers to the attributes of an airline itinerary such as fare, travel time, number of stops, legroom, and aircraft. The respondents were presented hypothetical choice situations and offered three alternatives as seen in Figure 3.1. The first is a non-stop itinerary, the second one is a one-stop itinerary with the same airline and the third is connecting with a different airline. By design, the data has enough variability in terms of price and other variables. The SP sample has 3609 observations.

3.4 The simultaneous estimation of RP and SP models

As mentioned previously the RP model presented in section 3.2 is simultaneously estimated with the SP model in order to take the advantage of its elasticity. The SP model is also a logit

[1] http://challenge.roadef.org/2009/en



Pick Your Preferred Flight

Three flight options are described for your trip from Chicago to San Diego . These are options that might be available on this route or might be new options actively being considered for this route as well as replacing some options that are offered now. The options differ from each other in one or more of the features described on the left.

Please evaluate these options, assuming that eveything about the options is the same except these particular features. Indicate your choices at the bottom of the appropriate column and press the Continue button.

FEATURES	Non-Stop (Option 1)	1 Stop (Option 2)	1 Stop (Option 3)
Departure time (local)	6:00 PM	4:30 PM	6:00 PM
Arrival time (local)	8:14 PM	8:44 PM	9:44 PM
Total time in air	4 hr 14 min	4 hr 44 min	4 hr 44 min
Total trip time	4 hr 14 min	6 hr 14 min	5 hr 44 min
Legroom	typical legroom	2-in more of legroom	4-in more of legroom
Airline [Airplane]	Depart Chicago Continental Airlines [B737] to San Diego	Depart Chicago Southwest Airlines [A320], connecting with Southwest Airlines [MD80] to San Diego	Depart Chicago Northwest Airlines [MD80], connecting with American Airlines [DC9] to San Diego
Fare	$565	$485	$620
1. Which is MOST attractive?	Option 1	Option 2	Option 3
2. Which is LEAST attractive?	Option 1	Option 2	Option 3

3. If these were the ONLY three options available, I would NOT make this trip by air. Yes No

Figure 3.1: An example page for the SP survey

model. The choice set consists of three alternatives. The first one is a nonstop itinerary. The second alternative is a one-stop itinerary both flights being operated by the same airline. The third alternative is also a one-stop itinerary where the connection is provided by another airline. The utilities for these alternatives are provided by the equations 3.2, 3.3, and 3.4 respectively.

Since the models for RP and SP datasets are estimated simultaneously, we need to define a scale variable, $scale_{SP}$. The the scale of the RP data is fixed to 1 and $scale_{SP}$ is to be estimated in order to capture the differences in the covariance structure of the error terms of the two models.

Similar to the RP model, the parameters are specified as economy and business. The parameters of the price variables for each of the alternatives ($\beta^{E,NS}_{\text{price}}$, $\beta^{B,NS}_{\text{price}}$, $\beta^{E,S}_{\text{price}}$, $\beta^{B,S}_{\text{price}}$) are constrained to be the same as the price parameters of the RP model presented in section 3.2. Similarly the parameters of the time variables ($\beta^{E,NS}_{\text{time}}$, $\beta^{B,NS}_{\text{time}}$, $\beta^{E,S}_{\text{time}}$, $\beta^{B,S}_{\text{time}}$) and the parameters of the morning variables ($\beta^{E}_{\text{morning}}$, $\beta^{B}_{\text{morning}}$) are also designed to be the same as the parameters of the RP model.

In the SP model, there are additional explanatory variables since it is based on a rich data set. For business passengers we have the information whether they pay their ticket or their company pay for that. Therefore there is an additional dummy variable, *business/others-pay* (denoted by OP), which is 1 if the business passenger's ticket is not paid by himself. There are other explanatory variables which are represented by vector v. These variables include the legroom provided in the airplane, the delay of the flight in case of late or early arrival and the variable representing whether the passenger is a frequent flyer or not.

$$\begin{aligned}
V_1 = \text{scale}_{SP} \times (&\beta^{E,NS}_{\text{price}} \times \ln(\text{price}_1/100) \times \text{economy} + \beta^{B,NS}_{\text{price}} \times \ln(\text{price}_1/100) \times \text{business} \\
&+ \beta^{B-OP}_{\text{price}} \times \ln(\text{price}_1/100) \times \text{business/others-pay} \\
&+ \beta^{E,NS}_{\text{time}} \times \text{time}_1 \times \text{economy} + \beta^{B,NS}_{\text{time}} \times \text{time}_1 \times \text{business} \\
&+ \beta^{E}_{\text{morning}} \times \text{morning}_1 \times \text{economy} \\
&+ \beta^{B}_{\text{morning}} \times \text{morning}_1 \times \text{business} \\
&+ \sum_i \beta^E_i \times v^i_1 \times \text{economy} + \beta^B_i \times v^i_1 \times \text{business})
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
V_2 = \text{scale}_{SP} \times (&\text{ASC}^E_2 \times \text{economy} + \text{ASC}^B_2 \times \text{business} \\
&+ \beta^{E,S}_{\text{price}} \times \ln(\text{price}_2/100) \times \text{economy} + \beta^{B,S}_{\text{price}} \times \ln(\text{price}_2/100) \times \text{business} \\
&+ \beta^{B-OP}_{\text{price}} \times \ln(\text{price}_2/100) \times \text{business/others-pay} \\
&+ \beta^{E,S}_{\text{time}} \times \text{time}_2 \times \text{economy} + \beta^{B,S}_{\text{time}} \times \text{time}_2 \times \text{business} \\
&+ \beta^{E}_{\text{morning}} \times \text{morning}_2 \times \text{economy} \\
&+ \beta^{B}_{\text{morning}} \times \text{morning}_2 \times \text{business} \\
&+ \sum_i \beta^E_i \times v^i_2 \times \text{economy} + \beta^B_i \times v^i_2 \times \text{business})
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
V_3 = \text{scale}_{SP} \times (&\text{ASC}^E_3 \times \text{economy} + \text{ASC}^B_3 \times \text{business} \\
&+ \beta^{E,S}_{\text{price}} \times \ln(\text{price}_3/100) \times \text{economy} + \beta^{B,S}_{\text{price}} \times \ln(\text{price}_3/100) \times \text{business} \\
&+ \beta^{B-OP}_{\text{price}} \times \ln(\text{price}_3/100) \times \text{business/others-pay} \\
&+ \beta^{E,S}_{\text{time}} \times \text{time}_3 \times \text{economy} + \beta^{B,S}_{\text{time}} \times \text{time}_3 \times \text{business} \\
&+ \beta^{E}_{\text{morning}} \times \text{morning}_3 \times \text{economy} \\
&+ \beta^{B}_{\text{morning}} \times \text{morning}_3 \times \text{business} \\
&+ \sum_i \beta^E_i \times v^i_3 \times \text{economy} + \beta^B_i \times v^i_3 \times \text{business})
\end{aligned} \tag{3.4}$$

3.5 Estimation results

From the RP data, 3 OD pairs are selected to be combined with the SP data. There are in total 30 alternative itineraries serving 904 passengers between these 3 OD pairs. These OD pairs are the ones with the most variability in the attributes. In Appendix A.1.1 we provide results with 24 OD pairs from the RP data where the lack of price elasticity can be observed.

The characteristics of the alternatives for the selected 3 OD pairs can be seen in Table 3.5. When there is a business itinerary it is in fact the same product with the subsequent economy itinerary. For example, alternative 7 and 8 of the first OD pair are the same product with different classes. In this section we provide results for the RP model since the focus of the study is to obtain an appropriate model for the RP data in order to be used in the framework of schedule planning models.

The estimation of the parameters for the joint RP/SP model is done with a maximum likelihood estimation using the software BIOGEME Bierlaire and Fetiarison (2009). In Table 3.3 we present the estimated parameters. In the following, we present the RP model parameters which are constrained to be common with those of the SP model. In addition to the common parameters we also present the scale parameter introduced in the SP model. The main observations can be listed as follows:

- The cost and time parameters have negative signs as expected since the increase in the price or the time of an itinerary decreases its utility.
- Economy demand is more sensitive to price and less sensitive to time compared to business demand as expected (Belobaba et al., 2009).
- For non-stop itineraries time and cost parameters are higher in absolute value compared to one-stop itineraries. Therefore, passengers on connecting itineraries are less affected by 1 € increase in price or 1 minute increase in travel time compared to non-stop itineraries. The reason is that, in our RP data the connecting itineraries are more expensive and by nature have longer travel time. Therefore we need to check the indicators of willingness to pay and elasticities to analyze these effects appropriately.
- Departure time of the day parameter, $\beta_{morning}$, is higher for business demand compared to the economy demand, which means that business passengers have a higher tendency to chose morning itineraries.
- Scale parameter for the SP model is significant and has a value of 4.32 which indeed confirms that the variability of the SP data is higher than the RP data.
- In the SP model there is an additional price parameter, $\beta_{\text{price}}^{B-OP}$, for the individuals who do not pay their ticket. This parameter has a positive sign which says that people have higher utility when their tickets are paid by their companies as expected.
- All the parameters are significant with a 90% confidence level except the time-of-day parameter for economy class.

In order to see the added value of the combination of the two datasets, in Table 3.4 we present the estimated values of the same parameters when using only the RP data. It is seen that the parameters are not significant which prevents us from drawing conclusions. Even the sign of the parameters are inconsistent with reality. Therefore, the model based on the RP data cannot be used for forecasting future market shares of the itineraries.

Table 3.2: The attributes of the alternative itineraries for the RP data

	alt.	stops	class	price (€)	time(min)	morning	Actual demand
OD1	1	one-stop	E	563.8	260	1	3
	2	one-stop	E	312.5	260	1	6
	3	one-stop	E	262.5	360	0	27
	4	non-stop	E	175	70	0	49
	5	non-stop	E	175	70	0	56
	6	non-stop	E	175	70	1	38
	7	non-stop	B	409.5	70	1	9
	8	non-stop	E	175	70	1	29
	9	non-stop	B	409.5	70	0	16
	10	non-stop	E	175	70	0	26
	11	non-stop	B	409.5	70	0	2
	12	non-stop	E	175	70	0	28
OD2	1	one-stop	E	250	175	1	17
	2	non-stop	E	150	60	1	29
	3	non-stop	E	150	60	0	2
	4	non-stop	E	150	60	0	19
	5	one-stop	B	953	235	1	1
	6	one-stop	E	601.2	235	1	2
	7	one-stop	B	701.8	235	1	2
	8	one-stop	E	350	235	1	3
OD3	1	one-stop	B	655.5	265	1	4
	2	one-stop	E	387.5	265	1	6
	3	non-stop	E	237.5	95	1	59
	4	non-stop	E	237.5	95	0	125
	5	one-stop	E	609.8	230	0	3
	6	one-stop	E	325	230	0	6
	7	non-stop	E	237.5	95	0	73
	8	non-stop	E	237.5	95	0	84
	9	non-stop	E	237.5	95	0	73
	10	non-stop	E	237.5	95	0	107

Table 3.3: Estimated parameters for the model with joint RP and SP data

	Parameters	Estimated value	t-test
RP & SP	$\beta^{E,NS}_{\text{price}}$	-2.23	-3.48
	$\beta^{B,NS}_{\text{price}}$	-1.97	-3.64
	$\beta^{E,S}_{\text{price}}$	-2.17	-3.48
	$\beta^{B,S}_{\text{price}}$	-1.97	-3.68
	$\beta^{E,NS}_{\text{time}}$	-0.102	-2.85
	$\beta^{B,NS}_{\text{time}}$	-0.104	-2.43
	$\beta^{E,S}_{\text{time}}$	-0.0762	-2.70
	$\beta^{B,S}_{\text{time}}$	-0.0821	-2.31
	$\beta^{E}_{\text{morning}}$	0.0283	1.21*
	$\beta^{B}_{\text{morning}}$	0.0790	1.86
SP	scale_{SP}	4.32	3.50
	$\beta^{B-OP}_{\text{price}}$	0.813	2.91

(* Statistical significance < 90%)

In Appendix A.1 in Table A.1 we present the results estimated with the SP data. In order to have a comparison, in Table A.2 we provide the scaled values for the joint estimation results. It is seen that the results of the joint dataset is close to that of the SP data. Therefore, when we have only 3 OD pairs for the RP data, the results are mainly guided by the SP data. Especially when we look at the price parameters, SP data is dominant since RP data does not have enough variability. For the time parameters the RP data has an effect on the results. However when more RP observations are included as presented in Appendix A.1.1 the results change significantly.

Since this is a complicated model with the combination of two datasets, it is better to analyze the demand indicators such as willingness to pay and elasticities rather than the parameter estimates themselves.

3.5.1 Value of time

Value of time (VOT) is the willingness of passengers to pay for one hour of travel. For each alternative i VOT is given by equation 3.5. Since the price is included as a log formulation in the utilities VOT formula includes the price.

$$\begin{aligned} VOT_i &= \frac{\partial V_i / \partial time_i}{\partial V_i / \partial price_i} \\ &= \frac{\beta_{time} \cdot price_i}{\beta_{price}} \end{aligned} \tag{3.5}$$

In Table 3.5 the VOT values for all the alternatives of the RP data are listed. VOT is higher for business itineraries compared to economy itineraries. For example, the itineraries 7 and 8

Table 3.4: Estimated parameters based on the RP data

Parameters	Estimated value	t-test
$\beta^{E,NS}_{\text{price}}$	0.0851	0.08*
$\beta^{B,NS}_{\text{price}}$	-0.451	-0.60*
$\beta^{E,S}_{\text{price}}$	-1.47	-0.88*
$\beta^{B,S}_{\text{price}}$	-3.19	-1.63*
$\beta^{E,NS}_{\text{time}}$	-0.0204	-0.31*
$\beta^{B,NS}_{\text{time}}$	-0.108	-1.04*
$\beta^{E,S}_{\text{time}}$	-0.0705	-0.12*
$\beta^{B,S}_{\text{time}}$	0.969	1.13*
$\beta^{E}_{\text{morning}}$	0.282	0.34*
$\beta^{B}_{\text{morning}}$	-0.700	-0.85*

(* Statistical significance < 90%)

of the first OD pair. This is also observed for the itineraries 9-10 and 11-12 for the first OD pair; itineraries 5-6 and 7-8 for the second OD pair; and itineraries 1-2 for the third. When we compare the VOT for non-stop and one-stop itineraries it seems as if the passengers are ready to pay more for the one-stop itineraries compared to non-stop itineraries. However this happens due to the fact that one-stop itineraries are more expensive.

Therefore in order to see the effect of the number of stops in VOT we consider two itineraries with the same price. As an example, let's take a non-stop and a one-stop itinerary which have the same price, 600 €. When we calculate the VOT, we observe that passengers are ready to pay 28 € for an hour reduction in the travel time of the non-stop alternative. For the one-stop itinerary this value is 21 € which is lower as expected.

3.5.2 Price and time elasticities of demand

Elasticities of demand give the sensitivity of passengers to the corresponding case. In this study we are interested in the price and time elasticities. They are given by the following equations:

$$E^{P_i}_{price_i} = \frac{\partial P_i}{\partial price_i} \cdot \frac{price_i}{P_i}$$

$$E^{P_i}_{time_i} = \frac{\partial P_i}{\partial time_i} \cdot \frac{time_i}{P_i}$$

Belobaba et al. (2009) provide a range of airline O-D market price elasticities from -0.8 to -2.0. For business demand the average is given as -0.8 which means that if there is a 1% increase in cost, business demand will decrease by 0.8%. For economy demand this value is provided as -1.6. For time elasticity they mention that business demand has a time elasticity < -1.0 and for economy demand it is > -1.0 meaning that business demand is more elastic to time compared to economy demand.

The price and time elasticities are presented in Table 3.5 for the alternatives of the RP data. Price elasticity is higher for economy alternatives compared to business itineraries. For example, for the first OD pair, for the alternatives 7-8, 9-10, and 11-12 economy demand is more elastic to price compared to the business demand. This phenomenon is also observed for the itineraries 5-6 and 7-8 of the second OD pair and for the itineraries 1-2 of the third OD pair. Furthermore, the elasticity is higher for the one-stop itineraries compared to non-stop itineraries. This means that, in case of an increase in price, passengers have a higher tendency to reject flying with a one-stop itinerary compared to a non-stop alternative. This is in line with the studies in literature Garrow (2010).

Time elasticities are low compared to the literature for the RP data since it includes European itineraries and the time attribute does not differ between different itineraries. However when we look at the relative elasticities for business and economy alternatives, it is seen that business demand is more elastic to time which is consistent with the empirical studies mentioned in Belobaba et al. (2009). Similarly, the time elasticity is higher for one-stop alternatives compared to non-stop ones which says that passengers are more sensitive to changes in the time for one-stop alternatives as expected.

3.5.3 Illustration for the application of the model

The developed demand model will be integrated in a schedule planning framework for airlines. Therefore in this section we illustrate how the model will be applied.

The alternative specific constants ASC_i for each itinerary i are not used for applying the model. The critical parameters for the application of the model are the price, the time and the time-of-day parameters which are kept the same for RP and SP models. The no-revenue itineraries, which are described in section 3.2 are introduced based on average market prices for competitor airlines.

For illustration purposes, we choose an arbitrary OD pair A-B. There are two alternatives of economy itineraries which are both nonstop itineraries. We include the no-revenue itinerary A-$\text{B}^{'}$. The values of attributes can be seen in Table 3.6. According to the attributes the resulting choice probability, which is referred as the *market share*, is presented in the last column. The itinerary 2 has the lowest price and is a morning itinerary. Therefore it attracts the biggest number of passengers.

3.6 Conclusions and future research directions

In the context of airline network and schedule planning, demand forecasting models lead to an increasing interest in order to better understand the underlying travel behavior of passengers. In this chapter, an itinerary choice model is developed based on a real dataset which is aimed

Table 3.5: Demand indicators for the alternatives for the 3 OD pairs

	alt.	stops	class	VOT($\frac{€}{h}$)	price elas.	time elas.
OD1	1	one-stop	E	19.79	-2.15	-0.33
	2	one-stop	E	10.97	-2.12	-0.32
	3	one-stop	E	9.22	-1.97	-0.41
	4	non-stop	E	8.01	-1.85	-0.10
	5	non-stop	E	8.01	-1.80	-0.10
	6	non-stop	E	8.01	-1.94	-0.10
	7	non-stop	B	21.68	-1.90	-0.12
	8	non-stop	E	8.01	-2.01	-0.11
	9	non-stop	B	21.68	-1.86	-0.11
	10	non-stop	E	8.01	-2.03	-0.11
	11	non-stop	B	21.68	-1.95	-0.12
	12	non-stop	E	8.01	-2.01	-0.11
OD2	1	one-stop	E	8.78	-1.69	-0.17
	2	non-stop	E	6.86	-1.37	-0.06
	3	non-stop	E	6.86	-2.17	-0.10
	4	non-stop	E	6.86	-1.67	-0.08
	5	one-stop	B	39.81	-1.93	-0.32
	6	one-stop	E	21.11	-2.11	-0.29
	7	one-stop	B	29.31	-1.91	-0.31
	8	one-stop	E	12.29	-2.08	-0.29
OD3	1	one-stop	B	27.38	-1.95	-0.36
	2	one-stop	E	13.60	-2.14	-0.33
	3	non-stop	E	10.87	-1.99	-0.14
	4	non-stop	E	10.87	-1.71	-0.12
	5	one-stop	E	21.41	-2.16	-0.29
	6	one-stop	E	11.41	-2.15	-0.29
	7	non-stop	E	10.87	-1.93	-0.14
	8	non-stop	E	10.87	-1.88	-0.14
	9	non-stop	E	10.87	-1.93	-0.14
	10	non-stop	E	10.87	-1.79	-0.13

Table 3.6: Attributes of the itineraries and the resulting market shares

OD	price	time of day	market share
A-B_1	225	0	0.26
A-B_2	203	1	0.44
A-B'	220	0	0.30

to be integrated in a schedule planning model in order to explicitly model supply-demand interactions.

A combined RP/SP dataset is utilized for the estimation of the parameters in order to take the advantage of the elasticity of the SP data. The combination is carried out by constraining a subset of the parameters of the two models to be the same and by introducing a scale parameter for the SP model. As a result, a price elastic demand model is obtained with the help of the combination of the two datasets.

As a future work, the prediction power of the model needs to be analyzed by applying the model on a validation data. The RP data used in this chapter is not very rich in terms of the available explanatory variables. In the existence of a richer dataset the model can be improved.

Integrated supply models Part II

Part II focuses on the integrated schedule planning models with explicit representation of demand models. Chapter 4 introduces an integrated scheduling, fleeting and pricing model. The pricing decision is integrated through the itinerary choice model presented in Chapter 3. Chapter 5 presents a local search heuristic for the integrated model. Different size of problem instances are generated and solved with three approaches. Chapter 6 presents a further analysis of the integrated model. Firstly, a reformulation of the model is presented and results are provided for comparison purposes. Then a sensitivity analysis is performed in order to address the robustness of scheduling decisions to demand model parameters.

4 An integrated airline scheduling, fleeting and pricing model

In airline schedule planning models, the demand and price information are usually taken as inputs to the model. Therefore schedule and capacity decisions are taken separately from pricing decisions. In this chapter we present an integrated scheduling, fleeting and pricing model for a single airline where these decisions are taken simultaneously. This integration enables to explicitly model supply and demand interactions and take superior decisions. The model refers to a monopolized market. However, competing airlines are included in the model as a reference for the pricing decisions. The pricing decision is formulated through an itinerary choice model which determines the demand of the alternative itineraries in the same market according to their price, travel time, number of stops, and the departure time of the day. The demand model is estimated based on real data as explained in Chapter 3. The seat allocation for these classes are optimized according to the demand model. The choice model is also used to appropriately model the spill and recapture effects. The resulting model is evaluated with different illustrations and the added value of the integrated approach is analyzed compared to a sequential approach. Results over a set of representative instances show that the integrated model is able to take superior decisions by jointly adjusting capacity and pricing.

4.1 Introduction and related literature

The increase in the mobility needs of individuals is an indispensable fact for the last decades. According to the statistics provided by the Association of European Airlines (AEA), air travel traffic has grown at an average rate of 5% per year over the last three decades. Similarly, Bureau of Transportation Statistics reports that the number of departures performed increased by 30% in the last decade. This increase in air travel demand justifies the need for improving the demand responsiveness of air transportation capacity. The underlying demand process should be understood and included in airline scheduling models for more profitable scheduling decisions. The air transportation capacity is determined by the fleet assignment process and is a good canditate to analyze the impacts of the integration of the demand models. In this chapter, we study the integrated fleet assignment and schedule design models where we further integrate pricing decisions to better represent the supply-demand interactions compared to state of the art studies. Before we provide the model we give reference to the relevant studies literature.

As a reference for the basic fleet assignment model (FAM) we refer to the models proposed by Abara (1989) and Hane et al. (1995). There are various extensions of the FAM as reviewed by Sherali et al. (2006). One of the extensions is the integration of schedule design decision in fleet assignment models. These integrated models are studied with the purpose of increasing the revenue by making simultaneous decisions on the schedule and the fleet assignment. Schedule design is handled in different ways according to the flexibility allowed for the changes in the schedule. Desaulniers et al. (1997) and Rexing et al. (2000) study the option of shifting departure and arrival times within a given time-windows. Lohatepanont and Barnhart (2004) work with sets of mandatory and optional flights where optional flights can be canceled when not sufficiently profitable.

The integration of schedule design and fleet assignment models necessitate the inclusion of supply-demand interactions in order to represent the interactions between the proposed schedule and the demand. Supply-demand interactions are considered in fleeting models from different perspectives. Yan and Tseng (2002) study an integrated schedule design and fleet assignment model in which the set of flight legs is built considering the itineraries under a given expected demand for every origin-destination pair. In the context of itinerary-based fleet assignment, spill and recapture effects can be integrated in the model. These effects represent the potential number of passengers that could be redirected to alternative itineraries in the market when there is a capacity restriction on their desired itinerary. This information can be considered by the airlines in the planning phase in order to more effectively decide on the capacity. Barnhart et al. (2002) consider the spill and recapture effects separately for each fare class resulting from insufficient capacity. Similarly, Lohatepanont and Barnhart (2004) study the network effects including the demand adjustment in case of flight cancellations and spill effects. More recently, Dumas et al. (2009) model the passenger flow which gives the distribution of demand for each itinerary. This passenger flow model is also used as an estimation for the recapture ratios between itineraries. Cadarsoa and Marín (2011) include passenger considerations through a schedule development based on passenger satisfaction. Their integrated schedule design and fleet assignment model takes into account the disrupted and misconnected passengers.

In the literature, supply-demand interactions are modeled in different ways. Interactions are either directly integrated into the decision model or external demand simulators are used to provide better inputs to the planning process. When there is a direct representation of supply-demand interactions in the planning problems it is assumed that the airlines have a control on the revenue side. On the other hand, researchers who believe that airlines can not have such a control prefer to keep the revenue related decisions external to the model. This enables to keep the stochastic nature of the demand and use advance demand modeling techniques. As an example for external supply-demand interactions, Jacobs et al. (2008) present a leg-based FAM where network effects are estimated with a passenger mix model. The passenger mix model is a nonlinear network flow model which estimates the total revenue given the capacity. Simplified network effects are included directly in FAM in order to keep a linear formulation. Dumas et al. (2009) present a framework where a passenger flow model and a leg-based FAM are iteratively solved. Their aim is to keep the stochastic nature of the demand and reflect the time dimension in the booking process. They assume that demand distributions of itineraries and recapture ratios are known.

Direct integration of supply-demand interactions lead to itinerary-based fleet assignment (IFAM) since the information on the demand is at the itinerary level (Barnhart et al., 2002). As already mentioned, Lohatepanont and Barnhart (2004) present an integrated schedule design and fleet assignment model with network effects. The considered effects are demand correction for the market demand in case of flight cancellations and recapture effects. Recapture ratios are estimated based on the Quality Service Index (QSI) and introduced as fixed inputs to the model. Sherali et al. (2010) also present an integrated schedule design and fleet assignment model where they work with itinerary-based demands for multiple fare classes. They optimize the allocation of seats for each fare class. However they do not include network effects in the model. Recently, Wang et al. (2012) use utility models similar to discrete choice modeling in order to represent the spill and recapture effects. They present the idea with a basic passenger mix model. Extensions to the model are proposed with fleet assignment and schedule design decisions as well as market and departure time selections.

Advanced supply and demand interactions can be modeled by letting the model to optimize itinerary's attributes (e.g., the price, departure time). There are studies in the context of schedule planning of airlines where utility of passengers are considered when deciding on the frequency (Brueckner and Zhang, 2001;Brueckner and Flores-Fillol, 2006). Similarly, Vaze and Barnhart (2010) work on a game theoretical framework where they include an S-curve demand model to represent the impact of frequency on the demand. When we move the focus back to fleet assignment literature, Talluri and van Ryzin (2004b) integrate discrete choice modeling into the single-leg, multiple-fare-class revenue management model that determines the subset of fare products to offer at each point in time. They provide the characterization of optimal policies under a general choice model of demand. To overcome the missing no-purchase information in airline booking data, they use expectation-maximization (EM) method. Schön (2006) develops a market-oriented integrated schedule design and fleet assignment model with integrated pricing decisions. It is assumed that customers can be segmented according to some characteristics and different fares can be charged for these segments. Schön (2008) gives several specifications for the inverse price-demand function described in Schön (2006) including logit and nested logit models where the explanatory variable is the price of the itinerary. Budhiraja et al. (2006) also work on a similar topic where the change in unconstrained itinerary demand is incorporated into the model as a function of supply.

In this chapter, we introduce an integrated scheduling, fleeting and pricing model in a monopolized market. We refer to this model as IFAM-PR. Integration of pricing decisions in schedule planning enables to capture supply-demand interactions and improve the profitability of the schedule plan. The pricing decisions are captured by the demand model introduced in chapter 3. This itinerary choice model provides profitable average prices for each cabin class. The developed itinerary choice model is adapted to model the spill and recapture effects. Since the demand model is explicitly included in the model, these effects are also elastic to the changes in the attributes of the itineraries. With all the listed considerations, the resulting model optimizes the schedule design, fleet assignment, average price, and seat allocation for each cabin class. The added value of the integrated model is analyzed through various illustrations and experiments. To the best of our knowledge the integrated model is not studied in literature. The schedule planning model is close to the work of Lohatepanont and Barnhart (2004). However they include the given demand as an input to the model so that the demand is inelastic to the attributes of the itineraries. Similarly, they use preprocessed recapture ratios to represent supply-demand interactions. A variant of the integration of pricing decision in schedule planning is presented by Schön (2008). However it is carried out with a demand model where the utility is defined by only the price of the itinerary. Spill and recapture effects are ignored. Moreover the demand model and the solution of the integrated model is based on synthetic data. In order to have a concave formulation Schön (2008) utilizes the inverse demand function rather than the logit formula itself. However this restricts the model for the inclusion of more policy variables and socio-economic characteristics. The presented model integrates the logit formula explicitly which brings flexibility for such extensions and allows for disaggregate models accounting for heterogeneity of behavior in the market.

The remainder of the chapter is organized as follows. In section 4.2 we briefly talk about the demand model and explain how it is integrated with schedule planning decisions. In section 4.3 we present our integrated model, IFAM-PR. Section 4.4 provides reference models based on the state-of-the-art models in order to be compared with the integrated model. In section 4.5 we illustrate the added value of the integrated model in comparison to the reference models and provide computational experiments. Finally we conclude the chapter in section 4.6.

4.2 Demand model

The logit demand model gives the market share (u_i) for each itinerary i in segment s and when multiplied with the total forecasted demand of the segment, D_s, it provides the estimated demand of each itinerary as represented by equation 4.1.

$$
\begin{aligned}
u_i &= \frac{\exp(V_i(p_i, z_i; \beta))}{\sum\limits_{j \in I_s} \exp(V_j(p_j, z_j; \beta))} \\
\tilde{d}_i &= D_s u_i \qquad \forall h \in H, s \in S^h, i \in I_s
\end{aligned}
\tag{4.1}
$$

The explanatory variables of the logit model include price, p_i, as a policy variable which can be controlled by the integrated model. The other explanatory variables given in chapter 3 are context variables which we denote by the vector z_i. These context variables provide

Table 4.1: Resulting recapture ratios

	A-B$_1$	A-B$_2$	A-B$^{'}$
A-B$_1$	0	0.552	0.448
A-B$_2$	0.487	0	0.513

information for the demand and improves the estimation of the market shares but can not be modified by the integrated model. In order to explicitly represent these variables we refer to the utilities V_i as $V_i(p_i, z_i; \beta)$, where β are parameters estimated from real data. For the specification of the utility function we refer to Table 3.1 in chapter 3 where further details on the demand model are presented.

The itinerary choice model is also used to model the interactions between the itineraries in case of capacity shortage. Passengers, who can not be accommodated on their desired itineraries, may be redirected to other available itineraries in the same market segment in case of such shortages. This effect is referred as spill and recapture effect and assumed to be controlled by the airline with associated decision variables. The response from the market is obtained through the recapture ratios from the choice model. It is important to note that, the spill effects are not considered in the day of operations but rather in the planning phase. Airlines can take advantage of this knowledge when planning for the schedule and the design of fleet capacity. They can keep their capacity at profitable levels by taking into account the possibility of redirecting passengers to the alternative itineraries. For example, for a flight with a forecasted demand of 100 passengers, the airline may investigate the option of assigning an aircraft with 70 seats. If there are similar alternatives in the same market by the same airline, the airline may assume that a portion of 30 spilled passengers will still fly on those itineraries. Therefore, the spill and recapture information is not communicated to the passengers but only investigated at the planning phase. The passengers will be aware of the capacity limits at the booking phase as usual.

We assume that the spilled passengers are recaptured by the other itineraries with a recapture ratio based on the logit formulation. Therefore the recapture ratio is represented by equation (4.2).

$$b_{i,j} = \frac{\exp\left(V_j(p_j, z_j; \beta)\right)}{\sum\limits_{k \in I_s \setminus \{i\}} \exp\left(V_k(p_k, z_k; \beta)\right)} \quad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}), j \in I_s. \tag{4.2}$$

The recapture ratios $b_{i,j}$ represent the proportion of recaptured passengers by itinerary j among $t_{i,j}$ number of redirected passengers from itinerary i. The recapture ratio is calculated for the itineraries that are in the same market segment where the desired itinerary i is excluded from the choice set. Therefore lost passengers may be recaptured by the remaining alternatives of the company or by the no-revenue options. Since the airline can not control the no-revenue itineraries, we assume that no spill exist from them.

In order to illustrate the spill and recapture effects we use the same example given in section 3.5.3. In Table 3.6 the resulting market shares are listed and here the resulting recapture ratios are presented in Table 4.1. For example, in case of capacity shortage for itinerary 1, the airline may decide to redirect the passengers to itinerary 2. However 55% of these redirected passengers will accept to fly on itinerary 2. Similarly, the recapture ratio for the competing itinerary is 45%. Since the price of itinerary 2 is lower than the price of competitor, the recapture ratio for itinerary 2 is higher.

4.3 Integrated scheduling, fleeting and pricing model: IFAM-PR

In this section, we introduce an integrated scheduling, fleeting and pricing model for a single airline. We explicitly model the demand and integrate it in the schedule planning which enables to make use of the interaction between supply and demand.

Let F be the set of flight legs, there are two subsets of flights: mandatory flights (F^M), which should be flown, and optional flights (F^O) which can be canceled. The included schedule design context is solely related to the optional flights, apart from that the schedule is known and assumed to be used without any change. A represents the set of airports and K is for the fleet where each type of aircraft in the fleet is indexed by k. The schedule is represented by time-space network such that $N(k,a,t)$ is the set of nodes in the time-line network for aircraft type k, airport a and time $t \in T$. $\text{In}(k,a,t)$ and $\text{Out}(k,a,t)$ are the sets of inbound and outbound flight legs for node (k,a,t).

Objective (4.3) is to maximize the profit calculated as revenue minus operating costs. The revenue is the sum of the revenues for business and economy passengers taking into account the lost revenue due to spill, where $t_{i,j}$ is the number of passengers the airline wants to redirect from itinerary i to j. The price of the itinerary i is represented by p_i. Operating cost for flight f when using aircraft type k is represented by $C_{k,f}$ which is associated with a binary variable of $x_{k,f}$ that is one if an aircraft of type k is assigned to flight f.

Firstly, we have the fleet assignment constraints. Constraints (4.4) ensure the coverage of mandatory flights which must be served according to the schedule development. Constraints (4.5) are for the optional flights that have the possibility to be canceled. Constraints (4.6) are for the flow conservation of fleet, where y_{k,a,t^-} and y_{k,a,t^+} are the variables representing the number of type k aircraft at airport a just before and just after time t. Constraints (4.7) ensure that for each fleet type k, the number of used aircraft does not exceed the number of available aircarft represented by R_k. minE_a^- represents the time just before the first event at airport a and CT is the set of flights flying at count time. It is assumed that the network configuration at the beginning and at the end of the day is the same in terms of the number of aircraft at each airport. This is ensured by the constraints (4.8) where maxE_a^+ represents the time just after the last event at airport a.

The relation between the supply capacity and the actual demand should be maintained.

Therefore we have the constraints (4.9) which maintain that the assigned capacity for a flight should satisfy the demand for the corresponding itineraries. The assigned capacity for flight f by an aircraft type k for class h passengers is represented by $\pi^h_{k,f}$. The actual demand is composed of the original demand of the itinerary minus the spilled passengers plus the recaptured passengers from other itineraries. The same constraints ensure that the itineraries do not realize any demand if any of the corresponding flight leg is canceled. $\delta_{i,f}$ is a binary parameter which is one if itinerary i uses flight f and enables us to have itinerary-based demand. We let the allocation of business and economy seats to be decided by the model as a revenue management decision. We assume that the capacity for business and economy classes can be arranged freely, i.e. there is no physical requirement for the business class seats. We work with an interval of 10%-30% for the percentage of business class seats. We need to make sure that the total allocated seats do not exceed the capacity of the aircraft. This is ensured by the constraints (4.10) where Q_k is the capacity of aircraft type k.

Demand related constraints include constraints (4.11) which maintain that the total redirected passengers from itinerary i to all other itineraries including the no-revenue options do not exceed its realized demand. We have already explained the constraints (4.12) and (4.13) in section 4.2.

Finally, we have the non-negativity constraints and upper bounds (4.14)-(4.20) for the decision variables. A demand variable d_i is defined in order to allow the airline to decide whether to lose the passengers or redirect them. This demand value cannot be greater than the demand value provided by the logit model, $\tilde{d}_i$. The price of each itinerary has an upper bound UB_i, which is assumed to be the average market price plus the standard deviation. Note that, the price is not a decision variable for the no-revenue options.

For airlines, it is a dream to have flight demands very close to the aircraft capacity. The load factors of 80-85% are already high for airlines. Therefore, some fleet assignment models take into account a maximum allowable load factor as presented by Dumas and Soumis (2008) and Cadarso et al. (2013). Mainly the demand capacity balance constraints (4.10) are modified by multiplying the actual capacity of the aircraft Q_k by the selected maximum load factor. In this thesis, it is not taken into account and left as a future work.

$$z^*_{IFAM-PR} = \max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I_s^{'})} t_{j,i}b_{j,i})p_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f} \tag{4.3}$$

$$\text{s.t.} \sum_{k\in K} x_{k,f} = 1 \qquad \forall f\in F^M \tag{4.4}$$

$$\sum_{k\in K} x_{k,f} \le 1 \qquad \forall f\in F^O \tag{4.5}$$

$$y_{k,a,t^-} + \sum_{f\in \mathrm{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in \mathrm{Out}(k,a,t)} x_{k,f} \qquad \forall [k,a,t]\in N \tag{4.6}$$

$$\sum_{a\in A} y_{k,a,\mathrm{minE}_a^-} + \sum_{f\in CT} x_{k,f} \le R_k \qquad \forall k\in K \tag{4.7}$$

$$y_{k,a,\mathrm{minE}_a^-} = y_{k,a,\mathrm{maxE}_a^+} \qquad \forall k\in K, a\in A \tag{4.8}$$

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})} \delta_{i,f}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I_s^{'})} t_{j,i}b_{j,i}) \le \sum_{k\in K}\pi^h_{k,f} \qquad \forall h\in H, f\in F \tag{4.9}$$

$$\sum_{h\in H}\pi^h_{k,f} \le Q_k x_{k,f} \qquad \forall f\in F, k\in K \tag{4.10}$$

$$\sum_{j\in I_s} t_{i,j} \le d_i \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \tag{4.11}$$

$$\tilde{d}_i = D_s \frac{\exp(V_i(p_i, z_i;\beta))}{\sum_{j\in I_s}\exp(V_j(p_j,z_j;\beta))} \qquad \forall h\in H, s\in S^h, i\in I_s \tag{4.12}$$

$$b_{i,j} = \frac{\exp(V_j(p_j,z_j;\beta))}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k(p_k,z_k;\beta))} \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}), j\in I_s \tag{4.13}$$

$$x_{k,f}\in\{0,1\} \qquad \forall k\in K, f\in F \tag{4.14}$$

$$y_{k,a,t}\ge 0 \qquad \forall [k,a,t]\in N \tag{4.15}$$

$$\pi^h_{k,f}\ge 0 \qquad \forall h\in H, k\in K, f\in F \tag{4.16}$$

$$0\le d_i\le \tilde{d}_i \qquad \forall h\in H, s\in S^h, i\in I_s \tag{4.17}$$

$$\mathrm{LB}_i \le p_i \le \mathrm{UB}_i \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \tag{4.18}$$

$$t_{i,j}\ge 0 \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}), j\in I_s \tag{4.19}$$

$$b_{i,j}\ge 0 \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}), j\in I_s \tag{4.20}$$

4.4 Reference models

In order to quantify the impact of the presented IFAM-PR, we consider the state-of-the-art models as reference models which are already cited in section 4.1. Firstly, we consider the model of Lohatepanont and Barnhart (2004). This model considers the demand and price as inputs to the schedule planning model. We compare our integrated model to a similar model in order to show the added-value of integrating explicit supply-demand interactions. Lohatepanont and Barnhart (2004) use QSI index to model the recapture ratios. Since we do not have access to the parameters of these ratios, we formulate spill and recapture with our itinerary choice model. Moreover we do not consider the demand correction terms that they have introduced. We do not have the decision variable on the cancellation of the itineraries. We decrease the demand for an itinerary to zero by spill and recapture in case of an associated flight cancellation. We refer to this model as **IFAM** (see the model in Appendix A.2.1).

Secondly, we consider the model of Schön (2008) which is an integrated schedule planning and pricing model. This model does not include spill variables since the demand model could be integrated without any limits on the price (we refer to the discussion in section 7.5.2). This is achieved with an inverse demand function which enables to have a mixed integer convex programming problem. In our case we need spill variables and in order to analyze the added value of spill and recapture, we compare our integrated model with a similar model, named **IFAM-PR w/o spill**. Schön (2008) uses a synthetic data and we do not have access to this data. Therefore, for this reference model we use our estimated parameters. Since our demand model is specific to the cabin class we keep the revenue management decision on the allocation of seats to the classes. Schön (2008) does not include this decision since she does not consider different cabin classes.

The presented integrated model includes explicit interactions between supply capacity and the demand. On the other hand, many revenue management models assume that the capacity is fixed and provided by the schedule planning process (Talluri and van Ryzin, 2004a). Therefore we consider this current practice of airlines through a **sequential approach** and compare it with the integrated model. In the sequential approach, firstly, the schedule planning model is optimized to obtain an optimal fleet assignment. Then as a sequential step, the revenue maximization is performed with this optimal fleet assignment. In other words, the fleet assignment is decided with an assumption of inelastic demand and then we expose the model to elastic demand. The integrated model is compared with the sequential approach in order to evaluate the advantage of simultaneously optimizing the schedule planning and revenue related decisions. A similar sequential approach is utilized by Lohatepanont (2002) for the comparison of leg-based fleet assignment and itinerary-based fleet assignment.

Table 4.2: The data instance used for the illustrations

Number of airports:	3
Number of flights:	26
Flight density:	4.33 flights per OD pair
Average demand:	56.12 passengers per flight
Number of itineraries:	36
Cabin classes:	Economy and business
Level of service:	All itineraries are nonstop
Available fleet:	3 types of aircraft (100, 50 and 37 seats)

4.5 Results

In this section we provide illustrations and results to quantify the impact of the integrated model. The data instances are based on the same RP data source introduced in section 3.3. We focus on a daily cyclic schedule.

The presented integrated model is a mixed integer nonlinear problem. The nonlinearity is due to the explicit integration of the demand model. The model is implemented in AMPL[1] and BONMIN[2] solver (Bonami et al., 2008) is used for the solution of the problem. BONMIN solver applies several algorithms depending on the nature of the problem including branch-and-bound, branch-and-cut and outer-approximation. It serves as a heuristic approach since we cannot guarantee the convexity of the problem.

4.5.1 Illustrative example for the impacts of the integrated demand model

In order to analyze the added value of the integrated scheduling, fleeting and pricing model, we compare IFAM-PR with IFAM that is defined in section 4.4. The two models are solved with the data instance provided in Table 4.2. In Table 4.3, we provide the results for IFAM and two sets of results for IFAM-PR. IFAM-PR has the flexibility to change the prices of the itineraries which might be higher than the average values used by IFAM. Therefore we first present the results of IFAM-PR where we constrain the itinerary prices not to be higher than the average prices used by IFAM (*IFAM-PR with limited prices*). The motivation for constraining the prices is to show that the strength of the integrated model is not only due to the ability to increase the prices, but also the simultaneous decisions which lead to superior schedule planning. Finally we present the results for IFAM-PR where the prices can be increased above the average prices. We arbitrarily select 2 of the 3 airports and present the realized price and demand values for the bi-directional flights (A-B and B-A) between these airports.

[1] `http://www.ampl.com`
[2] https://projects.coin-or.org/Bonmin

Table 4.3: Results on the impact of the integrated demand model

					IFAM			IFAM-PR - limited prices			IFAM-PR		
				Revenue			204,553			214,380			244,924
				Operating costs			150,603			160,003			173,349
				Profit			**53,949**			**54,377 (+ 0.8%)**			**71,575 (+ 32.7%)**
				Number of flights			**22**			**22**			**24**
				Transported passengers			**943**			**1031 (+ 9.3%)**			**1064 (+ 12.7%)**
				Economy-Business passengers			882 E - 61 B			970 E - 61 B			997 E - 67 B
				Allocated seats			**274**			**324**			**324**
	OD pair	**Class**	**Morning**	**Forecasted demand**	**Avg. price**	**Realized demand**	**Assigned capacity**	**Realized price**	**Realized demand**	**Assigned capacity**	**Realized price**	**Realized demand**	**Assigned capacity**
1	B-A	E	0	59	225	50	50	225	50	50	250	50	50
2	A-B	E	1	68	212.5	50	50	212.5	50	50	225	50	50
3	B-A	E	0	59	225	50	50	225	50	50	250	50	50
4	A-B	E	0	66	212.5	50	50	212.5	50	50	225	50	50
5	B-A	B	1	6	526.5	6	**50**	526.5	6	**37**	550	6	**37**
		E		60	**225**	**44**		**225**	**31**		**250**	**31**	
6	A-B	B	0	2	497.2	2	**50**	497.2	2	**37**	507.34	2	**37**
		E		66	**212.5**	**45**		**212.5**	**33**		**225**	**33**	
7	A-B	E	0	66	212.5	50	50	212.5	50	50	225	50	50
8	B-A	E	1	60	**225**	**50**	50	**182.4**	**100**	**100**	**195.15**	**100**	**100**
9	A-B	E	1	68	**212.5**	**50**	50	**194**	**100**	**100**	**201.56**	**100**	**100**
10	B-A	E	0	59	225	50	50	225	50	50	250	50	50
11	A-B	E	0	66	212.5	50	50	212.5	50	50	225	50	50
12	B-A	E	0	59	225	50	50	225	50	50	250	50	50

The results in Table 4.3 indicate that the integrated model has the flexibility to change the fleet assignment decisions simultaneously with the pricing decisions in order to have more profitable planning. For the case of limited prices, the integrated model decreases the prices of itineraries 8 and 9 and assigns larger capacity to them. It is observed that these itineraries are morning itineraries and therefore more attractive itineraries according to the logit model. The model decides to increase the capacity of these itineraries since it can be maintained without significant decrease in price. It is also observed that the decrease in the prices of itineraries 8 and 9 affects the demand of the itineraries 5 and 6 respectively and they are assigned smaller capacity. With similar decisions for the other OD pairs, that are not presented here, the resulting profit and the number of transported passengers are higher compared to IFAM. When the integrated model is allowed to increase the prices beyond the average prices (last column), the resulting profit and the served demand increases more significantly. The decisions on the prices of the itineraries show that the integrated model increases the prices whenever it sees a potential and decreases the prices when assigning a larger capacity is more profitable. We also observe that the decisions taken by the integrated model render additional optional itineraries profitable and therefore 24 flights are operated instead of 22.

In this example the available fleet is sufficient to serve the mandatory flights and extra aircraft to serve the optional flights when needed. The integrated model uses these extra aircraft and carries more passengers with an increased capacity. However, in real applications it is more typical to work with tight capacity. Therefore further analysis should be carried out in order to understand the impact of capacity limitations on the results. In section 4.5.4 we evaluate a few examples with tight fleeting capacity.

4.5.2 Illustrative example for the reaction of the integrated model to the market conditions

One of the most important factors for airlines in their revenue management is the alternative itineraries provided by competitive airlines. As explained in section 4.2, we introduce no-revenue options in our model to represent the attributes of the competitors' itineraries. Therefore the integrated model takes into account those competitive itineraries offered by other airlines, while optimizing the revenue decisions. In order to illustrate this phenomenon we compare IFAM with IFAM-PR in three different market conditions based on the data instance provided in Table 4.2. Compared to the actual itineraries, the competitors have lower, similar and higher prices respectively in the presented scenarios.

The results are provided in Table 4.4. For the scenario with similar prices, the scheduling decisions of IFAM and IFAM-PR are the same and therefore the realized demand is similar. In the scenario where the competitors are more expensive, IFAM keeps the same scheduling decisions which result with an improvement in the profit and realized demand since the competitors are less attractive. For the same scenario, IFAM-PR allocates higher capacity and operates one more flight which results with a significant increase in realized demand.

Table 4.4: The results with changing market conditions

Competitors with higher prices		
	IFAM	IFAM-PR
Revenue	206,001	247,269
Operating costs	150,604	173,349
Profit	**55,397**	**73,920 (+ 33%)**
Number of flights	**22**	**24**
Transported passengers	**951**	**1,076 (+ 13%)**
Economy-Business passengers	888 E - 63 B	1007 E - 69 B
Allocated seats	**274**	**324**
Competitors with similar prices		
	IFAM	IFAM-PR
Revenue	202,645	218,456
Operating costs	150,604	149,656
Profit	**52,401**	**68,800 (+ 31%)**
Number of flights	**22**	**22**
Transported passengers	**935**	**935**
Economy-Business passengers	876 E - 59 B	878 E - 57 B
Allocated seats	**274**	**274**
Competitors with lower prices		
	IFAM	IFAM-PR
Revenue	190,590	215,429
Operating costs	140,822	149,656
Profit	**49,768**	**65,773 (+ 32%)**
Number of flights	**20**	**22**
Transported passengers	**871**	**926 (+ 6%)**
Economy-Business passengers	815 E - 56 B	871 E - 55 B
Allocated seats	**274**	**274**

The advantage of IFAM-PR emerges from the fact that either it finds room to attract more passengers or it has a potential to increase the prices. When we analyze the results in the case of cheaper competitors, it is observed that IFAM operates less flights. Since it can not compete with the cheap prices it carries less passengers compared to the other scenarios. However, IFAM-PR can still accommodate a similar level of passengers thanks to the flexibility of decreasing the prices in order to attract passengers. It can be concluded that IFAM-PR is able to react to market changes which is an indication for increased robustness.

4.5.3 Illustrative example for the spill and recapture effects

We compare our integrated approach with *IFAM-PR w/o spill* described in section 4.4 in order to analyze the design flexibility of the schedule gained by airlines with the spill and recapture effects. In order to be able to see the impact of the spill more clearly, in this analysis the upper bound on the prices is set to the average price.

The results over the data instance provided in Table 4.2 are presented in Table 4.5. We select the OD pair C-D arbitrarily among 6 OD pairs to present the impact of spill from an itinerary level. Spill values with "+" sign means that the itinerary recaptures passengers that are spilled from other itineraries. On the other hand, spill values with "−" sign corresponds to the total redirected passengers from the itinerary to the remaining alternatives in the same market. It is observed that the integrated model with spill modifies the prices of the itineraries relatively to capture the passengers of the other itineraries for the same market. For example itinerary 4 attracts both economy and business passengers from other itineraries and therefore assigns a larger aircraft compared to *IFAM-PR w/o spill.* The flexibility of redirecting passengers to other itineraries enables to keep the prices higher. As an example, IFAM-PR has a higher price for itinerary 1 compared to *IFAM-PR w/o spill*, but realizes the same demand due to recaptured passengers. Furthermore it is observed that itinerary 5 is not operated since some of its passengers can be recaptured by other itineraries and it is more profitable to cancel it. With similar decisions for other OD pairs, IFAM-PR has higher profit and carries more passengers in the presence of spill and recapture. Note that for this particular OD pair it IFAM-PR results with less transported passengers. However in total, i.e. considering all OD pairs, the resulting number of transported passengers is 6.1% higher.

Table 4.5: Illustration for the spill and recapture effects

					IFAM-PR w/o spill			IFAM-PR			
Revenue							208,955				214,380
Operating costs							158,441				160,003
Profit							**50,514**				**54,377 (+ 7.7%)**
Number of flights							24				22
Transported passengers							**972**				**1031 (+ 6.1%)**
Economy-Business passengers							902 E - 70 B				970 E - 61 B
Allocated seats							**224**				**324**
	OD pair	**Class**	**Morning**	**Forecasted demand**	**Realized price**	**Realized demand**	**Assigned capacity**	**Realized price**	**Spill**	**Realized demand**	**Assigned capacity**
1	C-D	E	0	34	**160.2**	**37**	37	**169**	**+3**	**37**	37
2	C-D	E	0	32	-	0	0	-	-32	0	0
3	C-D	E	1	34	**158.6**	**39**	**50**	**171.4**	**+3**	**37**	**37**
4	C-D	B	1	7	**409.5**	**7**	**37**	**404.3**	**+3**	**10**	**50**
		E		35	**175**	**30**		**168.7**	**+5**	**40**	
5	C-D	B	0	7	409.5	**7**	**37**	-	**- 7**	**0**	**0**
		E		32	175	**30**		-	**-32**	**0**	
6	C-D	B	0	7	409.5	7	37	409.5		7	37
		E		32	175	30		175	-2	30	

Table 4.6: The experiments

No	Airports	Flights	Flight density	Average demand	Fleet composition	
1	3	10	1.67	51.9	2	50-37 seats
2	3	11	2.75	83.1	2	117-50 seats
3	3	12	2.00	113.8	6	164-146-128-124-107-100 seats
4	3	26	4.33	56.1	3	100-50-37 seats
5	3	19	3.17	96.7	3	164-117-72 seats
6	3	12	3.00	193.4	3	293-195-164 seats
7	3	33	8.25	71.9	3	117-70-37 seats
8	3	32	5.33	100.5	3	164-117-85 seats
9	2	11	5.50	173.7	3	293-164-127 seats
10	4	39	4.88	64.5	4	117-85-50-37 seats
11	4	23	3.83	86.1	4	117-85-70-50 seats
12	4	19	3.17	101.4	5	128-124-107-100-85 seats
13	4	15	1.88	58.1	5	117-85-70-50-37 seats
14	4	14	2.33	87.6	5	134-117-85-70-50 seats
15	4	13	2.60	100.1	5	164-134-117-100-85 seats

4.5.4 Experiments on the added value of IFAM-PR

In order to see the added value of the integration of the demand model we need to support our observations with a comprehensive set of experiments. For that purpose we identified 15 data instances with different characteristics that are listed in Table 4.6. For the experiments, we present the number of airports and the number of flights in the network. Moreover, the flight density stands for the average number of flights per OD pair. The average demand gives the average number of passengers per flight according to demand forecast. The fleet composition provides information on the number of different aircraft types in the fleet together with the seat capacity for each type.

For the considered data instances, we compare the sequential approach presented in section 4.4 and the integrated model, IFAM-PR. The comparative results are presented in Table 4.7. It is observed that for 8 of these 15 instances there is an improvement (of 3% on average) with the IFAM-PR in terms of the profit. These are the cases where the simultaneous optimization of the schedule planning and pricing lead to different scheduling decisions such as the operated number of flights or the number of allocated seats.

We observe that the improvement is higher for the experiments where the demand values of the itineraries are not close to the aircraft capacities. In those cases, IFAM-PR is able to adjust the capacity according to the demand and has significant improvement over the sequential approach. Experiment 2 is a good example for this phenomenon. There are 2 different fleet types with 50 and 117 seats. The sequential approach does not use the larger aircraft which is costlier to fly. On the other hand, IFAM-PR uses this large aircraft thanks to its flexibility in controlling the demand by pricing decisions. As a result, there is a 5.55% increase in profit and 33.5% more passengers are transported. Similarly, for the experiments 4 and 9, IFAM-PR

Table 4.7: The results of the experiments

	Sequential approach				IFAM-PR				Improvement	
No	Profit	Pax.	Flights	Seats	Profit	Pax.	Flights	Seats	Profit	Pax.
1	15,091	284	8	124	15,091	284	8	124	-	-
2	35,372	400	8	**150**	37,335	534	8	**217**	**5.55%**	**33.50%**
3	43,990	882	**10**	**331**	46,037	725	**8**	**207**	**4.65%**	**-17.80%**
4	69,901	931	**22**	**274**	70,904	1063	**24**	**324**	**1.43%**	**14.18%**
5	82,311	1145	16	333	82,311	1145	16	333	-	-
6	779,819	1448	10	1148	779,819	1448	10	1148	-	-
7	135,656	1814	32	498	135,656	1814	32	498	-	-
8	107,927	2236	26	691	107,927	2236	26	691	-	-
9	854,902	1270	10	**1016**	858,544	1344	10	**1090**	**0.43%**	**5.83%**
10	109,906	1448	**32**	391	112,881	1541	**34**	391	**2.71%**	**6.42%**
11	82,440	1135	20	387	85,808	1164	20	387	**4.09%**	**2.56%**
12	37,100	1067	12	377	38,205	1049	12	377	**2.98%**	**-1.69%**
13	27,076	448	10	207	27,076	448	10	207	-	-
14	44,339	599	**10**	267	45,070	699	**12**	267	**1.65%**	**16.69%**
15	26,486	504	6	185	26,486	504	6	185	-	-

decides to use more capacity with the knowledge on the demand response. On the other hand, in experiment 3, the integrated model allocates less capacity since it decides to increase the price levels in order to be more profitable. In this particular experiment, the available capacity is tight and the integrated model cannot freely increase the capacity as done for experiment 2. In addition to the decision on the allocated capacity, IFAM-PR may decide to operate less/more flights by changing the attractiveness of the corresponding itineraries as seen in experiments 3, 4, 10, and 14. For example, in experiment 14, IFAM-PR operates 2 more flights with the same overall capacity compared to the sequential approach. Furthermore in experiments 11 and 12 the improvement is due to the changes in the fleet assignments with the same overall capacity utilization and the same number of flights.

4.6 Conclusions and future research directions

In this chapter an integrated scheduling, fleeting and pricing model, IFAM-PR, is presented for a single airline, which enables to take the advantage of explicit supply-demand interactions in decision making. The novelty of the model is due to the modeling of the demand through an itinerary choice model based on a real data and the integration of this demand model in a scheduling and fleeting framework for airlines. The demand model is utilized for pricing as well as the spill and recapture effects which gives flexibility to airlines in determining their transportation capacity.

The impacts of the integrated model is evaluated on a European air transportation network with several illustrations. It is observed that the integrated model has more flexibility on the decisions thanks to the simultaneous optimization. The pricing is determined according to the market conditions and whenever there is a potential in increasing the profit by altering the price the integrated model benefits from it. Therefore the integrated model is elastic to the market conditions.

The added value of the integrated model is analyzed in comparison to a sequential approach which mimics the current practice of airlines. It is shown that the integrated model may decide on different scheduling and/or fleeting compared to the sequential approach by making use of the supply-demand interactions. These differences may be in terms of the number of flights operated or the assigned capacity which result with an improved profitability.

The presented analysis shows that the airlines should consider the demand related information earlier in their planning phase when deciding on the schedule and capacity. Our model is a proof of concept for the integration of scheduling, fleeting and pricing decisions which is expected to improve the efficiency of decision support tools for airlines. This effort can motivate the integration of more detailed demand information through disaggregate demand models as a future extension. Even though the analysis is done with pricing, the impacts are expected to be more general serving as an example for introducing the explanatory variables of demand models as decision variables in optimization models. Furthermore, the explicit integration of the demand model in the planning process is expected to provide valuable information to the actual disaggregate revenue management process.

The presented integrated model is a mixed integer non-convex problem which is highly complex. When we go beyond the instances provided in Table 4.6 in terms of size, the solvers can not provide good quality feasible solutions. In chapter 5 we address it with a heuristic algorithm and obtain solutions for data instances that are similar to real flight networks.

In the presented model, the maximum load factor limitation on seating capacity is not included. As a future work, this phenomenon could be analyzed. Furthermore in this model, the spill phenomenon is assumed to occur between the itineraries of the same market segment only. However, some flight legs may serve different market segments. In such cases, spill and recapture can occur between flight legs that serve different market segments and airlines can benefit from this flexibility. With an itinerary-based revenue management, leg-based spill and recapture cannot be taken into account in a straightforward way. The demand model development should be carried out accordingly and the integration of spill effects based on the demand model should be reconsidered. Therefore, the concept of leg-based revenue management embedded in itinerary-based fleet assignment models is an interesting future research direction.

The considered demand model has only one policy variable which is the price of the itinerary. In other words the integrated model can only control the price of the itineraries in order to maximize the profit. However there is a variable for the departure time of the day which indicates whether the itinerary is a morning itinerary or not. As a future work the departure time can be introduced as a policy variable in addition to the price. This will enable the integrated model to take the advantage of the flexibility in changing the departure time of the flights. As another promising research direction, the presented model can be embedded in a competitive framework with a game theoretical approach in order to represent the response of each airline in the market segment.

5 A local search heuristic for the integrated model

In this chapter we present a local search heuristic method for the integrated airline scheduling, fleeting and pricing model, IFAM-PR, presented in Chapter 4. The model is a non-convex mixed integer nonlinear problem (MINLP) where the non-convexity is due to the explicit representation of a demand model guiding the revenue management decisions. The local search heuristic tackles the complexity of the problem decomposing the problem into two simplified versions of the integrated model. The first model is a fleet assignment model where the pricing decision is fixed. The fleet assignment sub-model is a mixed integer linear problem. The second model is a revenue management model where the fleet assignment decision, i.e., the transportation capacity, is fixed. This revenue sub-model is a continuous nonlinear problem. The sub-models are solved in an iterative way with two local search mechanisms. Firstly, a price sampling procedure is used for a local search on price based on spill information in order to explore new fleet assignment solutions. Secondly, a subset of fleet assignment solutions are fixed in a variable neighborhood search framework where the number of fixed solutions is updated based on the quality of the solution. The selection of the subset to be fixed is also determined based on the spill information. These metaheuristic mechanisms permit to escape from local optima. The local search heuristic is presented in comparison to two other heuristic approaches: a heuristic procedure provided by an open-source generic MINLP solver and a sequential approach which mimic the current practice of airlines. The three approaches are tested on a set of experiments with different problem sizes. The local search heuristic outperforms the two other approaches in terms of the quality of the solution and computational time.

5.1 Introduction

In Chapter 4 the added value of the model is reported by solving the monolithic model with an open-source solver, BONMIN. However, the solver is designed for convex problems. Therefore it is computationally inefficient for solving the full integrated model and cannot provide good quality feasible solutions for medium size instances even in 24 hours. These limitations necessitate the development of a more efficient method. We present a local search heuristic based on two sub-models of the problem. Inspired by the idea of D'Ambrosio et al. (2012), we fix either the fleet or the revenue part of the integrated model in order to obtain simplified models. When we fix the pricing part, we obtain a mixed integer linear problem (MILP). This sub-model is a fleet assignment problem where the price and recapture ratios are inputs. When we fix the fleet assignment decisions, we obtain a non-convex nonlinear problem (NLP) which is a revenue management model with a given capacity. The two sub-models are solved in an iterative procedure where local search techniques are used to explore alternative feasible solutions. Local search techniques include price sampling that is used to visit new fleet assignment solutions with different price inputs. Furthermore a variable neighborhood search (Hansen and Mladenović, 2001) is developed for large size instances so that a sub-set of the fleet assignments are fixed and kept for the next iteration based on the quality of the incumbent solution. The main contribution is a local search heuristic which is designed to handle the difficulties of the model thanks to a combination of the above-mentioned techniques. The interactions between supply and demand models are exploited and as a result, this combination provides better quality feasible solutions compared to other two heuristic approaches: a MINLP solver (BONMIN, Bonami et al., 2008) and the sequential approach that is introduced in chapter 4, section 4.4 in order to represent the current practice of airlines. The presented local search heuristic can easily be used by practitioners for the solution of integrated scheduling, fleeting and pricing decisions.

The remainder of the chapter is organized as follows. In section 5.2 we introduce the three heuristic approaches for the integrated model. Section 5.3 describes the data instances used for the experiments throughout the analysis. In section 5.4 we provide experimental results on the performance of the three approaches. In section 5.5 we present the details on the performance on the heuristic and analyze the added value of the components of the heuristic. We analyze the results in terms of the quality of the solution and computational time with details on the complexity of the sub-problems. Finally we conclude the chapter and provide future directions in section 5.6.

5.2 Heuristic approaches

We consider three heuristic approaches for the solution of the integrated airline schedule planning model. The first two approaches serve as references for testing the performance of the local search heuristic.

5.2.1 BONMIN solver for the integrated model

BONMIN is an open-source solver proposed by Bonami et al. (2008) and designed to solve convex MINLPs. As it is designed to be an exact method for convex problems, it can be only considered as an heuristic for solving the integrated model. The main methods embedded in the solver are branch and bound and polyhedral outer approximation.

5.2.2 Sequential approach

As a second heuristic approach for the solution of the integrated model, we mimic the current practice of airlines where revenue management decisions are taken with a fixed capacity provided by the schedule planning process. This sequential approach is introduced in chapter 4, in section 4.4. A similar sequential approach is utilized by Lohatepanont (2002) in the context of a sensitivity analysis for an itinerary-based fleet assignment model.

We represent the sequential approach with two sub-models of the integrated model. The first sub-model is the itinerary-based fleet assignment model (IFAM), where the price of the itineraries are inputs and the remaining decisions are optimized with the given price and demand. The optimized decisions are the schedule design, fleet assignment, seat allocation and the number of spilled passengers. This model is indeed an extended version of the state-of-the-art fleet assignment models (Lohatepanont and Barnhart, 2004) with more advanced methodology on the spill and recapture effects. Since the pricing decision is excluded, the prices of the itineraries (p) are fixed. The demand given by the logit ($\tilde{d}$) and the recapture ratios (b) are also parameters that are calculated with the given price. Therefore we represent them by $\bar{p}$, $\bar{d}$, and $\bar{b}$ respectively for clarification purposes. IFAM is a MILP and given as follows:

$$
\begin{aligned}
& z^*_{\mathbf{IFAM}} = \\
& \max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I_s^{'})} t_{j,i}\bar{b}_{j,i})\bar{p}_i \\
& \quad - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f} && (5.1)\\
& \text{s.t.} \sum_{k\in K} x_{k,f} = 1 && \forall f\in F^M \quad (5.2)\\
& \sum_{k\in K} x_{k,f} \le 1 && \forall f\in F^O \quad (5.3)\\
& y_{k,a,t^-} + \sum_{f\in \mathrm{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in \mathrm{Out}(k,a,t)} x_{k,f} && \forall [k,a,t]\in N \quad (5.4)\\
& \sum_{a\in A} y_{k,a,\mathrm{minE}_a^-} + \sum_{f\in CT} x_{k,f} \le R_k && \forall k\in K \quad (5.5)\\
& y_{k,a,\mathrm{minE}_a^-} = y_{k,a,\mathrm{maxE}_a^+} && \forall k\in K, a\in A \quad (5.6)\\
& \sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})} \delta_{i,f}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I_s^{'})} t_{j,i}\bar{b}_{j,i}) \\
& \quad \le \sum_{k\in K} \pi^h_{k,f} && \forall h\in H, f\in F \quad (5.7)\\
& \sum_{h\in H} \pi^h_{k,f} \le Q_k x_{k,f} && \forall f\in F, k\in K \quad (5.8)\\
& \sum_{j\in I_s} t_{i,j} \le d_i && \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \quad (5.9)\\
& x_{k,f}\in\{0,1\} && \forall k\in K, f\in F \quad (5.10)\\
& y_{k,a,t}\ge 0 && \forall [k,a,t]\in N \quad (5.11)\\
& \pi^h_{k,f}\ge 0 && \forall h\in H, k\in K, f\in F \quad (5.12)\\
& 0\le d_i\le \bar{d}_i && \forall h\in H, s\in S^h, i\in I_s \quad (5.13)\\
& t_{i,j}\ge 0 && \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}), j\in I_s \quad (5.14)
\end{aligned}
$$

The second sub-model is a revenue management model with pricing (RMM-PR) given a fixed capacity. The available seat capacity for every flight is given as input. This model is a non-convex NLP. Since the fleet assignment decisions of x and y are fixed they are parameters for RMM-PR and represented by $\bar{x}$ and $\bar{y}$ for the ease of explanation. RMM-PR is provided as follows:

$$
\begin{aligned}
& z^*_{\textbf{RMM-PR}} = \\
& \max \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s')} (d_i - \sum_{j \in I_s} t_{i,j} + \sum_{j \in (I_s \setminus I_s')} t_{j,i} b_{j,i}) p_i && && (5.15) \\
& \text{s.t.} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s')} \delta_{i,f} (d_i - \sum_{j \in I_s} t_{i,j} + \sum_{j \in (I_s \setminus I_s')} t_{j,i} b_{j,i}) \\
& \quad \leq \sum_{k \in K} \pi^h_{k,f} && \forall h \in H, f \in F && (5.16) \\
& \sum_{h \in H} \pi^h_{k,f} \leq Q_k \bar{x}_{k,f} && \forall f \in F, k \in K && (5.17) \\
& \sum_{j \in I_s} t_{i,j} \leq d_i && \forall h \in H, s \in S^h, i \in (I_s \setminus I_s') && (5.18) \\
& \tilde{d}_i = D_s \frac{\exp(V_i(p_i, z_i; \beta))}{\sum_{j \in I_s} \exp(V_j(p_j, z_j; \beta))} && \forall h \in H, s \in S^h, i \in I_s && (5.19) \\
& b_{i,j} = \frac{\exp(V_j(p_j, z_j; \beta))}{\sum_{k \in I_s \setminus \{i\}} \exp(V_k(p_k, z_k; \beta))} && \forall h \in H, s \in S^h, i \in (I_s \setminus I_s'), j \in I_s && (5.20) \\
& \pi^h_{k,f} \geq 0 && \forall h \in H, k \in K, f \in F && (5.21) \\
& 0 \leq d_i \leq \tilde{d}_i && \forall h \in H, s \in S^h, i \in I_s && (5.22) \\
& LB_i \leq p_i \leq UB_i && \forall h \in H, s \in S^h, i \in (I_s \setminus I_s') && (5.23) \\
& t_{i,j} \geq 0 && \forall h \in H, s \in S^h, i \in (I_s \setminus I_s'), j \in I_s && (5.24) \\
& b_{i,j} \geq 0 && \forall h \in H, s \in S^h, i \in (I_s \setminus I_s'), j \in I_s && (5.25)
\end{aligned}
$$

The sequential approach first solves IFAM with the average price values provided in the dataset. It optimizes the schedule design and fleet assignment ($x_{k,f}$, $y_{k,a,t}$). These decisions on the capacity are given as inputs to the next step which is the solution of RMM-PR. It provides the price of each itinerary (p_i), the actual demand (d_i), the allocated seats to each class ($\pi^h_{k,f}$) and the number of spilled passengers ($t_{i,j}$).

5.2.3 Local search heuristic

The third heuristic approach is the main contribution of this chapter. It is based on the sequential approach and the use of appropriate local search mechanisms. The main shortcoming of the sequential approach is that the capacity provided by IFAM cannot make use of the information on the revenue since it runs with fixed price and demand for the itineraries. IFAM is not able to account for the potential in changing the pricing decisions in order to shape the demand and come up with more profitable schedule planning. Therefore a local search heuristic is developed answering to this lack of interaction between planning and revenue decisions. The neighborhood is defined by local search techniques which provide alternative schedule planning decisions. Namely, the alternative solutions for the $x_{k,f}$ variables constitute neighborhood solutions.

The first local search mechanism is *price sampling* which reveals the potential improvement on the revenue as a consequence of the adjustments on the price. The second mechanism is *variable neighborhood search* which keeps a varying subset of fleet assignment solutions fixed in the model based on the quality of the solution. Both of the local search procedures are based on the number of spilled passengers. This information is found to be important since the spilled passengers are potential revenue sources. The local search procedures are then capable of realizing the impact of planning decisions on the revenue and directing the algorithm towards good feasible solutions.

Price sampling

As mentioned previously, IFAM considers fixed price and fixed demand values. In order to visit alternative solutions, the model is iteratively solved drawing different price samples that result with different itinerary demands. The sampling procedure takes into account the rate of spilled passengers resulting from the solution of RMM-PR in the previous iteration. The spill rate of a flight is defined as the average number of spilled passengers divided by the total demand for the flight (McGill, 1989; Belobaba, 2006). Similarly, for every itinerary i, the SR_i^g rate is defined as the number of spilled passengers over the realized demand in iteration g as follows:

$$SR_i^g = \frac{\sum_{j \in I_s} t_{i,j}^g}{d_i^g} \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}) \tag{5.26}$$

In price sampling, according to the solution of RMM-PR, the price of an itinerary in iteration g is altered based on the number of spilled passengers in the previous iteration $g-1$. The price is decreased if that itinerary presents a lower SR_i^{g-1} rate compared to the average rate, which is denoted by SR_{mean}^{g-1}. The idea is that, if there is a low spill rate it means the capacity was enough in the previous iterations and lower prices can be tried in the current iteration in order to attract more passengers. In order to do that, a random price value is uniformly drawn between the lower bound and the current price value. On the other hand, the price is increased if the spill rate is higher than SR_{mean}^{g-1}, since the itinerary already has a high demand and price can be increased in order to increase the revenue. A random price value is uniformly drawn between the current price value and the upper bound. This price sampling is given as follows:

$$\bar{p}_i^g = \begin{cases} \text{unirand}(LB_i, p_i^{g-1}) & \text{if } SR_i^{g-1} \leq SR_{\text{mean}}^{g-1} \\ \text{unirand}(p_i^{g-1}, UB_i) & \text{otherwise} \end{cases} \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}) \tag{5.27}$$

Variable neighborhood search - VNS

While neighborhood schedule planning solutions are being explored, a subset of fleet assignments is fixed, i.e. some flights are kept assigned to the same aircraft, for a number of iterations in order to keep the good fleet assignment solutions and speed-up the solution of IFAM. The number of fixed assignments is represented by n_{fixed}. The variable neighborhood mechanism is embedded in such a way that n_{fixed} is altered according to the quality of the solution. If a better solution is obtained, n_{fixed} is increased by an increment of n_{inc} in the next iteration which is referred as *intensification*. On the other hand, when there is no improvement for a subsequent number of iterations, a *diversification* is applied, i.e. n_{fixed} is decreased by 1 in order to better explore the feasible region.

The set of fixed assignments is represented by Ł which has n_{fixed} elements. Each fixed assignment ℓ indicates a fleet type k_ℓ^{fixed} and a flight f_ℓ^{fixed}. This fixing is maintained by the constraint given by equation (5.28). Therefore IFAM considered for the local search heuristic is represented by (5.1)-(5.14) and (5.28).

$$x_{k_\ell^{\text{fixed}}, f_\ell^{\text{fixed}}} = 1 \qquad \forall \ell \in Ł \tag{5.28}$$

The decision to fix a fleet assignment is taken considering the number of spilled passengers. In other words, an aircraft type is assigned to the corresponding flight in the current solution with a probability which depends on the number of passengers spilled from itineraries involving that flight. Intuitively the smaller the spill from a flight, the higher the probability that the flight-aircraft pair is fixed in the current iteration. The set of flights which are flown at iteration g is represented by F_{flown}^g. The spill rate of a flight, SR_f^g, is the sum of the spill rates of all itineraries involving flight f as stated in equation (5.29).

$$SR_f^g = \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I_s')} \delta_{i,f} SR_i^g \qquad \forall f \in F_{\text{flown}}^g \tag{5.29}$$

The maximum SR_f^g rate among all the flights in F_{flown}^g is denoted by SR_{max}^g. The probability of fixing the assignment of flight f at iteration g, prob_f^g, is obtained according to the number of spilled passengers at iteration $g-1$ as provided in equation (5.30). It is proportional to the gap between the maximum spill rate and the spill rate of flight f. Therefore, the probability is higher when the number of spilled passengers is lower.

$$\text{prob}_f^g = \frac{SR_{\text{max}}^{g-1} - SR_f^{g-1}}{\sum\limits_{j \in F_{\text{flown}}^{g-1}} (SR_{\text{max}}^{g-1} - SR_j^{g-1})} \qquad \forall f \in F_{\text{flown}}^{g-1} \tag{5.30}$$

Algorithm 1 Local search heuristic

Require: x^0, y^0, d^0, p^0, t^0, b^0, π^0, time$_{max}$, n_{min}, n_{max}, n_{inc}, notImpr,
  $g := 0$, time := 0, $n_{\text{fixed}} := n_{\text{min}}$, notImpr := 0, $z^* := -\text{INF}$,
  repeat
    $\bar{p}^g$:= Price sampling(t^{g-1}, p^{g-1}, d^{g-1}) *[section 5.2.3]*
    $\{\bar{d}^g, \bar{b}^g\}$:= Logit models($\bar{p}^g$) *[demand and recapture ratios for the sampled price based on equations (4.12) and (4.13)]*
    Ł := VNS - Fixing(x^{g-1}, t^{g-1}, d^{g-1}, n_{fixed}) *[selection of fixed assignments - section 5.2.3]*
    $\{x^g, y^g, \pi^g, t^g\}$:= solve $z_{\text{IFAM}}(\bar{p}^g, \bar{d}^g, \bar{b}^g, \text{Ł})$ *[solve IFAM with the sampled price]*
    $\{p^g, d^g, b^g, \pi^g, t^g\}$:= solve $z_{\text{RMM-PR}}(\bar{x}^g, \bar{y}^g)$ *[solve RMM-PR with fixed capacity]*
    if ($z_{\text{RMM-PR}} \geq z^*$) *[if a better solution is obtained]* **then**
      Update z^*
      VNS - Intensification: $n_{\text{fixed}} := n_{\text{fixed}} + n_{\text{inc}}$ when $n_{\text{fixed}} \leq n_{\text{max}} - n_{\text{inc}}$, $n_{\text{fixed}} := n_{\text{max}}$ otherwise
      notImpr := 0
    else
      notImpr := notImpr + 1
      if (notImpr == 5) *[if no improvement is obtained in the last 5 iterations]* **then**
        VNS - Diversification: $n_{\text{fixed}} := n_{\text{fixed}} - 1$ when $n_{\text{fixed}} > n_{\text{min}}$
        notImpr := 0 *[reset the number of iterations without improvement]*
      end if
    end if
    $g := g + 1$
  until time ≥ time$_{max}$

The complete local search heuristic

The local search heuristic consists of iterations each of which solves IFAM and RMM-PR subsequently. As mentioned previously, IFAM is solved by fixing the revenue part and RMM-PR is solved by fixing the schedule planning decisions. This fixing is embedded in an iterative process similar to the idea of D'Ambrosio et al. (2012). The iterative process is carried out with the local search mechanisms defined above. These local search techniques enable to visit good quality neighborhood solutions.

The procedure is presented by Algorithm 1. The iterations continue until the time limit, time$_{\text{max}}$. The decision variables of the model are represented by the same notation in the algorithm. The price variables are initialized with the given price values in the data set. This implies that the first iteration of the local search heuristic is actually the sequential approach. However with the local search mechanisms this sequential approach solution is improved. Since we have a non-convex problem, the initial point is important for the performance of the heuristic approach. Therefore, the solution of the sequential approach is selected as the initial solution of the heuristic method.

n_{min} and n_{max} are defined as the minimum and maximum number of fixed assignments according to the data instance. n_{inc} is the increment in the number of fixed fleet assignments. It is added to the actual number of fixed assignments when the solution is improved in order to intensify the search. *notImpr* is the number of subsequent iterations where there was no improvement in the best objective function value, z^*.

5.3 The data instances

As done throughout the thesis, data instances are generated based on the data provided for ROADEF Challenge 2009. In order to test the heuristic, new data instances are generated in addition to the ones provided in Chapter 4 Table 4.6. These new instances include relatively larger flight networks (20-27). Instances 26-27 are the largest and similar to real life cases. Instance 26 has high demand values and 10 different aircraft types while instance 27 has 272 flights a day that generate 485 itineraries. For the sake of completeness we provide all the instances in Table 5.1. Real flight networks, especially in the US market, may count as much as 1,000 a day. According to AEA (2007) average number of daily flights is around 300-350 flights in the European context. Therefore the instances 26 and 27 can be considered as large enough to represent real life instances. Furthermore, the problem size for each data instance is presented in Table 5.2 as given by AMPL, with details on the number of variables and constraints.

5.4 Performance of the heuristic approaches

In this section we present results for the three heuristic approaches presented in section 5.2. We present the comparison between the three heuristic approaches and analyze the results. All the models are implemented in AMPL. BONMIN runs over the full integrated model presented in section 4.3. The sequential approach and the local search heuristic work with the models IFAM and RMM-PR. IFAM is a MILP and solved using the GUROBI[1] solver. RMM-PR is a non-convex NLP and therefore BONMIN solver is used as done for the integrated model. For all the RMM-PR's solved in the sequential approach and in the local search heuristic, BONMIN converges to a local optima.

The heuristic is implemented in C++ communicating with AMPL for the solution of the models. For the computations Intel Xeon 3.33 GHz CPU with 64 GB of RAM for a total of 24 threads is used. For a single thread RAM is 2.67 GB. BONMIN uses a single thread for all the computations. On the other hand, the maximum number of threads for GUROBI is limited to 20.

In order to test the performances of the three approaches we use the set of instances provided in Table 5.1. The time-limit for the solution of the integrated model with BONMIN is chosen as 24 hours in order to obtain feasible solutions to this highly complex problem. Maximum computational time allowed for the sequential approach and the local search heuristic is 1 hour. Sequential approach consists of one solution of IFAM and RMM-PR each and therefore does not need an excessive computational time. For the local search heuristic we also preferred to have a 1 hour limit in order to show that the resulting method is a practical method which

[1] `http://www.gurobi.com/`

Table 5.1: The data instances for the experiments

no	airports	flights	flights per route	demand per flight	OD pairs	itine-raries	fleet composition	
1	3	10	1.67	51.90	6	16	2	50-37
2	3	11	2.75	83.10	4	11	2	117-50
3	3	12	2.00	113.80	6	12	2	164-100
4	3	12	2.00	113.80	6	12	6	164-146-128-124-107-100
5	3	26	4.33	56.10	6	36	3	100-50-37
6	3	19	3.17	96.70	6	22	3	164-117-72
7	3	19	3.17	96.70	6	22	5	124-107-100-85-72
8	3	12	3.00	193.40	5	28	3	293-195-164
9	3	33	8.25	71.90	4	33	3	117-70-37
10	3	32	5.33	100.50	6	33	3	164-117-85
11	3	32	5.33	100.50	6	33	5	128-124-107-100-85
12	2	11	5.50	173.70	2	22	3	293-164-127
13	4	39	4.88	64.50	10	51	4	117-85-50-37
14	4	23	3.83	86.10	8	27	4	117-85-70-50
15	4	19	3.17	101.40	6	19	4	134-117-100-85
16	4	19	3.17	101.40	6	19	5	128-124-107-100-85
17	4	15	1.88	58.10	8	18	5	117-85-70-50-37
18	4	14	2.33	87.60	7	16	5	134-117-85-70-50
19	4	13	2.60	100.10	6	14	5	164-134-117-100-85
20	3	33	8.25	71.90	4	33	4	85-70-50-35
21	3	46	7.67	86.85	6	59	5	128-124-107-100-85
22	7	48	2.29	101.94	23	50	4	124-107-100-85
23	3	61	15.25	69.15	4	61	4	117-85-50-37
24	8	77	2.08	67.84	39	109	4	117-85-50-37
25	8	97	3.46	90.84	33	106	5	164-117-100-85-50
26	5	100	6.25	347.99	16	140	10	452-400-335-293-185
								174-150-146-128-124
27	33	272	1.3	148.25	157	485	5	293-195-146-117-100

Table 5.2: Details on the problem size

Instance no	Number of variables		Number of constraints	
	total	binary	total	nonlinear
1	282	20	282	63
2	279	22	238	68
3	267	24	241	59
4	551	72	481	59
5	1,016	78	875	247
6	602	57	529	130
7	830	95	727	130
8	616	36	558	177
9	1,280	99	971	354
10	1,254	96	959	340
11	1,632	160	1,274	340
12	577	33	502	175
13	1,681	156	1,399	375
14	930	92	765	191
15	693	76	593	118
16	808	95	689	118
17	643	75	591	85
18	605	70	523	87
19	563	65	484	78
20	1,471	132	1,130	354
21	2,975	230	2,215	841
22	1,663	192	1,435	262
23	3,559	244	2,490	1,082
24	3,091	308	2,749	593
25	4,632	485	3,560	772
26	9,514	1,000	7,453	1,831
27	13,459	1,360	12,331	2,434

can be used by practitioners. For all the approaches we report the time when the best solution is found. We note that since the considered revenue models are non-convex for all the approaches, the presented results are the best solutions obtained in the time limit and we can not provide any guarantee of optimality.

The comparative results of the three approaches are presented in Table 5.3. The analysis of the results enables us to distinguish the following three cases.

5.4.1 Case 1 - Easy instances with no improvement due to the integrated model

For the first 19 test cases, BONMIN converges to a local optima when solving the integrated model. For 9 of these instances (1, 3, 6, 8, 9, 10, 15, 17, 19), the integrated model does not improve the solution of the sequential approach. In other words, these instances do not show the superiority of simultaneous decision making on pricing and schedule planning. Therefore, these instances are not useful to validate the performance of the local search heuristic. They are signified by a gray row color in Table 5.3. Since the solution of the sequential approach is the same as the integrated model solution, the local search heuristic stops after one iteration. As mentioned earlier, the local search heuristic solves the sequential approach as the first iteration. The computational time needed is less than a second for those instances. This implies 2 orders of magnitude reduction for instances 3, 15, 17, 19. The gain of computational time is even more evident for instances 6, 8, 9, and 10 with 3 to 4 orders of magnitude.

5.4.2 Case 2 - Easy instances with an improvement due to the integrated model

Among the easy instances, the integrated model results with a superior solution compared to the sequential approach for instances 2, 4, 5, 7, 11, 12, 13, 14, 16, and 18. The computational time needed for the sequential approach is again less than a second. However it cannot reach the quality provided by the integrated model. The deviation of the sequential approach from the best solution can be up to 5.26% as observed for instance 2. We observe that the local search heuristic is able to find all but one best solutions provided by BONMIN in a significantly reduced computational time. This reduction is observed as 4 orders of magnitude for instances 5, 12, 14 and 3 orders of magnitude for instances 7 and 13. This shows that the local search heuristic is successful to improve the sequential approach solution in a reasonable computational time. There is only one instance, 5, where the solution of the local search heuristic deviates (0.78%) from the solution of the integrated model provided by BONMIN.

Table 5.3: Performance of the heuristic approaches

	BONMIN Integrated model		**Sequential approach (SA)**			**Local search heuristic** *Average over 5 replications*			
	Profit	Time (sec) *max 86,400*	Profit	% deviation from BONMIN	Time (sec) max 3,600	Profit	%deviation from BONMIN	%improvement over SA	Time (sec) *max 3,600*
1	15,091	2.35	15,091	0.00%	0.09	15,091	0.00%	0.00%	0.09
2	37,335	21.81	35,372	-5.26%	0.11	37,335	0.00%	5.55%	2.17
3	50,149	62.34	50,149	0.00%	0.11	50,149	0.00%	0.00%	0.11
4	46,037	2,807	43,990	-4.45%	0.22	46,037	0.00%	4.65%	68.75
5	70,904	1,580	69,901	-1.41%	0.33	70,348	-0.78%	0.64%	0.69
6	82,311	1,351	82,311	0.00%	0.23	82,311	0.00%	0.00%	0.23
7	87,212	32,400	84,186	-3.47%	0.37	87,212	0.00%	3.59%	15.20
8	779,819	8,137	779,819	0.00%	0.31	779,819	0.00%	0.00%	0.31
9	135,656	666	135,656	0.00%	0.41	135,656	0.00%	0.00%	0.41
10	107,927	482	107,927	0.00%	0.76	107,927	0.00%	0.00%	0.76
11	85,820	31,705	85,535	-0.33%	0.92	85,820	0.00%	0.33%	137.36
12	858,545	5,598	854,902	-0.42%	0.26	858,545	0.00%	0.43%	0.60
13	112,881	32,713	109,906	-2.64%	0.72	112,881	0.00%	2.71%	43.47
14	85,808	10,643	82,440	-3.93%	0.36	85,808	0.00%	4.09%	8.50
15	49,448	32.69	49,448	0.00%	0.33	49,448	0.00%	0.00%	0.33
16	38,205	240	37,100	-2.89%	0.34	38,205	0.00%	2.98%	66.93
17	27,076	34.80	27,076	0.00%	0.33	27,076	0.00%	0.00%	0.33
18	45,070	77.59	44,339	-1.62%	0.28	45,070	0.00%	1.65%	0.58
19	26,486	13.09	26,486	0.00%	0.32	26,486	0.00%	0.00%	0.32
20	146,773	30 846	146,464	-0.21%	0.55	147,506	0.50%	0.71%	25.9
21	194,987	4,963	210,134	7.77%	2.96	218,548	12.08%	4.00%	1,173
22	152,126	68,864	158,978	4.50%	1.03	159,258	4.69%	0.18%	1,355
23	227,643	40,862	226,615	-0.45%	3.92	230,305	1.17%	1.63%	1,354
24	153,384	59,708	154,301	0.60%	2.77	158,737	3.49%	2.87%	460
25	313,943	82,780	331,920	5.73%	8.44	333,978	6.38%	0.62%	1,433
26	*no feasible solution*		2,113,561	-	42.43	2,118,236	-	0.22%	3,013
27*	*no feasible solution*		3,804,603	-	495	3,867,410	-	1.65%	2,922

*The IFAM model is given an optimality gap allowance of 0.5%

5.4.3 Case 3 - Complex instances

The last 8 instances are larger compared to the first 19. The generic solver BONMIN does not converge for instances 20-25. Moreover, for experiments 26 and 27, BONMIN cannot provide any feasible solution in 24 hours.

When the results for experiments 20-25 are analyzed, it is observed that, the sequential approach runs less than 5 seconds and provides better feasible solutions in 4 of these instances. This means that with BONMIN the added value of the integrated approach cannot be shown. The local search heuristic performs better compared to the sequential approach in all the instances. The highest improvement is for experiment 21 with 4.00%. Similarly it outperforms the solutions provided by BONMIN on the integrated model. For instances 26 and 27 we observe that the local search heuristic again outperforms the two other approaches. For experiment 27, with 272 flights, the local search heuristic provides an improvement of 1.65% over the sequential approach. This supports that for relatively larger instances the heuristic enables us to evaluate the advantages of the integrated approach. Note that, in order to test the heuristic with instance 27, IFAM is solved with a gap allowance of 0.5% (the default gap of GUROBI is 0.01%).

The local search heuristic has a reasonable computational time with a time reduction of 1 to 3 orders of magnitude compared to BONMIN. The highest time reduction is observed for experiment 20 with 3 orders of magnitude. Even experiment 27 is solved to a better feasible solution compared to the sequential approach in less than 1 hour.

All in all, the local search heuristic provides better feasible solutions compared to BONMIN and the sequential approach. It can be used for flight networks similar to real networks, where available solvers cannot provide good quality solutions or even any feasible solution. Therefore the local search heuristic enables to understand the added value of the integrated modeling framework and can be used in decision making.

5.5 Details regarding the numerical performance

In this section we provide further results for the numerical performance of the heuristic in order to assess the added value of each of the components. We first evaluate the VNS procedure, i.e., the fleet assignment fixing in the IFAM model. Then we analyze the impact of the spill-based neighborhood search compared to a fully random search.

5.5.1 Added value of the VNS procedure

In order to assess the advantages and disadvantages of the VNS procedure, we compare two versions of the heuristic. In the first version, we disable the VNS procedure and therefore the fleet assignment is solved to optimality at each iteration. We refer to this algorithm as the *heuristic without VNS*. The second version is the original heuristic with the VNS procedure is referred as the *heuristic with VNS*.

In Table 5.4 we provide information for the number of iterations and the computational time of the experiments. Note that we only present results for 18 of the 27 instances where the integrated approach results are different from the sequential approach. The presented values are averages over 5 replications for both versions of the heuristic. In the first part of the table we provide the results for the heuristic without VNS and in the second part we have the heuristic with VNS. For both of the algorithms we provide the total number of iterations, the total computational time until the best solution is found, and the average computational time needed for IFAM and RMM-PR per iteration. For the case with VNS, we also provide the parameters selected for the VNS procedure. It is seen that as the data size is larger we increase the number of fixed fleet assignments ($n_{\min}$, $n_{\max}$) and the increment n_{inc}.

It is observed that when the size of the instance gets larger, RMM-PR can still be solved in reasonable time for both versions of the heuristic. Indeed, RMM-PR is kept the same regardless of the VNS procedure. On the other hand, the time needed for IFAM increases considerably as the size increases. As mentioned before, instance 27 is addressed with a gap allowance of 0.5% for the stand alone solution of IFAM by GUROBI. When the size is increased further, the gap allowance should be increased in order to obtain solutions in reasonable computational time. As an example, the IFAM model results with a gap of 3.10% after 24 hours for an instance of 554 flights and 5 fleet types. It is 5.26% for the same instance with 10 fleet types.

It is clear that for large size instances, IFAM is the bottleneck and solution methods can be developed for its solution rather than using available solvers directly. These solution methodologies could be heuristic approaches, column generation; or reformulations of the model could be considered. As an example, Lohatepanont and Barnhart (2004) propose a column generation approach for the solution of a similar IFAM. They have a running time of 19 hours for an instance of 1,988 flights. Furthermore, in the context of leg-based fleet assignment Lasalle Ialongo and Desaulniers (2012) propose a heuristic branch-and-bound procedure which could be exploited for the itinerary-based setting.

Our methodology to tackle with the computational time of IFAM is the VNS procedure where we fix a subset of fleet assignment solutions as explained in section 5.5.1. In general we see a time reduction in the computational time of IFAM with the VNS procedure. The reduction becomes more evident as the size of the instance increases, especially for the last two instances. The heuristic with the VNS procedure runs more number of iterations in the given time frame. In return, with a reduced computational time of IFAM, more neighborhood solutions can be explored. It is also observed that with the VNS procedure we are able to find better quality feasible solutions for the last three instances. When the total computational time values are analyzed, it is observed that in general for small instances the heuristic without VNS is faster since the optimal solution of IFAM without any fixed solutions leads to a more rapid improvement in the objective function. However for larger instances, as of instance 24, the version with VNS becomes faster. Note that the computational time for instance 27 seems increased with the VNS, however a better (+0.79%) solution is found.

We also demonstrate the % improvement in the objective function with respect to the sequential approach as a function of the computational time for experiment 27. In order to do that, we increase the time limit from 1 hour to 4 hours for the sake of a more thorough analysis. In Figure 5.1 we provide a comparison between the heuristic with VNS and without VNS. The solid lines correspond to the case with VNS and dashed lines are for the algorithm without VNS. The first 500 seconds represent the time spent for the first iteration, i.e. sequential approach. The presented results without VNS corresponds to 15 iterations on the average and average improvement of 1.23% is obtained over the sequential approach. On the other hand with VNS, heuristic runs on the average 133 iterations and results with an average improvement of 1.79%.

We observe that the heuristic without VNS needs more computational time to reach improved solutions. With a computational time limit of 1 hour, the improvement is 0.85% as reported in Table 5.3. In 4 hours this increases to 1.23%. On the other hand for the case with VNS, a good feasible solution is obtained more rapidly. The reduced computational time allows to run more iterations and to explore the feasible region more effectively. The improvement with respect to the sequential approach increases from 1.65% to 1.79% when we increase the time limit. We observe that the improvement in the objective function is in general more evident in the early iterations. However, improvements are observed also in the later iterations but are relatively smaller. We report that for most of the other instances the heuristic finds the best solution in much shorter computational time being clearly less than an hour.

Table 5.4: Details on the computational time

	Local search heuristic without VNS *Average over 5 replications*					**Local search heuristic with VNS** *Average over 5 replications*								
	Profit	Total ♯ of iterations	Total* time (sec)	IFAM time per iter.	RMM-PR time per iter.	% imp. in profit	Total ♯ of iterations	Total* time (sec)	abs. diff. in time (sec)	IFAM time per iter.	RMM-PR time per iter.	Parameters for the VNS procedure n_{min}	n_{max}	n_{inc}
2	37,335	65.8	7.28	0.07	0.04	-	19.4	2.17	-5.11	0.06	0.04	2	5	3
4	46,037	3.4	0.98	0.17	0.05	-	351.4	68.75	67.77	0.14	0.05	2	5	3
5	70,348	1.2	0.73	0.20	0.14	-	1.0	0.69	-0.04	0.22	0.14	3	10	3
7	87,212	39.6	14.86	0.22	0.14	-	46.2	15.20	0.34	0.18	0.13	3	10	3
11	85,820	15.0	14.74	0.34	0.58	-	151.6	137.36	122.62	0.32	0.55	5	15	3
12	858,545	1.2	0.58	0.15	0.12	-	1.4	0.60	0.02	0.12	0.12	3	10	3
13	112,881	103.6	75.31	0.50	0.22	-	69.4	43.47	-31.84	0.39	0.22	5	15	3
14	85,808	1.4	0.86	0.26	0.10	-	25.2	8.50	7.64	0.23	0.10	5	15	3
16	38,205	3.8	1.61	0.24	0.09	-	219.8	66.93	65.32	0.21	0.10	3	10	3
18	45,070	1.0	0.55	0.23	0.04	-	1.4	0.58	0.03	0.18	0.04	3	10	3
20	147,506	11.8	7.09	0.33	0.23	-	50.0	25.9	18.81	0.28	0.23	5	15	5
21	218,548	68.6	288	0.65	3.43	-	194.4	1,173	885	0.62	5.33	10	25	5
22	159,258	71.6	81	0.81	0.26	-	1269.3	1,355	1,274	0.78	0.24	10	25	5
23	230,305	178.6	600	0.64	2.66	-	275.0	1,354	754	0.61	4.25	10	25	5
24	158,737	292.2	799	2.01	0.53	-	190.2	460	-339	1.60	0.57	15	30	5
25	331,920	457.3	3,600	6.58	1.15	0.62%	373.3	1,433	-2,167	2.73	0.96	15	30	5
26	2,113,561	46.5	3,600	65.80	11.81	0.22%	116.0	3,013	-587	14.17	11.33	25	50	5
27**	3,837,039	2.3	2,524	885.56	14.08	0.79%	26.5	2,922	398	69.20	14.37	60	100	5

*Computational time is limited by 3600 seconds

**The IFAM model is given an optimality gap allowance of 0.5%

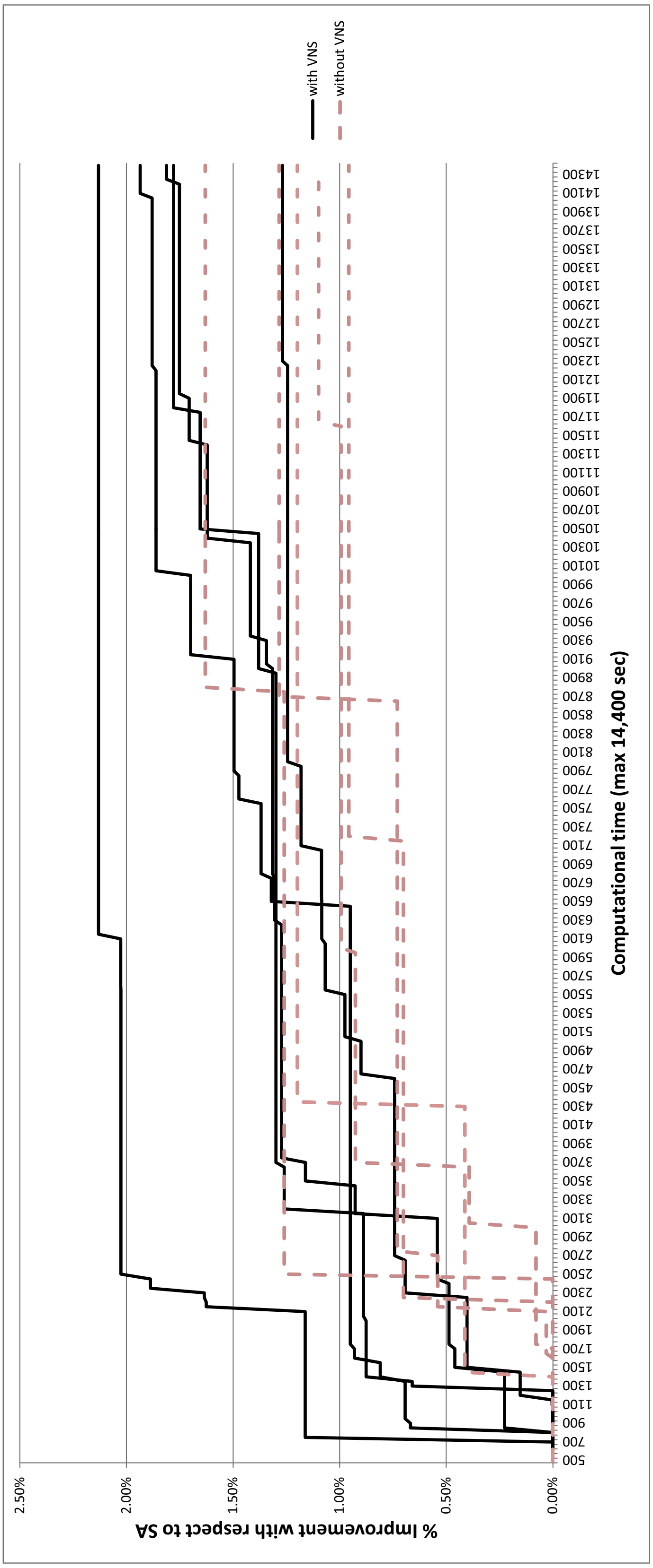


Figure 5.1: Instance 27, % improvement with respect to SA as a function of time for 5 replications of the heuristic without VNS (dashed lines) and with VNS (solid lines)

As a result, the heuristic with the VNS procedure is clearly better for dense flight networks consisting of around 100 or more flights. It reduces the computational time for the IFAM sub-problem and leads to significantly better solutions for the integrated model. On the other hand, for smaller instances the version without VNS can be preferred since IFAM can be solved to optimality without any fixed solutions in reasonable computational time.

5.5.2 Added value of the spill based local search

As mentioned in section 5.2, local search heuristic involves mechanisms which enable to visit neighborhood solutions based on the spilled number of passengers. The reasoning behind is that spill information provides the relation between the demand and capacity which could be used to create a communication between the two sub-problems. In order to quantify the advantage of using the spill information, the local search heuristic is tested against its counter part with a fully random local search. The prices of the itineraries are uniformly drawn between the lower and upper bounds. Similarly the fixing of assignments is done randomly regardless of the spill values in the context of the VNS procedure. For this particular experiment, the heuristic results are presented without the VNS procedure for the instances up to 24, and with the VNS procedure for the instances 25-27.

The comparative results between the random neighborhood and the one based on spill is presented in Table 5.5. The instances where the sequential approach and the integrated model result with the same solution are again omitted since in this case the local search heuristic is equivalent to the sequential approach.

Both versions of the local search heuristic have a time limit of 1 hour and the presented results are the average values for 5 replications of each. For 8 of the 18 instances, the random neighborhood does not improve the initial solution which is the same as the sequential approach in 1 hour. The neighborhood based on spill provides better quality solutions compared to the random neighborhood in 14 of the instances. The maximum improvement obtained in the profit is 2.87% (instance 24) and on the average this improvement is around 0.9%. In almost all of the instances the spill based local search reduces the computational time and the number of iterations considerably. The reduction in time can be up to 3 orders of magnitude as for instances 5 and 20. The advantage of using the spill information is more evident for instances with relatively high number of flights. For small instances, even a fully random setting may be able to explore the feasible region. However for larger instances a better guided local search is needed. Therefore, it is concluded that the information provided by the demand model on the spill guides the heuristic method in the right direction and generates better feasible solutions in less computational time.

Table 5.5: Improvement due to the neighborhood search based on spill

	Sequential approach (SA)	**Random neighborhood**			**Neighborhood based on spill**			**% Improvement over random neighborhood**	
		Avg. over 5 replications			*Avg. over 5 replications*			*in terms of*	
	Profit	Profit	Number of iterations	Time (sec)	Profit	Number of iterations	Time (sec)	Quality of the solution	Reduction in time
2	35,372	37,335	69	8	37,335	66	8	-	-
4	43,990	46,037	6	2	46,037	3	1	-	33.33%
5	69,901	69,929	1,907	3,220	70,348	1	1	0.60%	99.97%
7	84,186	87,179	2,744	961	87,212	40	15	0.04%	98.44%
11	85,535	*SA*	3,809	3,600	85,820	15	15	0.33%	99.57%
12	854,902	858,545	1,233	381	858,545	1	1	-	99.74%
13	109,906	110,798	773	1,222	112,881	104	75	1.88%	93.86%
14	82,440	85,694	1,217	862	85,808	1	1	0.13%	99.86%
16	37,100	38,205	4	2	38,205	4	2	-	-
18	44,339	45,070	216	59	45,070	1	1	-	98.31%
20	146,464	*SA*	689	3,600	147,506	12	7	0.71%	99.81%
21	210,134	215,550	231	918	218,548	69	288	1.39%	68.63%
22	158,978	*SA*	3,172	3,600	159,258	72	81	0.18%	97.76%
23	226,615	*SA*	658	3,600	230,305	179	600	1.63%	83.32%
24	154,301	*SA*	1120	3600	158,737	292	799	2.87%	77.81%
25	331,920	*SA*	1014	3,600	**333,978**	**373**	**1,433**	0.62%	60.19%
26	2,113,561	*SA*	165	3,600	**2,118,236**	**116**	**3,013**	0.22%	16.31%
27*	3,804,603	*SA*	27	3,600	**3,867,410**	**27**	**2,922**	1.65%	18.85%

*The IFAM model is given an optimality gap allowance of 0.5%

Boldface values are the results with VNS procedure

5.6 Conclusions and future research directions

In this chapter a local search heuristic method is presented for solving the integrated airline scheduling, fleeting and pricing model. The main motivation for the heuristic is to obtain good quality feasible solutions in a reasonable computational time for flight networks similar to real life instances. The iterative process is carried out over two simplified versions of the integrated model and local search mechanisms are employed to explore better feasible solutions. The local search mechanisms are based on the information provided by the demand model on spill. This is an important feature of the heuristic approach which explores better feasible solutions in less computational time compared to a fully random neighborhood search. The resulting heuristic is practical and provides insights about the added value of the integrated approach for data instances similar to real life networks.

The performance of the local search heuristic is compared to an available MINLP solver BONMIN and a sequential approach that represents the current practice of airlines. The local search heuristic outperforms the sequential approach when there is a potential gain from the simultaneous decision making. Otherwise, if there is no potential, it is equivalent to sequential approach. For instances that have more than 30-35 flights, the heuristic outperforms both of the other approaches. It is able to find better feasible solutions in a reasonable computational time.

As mentioned in section 5.5.1, the bottleneck of the heuristic is IFAM. Development of appropriate solution methodologies for IFAM is a promising research direction. The VNS procedure in this chapter where a subset of fleet assignment solutions is fixed based on the spill information reduces the computational time and enables to obtain better feasible solutions. Heuristic approaches, decomposition techniques or stronger reformulations of the model could be potential candidates for solution methodologies.

The performance of the presented heuristic is evaluated in terms of the best feasible solution found. Since the problem is non-convex, no evaluation can be done in terms of the duality gap. Therefore a potential future research is the extension of the study to obtain a valid upper bound through appropriate decomposition methods and/or transformations of the mathematical model. In the literature there are studies that come up with approximations to deal with the complexity of non-convex MINLPs. We refer to Nowak (2005) for a comprehensive set of methods for solving non-convex mixed integer nonlinear programs. Some studies present convex under estimation techniques for the non-convex functions in order to obtain valid bounds to the original problem (Gangadwala et al., 2006;Ballerstein et al., 2011). D'Ambrosio and Lodi (2011) present an overview on the available tools for convex and non-convex MINLPs. D'Ambrosio et al. (2012) develop an iterative technique for a non-convex MINLP based on a convex approximation of the model and a non-convex nonlinear program (NLP) that is obtained by fixing the integer part of the problem.

6 Analysis of the integrated model: reformulation and sensitivity analysis

The model presented in chapter 4 assumes that the realization of the demand is given by the expected demand and the spilled/recaptured passengers. It is assumed that the airline controls the number of spilled passengers with an associated decision variable. A recapture ratio is then considered over the number of spilled passengers based on the demand model. In case of a flight cancellation the only decision of the airline related to demand is to reduce it through lost and/or redirected passengers. There is no decision on the cancellation of the itineraries (which were actually introduced by Lohatepanont and Barnhart (2004)). Therefore, the initial expected demand ($\tilde{d}$) is a function of all the itineraries in the each market segment. This limits the potential revenue since the pricing decision and recapture ratios are restricted by the originally desired itineraries of the passengers.

In this chapter we reformulate the integrated model where the logit model on the recapture ratios is removed. Therefore the realized demand with the spill and recapture effects is based on a single logit model. As will be illustrated, the main difference between the reformulated model and the previous one is the impact of canceled flights on the prices of the operated itineraries. In the reformulated model, the attributes of the itineraries that are not operated do not play any role on the attractiveness of the remaining itineraries.

We first provide a reformulation of the logit model in section 6.1. The reformulated integrated model is presented in section 6.2 together with an illustrative example. The local search heuristic is adapted to the reformulated model in section 6.3. The local search heuristic has modifications due to nature of the reformulation. Secondly, we provide a sensitivity analysis in section 6.4 addressing the demand fluctuations and demand model parameters. The robustness of the integrated model solutions are tested in comparison to a leg-based FAM model.

6.1 Reformulation of the logit model

Here we recall the demand model presented in Chapter 3. As mentioned in Chapter 4 the only policy variable which could be controlled by the optimization model is the price (p_i). The other variables (trip length, departure time of day, number of stops) improve the estimation of choice probabilities but are only inputs to the optimization model. They are aggregated with their associated β coefficients as a constant and represented by z_i in the optimization model. For the specification of the utility function we refer to Table 3.1. With the simplification in the notation, the utility is rewritten as follows:

$$V_i = \beta \ln(p_i) + z_i \qquad \forall h \in H, s \in S^h, i \in I_s \tag{6.1}$$

Depending on the utility, the choice probability/market share and the demand associated with itinerary i in segment s is given by:

$$u_i = \frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)} = \frac{\exp(\beta \ln(p_i) + z_i)}{\sum_{j \in I_s} \exp(\beta \ln(p_j) + z_j)} \tag{6.2}$$

$$d_i = D_s u_i \qquad \forall h \in H, s \in S^h, i \in I_s \tag{6.3}$$

The denominator in equation (6.2) is same for all the itineraries in the segment. Similar to some studies in literature (e.g. Schön, 2008, Dong et al., 2009, Haase and Müller, 2013) a new variable v is defined as follows :

$$v_s = \frac{1}{\sum_{j \in I_s} \exp(\beta \ln(p_j) + z_j)} \qquad \forall h \in H, s \in S^h \tag{6.4}$$

Therefore the market share of each itinerary in the segment can be written as below:

$$u_i = v_s \cdot \exp(\beta \ln(p_i) + z_i) \qquad \forall h \in H, s \in S^h, i \in I_s \tag{6.5}$$

Since we do not use the full logit formula we need to make sure that the market shares sum up to 1. Therefore we need the following relation for each market segment:

$$\sum_{i \in I_s} u_i = 1 \qquad \forall h \in H, s \in S^h \tag{6.6}$$

The equations (6.5) and (6.6) together represent the market share equation given in (6.2). However this is not enough to represent the spill and recapture effects. Recall that the no-revenue options $(I_s^{'})$ have fixed price since the airline has no control on the pricing decision of these competitive itineraries. We represent the price of competitors' prices by $\bar{p}$. The market share of these options could be considered as a reference and the market share equation could be rewritten as follows:

$$u_i \leq v_s \cdot \exp(\beta \ln(p_i) + z_i) \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s^{'}) \tag{6.7}$$

$$u_j = v_s \cdot \exp(\beta \ln(\bar{p}_j) + z_j) \qquad \forall h \in H, s \in S^h, j \in I_s^{'} \tag{6.8}$$

As mentioned before v_s is common for the itineraries of the same market segment and will change depending on the utility of them. Inequality (6.7) imposes an upperbound on the market share of each itinerary with respect to its relative utility value compared to the utility of no-revenue options given by equation (6.8). The reason behind having an inequality for the market share is that the capacity is limited on each itinerary. Ideally, the demand model will find the best price value which exactly fits to the given capacity. However, in terms of computational purposes we introduce bounds on the price. As a result the demand model is not free to choose any price value. This phenomenon, which will be further discussed in section 7.5.2, necessitates the concept of spill on itineraries which is represented by the inequality constraints. Note that the introduction of constraints (6.6), (6.7) and (6.8) is sufficient and there is not need for the definition of v_s given in equation (6.4). A similar relation is used by Wang et al. (2012) in a setting where the utilities of the itineraries are fixed inputs for the optimization model.

6.2 Reformulated integrated model

Following the reformulation of the logit model, in this section we present the reformulated integrated model, which is referred as IFAM-PR$^{'}$. Here we mention the main differences compared to the model presented in section 4.3. The spill and recapture variables disappear from the model since the demand is based on a single logit model. The realized demand is represented by market share (u_i) times the total market demand (D_s). Therefore, the revenue in the objective function (6.9) and the realized demand in constraints (6.15) are updated accordingly. Constraints (6.17)-(6.19) represent the choice-based supply-demand interactions as mentioned in section 6.1.

The reformulated problem is again a mixed integer non-convex problem due to the utility functions which include the price as a decision variable and also the revenue in the objective function.

$$z^*_{IFAM-PR'} =$$

$$\max \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} u_i p_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f} x_{k,f} \tag{6.9}$$

$$\text{s.t.} \sum_{k\in K} x_{k,f} = 1 \qquad \forall f \in F^M \tag{6.10}$$

$$\sum_{k\in K} x_{k,f} \le 1 \qquad \forall f \in F^O \tag{6.11}$$

$$y_{k,a,t^-} + \sum_{f\in \text{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in \text{Out}(k,a,t)} x_{k,f} \qquad \forall [k,a,t] \in N \tag{6.12}$$

$$\sum_{a\in A} y_{k,a,\text{minE}_a^-} + \sum_{f\in CT} x_{k,f} \le R_k \qquad \forall k \in K \tag{6.13}$$

$$y_{k,a,\text{minE}_a^-} = y_{k,a,\text{maxE}_a^+} \qquad \forall k \in K, a \in A \tag{6.14}$$

$$\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} \delta_{i,f} u_i \le \sum_{k\in K} \pi^h_{k,f} \qquad \forall h \in H, f \in F \tag{6.15}$$

$$\sum_{h\in H} \pi^h_{k,f} \le Q_k x_{k,f} \qquad \forall f \in F, k \in K \tag{6.16}$$

$$\sum_{i\in I_s} u_i = 1 \qquad \forall h \in H, s \in S^h \tag{6.17}$$

$$u_i \le v_s \exp(V_i(p_i, z_i; \beta)) \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I'_s) \tag{6.18}$$

$$u_j = v_s \exp(V_j(\bar{p}_j, z_j; \beta)) \qquad \forall h \in H, s \in S^h, j \in I'_s \tag{6.19}$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \tag{6.20}$$

$$y_{k,a,t} \ge 0 \qquad \forall [k,a,t] \in N \tag{6.21}$$

$$\pi^h_{k,f} \ge 0 \qquad \forall h \in H, k \in K, f \in F \tag{6.22}$$

$$\text{LB}_i \le p_i \le \text{UB}_i \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I'_s) \tag{6.23}$$

$$u_i \ge 0 \qquad \forall h \in H, s \in S^h, i \in I_s \tag{6.24}$$

$$v_s \ge 0 \qquad \forall h \in H, s \in S^h \tag{6.25}$$

6.2.1 Illustration for the differences between the two formulations

In order to illustrate the differences between the reformulated model and the previous formulation, we consider experiment 2 that is already used in chapter 4 (see Table 4.6). The results for the two versions of the model are presented in Table 6.1. As mentioned before, the integrated model given in chapter 4 may underestimate the potential revenue as the demand and recapture ratios are computed considering all the available itineraries, including those that cannot be operated because of some flight cancellation. The results show that the profit is higher with the reformulated model as the prices are in general higher. Even if the number of assigned aircraft is the same for the two formulations, the selection of flights is different. Therefore, the operated itineraries are different for the two models (see itineraries 9 and 11). For the comparison between the two formulations over the whole set of experiments, we refer to the results that will be given in section 6.3.1. Furthermore, we provide a detailed illustration for the resulting market shares with the reformulation for a selected OD-pair of data instance

Table 6.1: An illustrative example for the reformulated model

	Previous model IFAM-PR		Reformulated model IFAM-PR$'$	
Revenue	118,729		125,527	
Operating costs	81,394		81,394	
Profit	37,335		44,133 (**+18.2%**)	
Number of flights	8		8	
Transported passengers	534		534	
Allocated seats	217		217	
Itineraries	demand	price	demand	price
1	*canceled*		*canceled*	
2	*canceled*		*canceled*	
3	50	250	50	250
4	50	225	50	225
5	50	250	50	250
6	50	225	50	225
7	**117**	**200**	**117**	**238.9**
8	**117**	**205.8**	**117**	**225**
9	**50**	**250**	*canceled*	
10	50	225	50	225
11	*canceled*		**50**	**250**

Boldface values show the differences between the results of the two formulations

2 in Appendix A.4

6.3 The local search heuristic for the reformulated problem

The local search heuristic proposed in chapter 5 can be adapted to the reformulated problem, IFAM-PR$'$. The fleet assignment problem used for the heuristic for the reformulated case is represented by IFAM$'$. It is the version of the reformulated integrated model (IFAM-PR$'$) presented in section 6.2 where the price variables are fixed to the given average prices in the dataset (see Appendix A.2.2). Similarly, the revenue sub-problem is replaced by RMM-PR$'$ (see Appendix A.3.3). The iterative process for the heuristic method is therefore carried out with these reformulated sub-problems.

An important feature of the reformulated problem is that we do not have the decision variables on the number of spilled passengers. The heuristic method on the other hand makes use of this information while exploring neighborhood solutions. Therefore, we need to provide this input to the procedure which can be represented by:

$$SR_i^g = D_s \left(\frac{\exp(V_i)}{\sum_{j \in I_s} \exp(V_j)} - u_i \right) \qquad \forall h \in H, s \in S^h, i \in (I_s \setminus I_s'). \tag{6.26}$$

This term represents the difference between the potential demand and the realized demand for each itinerary and it is used to replace the information carried by the $t_{i,j}$ variables in equation (5.26). With this adaptation, the price sampling procedure can be applied in a similar way for the reformulated problem.

At the flight level, it is found out that the gap between the assigned capacity and the realized demand provides a better guidance for the heuristic. Therefore, in the variable neighborhood search, a subset of fleet assignment solutions are fixed according to:

$$\sum_{k \in K} x^{g}_{k,f} - \sum_{h \in H} \sum_{s \in S^h} D_s \sum_{i \in (I_s \setminus I'_s)} \delta_{i,f} u_i, \tag{6.27}$$

where the first term gives the assigned capacity and the second term is the associated demand for the flight. Namely, it is a measure of how well the demand and capacity match. Since this result is obtained with a feasible solution of RMM-PR$'$, it is always positive. This measure is used for fixing the fleet assignment solutions in the context of VNS instead of the spill rate in equation (5.29) for the previous formulation of the model.

6.3.1 Heuristic results for the reformulated problem

The adapted heuristic procedure is tested in the same manner as done in section 5.4. We compare the results obtained by BONMIN for IFAM-PR$'$, the sequential approach and the local search heuristic. IFAM$'$ is solved by GUROBI and RMM-PR$'$ is solved by BONMIN as done for the previous case. For the solution of IFAM-PR$'$, BONMIN is given a time limit of 24 hours and the time limit for SA and the heuristic is 1 hour except the last 3 instances.

The same data instances are considered for the numerical experiments, that are already introduced in chapter 5, in Table 5.1. The results are presented in Table 6.2. As mentioned before the profit values are higher compared to the results obtained with previous the formulation given in chapter 5.

When solving the integrated model, BONMIN converges to a local optima for the first 20 instances. Note that BONMIN does not converge when solving instance 20 with the previous formulation. We again distinguish the cases where the sequential and integrated approaches converge to the same solution by gray color. It is observed that the number of such cases are fewer in this case (experiments 1, 6, 13, 15, 16, 17, and 19). When the integrated approach improves the solution of the sequential approach (in 20 out of 27 instances) there is an average improvement of 2%. The highest improvement is observed for experiment 4 with 7%. For all these instances, the local search heuristic finds the best solution provided by BONMIN in significantly less computational time. Only in instance 18, there is a deviation of 0.04% from the best solution given by BONMIN. The maximum gain on computational time is observed for experiment 7 with 4 orders of magnitude.

The experiments 21-24 are larger instances where BONMIN does not converge to a local optima and we present the best solution found until the time limit. The solution provided by BONMIN is inferior compared to the sequential approach for experiments 21 and 23. Therefore, with general purpose solvers the potential of the integrated approach is hindered and we need alternative solution methodologies. With the heuristic algorithm, better quality solutions are obtained compared to both BONMIN and the sequential approach with an average improvement of 2.02% and 1.75%, respectively. The highest improvement over the sequential approach is 3.25% for experiment 24 that is obtained in around 15 minutes. The time reduction with respect to BONMIN is observed as 1-2 orders of magnitude for these 4 instances. Note that the discrepancy between the sequential approach and the integrated model (when solved by BONMIN) is lower for IFAM-PR$^{'}$ compared to IFAM-PR given in chapter 4. The reason is, for these instances, more feasible solutions are explored and better quality solutions are obtained in the given time frame for the solution of IFAM-PR$^{'}$. When the size of the problem gets larger, this phenomenon goes in the opposite direction.

The reformulated model behaves similar to the previous formulation in terms of computational time for the first 24 instances. However, experiments 25-27 are difficult to handle, the complexity increases exponentially. As can be seen in the table, the computational time is significantly higher and the time limit is increased to 3-4 hours for these instances for the heuristic algorithm. BONMIN does not find any feasible solution for instances 26 and 27. On the other hand with the heuristic, we are able to obtain superior solutions compared to both BONMIN and the sequential approach. The improvement with respect to sequential approach is on the average 0.6% for these last two instances which are similar to real flight networks.

Table 6.2: Performance of the heuristic approaches for the reformulated integrated model

	BONMIN Integrated model		**Sequential approach (SA)**			**Local search heuristic** *Average over 5 replications*			
	Profit	Time (sec) *max 86,400*	Profit	% deviation from BONMIN	Time (sec) *max 3,600*	Profit	%deviation from BONMIN	%improvement over SA	Time (sec) *max 3,600*
1	15,091	3.27	15,091	0.00%	0.10	15,091	0.00%	0.00%	0.10
2	44,134	14.25	43,781	-0.80%	0.10	44,134	0.00%	0.81%	0.20
3	50,149	48.17	49,938	-0.42%	0.10	50,149	0.00%	0.42%	0.24
4	49,411	1,863.86	45,916	-7.07%	0.30	49,411	0.00%	7.61%	20.60
5	83,484	806.08	82,120	-1.63%	0.69	83,484	0.00%	1.66%	4.55
6	87,334	469.87	87,334	0.00%	0.27	87,334	0.00%	0.00%	0.27
7	90,973	16,399.52	88,049	-3.21%	0.38	90,973	0.00%	3.32%	3.58
8	1,232,790	171.48	1,227,459	-0.43%	0.30	1,232,793	0.00%	0.43%	1.59
9	149,520	1,462.41	147,869	-1.10%	0.48	149,520	0.00%	1.12%	4.52
10	130,145	1,243.91	127,748	-1.84%	0.84	130,146	0.00%	1.88%	11.77
11	119,767	942.44	119,206	-0.47%	0.94	119,767	0.00%	0.47%	4.35
12	1,230,770	63.15	1,198,134	-2.65%	0.16	1,230,766	0.00%	2.72%	10.68
13	137,200	1,306.31	137,200	0.00%	1.44	137,200	0.00%	0.00%	1.44
14	102,911	539.87	101,952	-0.93%	0.39	102,911	0.00%	0.94%	17.57
15	75,450	21.33	75,450	0.00%	0.19	75,450	0.00%	0.00%	0.19
16	60,485	80.26	60,485	0.00%	0.31	60,485	0.00%	0.00%	0.31
17	27,076	66.91	27,076	0.00%	0.25	27,076	0.00%	0.00%	0.25
18	56,417	121.97	54,022	-4.25%	0.27	56,394	-0.04%	4.39%	157.77
19	37,478	32.80	37,478	0.00%	0.22	37,478	0.00%	0.00%	0.22
20	155,772	1,429.00	154,322	-0.93%	1.61	155,772	0.00%	0.94%	31.00
21	303,726	84,872	303,469	-0.08%	10	307,479	1.24%	1.32%	281
22	161,197	18,440	163,324	1.32%	2	164,087	1.79%	0.47%	1,124
23	284,269	971	278,942	-1.87%	26	284,376	0.04%	1.95%	127
24	155,457	79,989	158,106	1.70%	9	163,247	5.01%	3.25%	938
25[1]	409,496	85,718	410,632	0.28%	4,438	414,273	1.17%	0.89%	5,846
26[2]	*no feasible solution*		2,265,013	-	47	2,271,312	-	0.28%	2,645
27[3]	*no feasible solution*		4,212,976	-	5,978	4,237,444	-	0.58%	10,072

[1] The time limit is set as 3 hours
[2] $IFAM'$ is given a gap allowance of 0.5%, time limit 3 hours
[3] $IFAM'$ is given a gap allowance of 1%, time limit 4 hours

Table 6.3: Details on the computational time

	Local search heuristic without VNS, Average over 5 replications					Local search heuristic with VNS, Average over 5 replications								
	Profit	Total ♯ of iter.	Total* time (sec)	IFAM′ time per iter.	RMM-PR′ time per iter.	% imp. in profit	Total ♯ of iter.	Total* time (sec)	abs. diff. in time (sec)	IFAM′ time per iter.	RMM-PR′ time per iter.	Parameters for the VNS procedure $n_{\min}$	$n_{\max}$	n_{inc}
2	44,134	2.0	0.30	0.08	0.02	-	1.0	0.20	-0.10	0.07	0.02	2	5	3
3	50,149	1.0	0.19	0.07	0.02	-	1.4	0.24	0.05	0.08	0.02	2	5	3
4	49,411	3.0	1.22	0.28	0.02	-	100.0	20.60	19.38	0.18	0.02	2	5	3
5	83,484	8.2	6.38	0.65	0.04	-	14.6	4.55	-1.83	0.22	0.04	3	10	3
7	90,973	1.0	0.75	0.35	0.03	-	17.6	3.58	2.83	0.15	0.03	3	10	3
8	1,232,793	5.8	2.01	0.26	0.04	-	7.0	1.59	-0.42	0.15	0.04	2	5	3
9	149,520	5.6	3.14	0.44	0.03	-	7.0	4.52	1.38	0.55	0.03	5	15	3
10	130,146	33.6	14.80	0.81	0.03	-	37.2	11.77	-3.03	0.27	0.03	5	15	3
11	119,767	11.8	12.06	0.91	0.03	-	8.2	4.35	-7.72	0.39	0.03	5	15	3
12	1,230,766	7.8	1.44	0.14	0.03	-	72.0	10.68	9.24	0.12	0.03	3	10	3
14	102,911	183.6	71.07	0.34	0.05	-	55.8	17.57	-53.50	0.26	0.05	5	15	3
18	56,417	10.6	3.07	0.22	0.04	-0.04%	661.8	157.77	154.70	0.20	0.04	3	10	3
20	155,772	5.8	4.80	0.67	0.10	-	65.4	31	26.20	0.42	0.03	5	15	5
21	307,479	146.6	717.40	4.77	0.05	-	162.0	280.6	-436.80	1.58	0.06	10	25	5
22	164,537	203.0	412.00	1.96	0.05	-0.27%	1,402.4	1,124.4	712.40	0.76	0.03	10	25	5
23	284,376	6.8	69.20	6.32	0.05	-	80.8	127	57.80	1.16	0.07	10	25	5
24	163,055	373.5	1,960.25	5.09	0.14	0.12%	557.8	937.8	-1,022.45	1.52	0.14	15	30	5
25[1]	410,632	1.0	10,419	5,981	0.13	0.89%	67.6	5,846	-4573	20.70	0.13	15	30	5
26[2]	2,267,825	43.2	5,361	122	0.25	0.15%	101.0	2,645	-2716	25.34	0.25	25	50	5
27[3]	4,212,976	1.0	14,299	4,159	1.33	0.58%	90.7	10,072	-4227	43.64	1.35	60	100	5

*Computational time is limited by 3600 seconds

[1] The time limit is set as 3 hours

[2] IFAM′ is given a gap allowance of 0.5%, time limit 3 hours

[3] IFAM′ is given a gap allowance of 1%, time limit 4 hours

The details on the performance of the heuristic algorithm are given in Table 6.3. Similar to the analysis performed in chapter 5, we present details on the computational time and the number of iterations. The last three columns provide the selected parameters for the VNS procedure. The number of fixed fleet assignment solutions is increased as the size of the instance increases. When the results are compared to that of the previous formulation given in Table 5.4, it is seen that the complexity of the sub-problems are different for the two cases. RMM-PR$'$ is easier to handle with the reformulation and we observe significant reduction in its computational time per iteration. Even for the instances that are relatively larger, it is at most few seconds. However IFAM$'$ is computationally expensive. Compared to the previous case, instances 25-27 need significantly more computational time. Instances 25 and 27 are very hard to solve as only two iteration can be performed within the time limit.

We also evaluate the advantages and disadvantages of the VNS procedure. In general, we observe that the introduction of VNS reduces the computational time needed for the IFAM$'$ sub-problem due to fixing of fleet assignment solutions. For small to medium size instances, VNS does not bring clear advantages. In instances 18 and 22 some quality is even lost on the resulting solution. There are instances like 4, 18, 20, 22 and 23, for which the total computational time increases with VNS. On the other hand, in experiments 14, 21, 24, 25, 26, and 27 VNS helps to reduce the computational time significantly. When we analyze larger instances, 24-27, the advantage of VNS can be clearly observed with an improvement in the quality of the solution. The time reduction when solving IFAM$'$ is significant which enables to run more iterations and explore the feasible region more efficiently. On the other hand, the heuristic without VNS is able to run only few iterations and gets stuck at the solution of the sequential approach for instances 25 and 27.

Furthermore, we present the % improvement in the objective function over the sequential approach as a function of time in Figure 6.1 for experiment 27. The considered algorithm is the one with the VNS procedure. We do not present the results without VNS since the algorithm gets stuck at the solution of the sequential approach. The figure illustrates the idea behind the selection of the time limit. The time needed for the first iteration which is the sequential approach is around 6000 seconds. Then, with the VNS, more rapid solutions are obtained. Significant improvements are obtained in the early iterations and very few improvements are observed close to the time limit. In the last hour there is almost no improvement for the replications of the heuristic. This supports the selection of the time limit as 4 hours.

Similar to the conclusions in chapter 5.2.3, we propose the use of the heuristic with the VNS procedure for data instances that have high number of flights, in order to reduce computational time and obtain good feasible solutions in reasonable computational time. For small instances on the other hand the heuristic without VNS can be chosen.

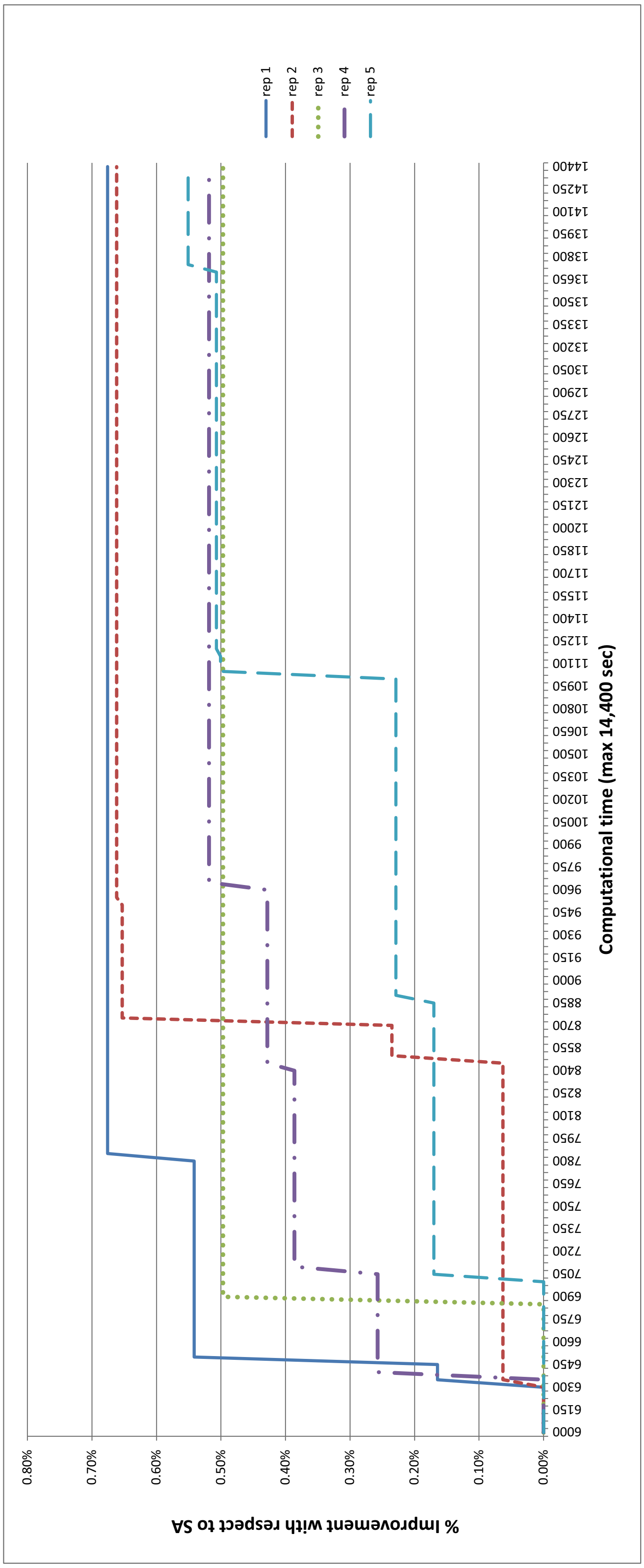


Figure 6.1: Instance 27, % improvement with respect to SA as a function of time for 5 replications of the heuristic with VNS

6.4 Sensitivity analysis

In this chapter we perform sensitivity analysis in order to address the key assumptions of the presented choice-based IFAMs. We have four different IFAMs: IFAM and IFAM-PR introduced in Chapter 4 where the spill decision is controlled by the airline; IFAM$^{'}$ (for the model see Appendix A.2.2) and IFAM-PR$^{'}$ introduced in Chapter 6 where the demand is given by a single logit model without any additional spill/recapture variables.

The sensitivity analysis addresses the assumptions related to the unconstrained demand, price parameter in the logit model and the assumptions regarding the competitors' offers. In section 6.4.1 we provide a description of the analysis with a reference FAM. The analysis for the demand uncertainty is presented in section 6.4.2. The impact of the price parameter is addressed in section 6.4.3 and analysis with different prices of competitors is carried out in section 6.4.4.

6.4.1 The framework for the sensitivity analysis

As a reference model we consider a leg-based FAM where itinerary level information is not considered. This leg-based FAM is similar to the model introduced by Hane et al. (1995).

$$z^*_{\mathbf{FAM}} = \min \sum_{k\in K}\sum_{f\in F} C_{k,f}x_{k,f} + \text{spill costs} \tag{6.28}$$

$$\sum_{k\in K} x_{k,f} = 1 \qquad \forall f \in F^M \tag{6.29}$$

$$\sum_{k\in K} x_{k,f} \le 1 \qquad \forall f \in F^O \tag{6.30}$$

$$y_{k,a,t^-} + \sum_{f\in \text{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in \text{Out}(k,a,t)} x_{k,f} \qquad \forall [k,a,t] \in N \tag{6.31}$$

$$\sum_{a\in A} y_{k,a,\text{minE}_a^-} + \sum_{f\in CT} x_{k,f} \le R_k \qquad \forall k \in K \tag{6.32}$$

$$y_{k,a,\text{minE}_a^-} = y_{k,a,\text{maxE}_a^+} \qquad \forall k \in K, a \in A \tag{6.33}$$

$$x_{k,f} \in \{0,1\} \qquad \forall k \in K, f \in F \tag{6.34}$$

$$y_{k,a,t} \ge 0 \qquad \forall [k,a,t] \in N \tag{6.35}$$

$$\tag{6.36}$$

A time-space network is used for the representation of the flight network. The objective (6.28) is to minimize the operating costs and the spill costs. Since the leg-based formulation does not carry information regarding demand, assumptions are made for the estimation of spill costs. First assumption is the *full fare allocation* where each flight in the itinerary is assigned the full fare of the itinerary. The spill estimation is performed in a deterministic way where the itineraries are listed in order of decreasing fare and the demand is assigned respecting this order. These assumptions on the fare allocation and spill estimation are explained by Kniker

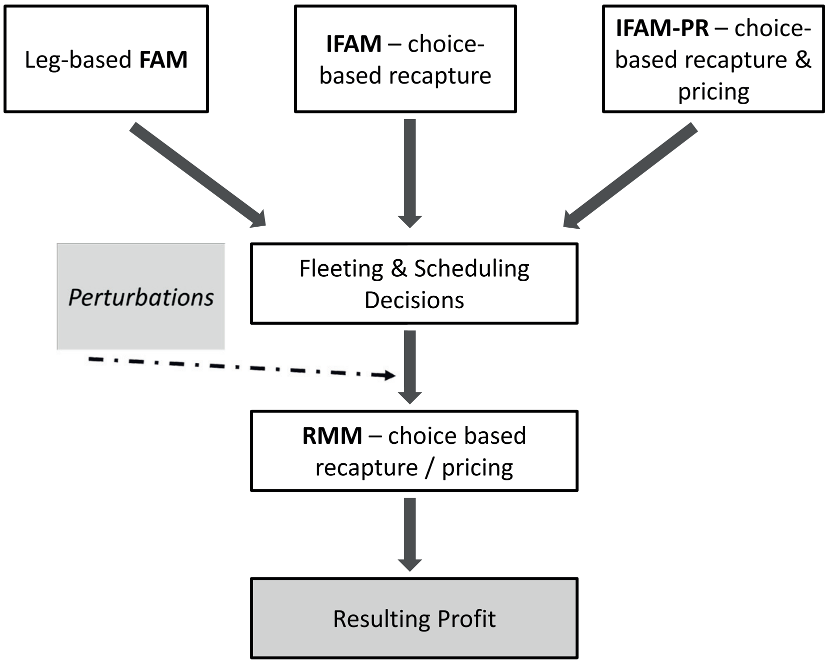


Figure 6.2: Framework for the sensitivity analysis

(1998). A similar approach is also considered by Lohatepanont (2002).

In addition to the fleet assignment decisions, the schedule design feature with mandatory and optional flights is integrated (6.29)-(6.30). The remaining constraints are classical FAM constraints: flow of aircraft should be conserved (6.31), the number of available aircraft for each type should be respected (6.32) and a cyclic schedule is maintained (6.33).

For the sensitivity analysis, we perform a comparative analysis between the leg-based FAM and itinerary based models that are introduced as IFAM and IFAM-PR in the thesis.

The sensitivity analysis for each of the assumptions starts with the solution of different FAM/IFAM/IFAM-PR models. The resulting fleeting and scheduling decisions are fixed. Using these decisions as an input, a passenger allocation model is employed to determine the quality of the planning decisions given by the models. The passenger allocation model is considered as the revenue management model, RMM, which consists of all the demand/revenue related equations of the IFAM-PR. In order to have a fair comparison for the models we have utilized the version with pricing (RMM-PR) and without pricing (RMM). These revenue subproblems are listed in Appendix A.3.

When solving the RMMs, perturbations on the demand model parameters are introduced and the profit obtained by the fleeting decision of different FAM/IFAM/IFAM-PR models are compared. The methodology for sensitivity analysis, that is illustrated in Figure 6.2, is inspired by the work of Lohatepanont (2002).

6.4.2 Demand uncertainty

In all the presented models we assume that the total demand for each market segment is known. The demand share of each itinerary in a market segment is proportional to this total demand given by the logit model. This is a strong assumption given the daily and seasonal fluctuations of the demand. In this section we address this assumption and test the added value of IFAMs and IFAM-PRs compared to FAM with simulated values of demand.

We randomly draw 500 realizations of the unconstrained itinerary demand from a Poisson distribution with a mean equal to the average demand value provided in the dataset. Furthermore, we introduce perturbations on the average demand value in a range of -30%-+30% for each market segment. Therefore for each perturbation on the average demand value we have 500 simulations. These 500 realizations of demand are meant to represent day-to-day variations on each average demand value. In order to decide on a meaningful range for the demand fluctuations we analyzed passenger flow statistics from The Airline Origin and Destination Survey (DB1B). For example for the OD pair JFK-FLL the total demand varies at most by around 15% from quarter to quarter in a year (data range 2005-2010). Similarly, a maximum of 20% variation is observed on the average itinerary demand from quarter to quarter. Since the daily fluctuation would be higher than the quarterly fluctuation we keep the range wider.

Analysis with IFAM and IFAM-PR

First of all, the added value of IFAM with choice-based recapture is tested compared to leg-based FAM using experiments 14 and 24 (see Table 5.1 for data instances). The considered RMM for passenger allocation is a model with choice-based recapture but without pricing (see Appendix A.3.1).

In experiment 14, supply-demand interactions embedded in IFAMs result with a higher capacity allocation since spilled passengers can be accommodated by other itineraries and the cost of larger capacity aircraft can be compensated. However, leg-based FAM decides to assign less capacity since it cannot make use of recapture effects. Therefore, as seen in Figure 6.3, when the demand is over-estimated FAM performs better in the range -30% to -5%. However starting with -5% IFAM performs better. When the demand is under-estimated the improvement due to IFAM can be clearly observed. After a level of underestimation (around 15%) leg-based FAM cannot change the revenue since the allocated capacity is not enough to accommodate all the demand.

Experiment 24 includes business class passengers which have higher average price compared to economy passengers. The existence of business passengers increases the revenue that could be obtained through recapture and therefore IFAM decides to allocate less capacity compared to leg-based FAM. As seen in Figure 6.4, IFAM performs better up to the perturbation level +20%. When the demand is under-estimated by more than 20% FAM performs better. The higher capacity allocation of FAM helps to recover the unexpected increase in the demand.

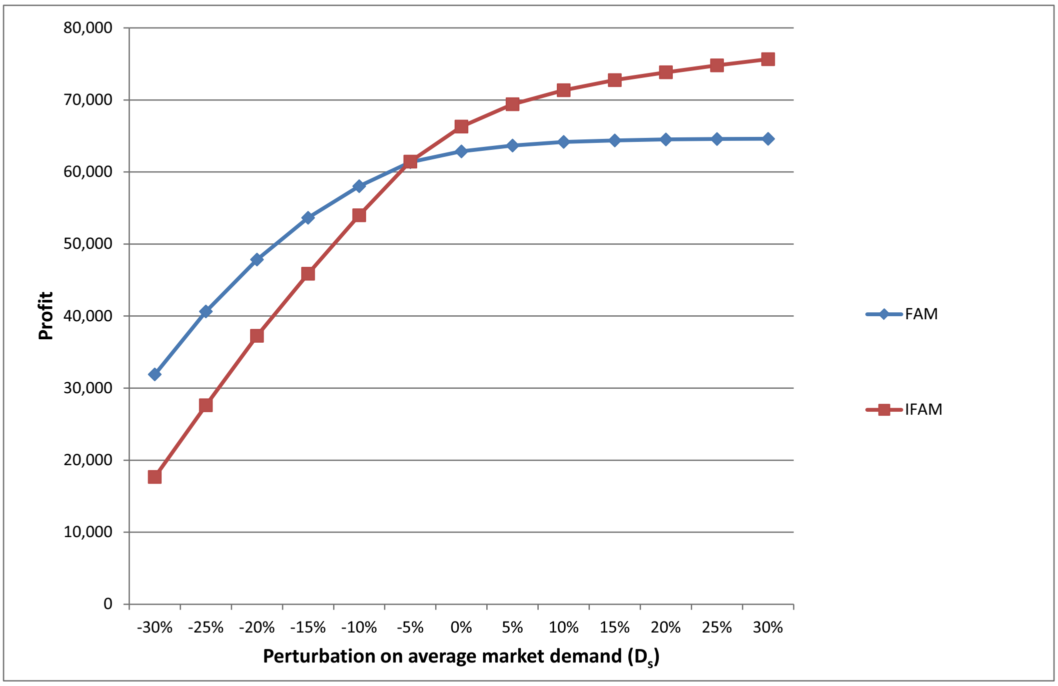


Figure 6.3: Sensitivity to demand fluctuations - IFAM vs FAM - Exp. 14

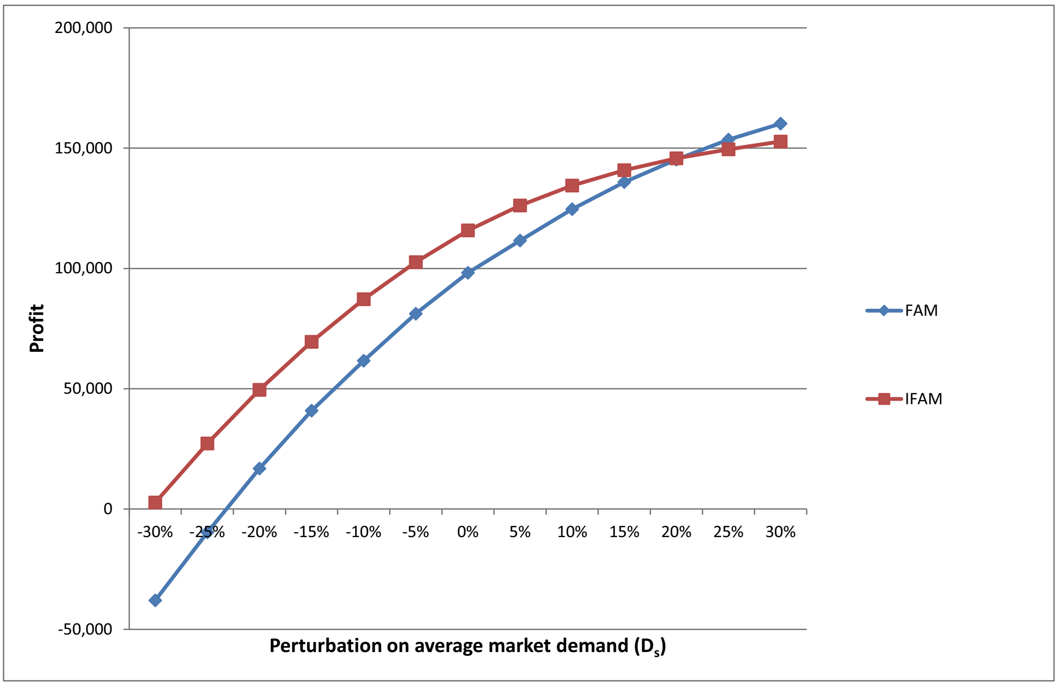


Figure 6.4: Sensitivity to demand fluctuations - IFAM vs FAM - Exp. 24

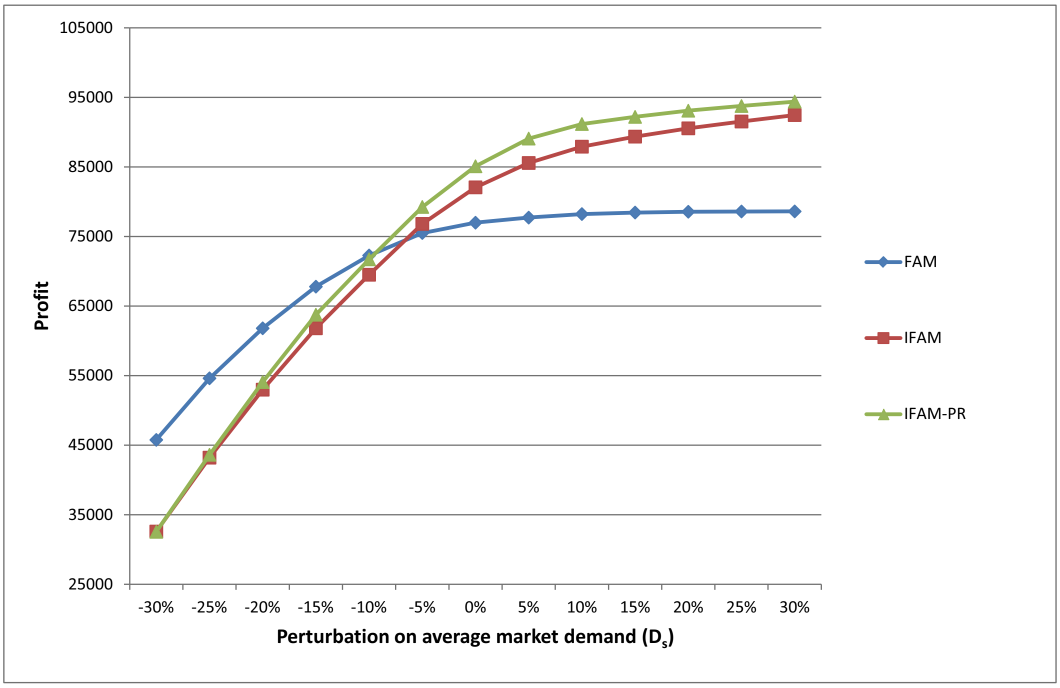


Figure 6.5: Sensitivity to demand fluctuations - IFAM/IFAM-PR vs FAM - Exp. 14

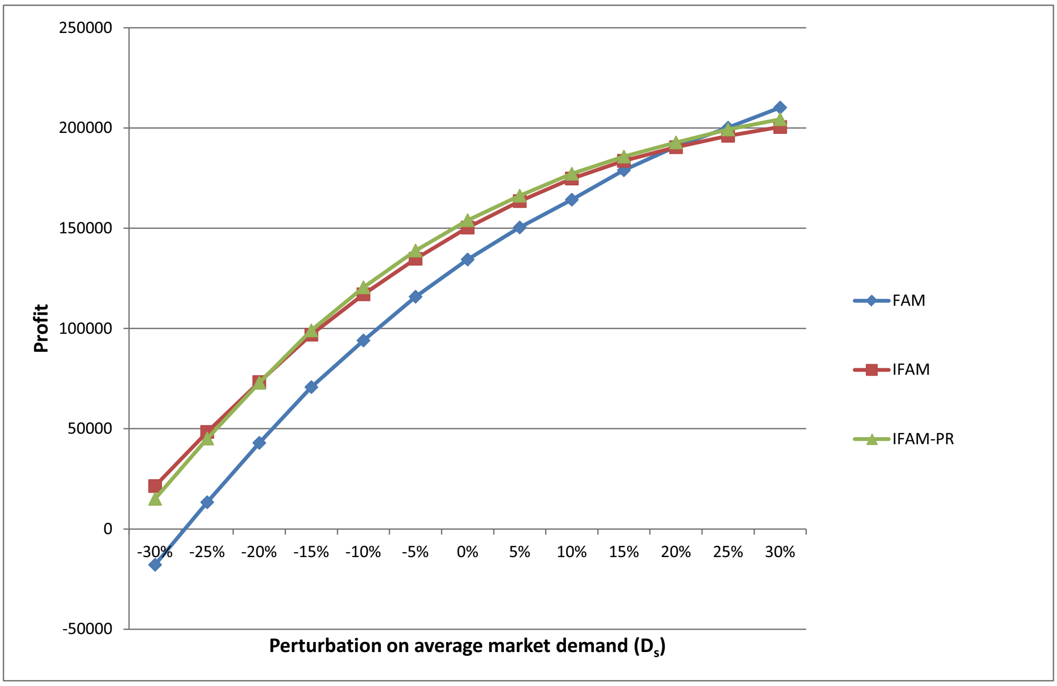


Figure 6.6: Sensitivity to demand fluctuations - IFAM/IFAM-PR vs FAM - Exp. 24

In order to test the sensitivity of IFAM-PR we performed the same analysis using the RMM-PR (see Appendix A.3.2). In Figures 6.5 and 6.6 the results for FAM, IFAM and IFAM-PR are presented. It is observed that the integration of pricing increases the robustness of the fleeting decisions. For experiment 14, IFAM is outperformed by FAM up to -5% fluctuation. However with IFAM-PR this is pulled back to -10%. A similar phenomenon is observed with experiment 24. IFAM is outperformed by FAM when demand is under-estimated by more than 20%. With IFAM-PR this is shifted towards 25%. This supports the fact that when the planning model has more flexibility and information from the revenue side, the perturbations can be absorbed better.

The presented results are generated with the average profit values over 500 simulations for each market demand value. In order to see the impact of day-to-day variations on the results we present box-plots in Figure 6.7. The box-plots are generated with 500 simulations for each market demand value. We compare the results of IFAM-PR and FAM for experiment 24. It is seen that the improvement provided by IFAM-PR is not significant when the demand is underestimated by more than 10%. We also analyze the deviation of the observed profit from the expected profit. For each model, we have a single expected profit value which is obtained with the expected demand value. The % absolute deviation is computed based on the following:

$$\%\text{deviation} = 100\frac{\left|\text{observed profit} - \text{expected profit}\right|}{\text{expected profit}}. \tag{6.37}$$

For each of the simulated demand values based on the perturbations, the deviation on the profit is calculated and boxplots are generated as seen in Figure 6.8. It is observed that the % deviation for IFAM-PR is less compared to the % deviation of FAM. The reduction in deviation is significant for an overestimation of more than 10% and for an underestimation of more than 5%. Furthermore, the variation on the % deviation of IFAM-PR is less than the variation observed for FAM as seen by the sizes of the boxes in the plot. This supports the fact that the fleeting and scheduling decisions obtained by IFAM-PR are more robust to the perturbations on the market demand.

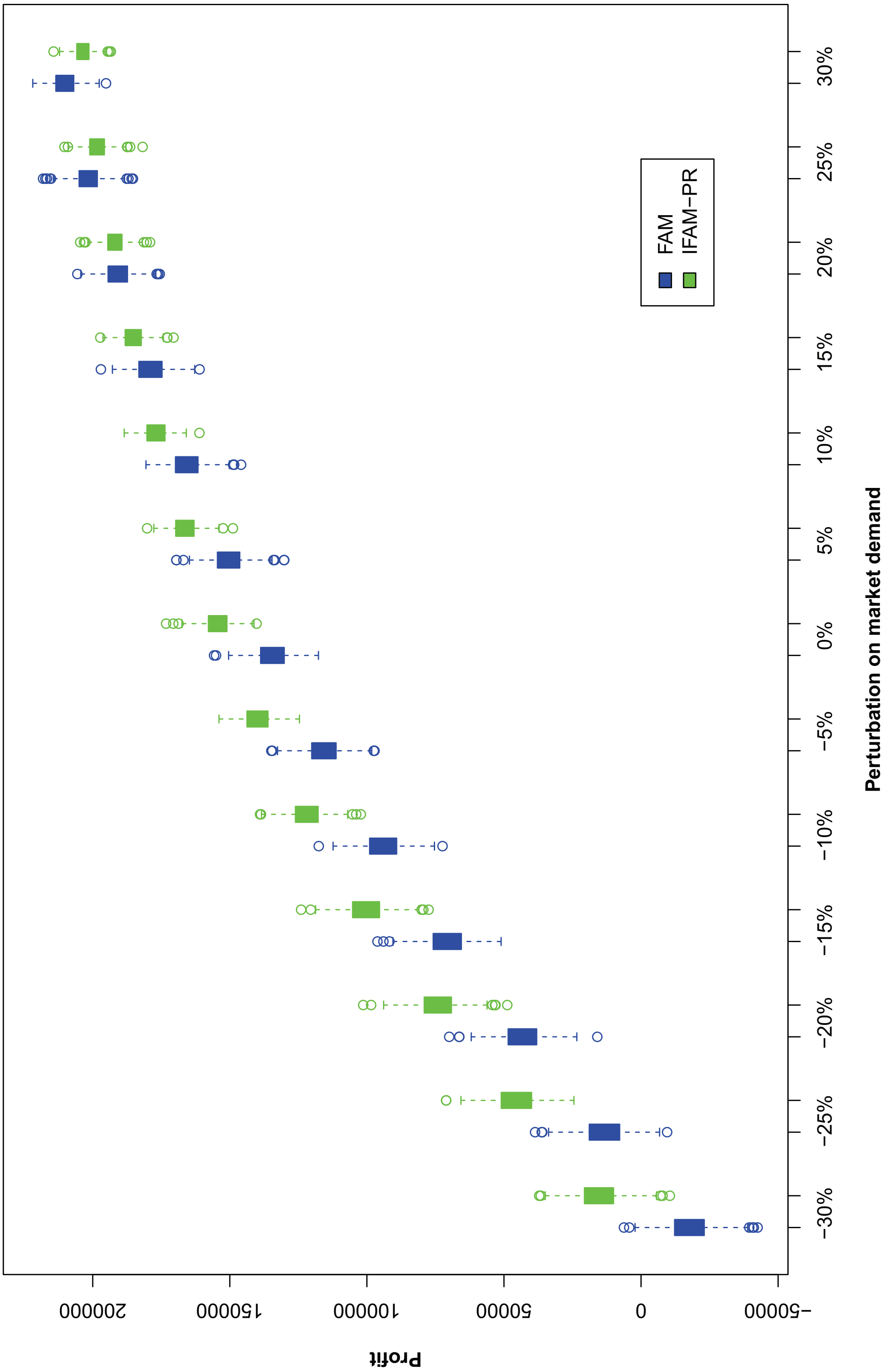


Figure 6.7: Boxplots for simulations of demand - IFAM-PR vs FAM - Exp. 24

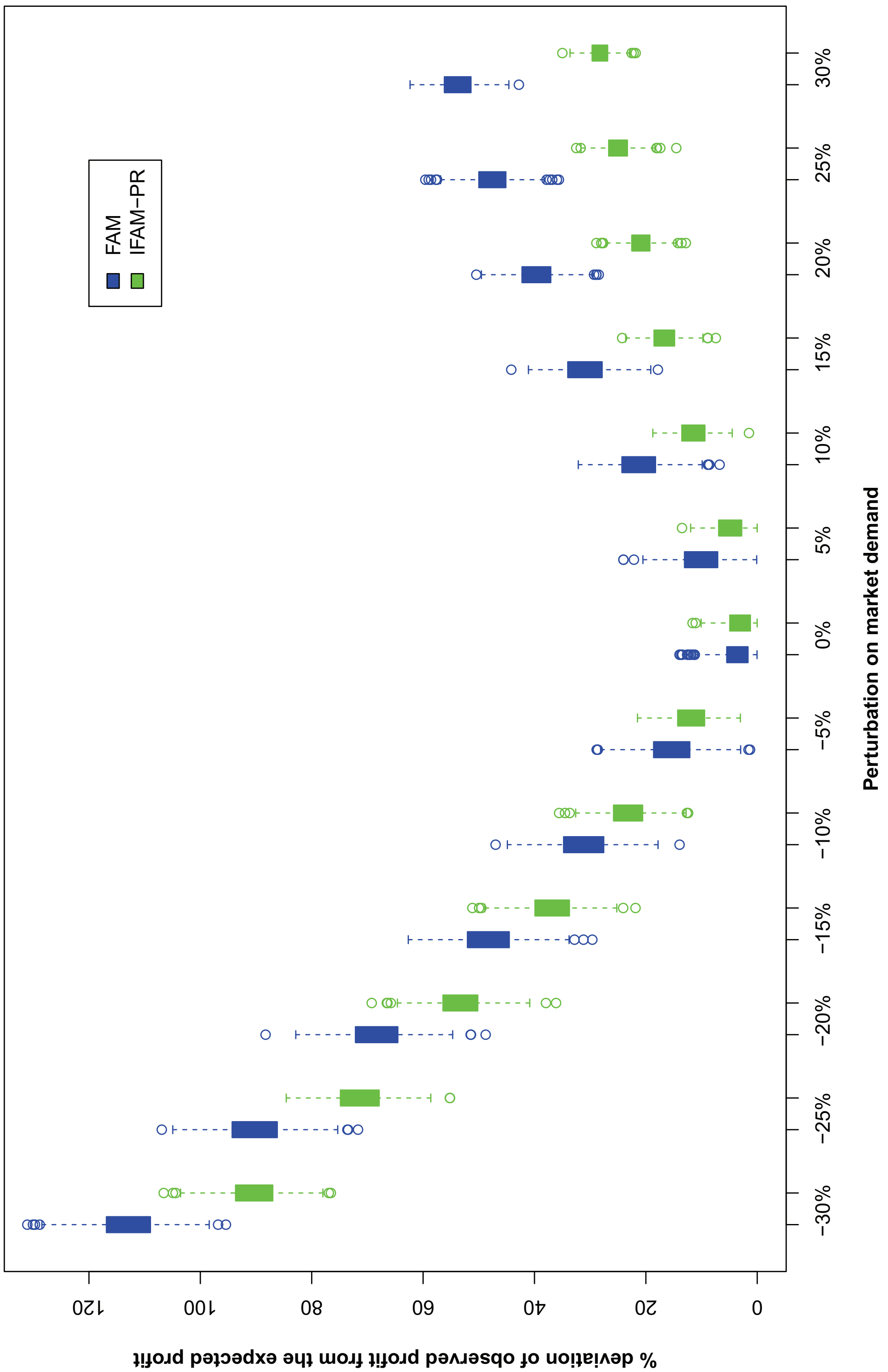


Figure 6.8: Boxplots for deviation from the expected revenue - IFAM-PR vs FAM - Exp. 24

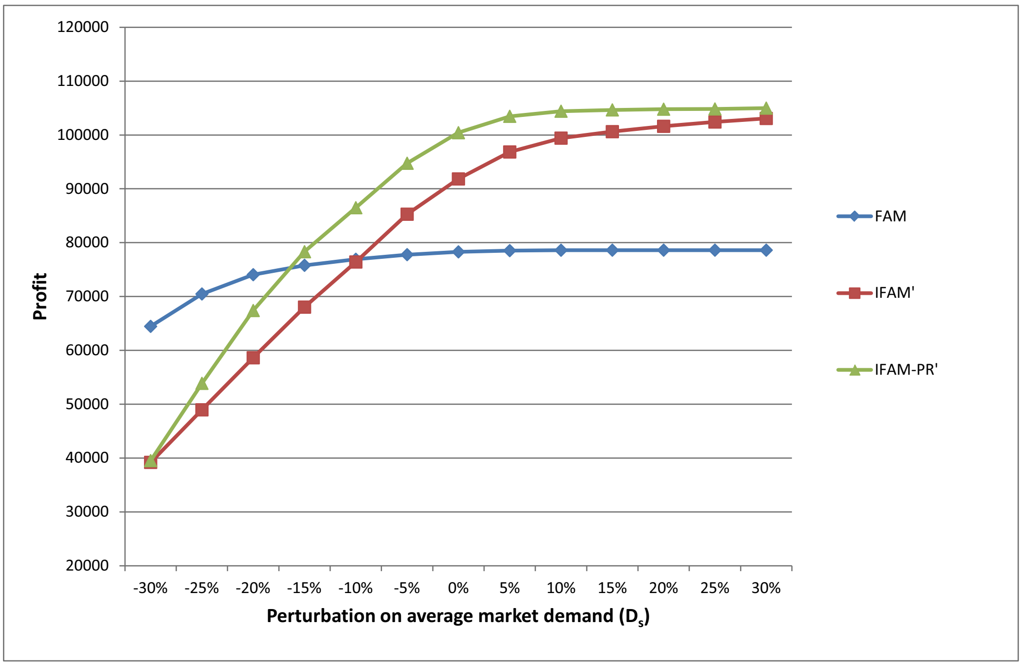


Figure 6.9: Sensitivity to demand fluctuations - IFAM$'$/IFAM-PR$'$ vs FAM - Exp. 14

Analysis with the reformulated model

A similar analysis is performed with the reformulated models IFAM$'$ and IFAM-PR$'$. It is preferred to keep the analysis for the models with pricing since the conclusions are similar. Throughout the analysis reformulated revenue model RMM-PR$'$ is used.

The results are provided in Figures 6.9 and 6.10 for experiments 14 and 24 respectively. The improvement due to pricing can be observed more obviously with the reformulated problem. Compared to IFAM$'$, IFAM-PR$'$ provides better profit in experiment 14 in almost all the cases except when the demand is overestimated more than 30%. The improvement over leg-based FAM can be observed in a wider range of demand when there is an integrated pricing decision. IFAM$'$ is outperformed by FAM when average demand is overestimated by more than 10%. However this value is between 15% and 20% for IFAM-PR$'$. Similary, in experiment 24, IFAM-PR$'$ results with a higher profit compared to IFAM$'$ except when the demand is overestimated more than 25%. There is a clear improvement with IFAM-PR$'$ compared to FAM even the demand is underestimated by up to 25%. However with IFAM$'$, the fleeting solution of FAM is not improved when there is an underestimation of 15% or more.

In order to see the impact of day-to-day variations on the results we present box-plots in Figure 6.11. As done before, the box-plots are generated with 500 simulations for each market demand value. We compare the results of IFAM-PR$'$ and FAM for experiment 24. It is seen that the improvement provided by IFAM-PR$'$ is not significant when the demand is underestimated by more than 15%. Furthermore, the % deviation on the profit is analyzed with the boxplots

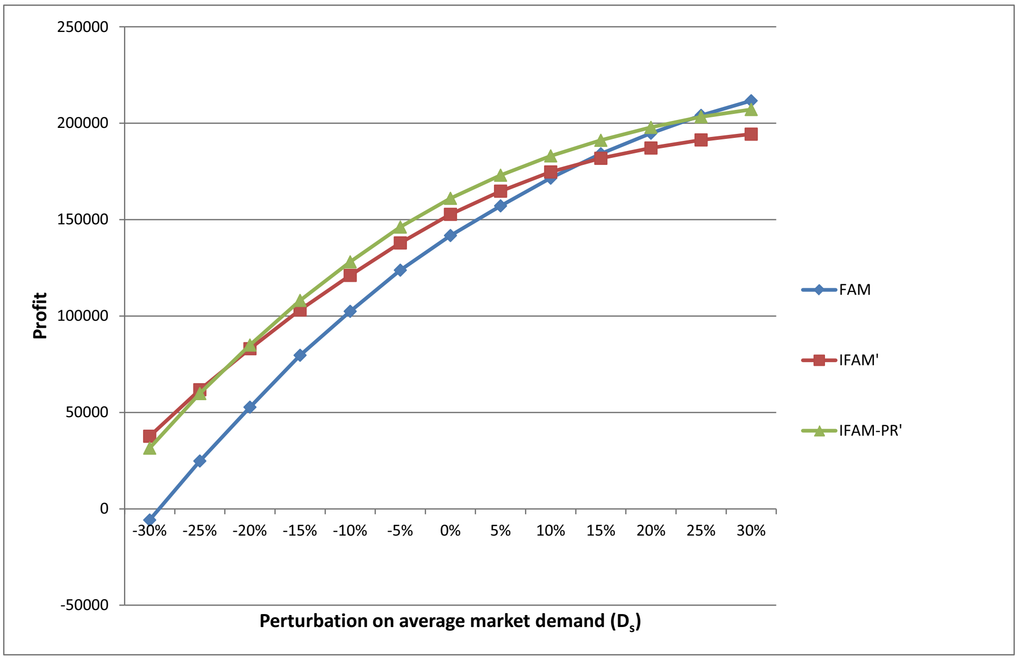


Figure 6.10: Sensitivity to demand fluctuations - IFAM$'$/IFAM-PR$'$ vs FAM - Exp. 24

presented in Figure 6.12. It is observed that the deviation on the profit is reduced with IFAM-PR$'$ compared to FAM. The reduction is significant for an underestimation of more than 10% and for an overestimation of more than 10%. The variation of the deviation is also reduced with IFAM-PR$'$ as seen from the sizes of the boxes in the plots.

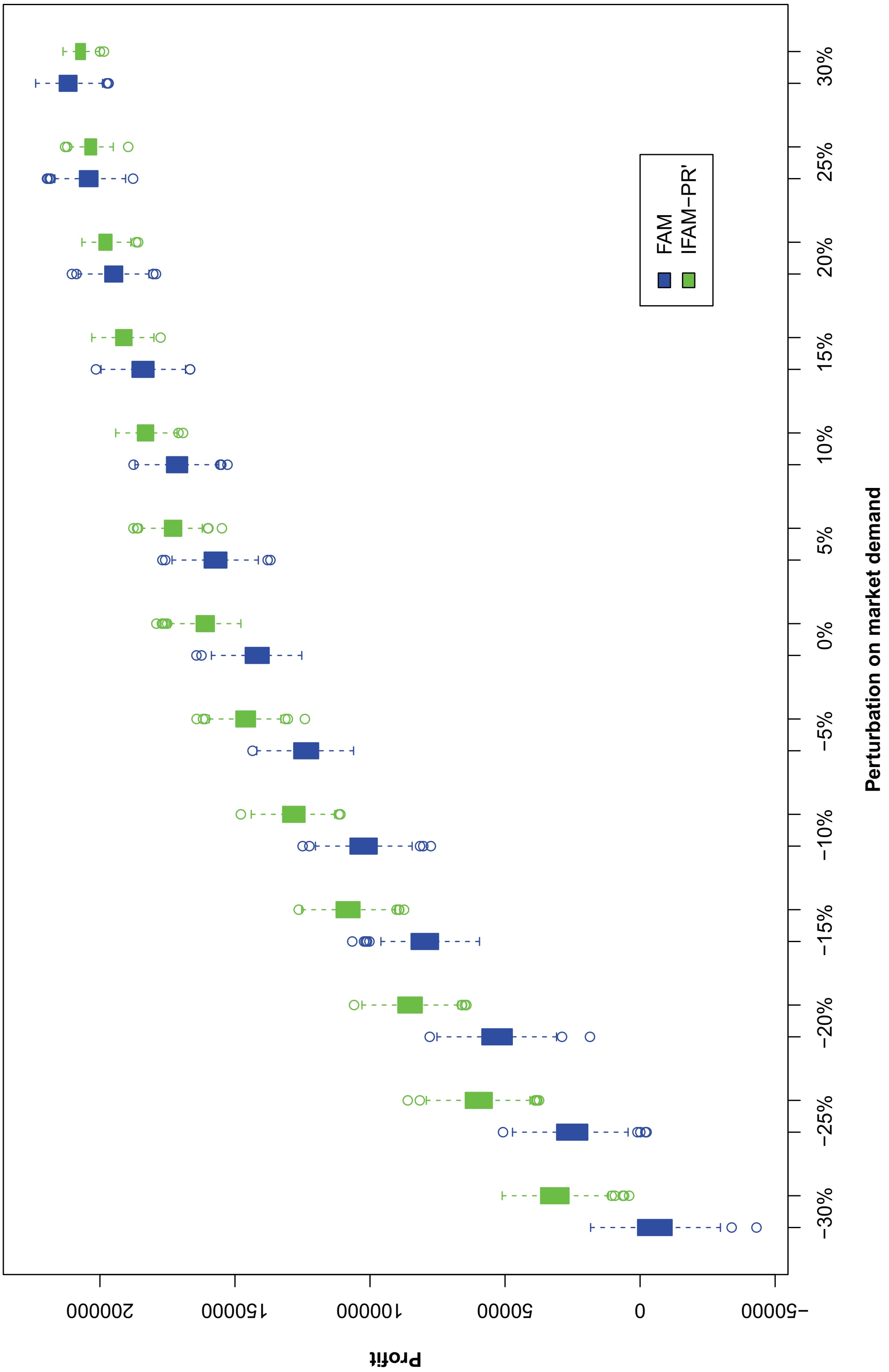


Figure 6.11: Boxplots for simulations of demand - IFAM-PR$^{'}$ vs FAM - Exp. 24

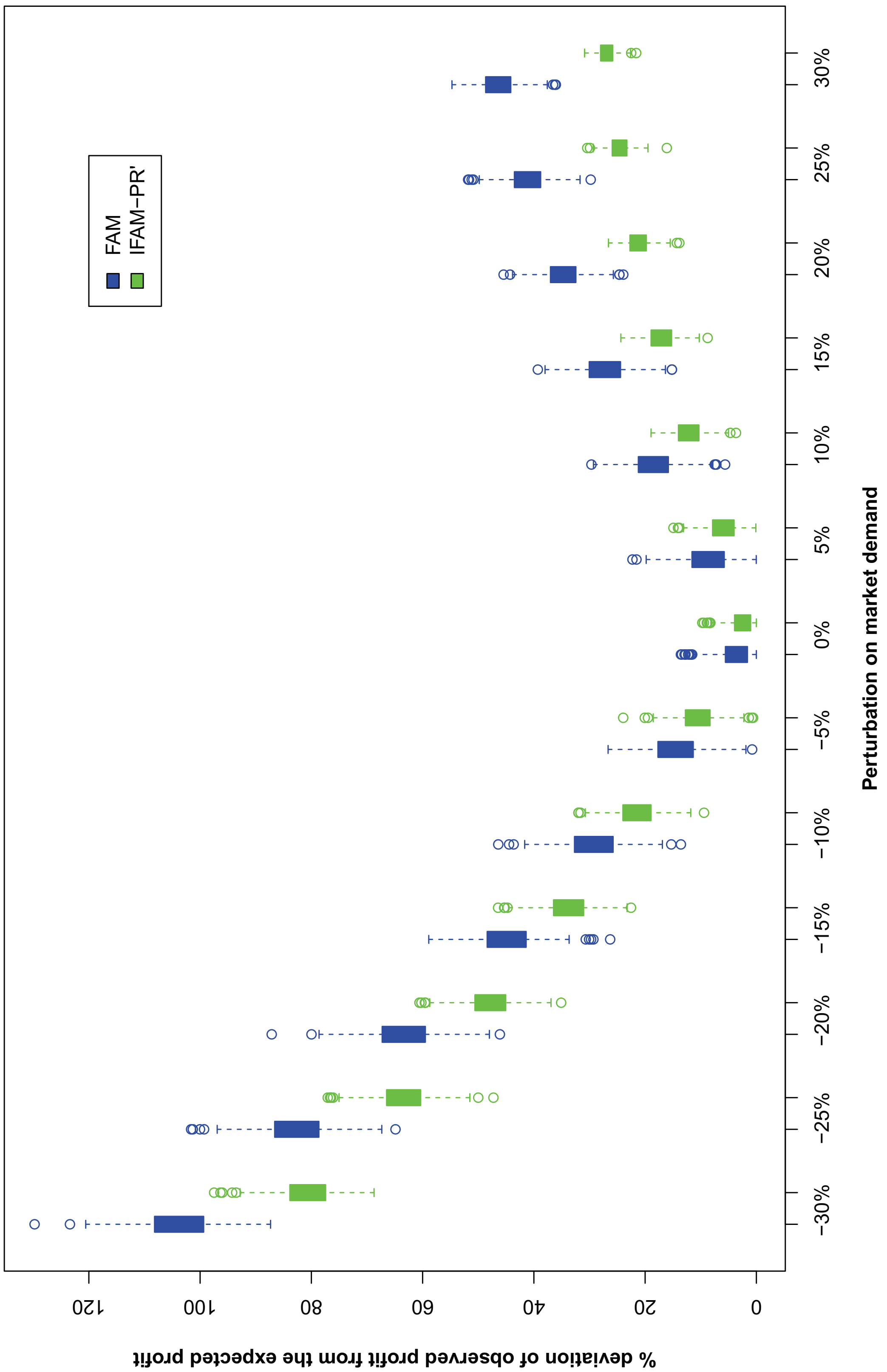


Figure 6.12: Boxplots for deviation from the expected revenue - IFAM-PR′ vs FAM - Exp. 24

Evaluation on the sensitivity to demand fluctuations

The analysis performed for the demand uncertainty shows that the performance of IFAMs are not affected by slight changes on the average demand. The sensitivity analysis performed by Lohatepanont (2002) concludes that the improvement due to IFAM is not clear even with a perturbation of 2-3% on average demand. In our case, the improvement provided by the IFAMs is more evident even when there is a deviation on the average demand by more than 10%. It can be concluded that a flexible planning model provides robust solutions since it can deal with more significant perturbations.

The integration of pricing decision introduces flexibility so that the robustness is increased. When IFAM-PRs and IFAMs are compared it is seen that the robustness to demand fluctuations is increased. In other words FAM solutions are outperformed in a wider range (around 10-15% wider) with the help of pricing decision. The significance of the improvement provided by the integrated models is analyzed with the box-plots of the simulations for each market demand value. It is observed that the improvement becomes insigificant when the perturbation on market demand is more than 10% and 15% for IFAM-PR and IFAM-PR$'$ respectively. It is also observed that the mean and variance of the % absolute deviation on the profit accross different realizations of the market demand are reduced with IFAM-PR and IFAM-PR$'$ compared to FAM. This supports the fact that the integrated models improve the robustness of fleeting and scheduling decisions.

It is important to note that experiment 14 is a smaller size instance compared to experiment 24 and BONMIN solver converges when solving instance 14. However for experiment 24 we work with the heuristic solution when solving IFAM-PR and IFAM-PR$'$. Therefore the improvement due to pricing is less evident for experiment 24 compared to experiment 14.

6.4.3 Price parameter of the itinerary choice model

The parameters of the demand model are estimated based on a mixed RP/SP dataset as explained in Chapter 3. Since the only policy variable is the price among the set of explanatory variables, its coefficient has a direct impact on the market shares. Depending on the data set used, the estimated value of the parameter will be different. Therefore in this section we perform a sensitivity analysis with perturbations on the price parameter.

As given in Table 3.3 in Chapter 3 the price parameter consists of 4 β parameters for non-stop/economy, nonstop/business, one-stop/economy, and one-stop/business itineraries. As a result of the estimation with BIOGEME we can obtain the variance-covariance matrix for the parameters. With the standard errors provided in this matrix we randomly generate 100 sets of price parameters from a standard normal distribution.

We perform the analysis on the price parameter with experiment 24. We present the analysis only with the reformulated models since the overall conclusions are the same. In Figure 6.13 we compare FAM, IFAM$'$, and IFAM-PR$'$. It is seen that IFAM-PR$'$ has the highest profit for

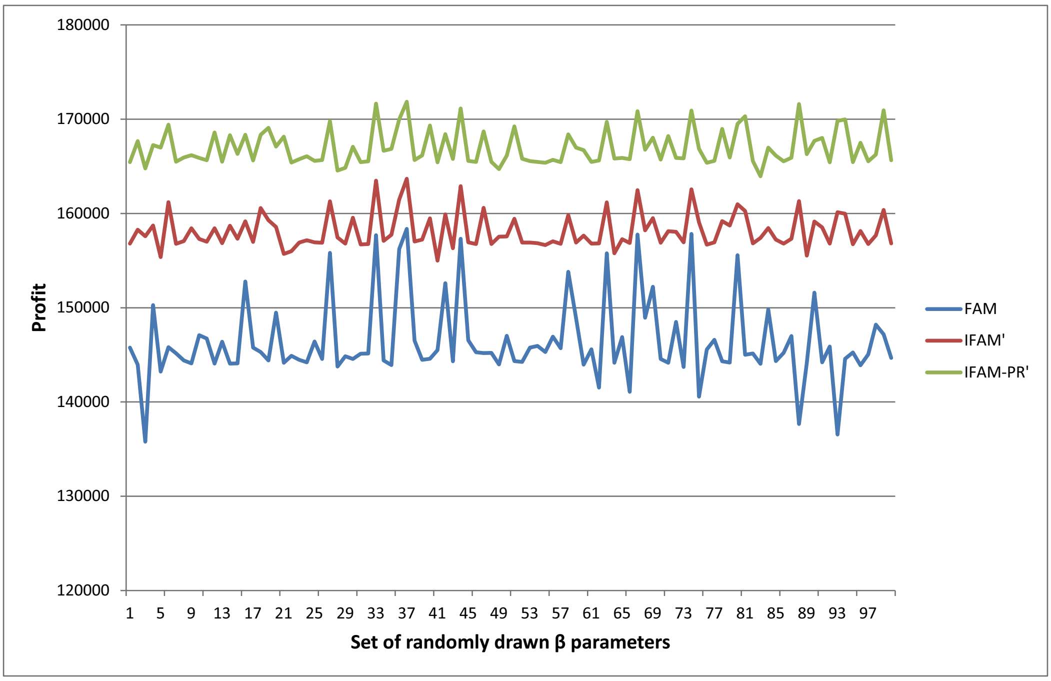


Figure 6.13: Sensitivity to price parameter - IFAM$'$/IFAM-PR$'$ vs FAM - Exp. 24

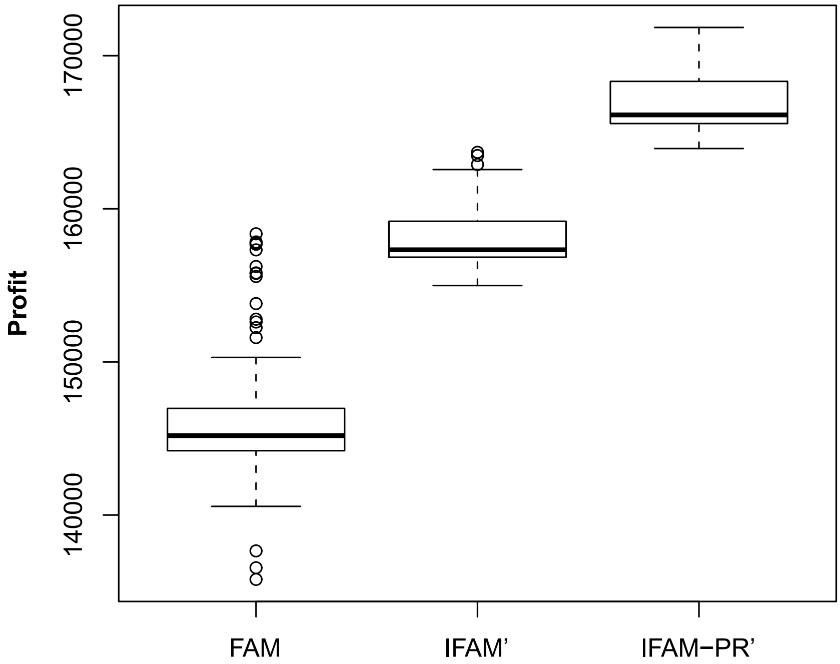


Figure 6.14: Box-plots with perturbed price parameters - Exp. 24

all of the cases and FAM has the lowest profit as expected. It is observed that FAM is more sensitive to the changes in the price parameter. In order to observe that box-plots are drawn as seen in Figure 6.14. The box-plots confirm that FAM is more sensitive to the perturbations.

This analysis concludes that the improvement provided by the supply-demand interactions through the itinerary choice model is not sensitive to the price parameter. There is a consistent improvement and the variability of the results with simulated price parameters is reduced thanks to supply-demand interactions.

6.4.4 Competitors' price

As mentioned in Chapter 3 in section 3.2 we introduce no-revenue options in the choice set. These alternative itineraries represent the alternatives offered by the competitors. The price of these alternatives are assumed to be known and even if there is a pricing decision they are kept fixed. Since the market share of the itineraries depends on the offer by the competitors, in this section we perform an analysis on the price of the competitor' itineraries. For each no-revenue option 100 prices are uniformly generated in a range of -50%- +50% with respect to the originally selected price. Similar to section 6.4.3, the analysis is carried out with reformulated models using experiment 24.

In Figure 6.15 we compare FAM, IFAM$'$, and IFAM-PR$'$ with uniformly drawn 100 sets of prices. It is observed that FAM has the lowest profit and IFAM-PR$'$ provides the highest profit in general. It is also concluded that FAM has a higher variation with competitors' prices compared to IFAMs. This can be confirmed by box-plots given in Figure 6.16. The difference between IFAM$'$ and IFAM-PR$'$ is relatively low in this case since the pricing decision is closely related to the price offered by the competitors and IFAM-PR$'$ is sensitive to high fluctuations.

This analysis shows that an itinerary-based setting is more robust to the changes in competitors' prices. Integrated supply-demand interactions with a choice model enables to react to market conditions which cannot be achieved through a leg-based FAM setting. Furthermore IFAM-PR$'$ provides the best profit among the three, given that the perturbation is not very high.

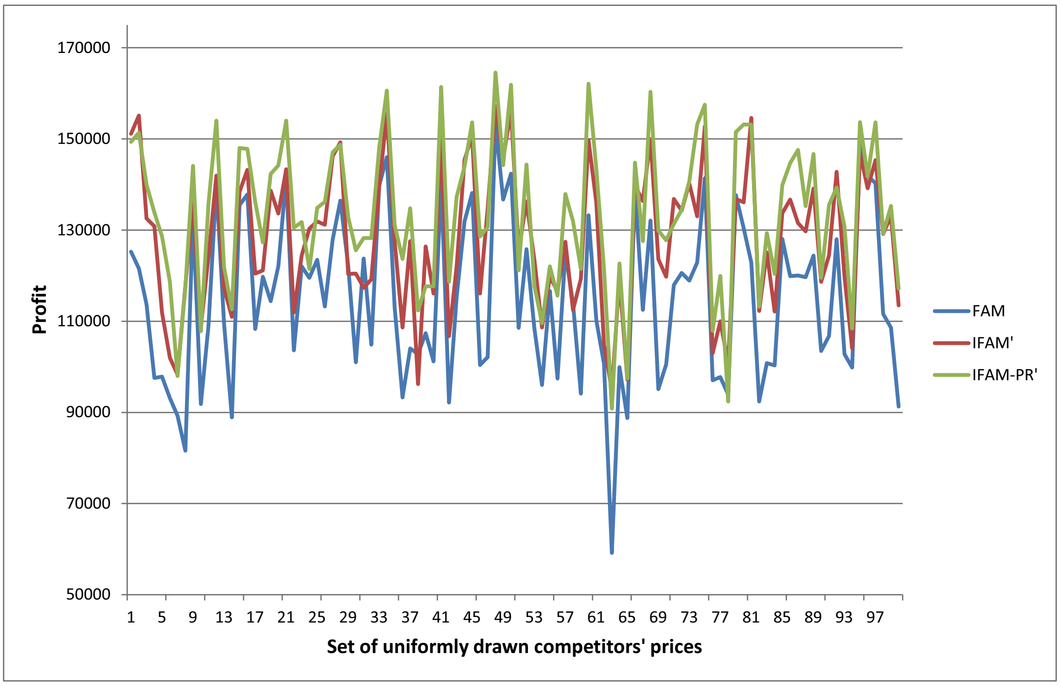


Figure 6.15: Sensitivity to competitors' prices - IFAM$'$/IFAM-PR$'$ vs FAM - Exp. 24

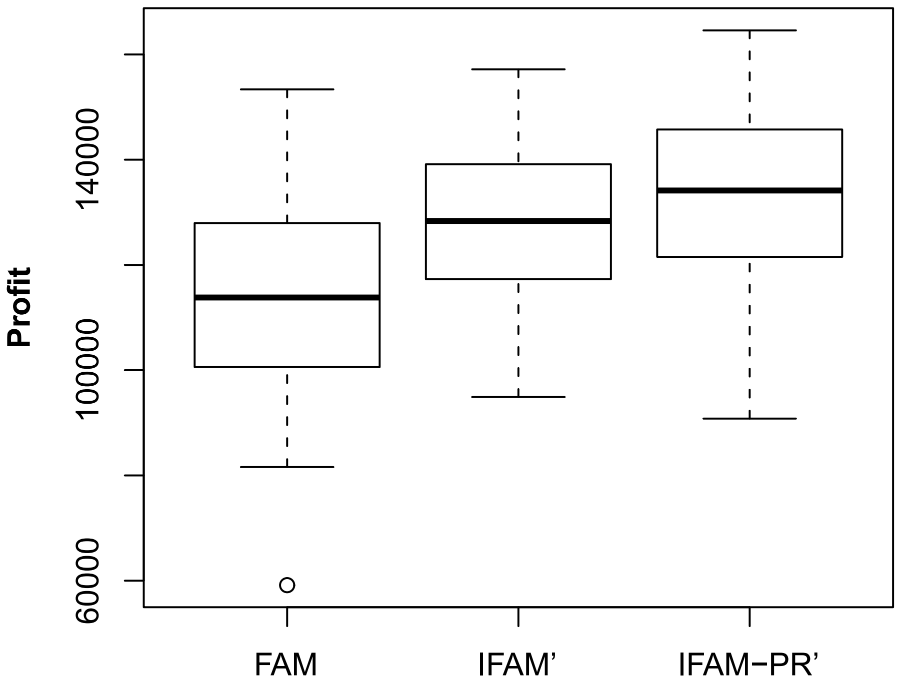


Figure 6.16: Box-plots with perturbed prices of competitors - Exp. 24

6.5 Conclusions

In this chapter we present a reformulation of the model which changes the way the demand model is introduced. In chapter 4 the spill and recapture is modeled with explicit variables as a control of the airline. In this chapter it is maintained by a single demand model and passengers settle to their desired itineraries without the spill and recapture variables. This reformulation is a relaxed version of the one in chapter 4 and results with higher profit.

The local search heuristic is adapted to the reformulated model. In general, a similar performance is observed which supports the use of the heuristic for different formulations of the model. It is observed that IFAM$'$ is a more difficult problem compared to IFAM. On the other hand RMM-PR$'$ is much simpler than RMM-PR with the new representation of the logit model. It is again observed that the IFAM$'$ is the bottleneck of the heuristic and a promising future research direction is the development of solution methodologies for the solution of it rather than using a MILP solver.

This chapter analyzes the sensitivity of the integrated models with respect to demand model parameters. It is observed that the itinerary-based models are not sensitive to slight changes in the inputs and provides a consistent improvement over the leg-based FAM. Furthermore the integration of pricing decision through IFAM-PRs provides an improved robustness of the solutions. The explicit representation of supply-demand interactions facilitate the simultaneous decision making in both demand and supply side. As a result better solutions are obtained.

7 Log transformation of the logit model

In this chapter we present a log transformation for revenue maximization models integrated with a demand model. The demand model integration alters classical optimization models since the demand becomes a variable of the optimization model rather than being an input parameter. As presented in the previous chapters, the integrated model is a complex problem due to the explicit demand model. Inspired by this complexity we present the transformation in a more general setting of revenue maximization models. The demand is represented by a logit model which is flexible in terms of the explanatory variables in the utility function. In other words, multiple explanatory variables and/or disaggregate variables such as socio-economic characteristics can be introduced in the utility of the alternatives. Moreover some explanatory variables of the demand model can be variables of the optimization model which increases the complexity. A logarithmic transformation is proposed for the logit model which facilitates a stronger reformulation of the optimization model. Illustrative examples are provided for aggregate and disaggregate demand models. As a case study, an airline revenue maximization model is presented, which is the sub-problem of the integrated model given in chapter 6. The proposed methodology is applied to realistic size instances.

7.1 Introduction and motivation

In classical operations research literature, demand is assumed to be an input parameter while optimizing planning decisions. Therefore demand is inelastic to the changes in supply (e.g. discussion in the context of revenue management by Kocabıyıkoğlu et al., 2013). Similarly, the demand forecast is usually done with fixed inputs from the planning decisions, typically with given capacity as mentioned by McGill and van Ryzin (1999). However supply and demand are highly interrelated; supply decisions change the attracted demand and vice versa. This motivates researchers for working on supply-demand interactions in various contexts.

In order to take into account supply-demand interactions, iterative/sequential methodologies are adapted (e.g. in airline optimization Dumas et al., 2009) where planning and demand problems are solved iteratively. The iterative process provides flexibility in terms of the considered demand forecasting such that complex statistical methodologies can be utilized without increasing the complexity of the planning problem. Also, there is an increasing interest for the explicit integration of demand models in planning problems which enables to better represent supply-demand interactions.

The considered demand models for the integration can be categorized as aggregate and disaggregate models. The demand-curve used in micro-economics (O'Sullivan and Sheffrin, 2006) is the basis for the aggregate demand modeling. The demand for a certain product is represented by the price of the product which is an average behavior of individuals across population. Disaggregate demand modeling through discrete choice methodology (McFadden, 2001) on the other hand provides flexibility in understanding the behavior of the choice maker. In the last decade there is an increasing interest for the integration of explicit demand models in optimization problems using discrete choice models. Examples from the literature include revenue management problems (Talluri and van Ryzin, 2004b; Dong et al., 2009; Zhang and Lu, 2013) and airline planning problems where the market shares are given by discrete choice models (Schön, 2008; Wang et al., 2012), facility location problems where the choice probability of each facility is expressed by a utility function depending on the location (Benati and Hansen, 2002;Haase, 2009), and the railway timetable design, where the quality of the timetable affects the attracted demand through a logit model (Cordone and Redaelli, 2011).

The integration of choice models generates an increased complexity. The level of complexity depends on the relation between the decision variables of the optimization problem and the explanatory variables of the choice model. If the explanatory variables of the demand model are input parameters to the optimization model it is easier to control the complexity. Wang et al. (2012) deal with choice-based spill and recapture in air transportation networks where the explanatory variables of the demand model are input parameters. They keep the model linear by representing the market shares relatively to the attractiveness rather than inserting the full logit formula. Haase and Müller (2013) present several reformulations of the logit embedded facility location problems which help to preserve linearity.

When an explanatory variable of the demand model is a variable of the optimization model, the complexity grows significantly. The nonlinearity is usually unavoidable and even convexity may be lost. Wang and Lo (2008), Cordone and Redaelli (2011), Atasoy et al. (forthcoming), and Mesa et al. (2013) present integrated models formulated as mixed integer non-convex problems, which can only treated by heuristic approaches. This type of integration is widely used in revenue management models where pricing decision is given by a demand model (e.g. Dong et al., 2009; Zhang and Lu, 2013). The non-convexity is avoided using an inverse-demand function so that price is written in terms of the market shares. Therefore a concave objective function is obtained with linear constraints. We provide the concavity analysis of this inverse-demand approach in Appendix A.6. This practical solution allows for a single explanatory variable of the demand model to be the decision variable of the optimization model. Otherwise the inverse function cannot be defined as proposed. Furthermore, in the existence of disaggregate level information such as socio-economic characteristics, the methodology can not be applied in a straightforward way. In case of non-convex formulations valid bounds can be obtained through approximations of the demand model. Wang and Lo (2008) make use of the relative market shares of the alternatives with log transformation. However the methodology is presented with two alternatives only. Cordone and Redaelli (2011) use the piecewise linear approximation of the logit model and adapt a branch and bound methodology in order to obtain valid bounds. In a game theoretical framework Cadarso et al. (2013) study an airline schedule planning model where the frequency of an airline is a decision variable of the optimization model and an explanatory variable of the demand model. They also use a piecewise linear approximation in order to deal with the complexity.

The mentioned studies in the literature clearly show the trade-off between an enhanced representation of demand in optimization models and the problem complexity. Therefore the integration of demand models in optimization problems is a fruitful and challenging research direction in terms of both mathematical modeling and solution methodologies.

In this chapter we propose a framework for revenue maximization models where a demand model is integrated as a logit model. A reformulation is proposed by a logarithmic transformation of the logit model. The framework is flexible for the introduction of multiple explanatory variables and disaggregate level information. Moreover, it is flexible for more than one explanatory variable of the demand model to be defined as a decision variable of the optimization model. The rest of the chapter is organized as follows: in section 7.2 we present a general revenue maximization model integrated with a logit model. The optimization model is reformulated in section 7.3 with a logarithmic transformation of the logit model which results with a non-convex formulation in general. Illustrative examples are provided in section 7.4. The methodology is presented with a case study in the context of airline optimization in section 7.5. This case study is linked to previous chapters having the same airline setting. However in this chapter we only focus on the revenue part which is similar to RMM-PR$'$ (see Appendix A.3.3). In section 7.6 we propose a piecewise linear approximation in order to have a valid upper bound as an ongoing work. The chapter is concluded together with future research ideas in section 7.7.

7.2 The revenue maximization model integrated with a logit model

We consider a competitive market where one player seeks to maximize its revenues by optimizing the price for the alternatives she offers. The market is composed of a number of market segments and the player offers a set of alternatives in each market segment. There are competitors which also offer alternatives in each segment. The reaction of competitors is ignored contrary to a game theoretical approach. Therefore, the prices of competing alternatives are assumed to be given and fixed. The relation between the attributes of the alternatives and the market share is given by a logit model. The total demand in each segment is assumed to be known and according to the logit model, a portion of demand is attracted by the player and the rest is lost to the competitors. The available capacity is assumed to be given so that we only focus on the demand related decisions.

We present the defined setting with a revenue maximization model where a demand model is explicitly integrated. The notation is similar to the previous chapters with few exceptions. The choice set is represented by C and the set of individuals in the market is represented by N. Since there are competitors in the market we define two sets $C^o \subset C$ and $C^c \subset C$ that represent the own alternatives of the company and the alternatives offered by the competitors, respectively. The choice probability for product i and individual n is denoted by $y_{i,n}$. The prices of competitive alternatives, $\bar{p}_i$, are fixed since the reaction of competitors is ignored. For the ease of notation, the model is defined for a single market segment. However, the extension is straightforward with the assumption of independent market segments. The general deterministic revenue maximization model P is given as follows:

$$z_{\text{P}} = \max \sum_{i \in C^o} \sum_{n \in N} y_{i,n} p_i \tag{7.1}$$

$$\text{s.t. } y_{i,n} = \text{logit}(p, \bar{p}, z; \beta) \qquad \forall i \in C^o, n \in N \tag{7.2}$$

$$g_{i,n}(y_{i,n}, p_i) \leq 0 \qquad \forall i \in C, n \in N \tag{7.3}$$

$$0 \leq y_{i,n} \leq 1 \qquad \forall i \in C, n \in N \tag{7.4}$$

$$p_i \geq 0 \qquad \forall i \in C^o \tag{7.5}$$

The decision variable of the optimization problem is the price, p. The objective function is the total revenue obtained with all the available products in the market (7.1). It is the product of the choice probability y and the price p. The demand model is embedded through constraints (7.2) which define the auxiliary variable y. The logit model gives the choice probability for a product i for choice-maker n as follows:

$$\begin{aligned} y_{i,n} &= \text{logit}(p, \bar{p}, z; \beta) \\ &= \frac{\exp\left(V_{i,n}(p_i, z_{i,n}; \beta)\right)}{\sum_{j \in C^o} \exp\left(V_{j,n}(p_j, z_{j,n}; \beta)\right) + \sum_{j' \in C^c} \exp\left(V_{j',n}(\bar{p}_{j'}, z_{j',n}; \beta)\right)} \qquad \forall i \in C^o, n \in N, \end{aligned} \tag{7.6}$$

where $V_{i,n}$ represents the deterministic utility for alternative i and individual n. Similar to the notation in chapter 4, z represents the set of explanatory variables of the demand model which are input parameters for the optimization model. β represents the set of estimated parameter values for the explanatory variables that are also input parameters for the optimization model. Here we provide a general representation for the utility function:

$$V_{i,n} = \beta_i^1 p_i + \beta_{i,n}^2 z_{i,n} \qquad \forall i \in C^o, n \in N, \tag{7.7}$$

where β^1 and β^2 represent the estimated coefficients for the explanatory variables. Since z is a vector of explanatory variables, the second part of the utility can be sum of several of them. For the ease of explanation we represented it with a single element of the vector z. In the following sections, different specifications will be provided for the illustrative examples and the case study. If the utility function does not have an individual dependent specification (e.g. socio-economic characteristics, individual specific coefficients), the choice probability for product i is the same for every individual, and is therefore equivalent to the market share:

$$u_i = \frac{\sum_{n \in N} y_{i,n}}{N} \quad \forall i \in C. \tag{7.8}$$

The set of constraints for the optimization problem (7.3) is represented by the function g. These constraints can be related to the capacity which limits the attracted demand. Furthermore, market conditions may constrain the attributes of the alternatives or the choice probabilities. Even if g is convex, this problem is typically a non-convex problem due to the explicit representation of the logit model and the revenue function in the objective.

The revenue maximization problem is presented with a single decision variable, price. However the methodology is valid for additional explanatory variables of the logit model to be defined as a decision variable of the optimization model. Depending on the structure of the model, i.e. the way these decision variables appear in the model the complexity may be increased. For revenue maximization models price is the critical variable as it appears in the objective function.

7.3 The log transformation

As done in section 6.1, for each choice-maker n we define a new variable v_n as follows:

$$v_n = \frac{1}{\sum_{j \in C} \exp(V_{j,n})} \quad \forall n \in N. \tag{7.9}$$

With this definition, the choice probability given in (7.6) can be represented by the following equations:

$$y_{i,n} = v_n \exp(V_{i,n}) \quad \forall i \in C, n \in N, \tag{7.10}$$

$$\sum_{i \in C} y_{i,n} = 1 \quad \forall n \in N. \tag{7.11}$$

Equation (7.11) is a linear constraint, however equation (7.10) is non-convex. We propose a logarithmic transformation over the choice probability equation (7.10) as follows:

$$y'_{i,n} = v'_n + V_{i,n} \quad \forall i \in C, n \in N, \tag{7.12}$$

where $y'_{i,n}$ represents $\ln(y_{i,n})$ and v'_n represents $\ln(v_n)$. Therefore, the choice probability definition is reformulated as a linear constraint given that the utility is linear in the decision variables of the optimization model. When the utility is nonlinear, some available linearizations may be applied, for example variable substitution. One example is illustrated in section 7.5.

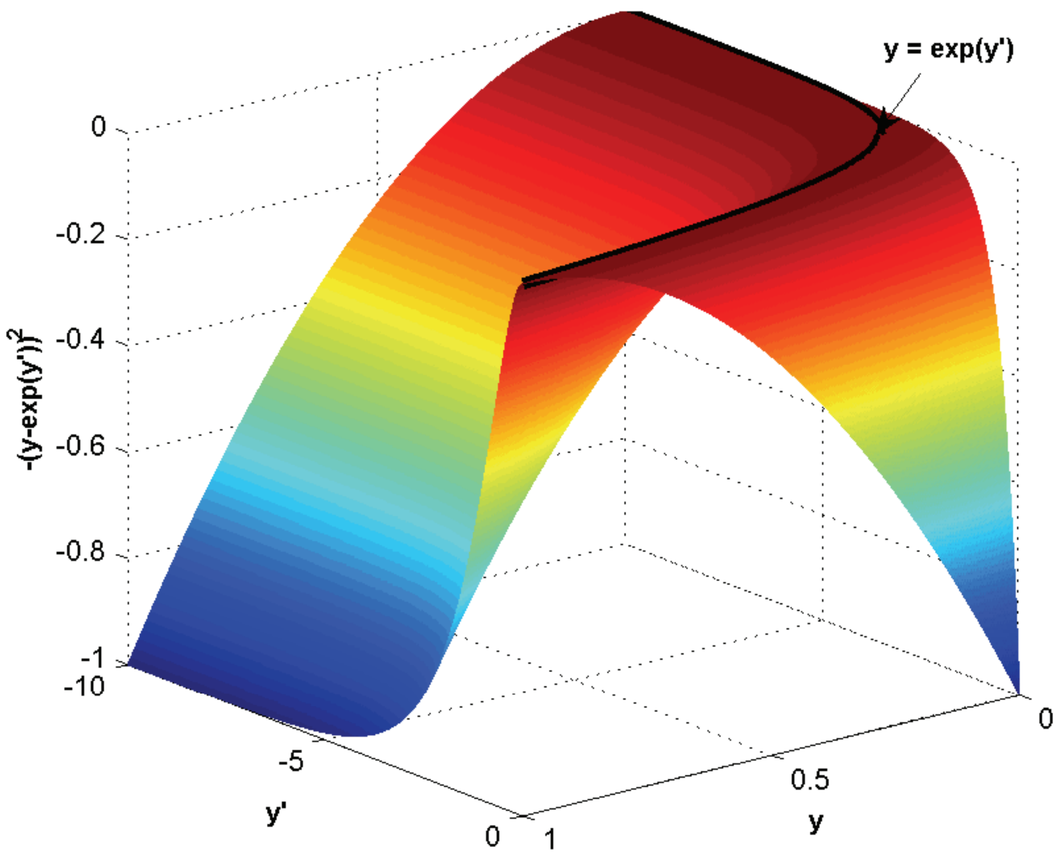


Figure 7.1: A plot for the penalty term

As function g is defined over the original choice probability variables y, we need to explicitly include the relation between the two variables. Adding a constraint $\exp(y') = y$ to impose equality destroys the convexity of the problem. Instead, we propose to penalize the deviation between $\exp(y')$ and y in the objective function. Since the problem is a maximization problem, the penalty on the deviation is introduced with a negative sign as follows:

$$f(y, y') = -M(y - \exp(y'))^2, \tag{7.13}$$

where $M \geq 0$ is the penalty value. The Hessian for the penalty term is given by:

$$H = \begin{pmatrix} \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial y'} \\ \frac{\partial^2 f}{\partial y' \partial y} & \frac{\partial^2 f}{\partial y'^2} \end{pmatrix} = \begin{pmatrix} -2M & 2M\exp(y') \\ 2M\exp(y') & 2M\exp(y')(y - 2\exp(y')) \end{pmatrix}.$$

The Hessian is negative semi-definite if the diagonals are ≤ 0 and the determinant is ≥ 0. First diagonal, $-2M$, is clearly negative. The second diagonal of the Hessian is $2M\exp(y')(y - 2\exp(y'))$. It is negative when $y \leq 2\exp(y')$. The determinant of the Hessian matrix is given by:

$$-4M^2\exp(y')(y - \exp(y')), \tag{7.14}$$

which is non-negative when $y \leq \exp(y')$. Therefore the penalty term is concave provided that $y \leq \exp(y')$. This can be illustrated with a 3D plot of the penalty term as given in Figure 7.1. The range for y is selected in $[0, 1]$ since it represents a probability and the range of y' is arranged accordingly to match the values. The function is flat along the curve $y = \exp(y')$ as seen in the figure. Taking this curve as a reference, when y decreases a concave subregion is reached as can be observed from the figure. Therefore, given that the condition $y \leq \exp(y')$ is guaranteed the penalty term is shown to be concave. However, the constraint $y \leq \exp(y')$ is a concave function and when introduced in the problem, renders the problem a non-convex programming problem.

A similar procedure can be adopted to reformulate the objective function. Define $R_{i,n}$ as the revenue obtained from individual n with alternative i ($R_{i,n} = y_{i,n} p_i$). Therefore the objective function given in (7.1) can be re-written as:

$$\begin{aligned} z_{\mathrm{P}} &= \max \sum_{i \in C^o} \sum_{n \in N} y_{i,n} p_i, \\ &= \max \sum_{i \in C^o} \sum_{n \in N} R_{i,n}. \end{aligned} \tag{7.15}$$

Note that $R_{i,n}$ is not defined for competitive alternatives. We consider a similar logarithmic transformation and define $R'_{i,n}$ as $\ln(R_{i,n})$. The deviation between $R_{i,n}$ and $\exp(R'_{i,n})$ also needs to be penalized in the objective function. $R'_{i,n}$ is considered by including the following constraint:

$$R'_{i,n} \leq y'_{i,n} + \ln(p_i) \qquad \forall i \in C^o, n \in N, \tag{7.16}$$

where the right hand side corresponds to the logarithm of $y_{i,n}p_i$. Instead of an equality constraint, we can safely formulate it as an inequality in order to define a convex relation. Indeed, this is the only constraint binding $R'_{i,n}$ from above and the objective direction is maximization. The right hand side is composed of a linear term and a concave function which shows that (7.16) is convex.

Therefore, the reformulated model with the logarithmic transformation ($\mathrm{P}^{\ln}$) and penalty terms can be written as:

$$\begin{aligned}
z_{\mathrm{P}^{\ln}} = \max & \sum_{i \in C^o} \sum_{n \in N} R_{i,n} - M1_{i,n}(R_{i,n} - \exp(R'_{i,n}))^2 \\
& - \sum_{i \in C} \sum_{n \in N} M2_{i,n}(y_{i,n} - \exp(y'_{i,n}))^2 && (7.17)\\
\text{s.t. } & R'_{i,n} \leq y'_{i,n} + \ln(p_i) & \forall i \in C^o, n \in N \quad & (7.18)\\
& y'_{i,n} = v'_n + V_{i,n}(p_i, z_{i,n}; \beta) & \forall i \in C^o, n \in N \quad & (7.19)\\
& y'_{j,n} = v'_n + V_{j,n}(\bar{p}_j, z_{j,n}; \beta) & \forall j \in C^c, n \in N \quad & (7.20)\\
& \sum_{i \in C} y_{i,n} = 1 & \forall n \in N \quad & (7.21)\\
& g_{i,n}(y_{i,n}, p_i) \leq 0 & \forall i \in C, n \in N \quad & (7.22)\\
& y_{i,n} \leq \exp(y'_{i,n}) & \forall i \in C, n \in N \quad & (7.23)\\
& y_{i,n} \geq 0 & \forall i \in C, n \in N \quad & (7.24)\\
& y'_{i,n} \in \mathbb{R} & \forall i \in C, n \in N \quad & (7.25)\\
& v'_n \in \mathbb{R} & \forall n \in N \quad & (7.26)\\
& R_{i,n} \leq \exp(R'_{i,n}) & \forall i \in C^o, n \in N \quad & (7.27)\\
& R_{i,n} \geq 0 & \forall i \in C^o, n \in N \quad & (7.28)\\
& R'_{i,n} \in \mathbb{R} & \forall i \in C^o, n \in N \quad & (7.29)\\
& p_i \geq 0 & \forall i \in C^o \quad & (7.30)
\end{aligned}$$

The set of penalty terms, $M1$, is introduced for the deviation of the revenue variable and set $M2$ is for the deviation of the choice probability. As a result, the objective function is concave; constraints (7.19)-(7.21) are linear constraints given that $V_{i,n}(p_i, z_{i,n}; \beta)$ is linear in p_i (or given that it can be linearized); constraints (7.22) are convex with a convex definition of g. However constraints (7.23) and (7.27) are non-convex functions.

With the proposed reformulation we do not obtain a convex programming problem. However, computational experiments suggest that this reformulation in much stronger and easier to solve. Intuitively, there is a subspace where the problem is convex, and where an algorithm may benefit from this convexity. Furthermore, for specific cases these constraints can be relaxed as described in section 7.4.1.

It is important to note that the conditioning of the Hessian matrix related to the full objective function depends on the values of $M1$ and $M2$. These values should be carefully selected according to the structure of the revenue function. As it is common with any penalty method the numerical stability of the problem depends on the values of the penalties, $M1$ and $M2$. A lower penalty value means a better conditioning of the Hessian and vice versa.

7.4 Illustrative examples

In this section, we illustrate the added value of the proposed transformation with two simple examples. The first one is carried out with an aggregate demand model. On the other hand the second example integrates a disaggregate demand model with two groups of individuals with different characteristics.

7.4.1 Aggregate demand model

Consider two alternative products with the following utility functions:

$$V_1 = \beta p_1, \qquad V_2 = \beta \bar{p}_2, \tag{7.31}$$

where there is no additional explanatory variable, z, and price is introduced with a generic coefficient, β in the utility function. The second product here represents the competing alternative and its price, $\bar{p}_2$, is fixed. Therefore the only decision is on p_1.

The utility is defined with aggregate variables and the market share for alternative 1 is given as:

$$u_1 = \frac{\exp(V_1)}{\exp(V_1) + \exp(V_2)} = \frac{\exp(\beta p_1)}{\exp(\beta p_1) + \exp(\beta \bar{p}_2)} \tag{7.32}$$

The objective is to maximize the revenue for alternative 1 and consider that there are 100 potential customers for the products. Therefore the model P given in (7.1)-(7.5) is formulated for this simple example as:

$$z_{\mathrm{P}_1} = \max\ 100 u_1 p_1 \tag{7.33}$$

$$\text{s.t. } u_1 = \frac{\exp(\beta p_1)}{\exp(\beta p_1) + \exp(\beta \bar{p}_2)} \tag{7.34}$$

$$u_2 = \frac{\exp(\beta \bar{p}_2)}{\exp(\beta p_1) + \exp(\beta \bar{p}_2)} \tag{7.35}$$

$$p_1 \geq 0 \tag{7.36}$$

The revenue for alternative 1 (7.33) is given by the product of the market share, the price, and the total demand 100. The market shares for the products are given in (7.34) and (7.35) respectively. It is assumed that there are no other constraints.

In this simple example we do not need to have a log transformation over the revenue function since there is only one revenue to be considered. Therefore, the objective function of this problem can be treated as below in order to have a concave function:

$$\max\ 100 u_1 p_1 \Leftrightarrow \max\ u_1^{'} + \ln(p_1), \tag{7.37}$$

which follows from the relation $100 u_1 p_1 = \exp(\ln(100) + \ln(u_1) + \ln(p_1))$. exp() can be ignored since it is strictly monotonic. Furthermore $\ln(u_1)$ is represented by $u_1^{'}$ and $\ln(100)$ can be removed since it is a constant. The objective function for the reformulated problem with the penalty terms can be written as:

$$u_1^{'} + \ln(p_1) - M_1(u_1 - \exp(u_1^{'}))^2 - M_2(u_2 - \exp(u_2^{'}))^2. \tag{7.38}$$

The Hessian for the objective function is given as follows:

$$H = \begin{pmatrix} -\frac{1}{p_1^2} & 0 & 0 & 0 & 0 \\ 0 & -2M_1 & 2M_1\exp(u_1^{'}) & 0 & 0 \\ 0 & 2M_1\exp(u_1^{'}) & 2M_1\exp(u_1^{'})(u_1 - 2\exp(u_1^{'})) & 0 & 0 \\ 0 & 0 & 0 & -2M_2 & 2M_2\exp(u_2^{'}) \\ 0 & 0 & 0 & 2M_2\exp(u_2^{'}) & 2M_2\exp(u_2^{'})(y_2 - 2\exp(u_2^{'})) \end{pmatrix},$$

which indicates the interrelation between the conditioning of the matrix and the values selected for M_1 and M_2.

The reformulated model for the problem can be given by:

$$z_{\mathrm{P}_1^{\ln}} = \max\ u_1^{'} + \ln(p_1) - M_1(u_1 - \exp(u_1^{'}))^2 - M_2(u_2 - \exp(u_2^{'}))^2 \tag{7.39}$$

$$\text{s.t. } u_1^{'} = v^{'} + \beta p_1 \tag{7.40}$$

$$u_2^{'} = v^{'} + \beta \bar{p}_2 \tag{7.41}$$

$$u_1 + u_2 = 1 \tag{7.42}$$

$$u_1 \leq \exp(u_1^{'}) \tag{7.43}$$

$$u_2 \leq \exp(u_2^{'}) \tag{7.44}$$

$$u_1, u_2 \geq 0 \tag{7.45}$$

$$u_1^{'}, u_2^{'} \in \mathbb{R} \tag{7.46}$$

$$v^{'} \in \mathbb{R} \tag{7.47}$$

$$p_1 \geq 0 \tag{7.48}$$

In this specific problem the objective function is written as a concave function. Furthermore since $u_1^{'}$ is in the objective rather than u_1, the constraint (7.43) is redundant in the formulation.

On the contrary, constraint (7.44) is necessary.

We present results with the original formulation P_1 given in (7.33)-(7.36) and the reformulated problem $\mathrm{P}_1^{\ln}$ given in (7.39)-(7.48). For the solution of the two formulations, BONMIN solver (Bonami et al., 2008) is used which is designed for convex problems and can treat non-convex problems as a heuristic. The price of the second alternative, $\bar{p}_2$, is assigned to 2 and two different β values are selected as scenarios.

Moderate price elasticity: $\beta = -1$

The model is illustrated in Figure 7.2. The change of the associated market shares and the revenues of the alternatives is given as a function of the price of alternative 1, p_1. It can be seen that the maximum value for the revenue is realized when $p_1 = 2$ which gives equal market shares for the alternatives. When the two versions of the model are solved, this solution is obtained for both of them.

This is an easy instance where the price elasticity is relatively low. The price elasticity for alternative 1 is given as:

$$E_{p_1}^1 = \frac{\partial u_1}{\partial p_1}\frac{p_1}{u_1} = (1 - u_1)p_1\beta \tag{7.49}$$

as explained in Ben-Akiva and Lerman (1985). For the price value of 2, the price elasticity is computed as -1 since the alternatives get a 0.5 market share each. For the reformulated model, experiments are done with penalty values in a range of $[1 - 1{,}000{,}000]$ and the optimal price is always found in this range.

High price elasticity: $\beta = -2.5$

According to (7.49), the price elasticity is computed as -2.5 when $p_1 = 2$. Therefore the price elasticity is higher compared to the previous case. The model is illustrated in Figure 7.3. It is observed that the maximum value for the revenue is realized when p_1 is around 1.55.

Numerical problems occur when solving the original formulation and 0 revenue is obtained as a result. However when we add the constraints $p_1 \leq 2$ and $p_1 \geq 1.5$ the optimum price of 1.57 is obtained. These additional constraints define a concave subregion as can be observed in Figure 7.3. If the shape of the objective function is easy to analyze, this kind of a treatment can be done. However, in real life problems it is not trivial to find the appropriate constraints on the variables.

When the reformulated model is solved, the optimum p_1 value is obtained with penalty values in a range of $[1 - 1{,}000{,}000]$. Even though the elasticity is higher, the reformulated model can be solved with BONMIN without any penalty related problems.

These two scenarios based on the aggregate demand model show that the non-convexity is manageable when the elasticity is relatively low. In such a case the original formulation and the reformulation can be solved to optimality without any problem. However when the price elasticity is higher the original problem cannot be solved with BONMIN. The reformulated problem on the other hand, can be solved to optimality for a wide range of penalty values. The same experiment is conducted with increased β values up to -10. The same observations are made; the original formulation does not allow to find the optimum, while the reformulation can be solved to optimality under a wide range of penalty values. It is concluded that with an aggregate setting as in this example, the penalty terms seem not to be critical. Even with low penalty values the optimal solution can be found. The flexibility in keeping the penalty value low improves the numerical stability of the problem.

7.4.2 Disaggregate demand model

In this example, individual specific coefficients are introduced through two groups of individuals. These two groups have different price elasticities. Consider that there are two alternative products in the market with the following utilities:

$$V_{i,n} = \beta_n p_i + z_i \qquad i, n \in \{1,2\}. \tag{7.50}$$

The index i is for the alternatives and n stands for the groups with different characteristics. Consider that there are 600 individuals in the first group and 400 individuals in the second group. For each group, the price parameter is generic across alternatives. An alternative specific constant, z_i, is introduced in the utility of the alternatives. In order to introduce such a constant we should fix the constant for one alternative (typically to zero) for identification purposes in the estimation process.

Similarly to the previous example, assume that the objective is to maximize the revenue resulting from the first alternative and that the price of the second alternative, $\bar{p}_2$ is fixed to 2. The choice probability of alternative i for each group n is represented by $y_{i,n}$ and the model P given in (7.1)-(7.5) is formulated for this example as:

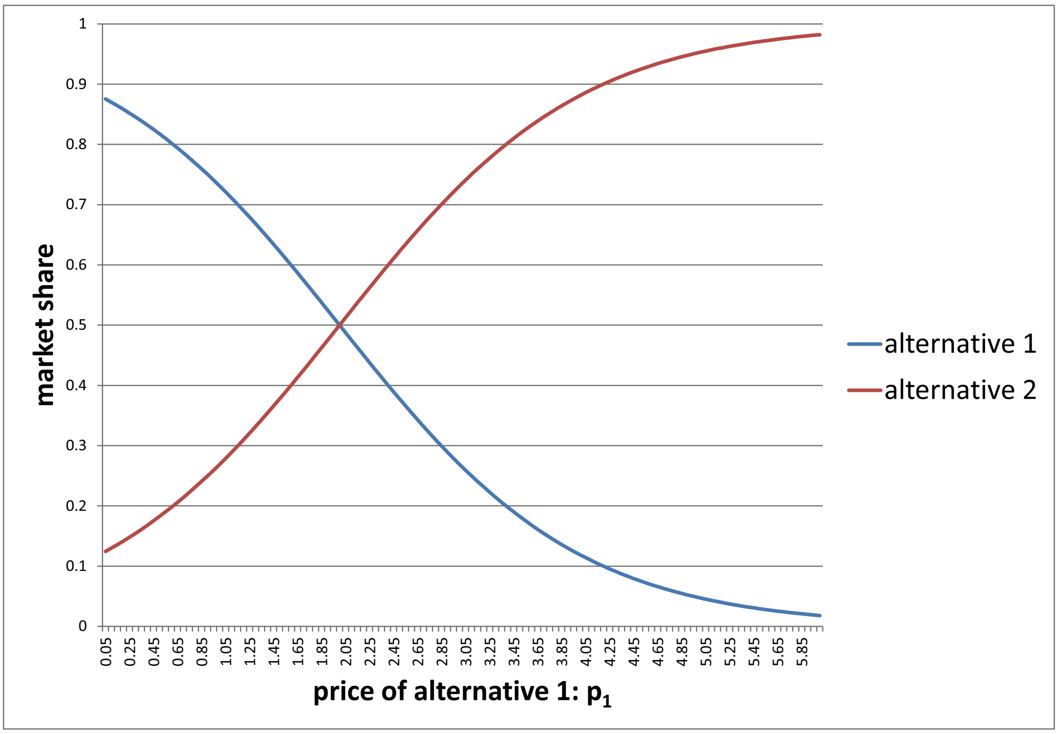
$$z_{P_2} = \max\ 600 y_{1,1} p_1 + 400 y_{1,2} p_1 \tag{7.51}$$

$$\text{s.t.}\ y_{1,n} = \frac{\exp(\beta_n p_1 + z_i)}{\exp(\beta_n p_1 + z_1) + \exp(\beta_n \bar{p}_2 + z_2)} \qquad n \in \{1,2\} \tag{7.52}$$

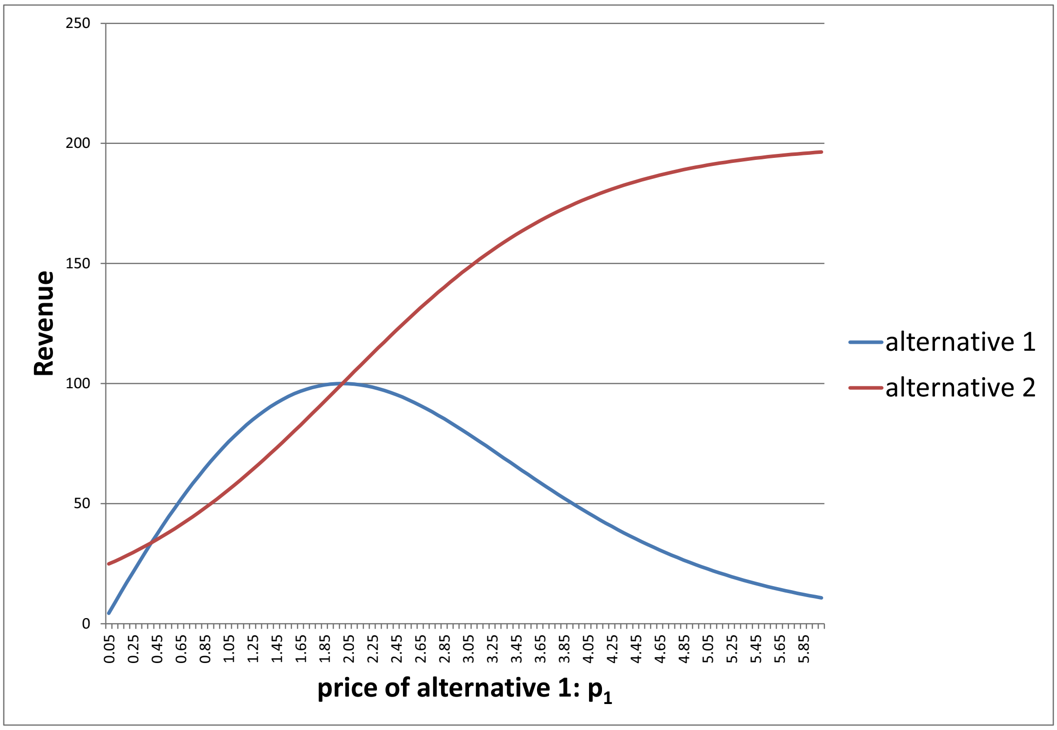
$$y_{2,n} = \frac{\exp(\beta_n \bar{p}_2 + z_2)}{\exp(\beta_n p_1 + z_1) + \exp(\beta_n \bar{p}_2 + z_2)} \qquad n \in \{1,2\} \tag{7.53}$$

$$p_1 \geq 0 \tag{7.54}$$

The revenue for the first alternative (7.51) is the sum of the revenues resulting from the attracted individuals in each group. In order to apply the proposed formulation given in section 7.3, the objective function of this problem should be modified with the penalty terms.

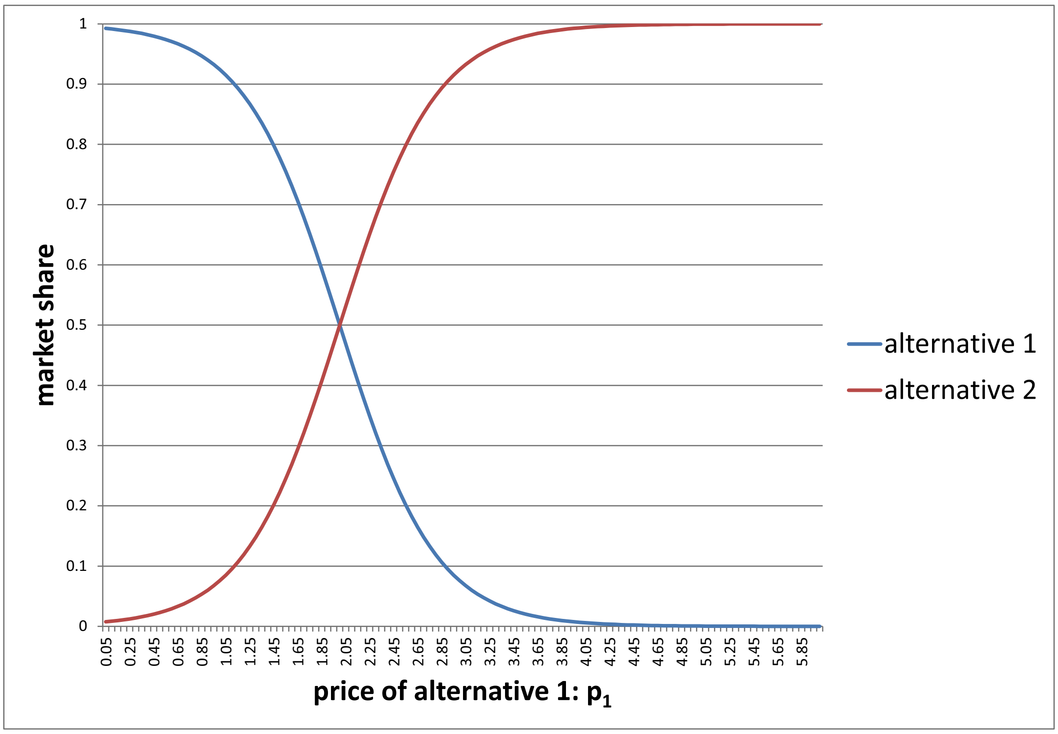


(a) Market share

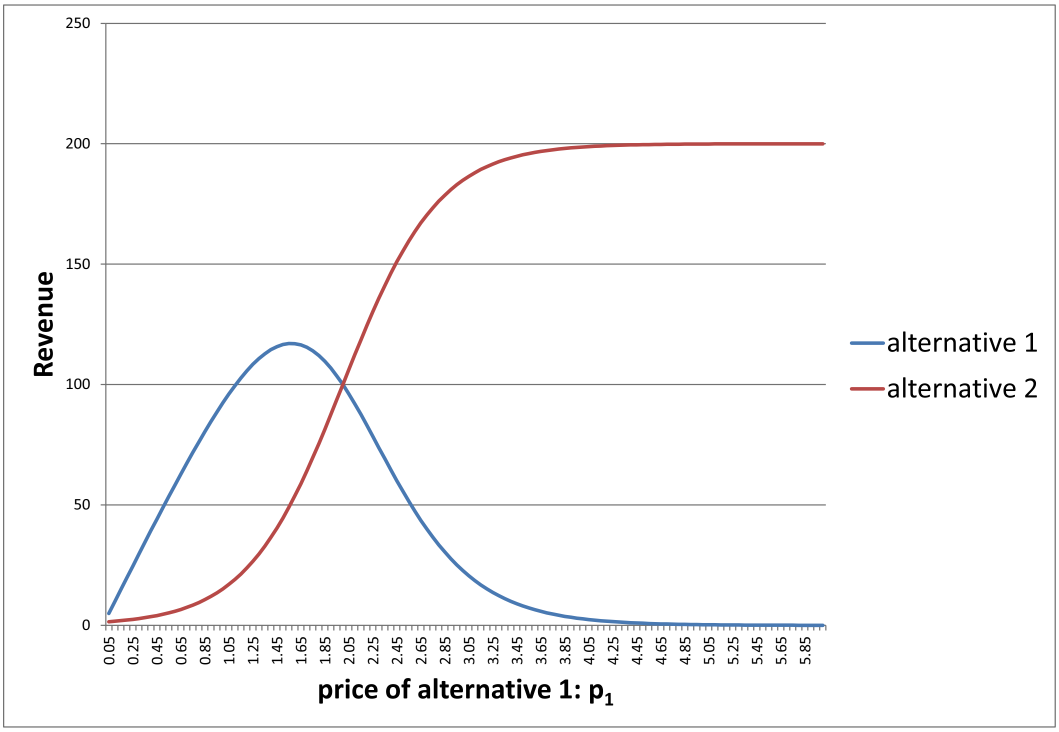


(b) Revenue

Figure 7.2: The market share and the revenue with $\beta = -1$



(a) Market share



(b) Revenue

Figure 7.3: The market share and the revenue with $\beta = -2.5$

The revenue for each group is given as:

$$R_{1,1} = 600 y_{1,1} p_1, \quad R_{1,2} = 400 y_{1,2} p_1 \tag{7.55}$$

As done before, we define $y'_{i,n}$ as the logarithm of the choice probabilities. Furthermore, as explained in section 7.3, $R'_{i,n}$ is defined as the logarithm of the revenue for alternative i and group n.

The reformulated model for the problem is given by:

$$\begin{aligned}
z_{\mathrm{P}_2^{\ln}} = \max\ & \sum_{n=1}^{2} R_{1,n} - \sum_{n=1}^{2} M1_n (R_{1,n} - \exp(R'_{1,n}))^2 \\
& - \sum_{n=1}^{2}\sum_{i=1}^{2} M2_{i,n}(y_{i,n} - \exp(y'_{i,n}))^2 && (7.56)\\
\text{s.t. } & R'_{1,1} \le \ln(600) + y'_{1,1} + \ln(p_1) && (7.57)\\
& R'_{1,2} \le \ln(400) + y'_{1,2} + \ln(p_1) && (7.58)\\
& R_{1,n} \le \exp(R'_{1,n}) \qquad n \in \{1,2\} && (7.59)\\
& y'_{1,n} = v'_n + \beta_n p_1 + z_1 \qquad n \in \{1,2\} && (7.60)\\
& y'_{2,n} = v'_n + \beta_n \bar{p}_2 + z_2 \qquad n \in \{1,2\} && (7.61)\\
& \sum_{i=1}^{2} y_{i,n} = 1 \qquad n \in \{1,2\} && (7.62)\\
& y_{i,n} \le \exp(y'_{i,n}) \qquad i,n \in \{1,2\} && (7.63)\\
& y_{i,n} \ge 0 \qquad i,n \in \{1,2\} && (7.64)\\
& y'_{i,n} \in \mathbb{R} \qquad i,n \in \{1,2\} && (7.65)\\
& R_{1,n} \ge 0 \qquad n \in \{1,2\} && (7.66)\\
& R'_{1,n} \in \mathbb{R} \qquad n \in \{1,2\} && (7.67)\\
& v'_n \in \mathbb{R} \qquad n \in \{1,2\} && (7.68)\\
& p_1 \ge 0 && (7.69)
\end{aligned}$$

As shown in section 7.3, penalty terms are concave when $R_{1,n} \le \exp(R'_{1,n})$ and $y_{i,n} \le \exp(y'_{i,n})$. Therefore constraints (7.59) and (7.63) are included in the model. Constraints (7.57)-(7.58) define the revenue in the logarithmic representation. They are introduced as ≤ constraints, in order to have convex representation. The remainder of the model is similar to the previous example. Since we cannot avoid (7.59) and (7.63), the reformulation is also non-convex. Since this problem is also easy, it is possible to identify the global optimum point.

We perform experimental analysis with the original representation of the problem, P_2, given in (7.51)-(7.54) and the reformulated problem $\mathrm{P}_2^{\ln}$ given in (7.56)-(7.69). As done before, BONMIN solver is used for solving the two formulations. Different scenarios are tested with different price parameters, β_n, and alternative specific constants, z_i. With the analysis of the aggregate

Table 7.1: Penalties resulting with global optimum, $\beta_n = (-0.75, -0.5)$, $z_i = (-0.5, 0)$

$M1_n$	$M2_{i,n}$
100,000	100,000
100,000	10,000
100,000	1,000
100,000	100
10,000	10,000

demand model example in section 7.4.1, it is concluded that high price parameter values result with higher elasticity and make the problem more difficult. For that reason lower β values are selected in this example, in order to see the impact of disaggregate demand model clearly.

Experiment with $\beta_1 = -0.75$, $\beta_2 = -0.5$, $z_1 = -0.5$, $z_2 = 0$

The revenue function with the selected parameters is presented in Figure 7.4. In this experiment, the elasticity of the two groups are -0.93 and -0.62 respectively when $p_1 = 2$ given by the equation (7.49). The elasticities are not very different and there is a unique optimum at around $p_1 = 2.3$. This experiment presents an easy setting since the elasticity around the optimum point is not high (it is -1.16 for the first group and -0.76 for the second).

The original formulation allows to find the optimum price value. Similarly, the reformulation is solved to optimality for a wide range of penalty values. In Table 7.1 we present the set of penalty values which allows to find the optimal price. It is observed that, in this example with lower penalty values we cannot find the optimum price value. However in the aggregate demand model example given in section 7.4.1, the penalty values could be chosen much lower.

Experiment with $\beta_1 = -0.75$, $\beta_2 = -0.1$, $z_1 = -0.5$, $z_2 = 0$

The revenue function with the selected parameters is presented in Figure 7.5. It is seen that there are two critical points of the function. One is at $p_1 = 3.2$ and the second is at $p_1 = 12.2$. Visually it is obvious that the second point is the global maximum. The price elasticities of the two groups are significantly different. It is calculated as -0.93 and -0.13 respectively when $p_1 = 2$. The first local optimum occurs when there are high number of attracted individuals from both of the groups with a low price. The second occurs when there is a very high price and a number of individuals from the second group are still attracted since they are less elastic to price compared to the first group.

Since this is a more critical experiment, the reformulation needs carefully selected penalty values. The penalty values which allow the solution of the problem to optimality are presented in Table 7.2. Note that the penalty values for the revenue resulting from the two groups, $M1_1$ and $M1_2$, are selected differently. With lower penalty values, BONMIN gets stuck at

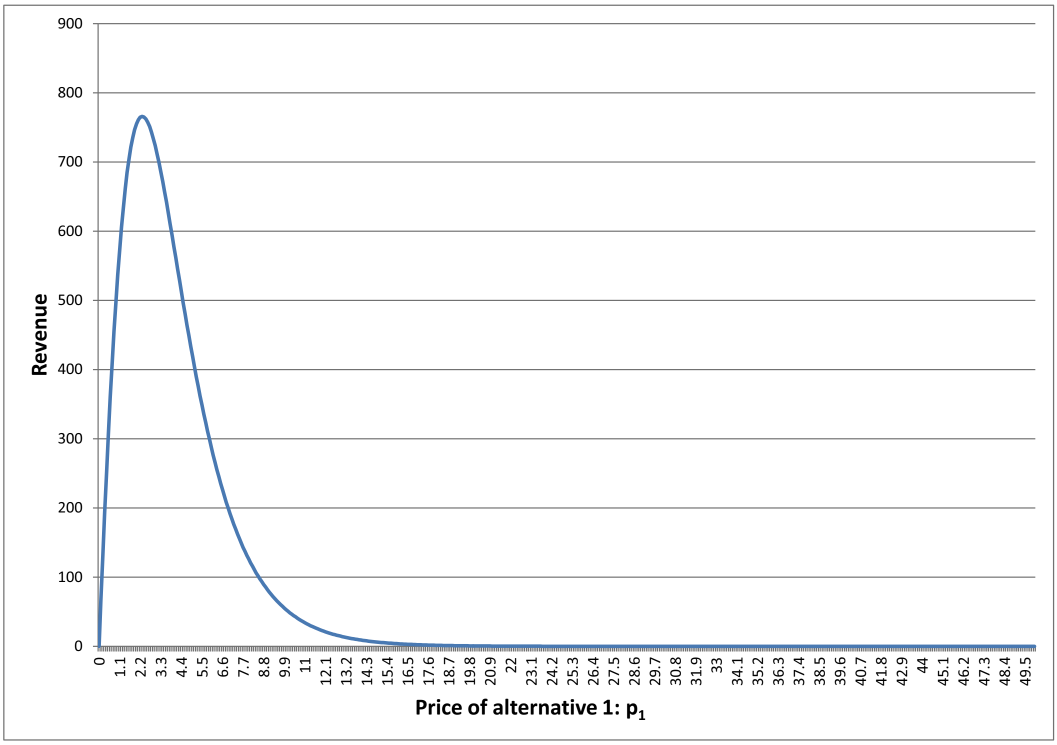


Figure 7.4: The revenue function with $\beta_n = (-0.75, -0.5)$, $z_i = (-0.5, 0)$

Table 7.2: Penalties resulting with global optimum, $\beta_n = (-0.75, -0.1)$, $z_i = (-0.5, 0)$

$M1_1$	$M1_2$	$M2_{i,n}$
100,000,000	1,000,000	1,000,000
10,000,000	100,000	1,000,000
1,000,000	10,000	1,000,000
1,000,000	10,000	100,000

the local optimum when solving the reformulated problem as happens with the original formulation. When higher penalty values are selected numerical issues may occur since the Hessian matrix becomes ill-conditioned. This example shows that when the problem becomes more difficult with significantly different elasticities, the reformulation enables us to obtain the global optimum with appropriate penalty values.

A further understanding can be obtained when the price-elasticity of demand is analyzed at the critical points. The elasticity around the first critical point is approximately -1.93 and -0.21 for the first and second group respectively. For the global maximum these values are around -9.00 and -0.98. The global optimum lies at a point with very high elasticity and makes the problem difficult to solve.

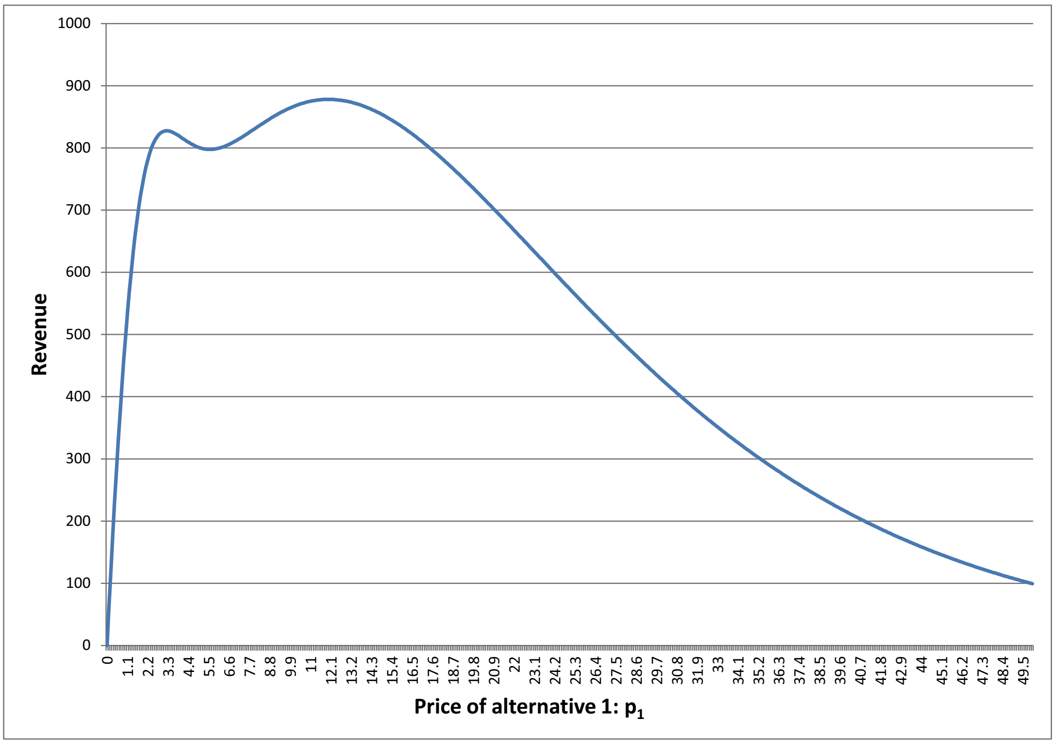


Figure 7.5: The revenue function with $\beta_n = (-0.75, -0.1)$, $z_i = (-0.5, 0)$

Experiment with $\beta_1 = -0.75$, $\beta_2 = -0.1$, $z_1 = 5$, $z_2 = 0$

In the previous experiment, different price elasticities of the two groups are analyzed. In this experiment, we also introduce a significant difference in the alternative specific constants. Having such a big constant results with an almost price-inelastic demand. The price elasticity of the two groups are calculated as -0.01 and -0.001 respectively for $p_1 = 2$. The revenue function is presented in Figure 7.6. It is observed that the optimum price is higher compared to the two previous cases and there are again two local optimum points: $p_1 = 7.8$ and $p_1 = 40.8$. The elasticity around the first critical point is approximately -2.0 and -0.009 for the first and second group respectively. For the global maximum these values are around -30.5 and -0.99. The global optimum lies at a point with extremely high elasticity and makes the problem difficult to solve. Note that, the elasticities observed in this example are not realistic and will not appear often in practice. Similarly, in the previous example the global optimum is at a point with very high elasticity for the first group of individuals. These examples are meant to investigate the role and limitations of the transformation rather than representing realistic illustrations.

When the original formulation is solved, infinite revenue is obtained due to numerical problems. Compared to the previous case, the solution of the original formulation creates more severe issues, even finding a local maximum is difficult.

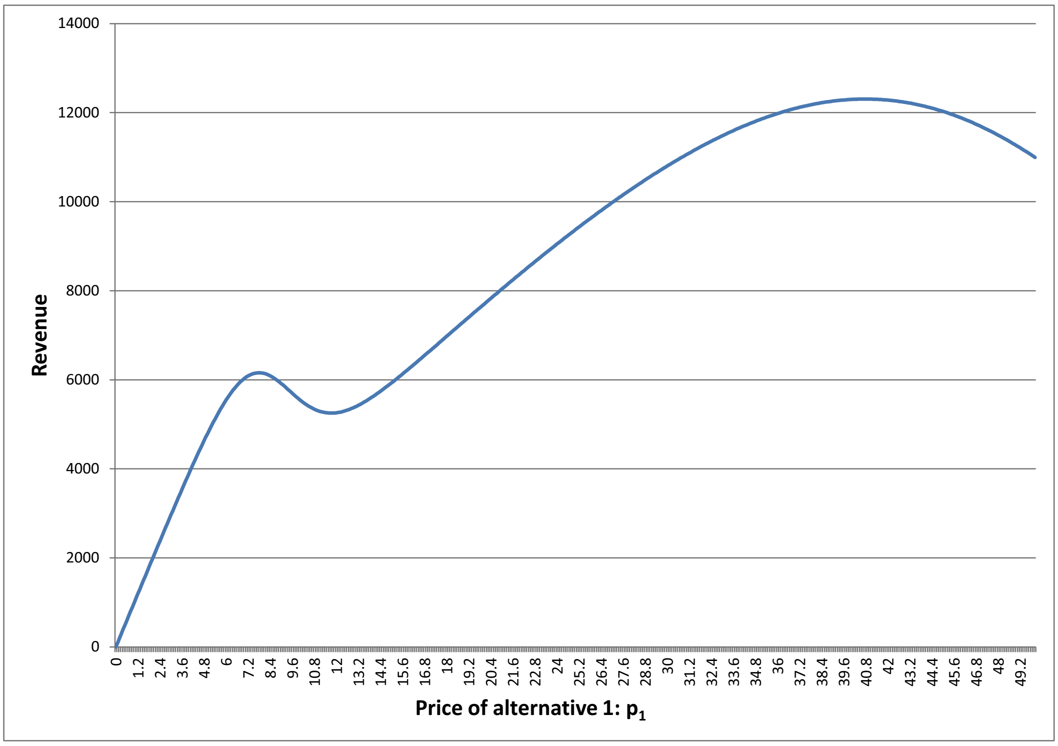


Figure 7.6: The revenue function with $\beta_n = (-0.75, -0.1)$, $z_i = (5, 0)$

Table 7.3: Penalties resulting with global optimum, $\beta_n = (-0.75, -0.1)$, $z_i = (5, 0)$

$M1_1$	$M1_2$	$M2_{i,n}$
1,000,000,000	10,000,000	1,000,000
100,000,000	1,000,000	1,000,000
1,000,000	10,000	100,000
100,000	1,000	100,000

The reformulation on the other hand, enables to obtain the global optimum with high enough penalty values. The penalty values that allow to solve the problem to optimality are presented in Table 7.3. The penalty values on the revenues for the two groups, $M1_1$ and $M1_2$, are again selected differently.

These three experiments with a disaggregate demand model show that, the individual characteristics increase the difficulty of the problem. The original formulation does not allow to find the global optimum when the population consists of significantly different characteristics. On the other hand, the reformulation can be solved to optimality given that penalty values are selected carefully. Compared to the case with aggregate demand model, higher penalty values and a finer calibration is needed for each penalty term.

As mentioned before, the constraints (7.59) and (7.63) are added in the formulation in order to have concave penalty terms. Even though these constraints themselves are not convex, they guide the algorithm for obtaining the global optimum solution. When these constraints are removed, BONMIN gets stuck at the local optimum values for the last two experiments. It is observed that, these non-convex constraints are handled without any problem in the illustrative experiments. Therefore, the reformulation is observed to be a stronger formulation compared to the original formulation. We presented the experiments only with BONMIN solver. However we expect similar results with other NLP solvers.

7.4.3 Conclusions on the illustrations

The illustrative examples provide valuable insights about the impact of the demand model parameters on the complexity of the optimization problem where price is an explanatory variable of the logit model and introduced as a decision variable in the optimization model. First of all, it is observed that the most critical parameter is the price parameter which directly affects the price elasticity. When the elasticity is high the original formulation can not be solved and the reformulation can be solved with higher penalty values. Secondly, having low price parameters is not a complete solution. If the demand model is disaggregate with individual-specific parameters, the optimal solution may be at a point where the elasticity is high. The shape of the objective function may change significantly in the existence of individuals with different elasticities. The reformulation of the model enables to obtain solutions with a careful selection of the penalty parameters for different specifications of the logit model.

7.5 A case study: airline revenue maximization

In this section we present the methodology with the revenue maximization model for airlines. The considered model is very similar to RMM-PR$^{'}$ which is the revenue sub-problem of the integrated model, IFAM-PR$^{'}$. It is given in Appendix A.3.3.

As explained in the first illustrative example in section 7.4.1, one way to deal with the non-convexity is introducing bounds on price. Therefore, in the integrated models presented in chapters 4 and 6, we impose bounds on prices with constraints (4.18) and (4.18) respectively. Since the considered demand model is aggregate, BONMIN is able to cope with the integrated model in the existence of these bounds. We acknowledge the potential use of bounds on the price in practical terms. There could be cases where airlines desire to keep the prices in pre-defined limits as a policy in specific market segments. Anyway, the existence of bounds on price may lead to sub-optimal solutions. With the proposed logarithmic transformation, the bounds can be removed in order to obtain superior decisions as the resulting formulation is stronger.

7.5.1 The model

In this section we present the original representation of the revenue management model without any bounds on the price. The objective function (7.70) is given by the sum of the revenues obtained for each itinerary operated by the airline. Competing itineraries ($i \in I_s^{'}$) do not contribute to the revenue. The market shares, u_i, are defined by the logit model as given in (7.71). The market shares for the competing itineraries are given by (7.72).

$$z_{\widetilde{\text{RMM}}} = \max \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I_s^{'})} u_i p_i \tag{7.70}$$

$$\text{s.t. } u_i = \frac{\exp(\beta_i \ln(p_i) + z_i)}{\sum_{j\in(I_s\setminus I_s^{'})}\exp(\beta_j \ln(p_j) + z_j) + \sum_{j^{'}\in I_s^{'}}\exp(\beta_{j^{'}} \ln(\bar{p}_{j^{'}}) + z_{j^{'}})} \quad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \tag{7.71}$$

$$u_j = \frac{\exp(\beta_j \ln(\bar{p}_j) + z_j)}{\sum_{\ell\in(I_s\setminus I_s^{'})}\exp(\beta_\ell \ln(p_\ell) + z_\ell) + \sum_{\ell^{'}\in I_s^{'}}\exp(\beta_{\ell^{'}} \ln(\bar{p}_{\ell^{'}}) + z_{\ell^{'}})} \quad \forall h\in H, s\in S^h, j\in I_s^{'} \tag{7.72}$$

$$\sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I_s^{'})} \delta_{i,f} u_i \le \text{Cap}_f \quad \forall f\in F \tag{7.73}$$

$$p_i \ge 0 \quad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \tag{7.74}$$

The considered logit model is the one estimated in Chapter 3. It is defined by attributes of the itineraries, i.e. there is no socio-economic characteristics or individual specific parameters. Therefore, the choice probability is equivalent to the market shares of the alternatives. The only policy variable is the price, p_i, i.e. it is defined as a decision variable of the optimization model. All the other explanatory variables are aggregated and represented by z_i which is a constant for every alternative i. For the specification of the utility function, we refer to Table 3.1. Constraints (7.73) ensure that the total demand for a flight does not exceed the allocated capacity, Cap_f. For this problem we assume that capacity is given typically by a decision problem on airline fleet assignment.

This model is a non-convex problem due to the objective function and the market share definition with logit formula. The same model can be represented in an easier formulation as done in Chapter 6, by replacing the market share constraints (7.71) with:

$$u_i = v_s \exp(\beta_i \ln(p_i) + z_i) \quad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}), \tag{7.75}$$

$$u_j = v_s \exp(\beta_j \ln(\bar{p}_j) + z_j) \quad \forall h\in H, s\in S^h, j\in I_s^{'}, \tag{7.76}$$

$$\sum_{i\in I_s} u_i = 1 \quad \forall h\in H, s\in S^h, \tag{7.77}$$

$$v_s \ge 0 \quad \forall h\in H, s\in S^h, \tag{7.78}$$

which is the first step towards the logarithmic transformation presented in section 7.3. Furthermore, this formulation, that is referred as $\widetilde{\text{RMM}}^{'}$, is numerically more stable. In the next section we apply the logarithmic transformation procedure given in section 7.3.

7.5.2 The reformulated model

For the reformulation of the problem, we define the logarithm of revenue and market share variables as $R_i^{'}$ and $u_i^{'}$ respectively. The objective function (7.79) represents the reformulated revenue function. The logarithmic transformation of the revenue and market share variables results with deviation from the original variables. Similar to the example in section 7.4.2, penalty terms are introduced for the for both of them.

$$
\begin{aligned}
z_{\widetilde{\mathrm{RMM}}^{\ln}} = \max & \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I_s^{'})} R_i - M1_i(R_i-\exp(R_i^{'}))^2 \\
& -\sum_{h\in H}\sum_{s\in S^h}\sum_{i\in I_s} M2_i(u_i-\exp(u_i^{'}))^2 && (7.79)\\
\text{s.t. } & R_i^{'}=\ln(D_s)+u_i^{'}+p_i^{'} && \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \quad (7.80)\\
& R_i\le\exp(R_i^{'}) && \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \quad (7.81)\\
& u_i^{'}=v_s^{'}+\beta_i p_i^{'}+z_i && \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \quad (7.82)\\
& u_j^{'}=v_s^{'}+\beta_j \bar{p}_j^{'}+z_j && \forall h\in H, s\in S^h, j\in I_s^{'} \quad (7.83)\\
& \sum_{i\in I_s} u_i=1 && \forall h\in H, s\in S^h \quad (7.84)\\
& u_i\le\exp(u_i^{'}) && \forall h\in H, s\in S^h, i\in I_s \quad (7.85)\\
& \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I_s^{'})}\delta_{i,f}u_i\le \mathrm{Cap}_f && \forall f\in F \quad (7.86)\\
& R_i\ge 0 && \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \quad (7.87)\\
& R_i^{'}\in\mathbb{R} && \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \quad (7.88)\\
& u_i\ge 0 && \forall h\in H, s\in S^h, i\in I_s \quad (7.89)\\
& u_i^{'}\in\mathbb{R} && \forall h\in H, s\in S^h, i\in I_s \quad (7.90)\\
& v_s^{'}\in\mathbb{R} && \forall h\in H, s\in S^h \quad (7.91)\\
& p_i^{'}\in\mathbb{R} && \forall h\in H, s\in S^h, i\in(I_s\setminus I_s^{'}) \quad (7.92)
\end{aligned}
$$

The considered logit model has the price as a logarithm in the utility function as described in chapter 3. Therefore we define $p_i^{'}$ in order to represent the logarithm of the price and we do not need to define the original price variables. As mentioned in section 7.3, the utility function can be introduced as a linear constraint if it is linear in decision variables of the optimization model, which is the price. However, it is possible to relax this condition whenever it is appropriate to define a new variable for the nonlinear relation. In this formulation, instead of working with original price variable, we define the logarithm of price and the utility can be represented as a linear constraint.

Constraints (7.80) give the logarithm of the revenue for each itinerary ($D_s u_i p_i$). Constraints (7.81) and (7.85) are introduced to ensure the concavity of the penalty terms. Constraints (7.82) and (7.83) provide the transformed market shares of the itineraries. $v_s^{'}$ is the logarithm of the variable v_s given in (7.9). Note that, as done for the price variable we work with the logarithm only and do not need the original variable v_s. The price of competing itineraries ($i\in I_s^{'}$) are given as parameters, $\bar{p}$, as usual.Constraints (7.84) maintain that the market shares of the itineraries in the same segment sum up to 1. Capacity constraints (7.86) are introduced as done for the original formulation $\widetilde{\mathrm{RMM}}$. The variable definitions are given in (7.89)-(7.92).

The presented reformulation can be simplified since there is indeed no need for the first set of penalty values, $M1_i$. The reasoning is that, the maximization will result with no slack on constraints (7.81) and R_i will be the best when it is equal to $\exp(R_i^{'})$ since there is no other constraint on R_i. Furthermore constraints (7.80) could be introduced as equality constraints since price is introduced as $p_i^{'}$. The only relaxed variable on the right hand side is the market shares. Therefore, if the deviation of the market share is penalized, there is not need for $M1_i$ and can simply be replaced by 0. The same simplification could not be done in the illustrative example of section 7.4.2 since the definition of revenue variables is given as ≤ constraints, in (7.57) and (7.58).

An important understanding is the concept of spill in case of integrated model with explicit demand representation. When the demand is introduced as a variable rather than an input parameter, the passengers should settle to the desired itineraries according to the demand model. This includes the concept of spill and therefore there is no need for an explicit modeling. However, this holds when explanatory variables are not constrained in the optimization model. In our context, there is no need for spill consideration when the price values are not limited. In other words, no spill occurs when the model is free to optimize the price without any bounds as the price will match the optimum given the available capacity. Since the non-convexity of the models do not allow for such a case due to numerical tractability, the integrated models in chapters 4 and 6 do consider spill. In chapter 4, the spill variables, $t_{i,j}$ are explicitly modeled as a control of the airline and recapture ratios $b_{i,j}$ are modeled with the logit used for the demand model. On the other hand in chapter 6, the spill is maintained by allowing the market share to be less than the potential market share given by the logit model (see constraints (6.7) and (6.18)). However, with the transformed model presented in this section, $\widetilde{\text{RMM}}^{\ln}$, we do not need to impose bounds on the price (7.92) and the market share equations can be given as equality (7.82). This is an added value of the logarithmic transformation. In this chapter the analysis is performed with the revenue maximization model only. However the same holds for the integrated model. The integrated model can be written without any bounds on the price. In Appendix A.7 we present the integrated model, that is given in chapter 6, with the log transformation and provide an illustrative example.

7.5.3 Application of the models

The application of the models is carried out with realistic data instances that are generated using the ROADEF Challenge 2009 dataset as done throughout the thesis. 3 data instances are selected with 10, 97 and 980 flights and referred as *small*, *medium* and *large* size instances respectively as seen in Table 7.4. The small and medium data instances are actually the experiments 1 and 25 introduced in Table 5.1. The large one was not considered for the integrated model throughout the thesis since it is not tractable in terms of computational time. However, if we consider the log transformation we can treat such large instances.

For the capacity related information, consider that there is an external decision tool which

Table 7.4: Air travel data instances

	Small	Medium	Large
Number of airports:	3	8	161
Number of flights:	10	97	980
Total demand (passengers):	519	8,811	164,137
Number of itineraries:	16	106	2,197
Level of service:	Nonstop	Nonstop/one-stop	Nonstop/one-stop
Assigned capacity for all flights:	50 seats	100 seats	195 seats

Table 7.5: Experimental results

		Small instance		**Medium instance**		**Large instance**	
$\widetilde{\textbf{RMM}}$	Revenue	*does not converge*		*does not converge*		*does not converge*	
$\widetilde{\textbf{RMM}}'$	Revenue	77,721		1,661,130		*does not converge*	
	Runtime	0.02 sec		0.04 sec		-	
$\widetilde{\textbf{RMM}}^{\ln}$		$M2_i$	Deviation*	$M2_i$	Deviation*	$M2_i$	Deviation**
	Experiments	10^4	220.7619%	10^6	36.1224%	10^8	0.7197%
		10^5	22.0762%	10^7	3.6126%	10^9	0.0720%
		10^6	2.2076%	10^8	0.3612%	10^{10}	0.0072%
		10^7	0.2208%	10^9	0.0361%	10^{11}	0.0008%
		10^8	0.0220%	10^{10}	0.0036%	10^{12}	0.0001%
		10^9	0.0022%	10^{11}	0.0006%	10^{13}	0.0000%
		10^{10}	0.0003%	10^{12}	0.0000%		
		10^{11}	0.0000%				
	Avg. runtime	0.02 sec		0.06 sec		4.87 sec	

(* Deviation from the revenue obtained with $\widetilde{\text{RMM}}'$)
(** Deviation from the revenue obtained with a penalty of 10^{13})

provides the capacity for each flight, Cap_f. For the presented data instances, the capacity of the flights in the network is assumed to be the same with 50, 100 and 195 seats for the small, medium and large size instances respectively.

In order to solve $\widetilde{\text{RMM}}$ and $\widetilde{\text{RMM}}'$, presented in section 7.5.1 and the transformed problem, $\widetilde{\text{RMM}}^{\ln}$, given in section 7.5.2, BONMIN is used as done for the illustrative examples.

The results for the three versions of the model are given in Table 7.5. It is observed that $\widetilde{\text{RMM}}$ cannot be solved for any of the instances. On the other hand $\widetilde{\text{RMM}}'$ can be solved for the small and medium instances, but not for the large instance. For the reformulated model several penalty values are tested. As mentioned before, when applying the logarithmic transformation, we do not need to penalize the deviation of the revenue variables. Therefore, the experiments are carried out with $M2_i$ only. For the small and medium instances, the deviation from the solution obtained by $\widetilde{\text{RMM}}'$ is reported. The penalty is increased by an order of magnitude until a deviation inferior to 1×10^{-4} is reached. For the large instance there is no reference point since we cannot solve the problem with the original formulations of the model. Therefore the reported deviations for the large instance are reported with respect to the solution obtained with a penalty of 10^{13}. For each instance, the same set of prices are found even if the final revenue is different due to deviation. It is observed that when the size of the instance increases, a higher penalty value is needed to converge. The runtime for the experiments is negligible even for the large instance which represents a large real network with 980 daily flights.

The case-study shows that, revenue maximization problem with explicit demand model is intractable to handle with the original formulation for realistic size data instances. With the logarithmic transformation, solutions can be obtained with available solvers. Note that we present results with the BONMIN solver only.

7.6 Obtaining a valid upper bound

The proposed formulation with logarithmic transformation is non-convex. The solution of this non-convex formulation provides a feasible solution which gives a lower bound to the revenue but no proof of optimality. In order to have a valid upper bound to the revenue, we propose an approximation in this section. We use the notation for the general case given in section 7.2.

The penalty terms in the previous section are introduced in order to penalize the deviation between the original variables and the logarithmic transformation, namely the deviation between $R_{i,n}$ and $\exp(R'_{i,n})$; $y_{i,n}$ and $\exp(y'_{i,n})$. Working with penalties is preferred rather than simply imposing $R_{i,n} = \exp(R'_{i,n})$ and $y_{i,n} = \exp(y'_{i,n})$ since the non-convexity with the equality constraints is more difficult to handle.

In order to obtain an upper bound on the revenue, we elaborate on the equality constraints. Let's consider the transformation of the choice probability. The equality $y_{i,n} = \exp(y'_{i,n})$ can be represented as the intersection of:

$$y_{i,n} \leq \exp(y'_{i,n}) \qquad \forall i \in C, n \in N, \tag{7.93}$$

$$y_{i,n} \geq \exp(y'_{i,n}) \qquad \forall i \in C, n \in N. \tag{7.94}$$

Constraints (7.93) are non-convex as mentioned in section 7.3. On the other hand, constraints (7.94) are convex and can be safely kept in the model. Constraints (7.93) can be approximated from above with a piecewise linear function as illustrated in Figure 7.7. It is observed that the feasible region in the existence of both constraints are the separable convex regions under each of the pieces. A similar approach is carried out by D'Ambrosio and Lee (2009) in an extended framework where feasible region is separable into convex and concave subregions. In each convex/concave region a further approximation is done with piecewise linear functions. In our case the function we address is concave and we can proceed with the second step directly.

The model with the piecewise linear approximation is given as:

$$z_{\text{pln}}^{\text{pwl}} = \max \sum_{i \in C^o} \sum_{n \in N} R_{i,n} \tag{7.95}$$

$$\text{s.t. } R'_{i,n} \leq y'_{i,n} + \ln(p_i) \quad \forall i \in C^o, n \in N \tag{7.96}$$

$$y'_{i,n} = v'_n + V_{i,n}(p_i, z_{i,n}; \beta) \quad \forall i \in C^o, n \in N \tag{7.97}$$

$$y'_{j,n} = v'_n + V_{j,n}(\bar{p}_j, z_{j,n}; \beta) \quad \forall j \in C^c, n \in N \tag{7.98}$$

$$\sum_{i \in C} y_{i,n} = 1 \quad \forall n \in N \tag{7.99}$$

$$g_{i,n}(y_{i,n}, p_i) \leq 0 \quad \forall i \in C, n \in N \tag{7.100}$$

$$y_{i,n} \geq \exp(y'_{i,n}) \quad \forall i \in C, n \in N \tag{7.101}$$

$$y_{i,n} \leq \tilde{y}_b + (y'_{i,n} - \tilde{y}'_b)\frac{\tilde{y}_{b+1} - \tilde{y}_b}{\tilde{y}'_{b+1} - \tilde{y}'_b} + M(1 - \omega 1_{b,i,n}) \quad \forall b \in B1, i \in C, n \in N \tag{7.102}$$

$$y'_{i,n} \leq \tilde{y}'_{b+1} + M(1 - \omega 1_{b,i,n}) \quad \forall b \in B1, i \in C, n \in N \tag{7.103}$$

$$y'_{i,n} \geq \tilde{y}'_b - M(1 - \omega 1_{b,i,n}) \quad \forall b \in B1, i \in C, n \in N \tag{7.104}$$

$$\sum_{b \in B1} \omega 1_{b,i,n} = 1 \quad \forall i \in C, n \in N \tag{7.105}$$

$$y_{i,n} \geq 0 \quad \forall i \in C, n \in N \tag{7.106}$$

$$y'_{i,n} \in \mathbb{R} \quad \forall i \in C, n \in N \tag{7.107}$$

$$R_{i,n} \geq \exp(R'_{i,n}) \quad \forall i \in C^o, n \in N \tag{7.108}$$

$$R_{i,n} \leq \tilde{R}_b + (R'_{i,n} - \tilde{R}'_b)\frac{\tilde{R}_{b+1} - \tilde{R}_b}{\tilde{R}'_{b+1} - \tilde{R}'_b} + M(1 - \omega 2_{b,i,n}) \quad \forall b \in B2, i \in C^o, n \in N \tag{7.109}$$

$$R'_{i,n} \leq \tilde{R}'_{b+1} + M(1 - \omega 2_{b,i,n}) \quad \forall b \in B2, i \in C^o, n \in N \tag{7.110}$$

$$R'_{i,n} \geq \tilde{R}'_b - M(1 - \omega 2_{b,i,n}) \quad \forall b \in B2, i \in C^o, n \in N \tag{7.111}$$

$$\sum_{b \in B2} \omega 2_{b,i,n} = 1 \quad \forall i \in C^o, n \in N \tag{7.112}$$

$$R_{i,n} \geq 0 \quad \forall i \in C^o, n \in N \tag{7.113}$$

$$R'_{i,n} \in \mathbb{R} \quad \forall i \in C^o, n \in N \tag{7.114}$$

$$v'_n \in \mathbb{R} \quad \forall n \in N \tag{7.115}$$

$$p_i \geq 0 \quad \forall i \in C^o \tag{7.116}$$

$$\omega 1_{b,i,n} \in \{0,1\} \quad \forall b \in B1, i \in C, n \in N \tag{7.117}$$

$$\omega 2_{b,i,n} \in \{0,1\} \quad \forall b \in B2, i \in C^o, n \in N \tag{7.118}$$

The piecewise linear approximation for the choice probability is given by (7.102)-(7.105). Consider that there is a set of breakpoints B1 for the piecewise function. $(\tilde{y}'_b, \tilde{y}_b)$ represents the breakpoint b where $\tilde{y}_b = \exp(\tilde{y}'_b)$. Binary variable $\omega 1_{b,i,n}$ is 1 if the constraint defined by the piece starting with breakpoint b is active for $y'_{i,n}$ and $y_{i,n}$. Constraints (7.102) actually define the line segment between b and $b+1$ and relate $y'_{i,n}$ and $y_{i,n}$ accordingly if this line is active. Constraints (7.103)-(7.104) restrict the variable $y'_{i,n}$ to be in the correct interval. Finally, constraints (7.105) ensure that only 1 piece is active for each alternative i and individual n. The piecewise linear approximation related to the revenue variable is similarly given by the constraints (7.109)-(7.112). The associated binary variables are called $\omega 2_{b,i,n}$ for the set of

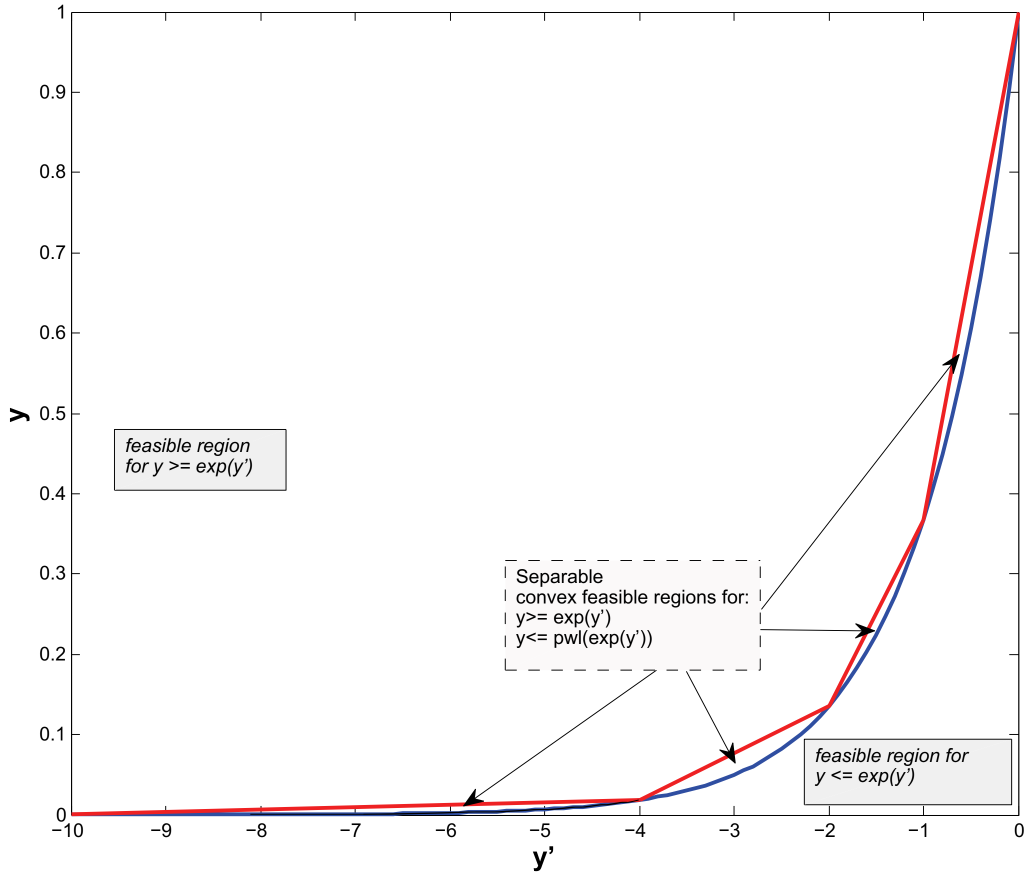


Figure 7.7: Piecewise linear approximation

breakpoints $b \in B2$. $(\tilde{R}_b^{'}, \tilde{R}_b)$ similarly represents the breakpoint b with $\tilde{R}_b = \exp(\tilde{R}_b^{'})$.

Therefore the resulting model consists of separable convex programming problems; the objective function is linear, the constraints (7.96), (7.100), (7.101), and (7.108) are convex constraints and the remaining constraints are linear. The binary variables for the piecewise linear approximation can be considered in a branch-and-bound framework as a solution methodology.

7.7 Conclusions and future research

In this chapter we present a general framework for the integration of explicit demand models in revenue maximization problems. The considered demand model is formulated as logit and the presented methodology is flexible for a wide range of model specifications. The utility function may include any number of explanatory variables some of which can be defined as the decision variables of the optimization model. The explanatory variables can be at a disaggregate level such as socio-economic characteristics.

The presented methodology is analyzed with illustrative examples in order to show the advantages and disadvantages of the approach. As a case study, the airline revenue optimization problem is presented. It is shown that the reformulation enables to obtain solutions in reasonable computational time for real flight networks where no solution can be obtained with the original formulation.

In this chapter the results with the reformulated model are provided in an experimental setting. The penalty values are selected as high as possible at the beginning which do not create any numerical issues. Note that high penalty values lead to numerical instability. The values of the penalties are then lowered until the pricing decision starts to be different. Therefore the set of penalty values with which the same pricing decision is obtained are presented. An interesting future work would be an algorithmic framework for the selection of penalty parameters which provides the appropriate penalties depending on the problem setting in an efficient way.

The presented idea for obtaining a valid upper bound in section 7.6 is an imminent future research direction. Since the logarithmic transformation significantly improves the formulation, we believe that a future direction in approximating the model to obtain valid bounds is promising. Moreover, an interesting research topic would be the design of an algorithm which stays in the convex part of the feasible region. As mentioned before, the integrated models in chapters 4 and 6 are studied with bounds on price which help for the convergence. These cuts could be studied for obtaining such an algorithmic framework.

The considered optimization problem assumes that supply information is given, i.e. supply decisions on capacity are already taken. The presented methodology is expected to facilitate the solution of integrated models where supply and demand decisions are taken simultaneously. Appropriate decomposition methods can be developed for the solution of such integrated models since the revenue sub-problem can be formulated as a convex problem. We demonstrate the solution of the integrated model in a Lagrangian relaxation framework with subgradient optimization in Appendix A.8, and in a Generalized Benders' Decomposition framework in Appendix A.9. Given that an efficient algorithm is obtained for the solution of the piecewise linear approximation, the presented decomposition methods will provide valid bounds.

Innovative application Part III

Part III provides an application with the new design of aircraft: Clip-Air. This part analyses Clip-Air that changes the concept of traveling and approaches flexibility from a different angle. The general concept is described with a focus on its impact in transportation systems. The advantages of this flexible system is analyzed in comparison to standard aircraft in several scenarios. For the analysis, the models developed in Part II are used.

8 Impact analysis of a flexible air transportation system

The chapter provides analytical evidence of the added-value of flexibility for air transportation systems. More specifically, the impact of a new innovative modular aircraft on the operations of an airline is deeply analyzed. The impact analysis is carried out with an integrated schedule planning model which presents a combination of appropriate optimization and behavioral modeling methodologies. The results show that the flexible system uses the transportation capacity more efficiently by carrying more passengers with less overall capacity. Moreover, it is observed that the flexible system deals better with insufficient transportation capacity. Furthermore, the scheduling decisions are robust to the estimated cost figures of the new system. For the analyzed range of costs, it is always carrying more passengers with less allocated capacity compared to a standard system.

8.1 Introduction and related literature

According to the statistics provided by the Association of European Airlines (AEA), air travel traffic has grown at an average rate of 5% per year over the last three decades (AEA, 2007)[1]. Consequently, sustainability of current transportation systems is threatened by increased energy consumption and its environmental impacts. Moreover, the increased mobility needs are inducing major disruptions in operations. Regarding air transportation, there is an increased number of landings and takeoffs from airports, resulting in frequent congestion and delays. The trade-off between the sustainability of transportation and the mobility needs justifies the investigation of new concepts and new solutions that can accommodate the increased demand with a minimal impact on the environment and the economy. The building stone of such new concepts is the introduction of various aspects of flexibility in transportation systems in general, and in air transportation systems in particular.

[1] The source is included as an example for year 2007 but there are yearly releases available

8.1.1 Flexibility in transportation systems

'Flexibility' is defined as 'the ability of a system to adapt to external changes, while maintaining satisfactory system performance.' (Morlok and Chang, 2004). Flexibility is a key concept for the robustness of transportation systems and studies on flexible transportation systems have an increased pace during the last decade. We refer to the work of Morlok and Chang (2004) for the techniques to measure the flexibility with a focus on capacity flexibility. Similarly, Chen and Kasikitwiwat (2011) develop network capacity models for the quantitative assessment of capacity flexibility.

Flexibility is studied for different transportation systems including land, rail, ship and air transportation. Brake et al. (2007) provide examples of Flexible Transportation System (FTS) applications that aim to improve the connectivity of public transport networks in the context of land transportation. Crainic et al. (2010) work on the flexibility concept with Demand-Adaptive Systems which combine the features of traditional fixed-line services and purely on-demand systems. Errico et al. (2011) provide a review on the semi-flexible transit systems where different flexibility concepts are introduced on the service areas and the time schedule. Zeghal et al. (2011) studies flexibility for airlines in terms of the active fleet and departure time of flights. An airline can increase or decrease the fleet size renting or renting out planes. Departure times can be adjusted within a given time-window. These flexibilities facilitate the integration of schedule design, fleet assignment, and aircraft routing decisions.

The nature of flexibility already embedded in transportation systems differs considerably. For example, in rail transportation, there is a natural capacity flexibility which rises from the modularity in fleet. In maritime transportation, the usage of standard unit load facilitates a more efficient practice of multi-modality with an efficient transfer between ships, trucks and trains. In this chapter we are investigating what impacts such flexibility may have in air transportation.

Rail transportation

Flexibility in rail transportation rises from modular carrying units and several operations research techniques are applied to improve this flexibility. We refer to Huisman et al. (2005) for a review on the models and techniques used in passenger railway transportation for different planning phases. Kroon et al. (2009) discuss the construction of a new timetable for Netherlands Railways which improves the robustness of the system decreasing the delays. Similarly, Jespersen-Groth et al. (2009) study the disruption management problems in passenger railway transportation drawing the analogies with airline disruption management.

Maritime transportation

Multi-modality is widely studied in the context of freight transportation where standard unit loads are transferred between maritime, land and rail transportation systems. In freight transportation, each movement of a loaded vehicle generates an empty flow and for the efficient use of the transportation system these empty flows need to be taken care of. We refer to Dejax and Crainic (1987) for a review of empty vehicle flow problems and proposed models on the subject. They also point out the potential advantages of an integrated management of loaded and empty vehicle movements. In maritime transportation Crainic et al. (1993) present models for the repositioning of empty containers in the context of a land transportation system. Olivo et al. (2005) study the repositioning problem in a multi-modal network where empty containers are transported by both maritime and land transportation. Di Francesco et al. (2009) consider empty container management problem under uncertainty and present a multi-scenario formulation regarding different realizations of uncertain parameters.

Air transportation

In the context of air transportation, airlines have dedicated a lot of efforts in increasing the flexibility through demand and revenue management (Talluri and van Ryzin, 2004a). Flexibility is obtained namely from differentiated fare products offered to different customer segments with the objective to increase the total revenue. Recently, additional attention has been paid to better represent the demand through advanced demand models. Coldren et al. (2003) work on logit models for travel demand, Coldren and Koppelman (2005) extend the models of the previous work using GEV, particularly nested logit model. Koppelman et al. (2008) apply logit models to analyze the effect of schedule delay by modeling the time of day preferences. Carrier (2008) and Wen and Lai (2010) work on advance demand modeling that enable customer segmentation with the utilization of latent class choice modeling. We refer to the work of Garrow (2010) for a comprehensive presentation of different specifications of choice models.

Advanced demand models are integrated into optimization models in different levels of the airline scheduling process. Talluri and van Ryzin (2004b) integrate discrete choice modeling into the single-leg, multiple-fare-class revenue management model. Authors provide characterization of optimal policies for the problem of deciding which subset of fare products to offer at each point in time under a general choice model of demand. Schön (2006) develops a market-oriented integrated schedule design and fleet assignment model with integrated pricing decisions. In order to deal with the non-convexity that is brought by the pricing model, an inverse demand function is used. The final model is a mixed integer convex problem and preliminary results are provided over a synthetic data. More recently Atasoy et al. (2012) introduces an integrated scheduling, fleeting and pricing model where a demand model, which is estimated on a real data, is explicitly included in the optimization model. The explicit representation of the demand model allows for further extensions of the framework with disaggregate passenger data. They also consider spill and recapture effects based on the demand model.

In addition to revenue management, schedule planning of airlines are more and more designed to be robust to unexpected disruptions, such as aircraft breakdowns, airport closures, or bad weather conditions (Lan et al., 2006; Gao et al., 2009), and associated recovery strategies are applied after the occurrence of these disruptions (Lettovsky et al., 2000; Eggenberg et al., 2010). The application of robust schedule planning models increases the profitability of airlines introducing flexibility to adapt to unexpected disruptions. In the literature, robustness is introduced for different subproblems of airline scheduling. Rosenberger et al. (2004) study a robust fleet assignment model that reduces the hub connectivity and embeds cancellation cycles in order to decrease the sensitivity to disruptions and they obtain a better performance compared to traditional fleet assignment models. Shebalov (2006) work on robust crew scheduling models where they introduce robustness by maximizing the number of crew pairs that can be swapped in case of unexpected situations. Lan et al. (2006) present two approaches to minimize passenger disruptions: a robust aircraft maintenance routing problem where they aim to reduce the delay propagation and a flight schedule re-timing model where they introduce time windows for the departure times of flight legs. Similarly, Weide (2009) studies an integrated aircraft routing and crew pairing model where the departure time of flights are allowed to vary in a time window. Inclusion of time windows in the schedule is shown to increase the flexibility of the model having improved results.

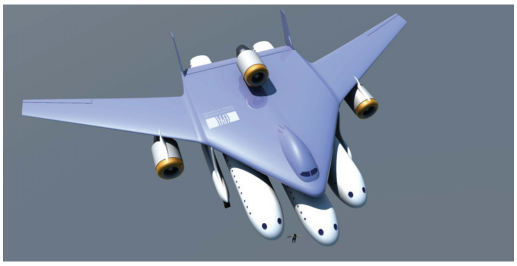
As mentioned previously, in air transportation the improvements are mostly investigated through decision support systems. Although these efforts are promising it is limited to the definition of the system itself. In this chapter we introduce and analyze a new way to bring flexibility into air transportation, based on the concept of a modular aircraft, called Clip-Air. The objective is to provide analytical evidences of the added-value of flexibility for air transportation systems.

8.1.2 A modular flexible aircraft: Clip-Air

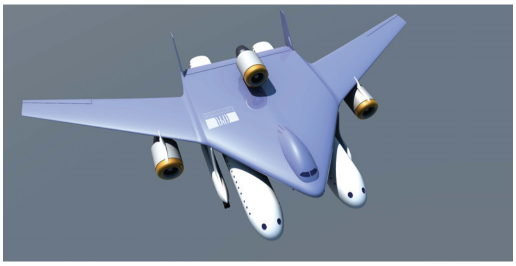
A new family of modular aircraft, called Clip-Air, is being designed at the Ecole Polytechnique Fédérale de Lausanne (EPFL, Leonardi and Bierlaire, 2011). Figures 8.1 and 8.2 illustrate the new design. Clip-Air is based on two separate structures: a *flying wing*, designed to carry the engines and the flight crew, and *capsules*, designed to carry the payload (passengers and/or freight). The wing can carry one, two or three capsules with a clipping mechanism which facilitates the separate handling of capsules. This modularity is the foundation of the Clip-Air concept for flexible transportation.

The Clip-Air project started in 2010. The project is now in its second phase called "feasibility studies" which is planned to be finished in 2013. The feasibility studies involve various research groups from EPFL that work on the aerodynamic structure, the energy aspects, the tests of Clip-Air in a simulation environment etc. Our research group is interested in the impact of the flexibility of Clip-Air on transportation systems. This impact analysis is important for understanding the potential of introducing flexibility and is expected to motivate the studies on various aspects of flexibility in other transportation systems, such as railways and transit systems.

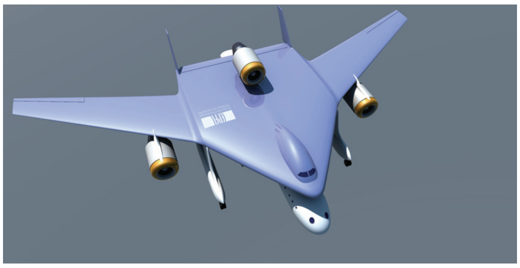
The Clip-Air project introduces a new concept in aircraft design. But its potential impact is



(a) Three capsules



(b) Two capsules



(c) One capsule

Figure 8.1: Modularity of Clip-Air

Figure 8.2: Clip-Air at an airport

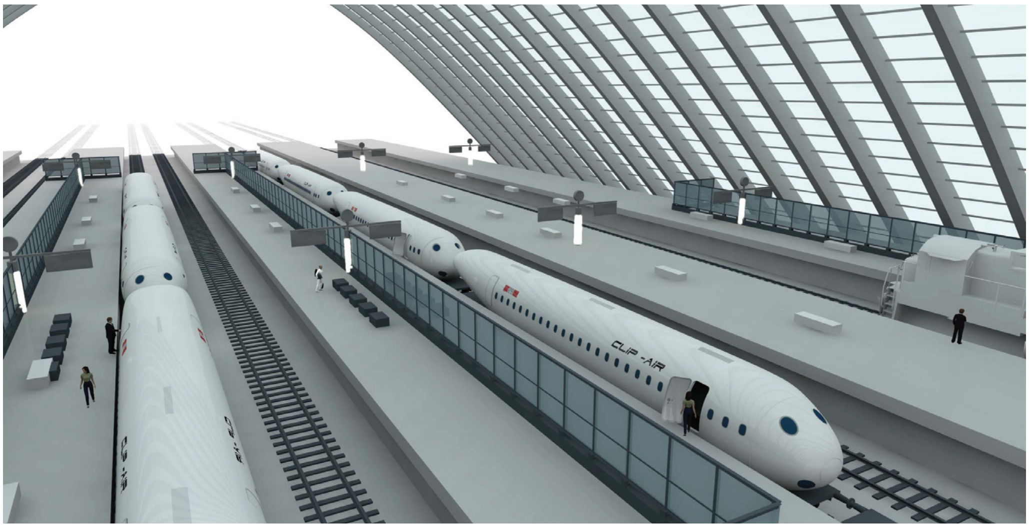
significantly more far-reaching. Indeed, the flexibility provided by the new aircraft modifies the fundamental operations of multi-modal transportation systems.

Clip-Air broadens the flexibility with its innovative design. In the first place, the decoupling of the wing and capsules brings the modularity of railways to airline operations. This decoupling provides several advantages in terms of operations. The capacity of Clip-Air can be adjusted according to the demand by changing the number of capsules to be attached to the wing. This flexibility in transportation capacity is highly important in case of unbalanced demand between airports. As another example, Clip-Air's modularity is expected to significantly improve the operations in hub-and-spoke networks where the itineraries connect through the hub airport. The flexibility of interchanging the capsules attached to the wings at the hub airport provides a better utilization of the capacity and simplifies the fleeting operations.

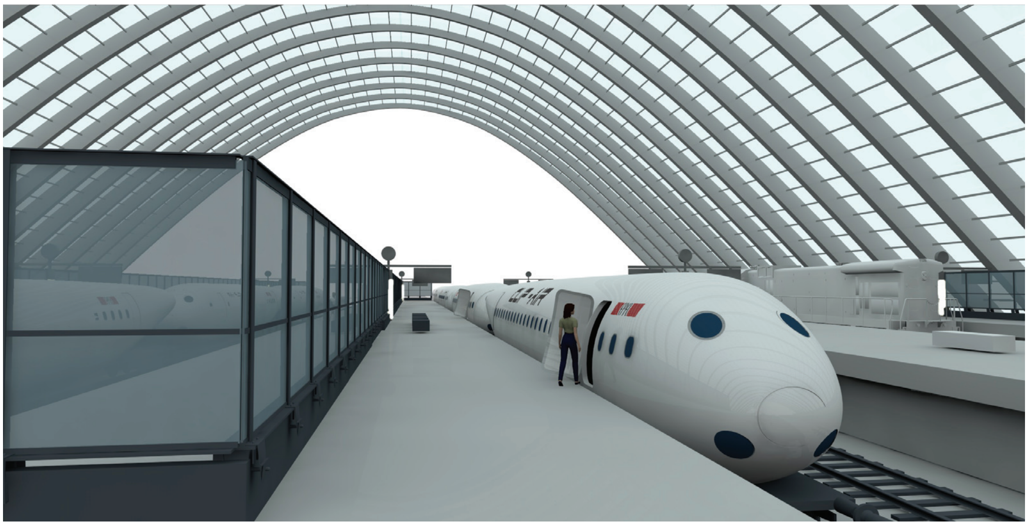
Secondly, Clip-Air imports the concept of standard unit loads from freight to passenger transportation thanks to the structure of the capsules. The capsules are easy to transfer and store which facilitates their move by other means of transportation. As an illustration, in case of unbalanced demand in the flight network, the empty capsules can be transfered by railways in order to better respond to the demand in busy airports. A similar notion is also provided for passenger transportation by the design of Clip-Air. A passenger can board the capsule at a railway station (Figure 8.3), and the loaded capsule is attached to the wing at the airport. Such a concept brings new dimensions for multi-modal transportation. Furthermore, Clip-Air is designed for both passenger and freight transportation (Figure 8.4). A capsule containing freight can fly under the same wing with passenger capsules so that mixed passenger and freight transportation can be operated without any compromise in comfort. This flexibility enables airlines to better utilize their capacity according to the variable demand pattern they are facing. All in all, the integration of air transportation in multi-modal networks, for both passenger and freight transportation, is expected to be strengthened by the design of Clip-Air.

The dedication of Clip-Air capsules to different commodities can be extended through special utilization of the capsules. Energy efficient and environmental friendly solutions are expected to be provided by dedicating one of the capsules to other sources of energy such as hydrogen (Figure 8.5). Normally, storing hydrogen is not efficient. However with Clip-Air this can be achieved since the capsule provides a substantial volume and since it is completely separated from other capsules with passengers. In addition to energy, capsules with special functions (Figure 8.6) can be attached to the wing such as capsules with medical personnel and equipment. All these possibilities provide flexibility in terms of the allocation of capsules for multiple purposes depending on the need.

The Clip-Air system combines the mentioned flexibility aspects, modularity and multi-modality, with the efficient demand management and robust scheduling methods of airlines. Therefore, the four types of flexibility (demand management, robustness and recovery, modular capacity, and multi-modality) are brought together in an integrated transportation system.

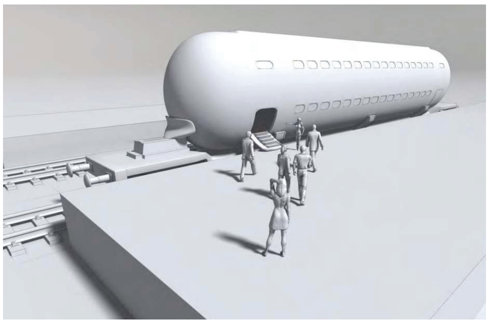


(a) Clip-Air capsules at a railway station

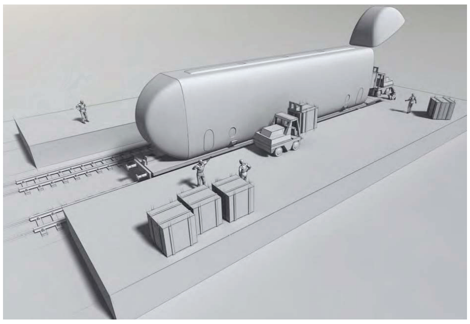


(b) A boarding passenger

Figure 8.3: Multi-modal transportation with Clip-Air



(a) passengers



(b) freight

(c) mixed

Figure 8.4: Mixed passenger and freight transportation with Clip-Air

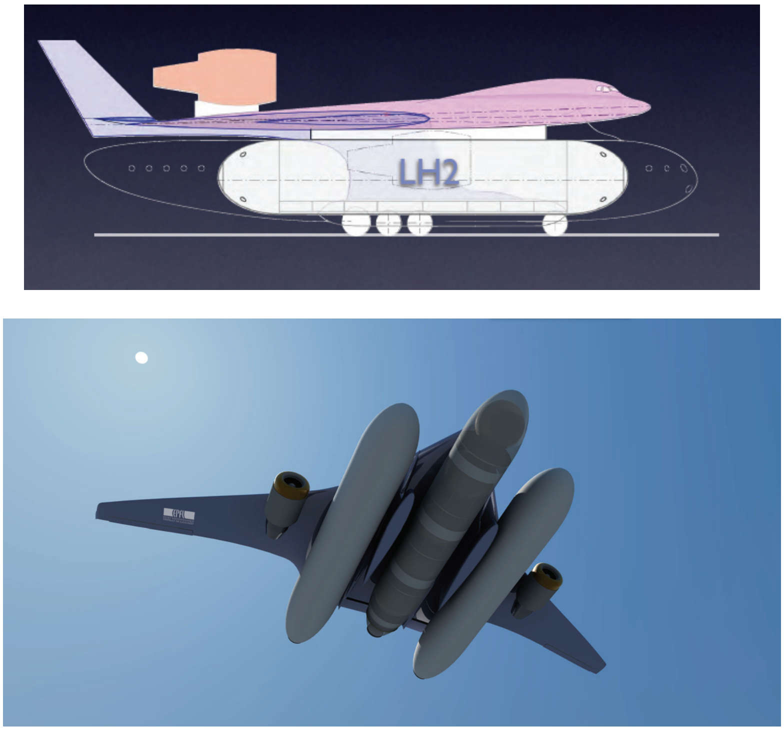


Figure 8.5: Energy solutions with Clip-Air

Figure 8.6: Multi-purpose Clip-Air capsules

8.1.3 Impact analysis of the flexibility of Clip-Air

The objective is to analyze the impact of Clip-Air's flexibility from an airline's perspective through the application of appropriate methodologies. For the concept of flexibility we focus on modularity and demand management. The design of Clip-Air has impact on many processes of air transportation. We focus on fleeting since Clip-Air's modularity alters the fleet assignment process considerably and the impact of flexibility can be directly observed through fleeting.

The novelty of the presented model is that it captures the modularity of Clip-Air by a simultaneous decision on the two levels of assignments: the assignment of wing to the flights and the assignment of capsules to the wing. This model is indeed the extension of IFAM presented in section 5.2.2 to the Clip-Air with two level of fleet assignments.

Beyond the analysis of Clip-Air itself, the contribution is the analysis of flexibility in transportation systems in general based on real data and through optimization models that integrate supply demand interactions. The non-trivial integration of the models is used to carry out a comparative analysis between a standard and a flexible system. In return, the introduction of flexibility provides promising advantages and motivates the analysis of flexibility in other modes of transportation as well as the analysis of other flexibility notions. All conservative assumptions and the design of experiments are detailed constituting a valuable reference for flexible transportation systems to be designed in the future.

8.2 Integrated schedule planning

As mentioned at the end of section 8.1.3 we focus on the aspects of modular capacity and demand management in the context of airline operations.

Modular capacity is provided by the design of Clip-Air and we analyze the impacts of modularity on fleet assignment process. As illustrated in section 8.1.2 capsules can be detached from the wing. This feature generates an additional level of assignment decisions to be made in comparison to the assignment problem of standard planes. Therefore we build an integrated schedule design and fleet assignment model which enables the appropriate assignment of wing and capsules (section 8.2.1).

As for the demand management dimension, we integrate supply-demand interactions into the fleet assignment problem through spill and recapture effects as already explained in section 4.2. The logit model for the estimation of recapture ratios is estimated based on a dataset where the flights are flown by standard aircraft. For the comparative analysis between standard aircraft and Clip-Air we assumed that the utilities would be the same for the flights regardless of the considered fleet. For the passenger acceptance of Clip-Air, a further study should be carried out with the help of a stated preferences survey. The data provided by such a survey would enable to extend the demand model in order to take into account the potential impact of Clip-Air on the demand.

8.2.1 Integrated schedule design and fleet assignment model

We present an integrated schedule design and fleet assignment model which facilitates the modularity of Clip-Air. The model optimizes the schedule design, the fleet assignment, the number of spilled passengers and the seat allocation to each class. The considered model is the extension of IFAM which is presented in section 5.2.2. The model with the pricing decision, IFAM-PR, is not considered mainly because the objective is to show how the fleeting operations are altered with Clip-Air. Furthermore, we aim at presenting experimental results with relatively large instances with around 100 flights and 10 aircraft types which is difficult to address with the integrated model. However we present results for IFAM-PR for one data instance in section 8.4.

The most important difference of Clip-Air from standard planes is that the fleet assignment includes both the assignment of wing and capsules. A flight can not be realized if there is no wing assigned to that flight. When a wing is assigned there is another decision about the number of capsules to be attached to the wing. Secondly, the operating cost allocation is different such that the costs are decoupled between wing and capsules. Flight crew cost is related only to the wing and cabin crew cost is related to the capsules. As will be explained in section 8.3.1, some other cost figures are also decoupled according to the weights of wing and capsules.

In this section we present the model for a fleet composed of Clip-Air wings and capsules, which considers a single airline. Schedule design is modeled with two sets of mandatory and optional flights such that schedule design decision is to operate the optional flights or to cancel them. The decision about the subset of flights to be flown could be integrated with a different convention based on the importance of flights. The proposed demand model is flexible to take into account different level of priorities for flights provided that the data is available to estimate the associated parameters. In case of such an extension, the schedule planning model will decide on the flights to be flown based on this additional information.

$$
\begin{aligned}
& z^*_{\text{IFAM}^{\text{Clip-Air}}} = \\
& \min \sum_{f \in F} (C^w_f x^w_f + \sum_{k \in K} C_{k,f} x_{k,f}) \\
& \quad + \sum_{h \in H} \sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} (\sum_{j \in I_s} t_{i,j} - \sum_{j \in (I_s \setminus I'_s)} t_{j,i} b_{j,i}) p_i && (8.1) \\
& s.t. \sum_{k \in K} x_{k,f} = 1 && \forall f \in F^M \quad (8.2) \\
& \sum_{k \in K} x_{k,f} \le x^w_f && \forall f \in F \quad (8.3) \\
& y^w_{a,t^-} + \sum_{f \in In(a,t)} x^w_f = y^w_{a,t^+} + \sum_{f \in Out(a,t)} x^w_f && \forall [a,t] \in N \quad (8.4) \\
& \sum_{a \in A} y^w_{a,\text{minE}^-_a} + \sum_{f \in CT} x^w_f \le R_w && (8.5) \\
& y^w_{a,minE^-_a} = y^w_{a,maxE^+_a} && \forall a \in A \quad (8.6) \\
& y^k_{a,t^-} + \sum_{\substack{f \in In(a,t) \\ k \in K}} k\, x_{k,f} = y^k_{a,t^+} + \sum_{\substack{f \in Out(a,t) \\ k \in K}} k\, x_{k,f} && \forall [a,t] \in N \quad (8.7) \\
& \sum_{a \in A} y^k_{a,\text{minE}^-_a} + \sum_{\substack{f \in CT \\ k \in K}} k\, x_{k,f} \le R_k && (8.8) \\
& y^k_{a,minE^-_a} = y^k_{a,maxE^+_a} && \forall a \in A \quad (8.9) \\
& \sum_{s \in S^h} \sum_{i \in (I_s \setminus I'_s)} \delta^i_f D_i - \sum_{j \in I_s} \delta^i_f t_{i,j} + \sum_{j \in (I_s \setminus I'_s)} \delta^i_f t_{j,i} b_{j,i} \\
& \quad \le \pi_{f,h} && \forall f \in F, h \in H \quad (8.10) \\
& \sum_{h \in H} \pi_{f,h} \le \sum_{k \in K} Q\, k\, x_{k,f} && \forall f \in F \quad (8.11) \\
& \sum_{j \in I_s} t_{i,j} \le D_i && \forall h \in H, s \in S^h, i \in (I_s \setminus I'_s) \quad (8.12) \\
& x^w_f \in \{0,1\} && \forall f \in F \quad (8.13) \\
& x_{k,f} \in \{0,1\} && \forall k \in K, f \in F \quad (8.14) \\
& y^w_{a,t} \ge 0 && \forall [a,t] \in N \quad (8.15) \\
& y^k_{a,t} \ge 0 && \forall [a,t] \in N \quad (8.16) \\
& \pi_{f,h} \ge 0 && \forall f \in F, h \in H \quad (8.17) \\
& t_{i,j} \ge 0 && \forall h \in H, s \in S^h, i \in (I_s \setminus I'_s), j \in I_s \quad (8.18)
\end{aligned}
$$

The notation is similar to the notation provided in Chapters 4 and 5 except the definition of fleet with Clip-Air. However we prefer to keep the explanation on the notation for the sake of completeness. Let F be the set of flights, mandatory flights and optional flights are represented by the sets of F^M and F^O. A represents the set of airports and K represents the set of aircraft types which can be a Clip-Air wing with one, two or three capsules. The schedule is represented by time-space network such that $N(a,t)$ is the set of nodes in the time-line network, a and t being the index for airports and time respectively. In(a,t) and Out(a,t) are the sets of inbound and outbound flight legs for node (a,t). H represents the set of cabin classes which is assumed to consist of economy and business classes. S^h is the set of market segments for class h, which is taken as distinct origin and destination pairs in this study. For example, all the available business class itineraries for Geneva-Paris represent a market segment. I_s represents the set of itineraries in segment s. We include a set of no-revenue itineraries $I_s^{'} \in I_s$ for each segment s which stands for the itineraries offered by other airlines. This set of itineraries is included in order to better represent the reality by considering the lost passengers to competitive airlines.

The objective (8.1) is to minimize the operating cost and loss of revenue due to unsatisfied demand. Operating cost for each flight f, has two components that correspond to operating cost for wings and capsules which are represented by C_f^w and $C_{k,f}$ respectively. These are associated with binary decision variables of x_f^w and $x_{k,f}$. x_f^w equals one if there is a wing assigned to flight f. $x_{k,f}$ represents the number of capsules assigned to flight f in such a way that it is one if there are k capsules assigned to flight f. The decision variable on the number of capsules could also have been defined as an integer variable. However the proposed formulation allows for more modeling flexibility. For example, it would allow to extend the model to capture the possible nonlinear relation between cost and the number of capsules. $t_{i,j}$ is the decision variable for the number of passengers redirected from itinerary i to itinerary j typically when there is insufficient capacity. $b_{i,j}$ is the proportion of passengers who accept to be redirected from itinerary i to j. The price of itinerary i is represented by p_i.

Constraints (8.2) ensure that every mandatory flight should be assigned at least one capsule. Optional flights are not exposed to such a constraint which forms the decision on the schedule design. Constraints (8.3) maintain the wing capsule relation such that if there is no wing assigned to a flight, there can be no capsule assigned to that flight. On the other hand if there is a wing assigned there can be up to three capsules flying. Constraints (8.4) and (8.7) are for the flow conservation of wings and capsules. y_{a,t^-}^w and y_{a,t^-}^k represent the number of wings and capsules at airport a just before time t respectively. Similarly y_{a,t^+}^w and y_{a,t^+}^k stand for the number of wings and capsules just after time t respectively. Constraints (8.5) and (8.8) limit the usage of fleet by the available amount which is represented by R_w and R_k for wings and capsules respectively. minE_a^- represents the time just before the first event at airport a and CT is the set of flights flying at count time. In this study it is assumed that the number of wings and capsules at each airport at the beginning of the period, which is one day, is the same as the end of the period. Constraints (8.6) and (8.9) ensure this cyclic schedule property, where maxE_a^+ represents the time just after the last event at airport a.

Constraints (8.10) ensure the relation between supply and capacity. Decision variables $\pi_{f,h}$ represent the allocated seats for flight f and class h. δ_f^i is a binary parameter which is one if itinerary i uses flight f and enables us to have itinerary-based demand. The left hand side represents the actual demand for each flight taking into account the spilled and recaptured passengers, where D_i is the expected demand for each itinerary i. Therefore, the realized demand is ensured to be satisfied by the allocated capacity. Similarly, these constraints maintain that when a flight is canceled, all the related itineraries do not realize any demand.

We let the allocation of business and economy seats to be decided by the model as a revenue management decision. Therefore we need to make sure that the total allocated capacity for a flight is not higher than the physical capacity of Clip-Air and this is represented by the constraints (8.11). The capacity of one capsule is represented by Q and the total capacity can be up to $3 \times Q$. Constraints (8.12) are for demand conservation for each itinerary saying that total redirected passengers from itinerary i to all other itineraries in the same market segment should not exceed its expected demand.

8.3 Results on the potential performance of Clip-Air

For carrying out the comparative analysis between standard planes and the Clip-Air fleet we work with a dataset from a major European airline which is the same dataset used throughout the thesis. Note that the tests used in chapters 4, 5 and 6 are not appropriate to show the performance of Clip-Air since the majority of them have not high number of passengers per flight (see Table 5.1). Since Clip-Air has minimum 150 seats (case with one capsule) we work with additional instances. Only experiment 26 is common with the previous chapters, which is used as a realistic size instance for the case of Clip-Air.

In addition to the data on the flight network, we need the estimated cost figures for Clip-Air wings and capsules which are explained in section 8.3.1.

As Clip-Air exists only in a simulated environment we make the following assumptions for the comparison with standard planes:

- The results for the standard fleet have been obtained by letting the model select the optimal fleet composition from a set of different available plane types. On the other hand Clip-Air capsules are of the same size. This is an advantage for standard fleet since it is able to adjust the fleet composition according to the characteristics of the network. We only impose that the overall capacity is the same for both standard fleet and Clip-Air.
- In the set of different fleet types, the aircraft that are close to the capacities of 1 capsule, 2 and 3 capsules are kept present in the experiments (A320 - 150 seats, A330 - 293 seats, B747-200 - 452 seats). As mentioned in section 8.3.1, Clip-Air is more expensive compared to these aircraft except when flying with 3 capsules. Standard fleet and Clip-Air have almost the same set of aircraft sizes. This experimental design is meant to minimize the impacts of the differences in size and to reveal to a larger extent the impact of modularity. This is clearly in favor of the standard fleet. Having higher costs, Clip-Air can only compete with its modularity and flexibility.
- Total available transportation capacity in number of seats is sufficient to serve all the demand in the network for all the analyzed instances. It is explained in section 8.3.5 that this is in favor of the standard fleet and whenever the capacity is restricted, Clip-Air performs significantly better than the standard fleet in terms of the number of

transported passengers.

- The schedule is assumed to be cyclic so that the number of aircraft/wings/capsules at each airport is the same at the beginning and at the end of the period, which is one day. This a limiting factor for Clip-Air since the modularity of the capsules is not efficiently used in such a case. The repositioning of the capsules by other means of transport modes could lead to more profitable and efficient schedules. However, we do not take into account the repositioning possibility in this study.

- As explained in section 8.3.1, we adjust only the fuel costs, crew costs and airport navigation charges. However the design of Clip-Air is expected to considerably decrease the maintenance costs due to the simple structure of the capsules. The capsules do not necessitate critical maintenance since all the critical equipments are on the wing. Furthermore, the overall number of engines needed to carry the same amount of passengers is reduced. Consequently, maintenance costs can be further reduced. These potential savings are ignored in this study.

- We challenge Clip-Air against a schedule conceived for a standard fleet. However the decoupling of wing and capsules is expected to reduce the turn around time and this advantage is ignored in this study.

- Clip-Air is designed for both passenger and cargo transportation. When the demand is insufficient to fill three capsules, additional revenue can be generated by using a capsule for freight. This is not considered in this study.

- As shown in sections 8.3.2-8.3.5, Clip-Air is found to allocate less capacity to carry the same amount of passengers compared to standard fleet. In other words, the flight network is operated with less number of aircraft due to the modularity of Clip-Air. It means that the total investment for the airline is potentially less important for a Clip-Air fleet than for a standard fleet. In this study we do not take this into account. Therefore the potential of Clip-Air in reducing the investment costs is ignored.

- Finally, we assume that the unconstrained demand for the itineraries (D_i) and the demand model for the recapture ratios are the same when the fleet is changed to Clip-Air. The overall impacts of the new system on passenger demand is not analyzed being out of scope of this study.

The assumptions above lead to a conservative comparison between Clip-Air and standard fleet. Therefore, the results presented below provide lower bounds on the expected gains that a Clip-air fleet may provide to the airline.

We have implemented our model in AMPL and the results are obtained with the GUROBI solver. We first present a small example to illustrate the advantages of the enhanced flexibility of the Clip-Air system. Then we present the results for different scenarios about the network

Table 8.1: Clip-Air configuration

		Clip-Air	**A320**
Maximum Capacity		3x150 (450 seats)	150 seats
Engines		**3** engines	**2** engines
Maximum	1 (plane/capsule)	139t (+78%)	78t
Aircraft Weight	2 (planes/capsules)	173.5t (+11%)	2x78t (156t)
	3 (planes/capsules)	208t (-11%)	3x78t (234t)

configuration, fleet size, fleet type and the costs of the Clip-Air fleet. The presented results include productivity measures in order to show the efficiency of the utilization of the capacity:

- Available seat kilometers (ASK): The number of seats available multiplied by the number of kilometers flown. This is a widely used measure for the passenger carrying capacity. Since our data does not provide information on the kilometers flown for the flights, we convert the total flight duration to kilometers with a speed of 850 kilometers per hour.

- Transported passengers per available seat kilometers (TPASK): A productivity measure which we adapt to compare the standard fleet and Clip-Air. It is the total number of transported passengers divided by the available seat kilometers and measures the productivity of the allocated capacity.

8.3.1 Cost figures for Clip-Air

As mentioned previously Clip-Air exists only in a simulated environment. Therefore estimated values are used for the operating cost of Clip-Air using analogies with the aircraft A320. The capacity of Clip-Air is designed to be 150 seats, the same as the capacity of an A320. In Table 8.1 we present the weight values for Clip-Air flying with one, two and three capsules in comparison to one, two and three aircraft of type A320. As seen from the Table, Clip-Air is 78% heavier than one A320 plane when it is flying with one capsule, and 11% heavier than two A320 planes when flying with two capsules. However when flying with three capsules Clip-Air is 11% lighter than three A320 planes. We use these weight differences to proportionally decrease/increase the fuel cost and air navigation charges since both depend on the aircraft weight. The airport charges are usually applied depending on the weight class of the aircraft rather than being directly proportional (ICAO, 2012). However to be on the conservative side we apply an increase which is proportional to the weight.

Furthermore we make adjustment on the crew cost due to the decoupling of wing and capsules. Flight (cockpit) crew cost is associated with the wing, and the cabin crew cost is associated with the capsules. Clip-Air flies with one set of flight crews regardless of the number of capsules used for the flight. It is given by the study of Aigrain and Dethier (2011) that flight crew constitutes 60% of the total crew cost for the A320. Therefore Clip-Air decreases the total crew cost by 30% and 40% when flying with two and three capsules respectively. Note that, the

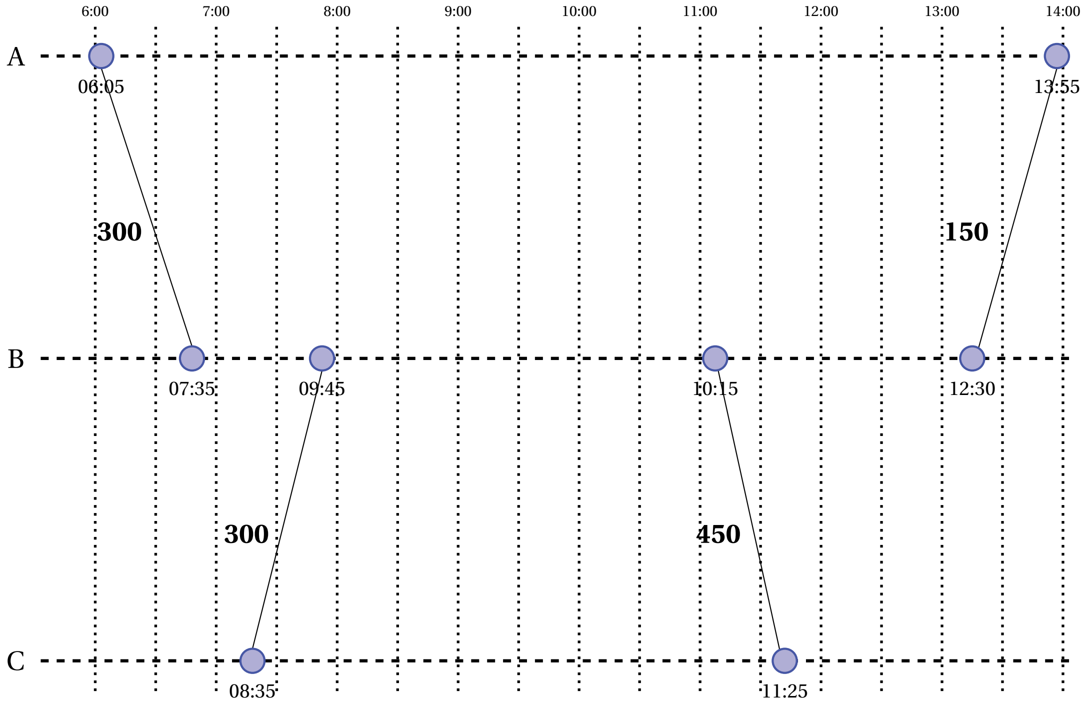


Figure 8.7: Time-line network for the illustrative example

modification on the flight crew cost is kept linear with seating capacity for standard fleet. In other words, having the flight crew cost for A320 as a basis, the cost is proportionally increased with increasing seating capacity.

The adjusted cost figures sum up to 56% of the total operating cost of European airlines: fuel cost 25.3% (IATA, 2010), crew cost 24.8% (IATA, 2010), airport and air navigation charges 6% (Castelli and Ranieri, 2007). The remaining operating cost values are assumed to be the same as the A320 for the utilization of each capsule.

8.3.2 An illustrative example

We present results for a small data instance to illustrate the flexibility provided by the Clip-Air system. The network consists of four flights with the demand and departure-arrival times given in Figure 8.7. There is an expected demand of 1200 passengers which is generated by 4 itineraries between airports A-C, B-C, C-A and C-B. The available fleet capacity is not limited and the circular property of the schedule is ignored for this example. For the standard fleet, it is assumed that there are three types of planes which have 150, 300 and 450 seats. Clip-Air capsules are assumed to have a capacity of 150 seats as presented in Table 8.1.

In order to fully satisfy the demand with standard planes, 2 aircraft with 300 seats each should depart from the airports A and C. At airport B an aircraft with 450 seats is needed for the departure to airport C and an aircraft with 150 seats for the departure to airport A. Therefore 4 aircraft are used with 1200 allocated seats. Clip-Air is able to cover the demand with 2 wings. The wings depart from airport A and C with 2 capsules each. At airport B, 1 capsule is transfered to the flight that departs to airport C. Therefore the flight B-C is operated with 3 capsules and the flight B-A is operated with 1 capsule. The total number of allocated seats is 600 which means that Clip-Air is able to transport the same number of passengers with 50% of the capacity of the standard fleet. This change in the fleet assignment operations leads to several simplifications in the operations. Since the same type of aircraft is used for all the flights the type of crew does not need to be changed for different flights. The airport operations are also simplified since the same type of aircraft can be assigned to the flights with necessary adjustments in the number of clipped capsules.

We can analyze the same data instance with a limited capacity of 600 seats for standard planes and Clip-Air. In that case 2 aircraft with 300 seats each will be operated from the airports A and C to airport B. The same aircraft will depart from airport B which will result with a loss of 150 passengers on the flight B-C and with an excess capacity of 150 seats on the flight B-A. However Clip-Air covers the demand without any loss or excess capacity with its flexible capacity.

This illustrative example gives the idea of the potential savings with Clip-Air which is quantified with the experiments presented in the continuation of this section.

8.3.3 Network effect

The type of the network is an important factor that needs to be analyzed for quantifying the performance of Clip-Air. For this matter, we present results for three different network structures: airport pair, hub-and-spoke network with single hub and peer-to-peer well connected network. Flight densities of these networks are different from each other which affects the performance of Clip-Air.

Airport-pair network

We present a network with 2 airports and 38 flights which are balanced for the two routes. The description of the data set is given in Table 8.2 and the results are provided in Table 8.3. It is observed that Clip-Air carries 7% more passengers compared to a standard fleet. The increase in the number of transported passengers is also reflected by the spill cost which is higher for standard fleet. Therefore the profit is 5% higher when flying with Clip-Air. The allocated capacity is similar for the two cases. The average demand per flight does not favor the usage of 3 capsules therefore the operating cost for Clip-Air is higher. This is compensated by the increased revenue due to the flexibility of Clip-Air on the allocated transportation capacity.

Table 8.2: Data instance for the airport-pair network

Airports	2
Flights	38
Density (Flights/route)	19
Passengers	13,965
Itineraries	45
Standard fleet types	A320(150), A330(293), B747-200(452)

Table 8.3: Results for the airport-pair network

	Standard fleet	Clip-Air
Operating cost	1,607,166	1,725,228
Spill costs	604,053	448,140
Revenue	2,419,306	2,575,219
Profit	**812,140**	**849,991 (+4.66 %)**
Transported pax.	**10,276**	**11,035 (+7.39 %)**
Flight count	38	38
Total flight duration	3135 min	3135 min
Used fleet	2 A320	7 wings
	5 A330	12 capsules
Used aircraft	7	7
Used seats	1765	1800
ASK	78,388,063	79,942,500
TPASK ($\times 10^{-}5$)	13.11	13.80

Table 8.4: Data instance for the hub-and-spoke network

Airports	5
Flights	26
Density (Flights/route)	3.25
Passengers	9,573
Itineraries	37
Standard fleet types	A320(150), A330(293), B747-200(452)

Table 8.5: Results for the hub-and-spoke network

	Standard fleet	Clip-Air
Operating cost	817,489	938,007
Spill costs	484,950	393,677
Revenue	1,247,719	1,338,992
Profit	**430,230**	**400,985 (- 6.80 %)**
Transported pax.	**5,031**	**5,721 (+ 13.71 %)**
Flight count	**24**	**22**
Total flight duration	**1850 min**	**1700 min**
Used fleet	5 A320	6 wings
	2 A330	12 capsules
	1 B747	
Used aircraft	**8**	**6**
Used seats	1788	1800
ASK	**46,860,500**	**43,350,000**
TPASK ($\times 10^-5$)	**10.74**	**13.20**

Hub and spoke network with a single hub

The Clip-Air system is analyzed for a hub-and-spoke network with a single hub where all the flights need to connect through the hub. Details for the data instance are given in Table 8.4. With Clip-Air, less flights are operated and there is a 14% increase in total transported passengers allocating a similar capacity as the standard fleet. The increase in the transported passengers with less number of flights is reflected through the TPASK measure. Since the flight density is low, which is 3.25 flights per OD pair, and since the connections are only possible through the hub, the profit with Clip-Air is 7% less compared to the standard fleet. However we are still using two aircraft less with Clip-Air which will reduce the number of flight crews and simplify the ground operations for airports. We need to mention that in this particular instance the incoming and outgoing flights from the hub are balanced in terms of the demand for each spoke airport. Therefore a standard fleet can also perform well in this situation.

Well connected peer-to-peer network

In this section we present results for a peer-to-peer network where the airports are well connected with 98 flights and 28,465 expected passengers as seen in Table 8.6. Clip-Air

Table 8.6: Data instance for the peer-to-peer network

Airports	4
Flights	98
Density (Flights/route)	8.17
Passengers	28,465
Itineraries	150
Standard fleet types	A320(150), A330(293), B747-200(452)

Table 8.7: Results for the peer-to-peer network

	Standard fleet	Clip-Air
Operating cost	3,189,763	3,117,109
Spill costs	982,556	978,683
Revenue	5,056,909	5,060,782
Profit	**1,867,146**	**1,943,673 (+ 4.1 %)**
Transported pax.	**20,840**	**21,424 (+ 2.8 %)**
Flight count	91	84
Total flight duration	6650 min	6160 min
Used fleet	7 A320	13 wings
	10 A330	28 capsules
	3 B747	
Used aircraft	20	13
Used seats	**5336**	**4200 (- 21.3 %)**
ASK	**502,695,667**	**366,520,000**
TPASK ($\times 10^-5$)	**4.15**	**5.85**

transports 2.8% more passengers with a 21.3 % reduction in the allocated capacity compared to the standard fleet. This means that Clip-Air uses the capacity more efficiently which is also supported by the increased TPASK measure. When we look at the used number of aircraft we see that there is a clear difference between standard fleet and Clip-Air. Therefore the minimum number of flight crews is 35% less for Clip-Air which is important for the crew scheduling decisions. The density of the network is higher compared to the hub-and-spoke instance and all the airports are connected pairwise. The possibility to change the number of capsules at airports is utilized more efficiently. Therefore this type of network reveals more prominently the advantages of the flexibility of Clip-Air.

8.3.4 Effect of the standard fleet configuration

Clip-Air is composed of modular capsules, the standard fleet can be composed of any aircraft type and the model has the opportunity to select the best fleet composition. Therefore it is important to see the effect of the fleet configuration when comparing with the performance of Clip-Air. This analysis enables us to figure out which type of airlines may profit better from the Clip-Air system.

We use the same data instance as the peer-to-peer network given in Table 8.6. We change the available standard fleet configuration by gradually decreasing the fleet heterogeneity. The total transportation capacity is kept high enough to serve the whole demand for all the tested instances. The first scenario is designed to be composed of a highly heterogeneous fleet which is representative of the existing aircraft types in the European market. The gradual decrease afterwards is carried out in such a way that the remaining set of aircraft have enough variation in terms of size. Therefore the aircraft which have a similar counterpart in the fleet are selected to be removed which is done to have a fair comparison between Clip-Air and standard fleet.

The results for Clip-Air and standard fleet with different fleet configurations are provided in Table 8.8. It is observed that the richer the fleet configuration, the better the performance of standard fleet. When the standard fleet has 10 or 7 plane types available, the profit is higher compared to Clip-Air. However the transported number of passengers is always higher for Clip-Air although it is allocating less capacity. The profit and the transported passengers dramatically decrease when the fleet configuration is highly restricted with one type of plane. When we look at the results with 1 plane type, which has the same capacity as 2 capsules, the decrease in profit is 12.8% and 8.8% less passengers are carried. The change of profit and total number of transported passengers with the fleet configuration can be seen more clearly in Figure 8.8. Furthermore, the measure of TPASK is better for Clip-Air for all the cases except the last case where the utilization of the capacity is very high due to the insufficient capacity allocation. In this last case, standard fleet operates significantly less flights since the flights are not profitable with a single type of aircraft.

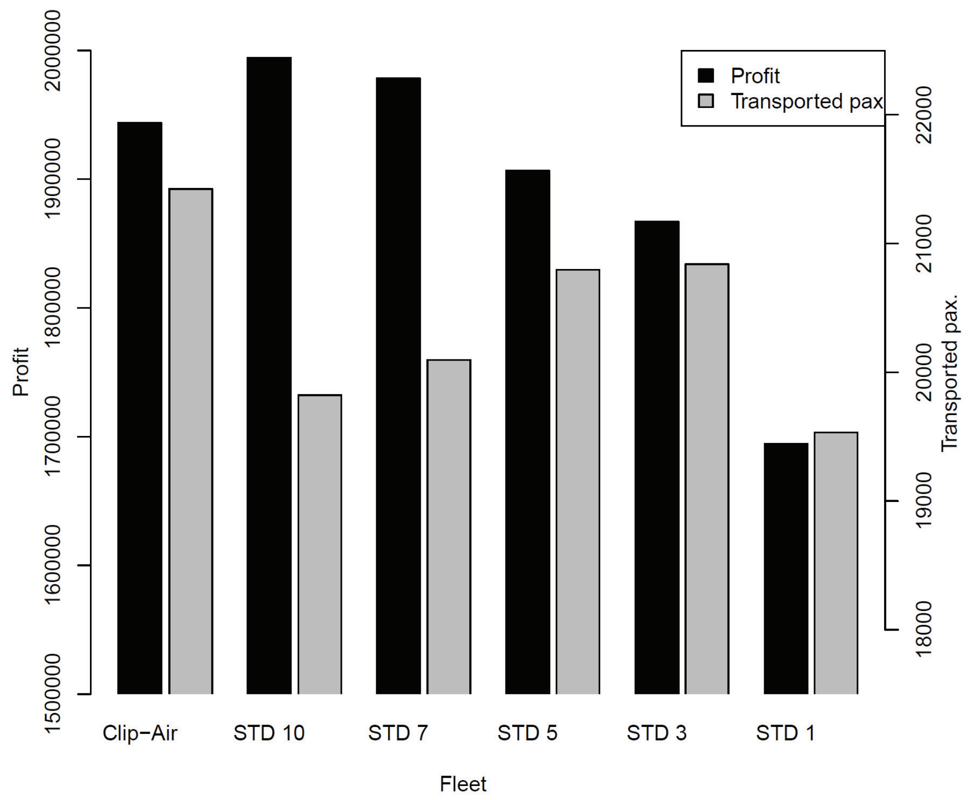


Figure 8.8: Profit and transported passengers for different fleet configurations

Table 8.8: Results with varying standard fleet configuration

		Standard fleet				
	Clip-Air	10 plane types	7 plane types	5 plane types	3 plane types	1 plane type
Operating cost	3,117,109	2,950,195	2,994,783	3,174,240	3,189,763	2,949,697
Spill costs	978,683	1,094,892	1,066,190	958,428	982,556	1,395,316
Revenue	5,060,782	4,944,573	4,973,275	5,081,038	5,056,909	4,644,150
Profit	**1,943,673**	**1,994,378 (+2.6%)**	**1,978,492 (+1.8%)**	**1,906,798 (-1.9%)**	**1,867,146 (-3.9%)**	**1,694,453 (-12.8%)**
Transported pax.	**21,424**	**19,823 (-7.5%)**	**20,096 (-6.2%)**	**20,796 (-2.9%)**	**20,840 (-2.7%)**	**19,533 (-8.8%)**
Flight count	84	93	94	93	91	77
Total flight duration	6160	6,780	6,875	6,780	6,650	5,705
Used fleet	13 wings 28 capsules	1 A318(107) 2 A319(124) 3 A321(185) 8 A330(293) 5 A340(335) 2 B737(128) 2 B777(400) 1 B747-400(524) 3 ERJ135(37) 2 ERJ145(50)	1 A319(124) 4 A321(185) 9 A330(293) 5 A340(335) 2 B737(128) 4 B777(400) 4 ERJ145(50)	5 A319(124) 2 A320(150) 10 A330(293) 5 A340(335) 2 B747-200(452)	7 A320(150) 10 A330(293) 3 B747-200(452)	12 A330(293)
Used aircrafts	**13**	**29**	**29**	**24**	**20**	**12**
Used seats	**4200**	**6720 (+60%)**	**7232 (+72%)**	**6429 (+53%)**	**5336 (+27%)**	**3516 (-16%)**
ASK	**366,520,000**	**645,456,000**	**704,366,667**	**617,505,450**	**502,695,667**	**284,166,050**
TPASK (×10⁻5)	**5.85**	**2.99**	**2.85**	**3.37**	**4.15**	**6.87**

Table 8.9: Data instance for the tests with different available capacity

Airports	5
Flights	100
Density (Flights/route)	6.25
Passengers	35,510
Itineraries	140
Standard fleet types	A319(124), A320(150), A321(185), A330(293), A340(335), B737-300(128), B737-400(146), B737-900(174), B747-200(452), B777(400)

8.3.5 Effect of the available seating capacity

All the previous results are obtained without any limit on the total capacity so that it is enough to cover the total expected demand. However in reality there may be capacity shortage in case of unexpected events, weather conditions or in high season. Therefore it is important to test the performance of Clip-Air compared to standard fleet when there is limited capacity. The data instance seen in Table 8.9, that consists of 100 flights, is used for the tests. Available capacity is decreased gradually and the results corresponding to each level of capacity is presented in Table 8.10.

For the unlimited capacity case, Clip-Air is able to carry 7% more passengers with 25% less transportation capacity. In all of the cases Clip-Air is able to carry more passengers compared to the standard fleet. In case of capacity restrictions, this advantage of Clip-Air over a standard fleet becomes more evident as the restriction becomes harder to overcome. This can also be observed from the TPASK measures which state that the productivity is higher for the allocated capacity compared to standard fleet.

As mentioned previously, there are mandatory flights which need to be served. Our dataset does not include information about the mandatory flights and to be able to represent the schedule design decision we randomly select a percentage of the flights to be mandatory. In this instance 50% of the flights are assumed to be mandatory. As the capacity restriction becomes more severe, Clip-Air flies with one capsule in order to operate these mandatory flights. This significantly increases the operating cost of Clip-Air and decreases the resulting profit. In the last case in Table 8.10 the standard fleet has 16% more profit due to the explained phenomenon. In order to see the effect of the mandatory flights, the same instance with an available capacity of 1950 seats is analyzed, where all the flights are assumed to be optional. In such a case Clip-Air has 9% more profit and carries 5% more passengers compared to a standard fleet. Indeed, when all the flights are optional, Clip-Air can select the most profitable flights where the level of demand enables to avoid the usage of one capsule flights.

When the available capacity is decreased further neither the standard fleet nor Clip-Air can serve the mandatory flights which makes the problem infeasible.

Table 8.10: Results with varying available capacity

	Clip-Air				
	Not limited	4500 seats	3750 seats	3000 seats	1950 seats
Operating cost	3,737,841	3,547,651	3,321,567	2,837,159	2,063,607
Spill costs	764,078	1,028,581	1,420,982	2,201,731	3,801,355
Revenue	6,120,255	5,855,752	5,463,351	4,682,602	3,082,978
Profit	**2,382,414**	**2,308,101**	**2,141,783**	**1,845,443**	**1,019,371**
Transported pax.	**27,061**	**25,682**	**23,722**	**19,851**	**12,810**
Flight count	93	93	89	82	72
Total flight duration	7110	7110	6780	6240	5460
Used fleet	18 wings	17 wings	17 wings	16 wings	14 wings
	39 capsules	30 capsules	25 capsules	20 capsules	13 capsules
Used aircrafts	18	17	17	16	14
Used seats	**5850**	**4500**	**3750**	**3000**	**1950**
ASK	**589,241,250**	**453,262,500**	**360,187,500**	**265,200,000**	**150,832,500**
TPASK ($\times 10^{-}5$)	**4.59**	**5.67**	**6.59**	**7.49**	**8.49**
	Standard Fleet				
	Not limited	4500 seats	3750 seats	3000 seats	1950 seats
Operating cost	3,656,793	3,510,037	3,168,626	2,651,208	1,741,825
Spill costs	1107237	1,326,018	1,787,240	2,526,149	3,958,092
Revenue	5,777,096	5,558,315	5,097,093	4,358,184	2,926,241
Profit	**2,120,303 (-11%)**	**2,048,278 (-11%)**	**1,928,467 (-10%)**	**1,706,976 (-8%)**	**1,184,416 (+16%)**
Transported pax.	**25,136 (-7%)**	**23,926 (-7%)**	**21,647 (-9%)**	**17,794 (-10%)**	**11,294 (-12%)**
Flight count	93	93	91	87	88
Total flight duration	7110	7110	6945	6585	6,700
Used aircrafts	26	17	16	15	14
Used seats	**7832**	**4498**	**3750**	**3000**	**1949**
ASK	**788,878,200**	**453,061,050**	**368,953,125**	**279,862,500**	**184,992,583**
TPASK ($\times 10^{-}5$)	**3.19**	**5.28**	**5.87**	**6.36**	**6.11**

8.3.6 Sensitivity analysis on the costs of Clip-Air

Since the Clip-Air system does not exist yet, sensitivity analysis needs to be carried out for the assumed operating cost of Clip-Air. As mentioned in section 8.3.1, we estimate the crew cost, fuel cost, airport and air navigation charges for Clip-Air. Therefore we present a sensitivity analysis of these cost figures. For example, for the fuel cost we analyze what happens if Clip-Air consumes more than our expectation. For this purpose we have analyzed 5 different scenarios: base case which is the expected cost and cases with an increase of 10%, 20%, 30%, 50% in fuel consumption. Similarly, airport and air navigation charges are analyzed with the cases of 10%, 20%, 30% and 50% higher values compared to the base values we have initially used.

The crew cost does not depend on the weight of the aircraft. Clip-Air always flies with one set of flight crews regardless of the number of capsules used. Therefore, Clip-Air crew cost savings depend on the repartition of overall crew costs between flight and cabin and we analyze the sensitivity of the results to this repartition. As mentioned in section 8.3.1, we assume that flight crew and cabin crew constitute 60% and 40 % of the total crew cost respectively. Therefore 60% represents the base case for the flight crew cost throughout the analysis. We consider two other cases where flight crew constitutes the 50% and 70% of the total crew cost. The 50% case implies a reduction in the potential savings of Clip-Air and the 70% case is in favor of Clip-Air where the crew cost is further decreased.

The analysis is carried out for the same data instance used for the analysis of the effect of transportation capacity in section 8.3.5. The results in Table 8.11 are presented in comparison to the results for standard fleet given in Table 8.10 for the case of unlimited capacity.

It is observed that the scheduling decisions are the same for almost all of the cases having 18 assigned aircraft and allocating 25% to 29% less capacity compared to the standard fleet. This is a good indicator which says that our model is robust in the analyzed range and the general conclusions remain valid. The number of transported passengers is higher for Clip-Air for all the analyzed cases and the range of this increase is between 4.5%-8.3%. The highest increase in profit is 14.8% which occurs when all the cost values are in favor of Clip-Air. On the other hand, the lowest profit of Clip-Air (20.9% lower than standard fleet) is observed when all the cost figures are in favor of the standard fleet.

Furthermore, we can draw conclusions on the relative impacts of each cost figure on the resulting profit and transported passengers. When all the other cost values are at their base levels, even a 50% increase in airport and air navigation charges does not affect the superiority of Clip-Air over a standard fleet. A 30% increase in the fuel cost decreases the profit of Clip-Air below that of a standard fleet even when all other costs are at their base levels. The impact of different percentages for flight crew cost is more evident when the fuel cost is increased. For example, when there is a 20% increase in fuel cost, the profit of Clip-Air may become inferieur to a standard fleet depending on the flight crew percentage. When it is 70% Clip-Air is still more profitable even for a 30% increase in airport and air navigation charges. However when the flight crew percentage is 50% Clip-Air is less profitable even for the base case.

Table 8.11: Sensitivity analysis for the cost figures of Clip-Air

Fuel cost			**Base**			**+10%**			**+20%**			**+30%**			**+50%**		
Flight crew %			**50%**	**60%**	**70%**	**50%**	**60%**	**70%**	**50%**	**60%**	**70%**	**50%**	**60%**	**70%**	**50%**	**60%**	**70%**
airport & air navigation charges	**Base**	**Profit:**	+9.9%	+12.4%	+14.8%	+4.9%	+7.3%	+9.8%	**-0.1%**	+2.3%	+4.7%	**-5.1%**	**-2.7%**	**-0.3%**	**-15.0%**	**-12.8%**	**-10.4%**
		Pax.	+6.9%	+7.7%	+8.3%	+6.9%	+6.9%	+7.7%	+6.2%	+6.9%	+7.7%	+6.2%	+6.9%	+6.9%	+4.5%	+6.2%	+6.9%
	+10%	**Profit:**	+8.7%	+11.2%	+13.6%	+3.7%	+6.1%	+8.6%	**-1.3%**	+1.1%	+3.5%	**-6.3%**	**-3.9%**	**-1.5%**	**-16.2%**	**-14.0%**	**-11.6%**
		Pax.	+6.9%	+7.7%	+8.3%	+6.9%	+6.9%	+7.7%	+6.2%	+6.9%	+7.7%	+6.2%	+6.9%	+6.9%	+4.5%	+6.2%	+6.9%
	+20%	**Profit:**	+7.5%	+10.0%	+12.4%	+2.5%	+4.9%	+7.4%	**-2.5%**	**-0.1%**	+2.3%	**-7.5%**	**-5.1%**	**-2.7%**	**-17.4%**	**-15.2%**	**-12.8%**
		Pax.	+6.9%	+7.7%	+7.7%	+6.9%	+6.9%	+7.7%	+6.2%	+6.9%	+7.7%	+5.7%	+6.9%	+6.9%	+4.5%	+6.2%	+6.9%
	+30%	**Profit:**	+6.3%	+8.7%	+11.2%	+1.3%	+3.7%	+6.2%	**-3.7%**	**-1.3%**	+1.1%	**-8.7%**	**-6.3%**	**-3.9%**	**-18.6%**	**-16.4%**	**-14.0%**
		Pax.	+6.9%	+7.7%	+7.7%	+6.9%	+6.9%	+7.7%	+6.2%	+6.9%	+7.7%	+4.5%	+6.9%	+6.9%	+4.5%	+5.7%	+6.9%
	+50%	**Profit:**	+4.0%	+6.3%	+8.8%	**-1.1%**	+1.3%	+3.7%	**-6.1%**	**-3.7%**	**-1.3%**	**-11.0%**	**-8.7%**	**-6.3%**	**-20.9%**	**-18.7%**	**-16.4%**
		Pax.	+6.9%	+6.9%	+7.7%	+6.2%	+6.9%	+7.7%	+6.2%	+6.9%	+6.9%	+4.5%	+6.2%	+6.9%	+4.5%	+4.5%	+6.2%

It is observed that both the increase in the fuel cost and the increase in airport and air navigation charges decrease the profit as expected. However the total number of transported passengers is not considerably affected by the change of the costs. When the percentage of the flight crew cost increases, Clip-Air uses the advantage of the decoupling of wing and capsules and reduces the crew cost considerably. Although the number of carried passengers is not highly affected, it is increased when the flight crew percentage is high. It can be concluded that crew cost and fuel cost are more critical compared to airport and air navigation charges in terms of the profit and the number of transported passengers, although there is not a significant effect on the scheduling decisions.

8.4 Integrated airline scheduling, fleeting and pricing model for the case of Clip-Air

Computational results in the previous sections are obtained with IFAM which does not have the pricing decision and average price/demand values are used as given in the dataset. This is preferred in order to present the comparative analysis based on provable optimal solutions obtained with a MILP model. In this section we present results with the integrated scheduling, fleeting and pricing model. For that purpose the formulation given in chapter 6, IFAM-PR$'$, is used.

The extension to the Clip-Air case is done similarly by modifying the fleet assignment related parts of the model. The model and the main differences to the standard aircraft case is given in Appendix A.5.

The results are presented in Table 8.12 for data instance 26 which is also used for the results in sections 8.3.5 and 8.3.6. The details on the data instance are provided in Table 8.9. We provide results for the application of sequential approach and the local search heuristic presented in chapter 5. Since we work with the reformulated model the adapted heuristic method is used (see chapter 6, section 6.3). We do not report any results by BONMIN solver as it does not converge to any feasible solution in 24 hours of computation. The results show that for both sequential approach and the integrated approach solved by the heuristic, Clip-Air has a significantly higher profit compared to standard aircraft. Furthermore it is carrying more passengers. The improvements are higher with the heuristic method compared to the sequential approach. The improvement in the profit is also higher compared to the results with IFAM presented in Table 8.10. This means that the potential of Clip-Air becomes more evident with the integrated model.

When the results of the heuristic are compared to the sequential approach, it is seen that the improvement is higher with Clip-Air. IFAM$'$ is computationally easier in the case of Clip-Air since it has 3 aircraft options whereas standard fleet has 10 different aircraft sizes. Therefore more iterations are realized in the case of Clip-Air and better solutions are explored.

This analysis shows that bringing several flexibility notions together (integrated modeling of

Table 8.12: Comparative results with the integrated model

		Standard aircraft	Clip-Air	% Improvement with Clip-Air
Sequential approach (SA)	Profit	2,265,013	2,688,751	**18.71%**
	Pax.	26,644	26,952	**1.16%**
Heuristic * (*avg. over 5 replications*)	Profit	2,271,312	2,735,556	**20.44%**
	Pax.	26,512	27,981	**5.54%**
	Number of iterations	101.0	632.6	
	Time (sec)	2,645.25	2613.60	
	IFAM$'$ time per iter	25.34	3.56	
	RMM-PR$'$ time per iter	0.25	0.38	
	% Impr. over SA	**0.28%**	**1.74%**	

*Heuristic is run with VNS for both of the cases where $n_{min} = 60$ and $n_{max} = 100$

supply and demand, flexible capacity of Clip-Air) provides superior solutions for the airline case study. It motivates the further analysis of flexible transportation systems with better decision support tools thanks to integrated supply-demand models and with new transportation alternatives that provide flexibility by design.

8.5 Conclusions and future research directions

In this chapter, the added value of flexibility in air transportation systems is analyzed. We have focused on the flexibility brought by the modularity of a new type of aircraft, Clip-Air, which is currently being designed. It is clearly shown that bringing flexibility helps to both better respond to the network demand and to increase revenues. The analysis of flexibility is not limited to Clip-Air and can be a reference for future studies on flexible transportation systems. This study is a promising step towards the integration of different types of flexibility in various transportation systems.

In order to quantify the added value of flexibility, a comparative analysis is carried out between the Clip-Air system and an existing standard configuration. For this purpose an integrated schedule design and fleet assignment model is developed for both Clip-Air and a fleet with standard planes. Sustainability of transportation systems is closely related to the demand responsiveness and this can not be achieved without introducing demand orientation in transportation models. For that matter, supply-demand interactions are integrated in the model through an itinerary choice model which represents spill and recapture effects. Therefore the presented methodology is an integration of advanced optimization and demand modeling methods for airlines.

Since the Clip-Air system does not exist yet, the estimation of the cost is based on reasonable assumptions. In order to perform a conservative comparison, our scenarios include some advantages for the standard fleet compared to Clip-Air. For instance, we do not allow Clip-Air to use different types of capsules, while the standard fleet can rely on different plane types.

Different scenarios are analyzed to quantify the performance of Clip-Air. The scenarios are designed to test the effects of the network type, fleet size, fleet configuration and the estimated cost of the Clip-Air system. In all analyzed cases, Clip-Air is found to carry more passengers allocating less capacity compared to the standard fleet. This is supported by the high TPASK measures which means that Clip-Air uses the available capacity more efficiently than the standard fleet. The scenarios show that the potential advantages of Clip-Air are more evident in a large network where the flight density is high and the airports are well connected. In such a network, airlines fly with different types of aircraft as a strategy to capture various demand patterns. Clip-Air is more efficiently responding to the demand with a single capsule type due to its flexibility. Therefore, airlines that operate over a large network with a high density of flights are expected to gain the most by switching to a Clip-Air fleet.

As mentioned previously, the cost estimation for the Clip-Air system is based on various assumptions. Therefore a sensitivity analysis is presented for crew cost, fuel cost and airport and air navigation charges. It is seen that scheduling decisions are not sensitive to the cost in the range of our analysis. Clip-Air is found to always perform better in terms of the number of carried passengers. In terms of profit, Clip-Air becomes less advantageous mainly when the fuel costs are increased above 20%.

The overall results show that Clip-Air has a significant potential for an efficient use of the capacity, as well as an increase of the airline profits. The conservative nature of the scenarios and the sensitivity analysis suggest that these reported improvements will be outperformed by a real implementation of the system. The presented model can simply be used for further generation of results when cost figures of Clip-Air are updated.

It has been mentioned that the investment cost for the purchase of aircraft is ignored in this study. However, the results suggest that a better utilization of the transportation capacity is provided by Clip-Air. Therefore we believe that the potential of Clip-Air will be better shown if the investment cost is included. The presented model can easily be extended with the adjustment of the objective function. The model already tracks the number of used aircraft for each type and this number can be used to take into account the investment cost.

The Clip-Air system can be analyzed from different perspectives thanks to its design. For instance, a standardization of the Clip-Air capsule would give a multi-modal dimension to the system. The capsules could be carried on railways and on trucks, allowing passengers to board outside of the airport. Since the capsules are of simple structure, their storage and transfer is relatively easy. We believe that the repositioning possibility will increase the flexibility of Clip-Air and help to show more clearly how it can adapt to different situations of the capacity and demand. A preliminary analysis on the repositioning of empty capsules is carried out by Blaiberg (2012). Moreover, the modularity of Clip-Air allows to have freight and passenger loaded capsules on the same flight which opens up new frontiers to mixed passenger and cargo transportation. Furthermore, it is more realistic for an airline company to have only part of the fleet composed of Clip-Air wings and capsules the rest being composed of standard aircraft. Therefore, a model with mixed fleet is crucial to see what types of aircraft should be replaced by Clip-Air. A dynamic business plan for companies can be obtained with the inclusion of the fixed cost for the purchase of the Clip-Air wings and capsules. Furthermore, a business model where the companies operating the wings are different from the companies operating the capsules should be analyzed.

The considered flexibilities share the viewpoint of airlines. However, Clip-Air is expected to change the airport operations as well. The ground operations will alter with the easy transfer of Clip-Air capsules. The needed turn-around time is expected to be reduced. The presented model can be used for such modifications on the schedule. Furthermore, this study considers that the schedule is given and the only option is the cancellation of optional flights. However, Clip-Air might be more profitable with an adapted flight schedule. As a future work, the presented methodology can be used with more flexibility in the schedule.

The Clip-Air concept opens the door to a wide range of new research opportunities in the context of flexible transportation. Analogies and differences among the existing transportation modes can be utilized better in order to design new concepts. Modularity, which is a flexibility we are used to see in railways, is shown to be significantly advantageous in airline operations. Therefore, the presented analysis is a promising step towards the new flexibility concepts without being confined in the boundaries of the existing systems.

9 Conclusions and future research directions

This thesis combines together methodologies in the context of demand, supply and integrated supply-demand models towards flexible transportation systems. A better understanding of the travel behavior and an integrated supply-demand modeling framework provide a more efficient use of the current transportation systems through better decision support tools. The methodologies not only improve the current systems but also motivate the design of new transportation alternatives thanks to the gained insights. Furthermore the presented methodologies facilitate the planning and analysis of these flexible transportation systems with the integrated decision mechanisms.

In this chapter we provide the main conclusions and future research directions for the thesis in three sections parallel to the overall presentation.

Advanced demand models

Advanced demand models presented in this thesis enable to better understand the underlying travel behavior of individuals. The integration of attitudes and perceptions through latent variable and latent class models is shown to be a promising research field. The results motivate the collection of comprehensive datasets including the attitudes and perceptions of individuals. With such rich datasets the preferences of passengers can be more deeply analyzed. The results of such analysis generates valuable insights for the improvements in the current public transport modes and for the design of more flexible transportation alternatives. Indeed the presented study is followed by further research, outside this thesis, where the results obtained with the revealed and stated preferences datasets are used by PostBus for improvements in their services and for the launch of new flexible services (Bierlaire et al., 2011, Danalet and Sahaleh, 2012, Schuler et al., 2012).

Advanced demand models provide important outputs and they can be further exploited in the planning phase of transportation systems. In order to achieve that, the reaction of demand to supply decisions should be modeled explicitly. Available datasets in the context of transportation are usually restricted having no access to the non-chosen alternatives. This is more evident in the case of airlines. In this thesis, a joint RP/SP dataset is used in order to obtain a price-elastic air itinerary choice model. The itinerary choice model is then used in the development of an integrated model where the revenue related decisions are given based on the choice of travelers. We believe that in the existence of more detailed datasets, advanced demand models can be estimated and used for a better understanding of travel behavior and for a more efficient planning of supply.

Integrated supply models

The thesis presents two formulations for the integrated airline scheduling, fleeting and pricing problems. The integrated models explicitly represent supply-demand interactions through the air itinerary choice model. Several tests are performed with a real European dataset in comparison to state-of-the-art models. It is shown that the integrated approach provides superior planning decisions due to the simultaneous decisions on the planning and revenue management. The airline profit is increased when the information from the demand model alters the planning decisions. The increase in profit is usually a few percents but it is important considering the tight profit margins of airlines.

A comprehensive sensitivity analysis is performed addressing the uncertainty on the demand model parameters. The analysis supports that the simultaneous decision making improves the robustness of the solution with respect to the fluctuations on the demand side. Even though the presented models are in the context of air transportation, they motivate the development of integrated supply-demand models in other contexts. The ability to react to alternating market conditions is an asset and provides flexibility in decision making in the context of all service systems. Indeed the use of such integrated models is a very recent trend. There are ongoing research projects worldwide where advance demand models are integrated for improving the planning problems.

The integration of explicit demand models in optimization problems is not only interesting in terms of modeling but also challenging in terms of solution methodologies. Integration of a demand model in an optimization framework brings nonlinearities which often lead to non-convexity. The presented formulations in this thesis are mixed integer non-convex problems. The complexity in terms of nonlinearity arises since price is a decision variable of the optimization model. With the logit model, the price directly affects the demand/market share which is another decision variable. Moreover the combinatorics in the problem due to the fleet assignment binary variables increases the problem complexity exponentially for large flight networks.

We propose a local search heuristic for the solution of the integrated models where the fleet assignment and revenue sub-problems are solved in an iterative way. Local search mechanisms are employed in order to explore the feasible region during the iterative process. The performance of the heuristic is tested compared to two other heuristics: MINLP solver BONMIN and a sequential approach which is a single iteration for the solution of the sub-problems. The heuristic is found to provide better quality solutions in reasonable computational time compared to the two other heuristics. The heuristic is tested for data instances up to around 300 daily flights and it provides better solutions for both of the formulations of the integrated model. The same heuristic is also utilized in the case of Clip-Air which shows that it can be used as a solution method for a range of integrated models with explicit demand representations.

The quality of the results provided by the heuristic method should be evaluated with valid bounds. The heuristic provides a lower bound on the profit and approximation methods can be considered in order to obtain upper bounds. This is a challenging task due to the non-convexity of the model. In the thesis we present a logarithmic transformation of the logit model which significantly improves the formulation. The strength is supported by illustrative examples and the revenue management sub-problem of the integrated model. Furthermore a piecewise linear approximation is proposed in order to obtain a convex programming problem. This methodology is valid theoretically however in order to obtain tight bounds, a very fine approximation should be used with several pieces which makes the problem intractable in terms of computational time even for small size instances. A promising research direction is the design of efficient approximation methods which provides tight upper bounds on the profit for realistic size instances. In this thesis, it is shown that it is not trivial to handle the non-convexities in the model. Therefore we believe that there are many potential developments in this area. The recent studies in mixed integer nonlinear programming (e.g. D'Ambrosio et al., 2012) can be exploited as a further research of this thesis.

The development of an efficient solution methodology for the integrated models with explicit supply-demand interactions is critical for a better use of such advanced planning models. We believe that it will lead to the development of a general framework for the integration of advanced demand models in optimization problems. As already mentioned, such models are attracting an increased attention in literature with applications in real life problems. Pioneering studies including this thesis show the potential of integrated approaches over classical planning models. The considered demand models to be integrated in planning models in the literature are so far basic models having mostly an aggregate nature with few explanatory variables. However, as observed from the estimation results of advanced demand models with the integration of individuals' attitudes and perceptions, there is a high potential towards a better understanding of travel behavior. Therefore a general framework with integrated modeling features and efficient solution methodologies which can deal with generalized models is a promising research direction with methodological contributions and crucial real-life implications.

Innovative application

The presented models in the thesis are applied to the case of Clip-Air which is an innovative flexible air transportation system. The models are extended in order to account for two levels of fleet assignment brought by the decoupling of wing and capsules. The advantages of Clip-Air is studied in a comparison to standard aircraft in several scenarios. The scenarios are always kept conservative with realistic and fair assumptions. Many of the potential advantages of Clip-Air are ignored. The main focus is the flexibility to adjust the number of attached capsules according to the demand. With this main feature Clip-Air is shown to outperform standard fleet in terms of profit and number of transported passengers. Since the cost figures of Clip-Air are based on preliminary investigations, a sensitivity analysis is performed and it is seen that Clip-Air has a better performance compared to standard fleet except the cases with very high perturbations.

This thesis shows the potential of such a flexible system from an airline point of view. As future work, the Clip-Air system should be analyzed in various aspects in order to evaluate the advantages and disadvantages more globally. Airport operations is one of such future work areas. The airport design should be studied with the needs of the Clip-Air system. Specifically, the flexibility to attach and detach capsules will create a flow in the airport and operations should be re-considered in this aspect. Furthermore, the integration of Clip-Air in multi-modal transportation networks due to its transferable capsules is a very fruitful research direction. Multi-modal networks are studied in the literature with an increasing pace and an efficient integration of air transportation in such networks is a key innovation. Moreover, Clip-Air brings a new dimension to mixed passenger and cargo transportation with a flexibility in completely separating passenger and cargo capsules. This is an interesting research direction with potential real-life benefits. These are some main future research directions which can be extended with several others.

Furthermore, the presented study is important as a motivation towards flexible systems in other transportation modes. With a single flexibility dimension in transportation capacity, significant advantages are obtained as supported by the experimental analysis. The conservative nature of the experimental analysis enables us to confidently state the potential of Clip-Air. We believe that introducing flexibility in any dimension will be followed by improvements in all aspects given that it is based on the needs of the current transportation systems. Combined with an appropriate analysis of the travel behavior with advance demand models, several flexibility notions can be brought together in new transportation alternatives that better meet the needs of today's mobility needs.

A Appendix

A.1 Supplementary results for the air itinerary choice model

In this section we provide supplementary results for the air itinerary choice model presented in chapter 3.

A.1.1 Estimation with 24 OD pairs from the RP dataset

We provide the estimation results with 24 OD pairs of RP data is included in the mixed RP/SP dataset. There are 165 alternative itineraries in total serving 5503 passengers between these 24 OD pairs.

In Table A.3 we see the results for RP and SP observation obtained with the joint estimation. The presented results are in this case are significantly different compared to the estimation results from the SP data (Table A.1). In Table A.4 we present the resulting demand elasticities and value of time for OD1 when the estimation is carried out with 24 OD pairs. It is observed that the elasticity of demand is reduced significantly compared to the results provided in Table 3.5. The demand model parameters in this case do not reflect the behavior of passengers and when integrated into the planning model the price of the itineraries are allowed to increase unrealistically because of the inelasticity.

Table A.1: Estimated parameters based on the SP data

Parameters	Estimated value	t-test
$\beta^{E,NS}_{\text{price}}$	-9.63	-24.05
$\beta^{B,NS}_{\text{price}}$	-8.50	-10.21
$\beta^{E,S}_{\text{price}}$	-9.37	-24.89
$\beta^{B,S}_{\text{price}}$	-8.51	-10.63
$\beta^{B-OP}_{\text{price}}$	3.52	3.52
$\beta^{E,NS}_{\text{time}}$	-0.439	-4.91
$\beta^{B,NS}_{\text{time}}$	-0.456	-2.99
$\beta^{E,S}_{\text{time}}$	-0.328	-4.23
$\beta^{B,S}_{\text{time}}$	-0.361	-2.76
$\beta^{E}_{\text{morning}}$	0.122	1.28*
$\beta^{B}_{\text{morning}}$	0.341	2.10

(* Statistical significance < 90%)

Table A.2: Estimated parameters for the model with joint RP and SP data

Parameters	Estimated value	scaled value for SP	t-test
$\beta^{E,NS}_{\text{price}}$	-2.23	-9.63	-3.48
$\beta^{B,NS}_{\text{price}}$	-1.97	-8.49	-3.64
$\beta^{E,S}_{\text{price}}$	-2.17	-9.37	-3.48
$\beta^{B,S}_{\text{price}}$	-1.97	-8.49	-3.68
$\beta^{B-OP}_{\text{price}}$	0.813	3.52	2.91
$\beta^{E,NS}_{\text{time}}$	-0.102	-0.440	-2.85
$\beta^{B,NS}_{\text{time}}$	-0.104	-0.449	-2.43
$\beta^{E,S}_{\text{time}}$	-0.0762	-0.329	-2.70
$\beta^{B,S}_{\text{time}}$	-0.0821	-0.354	-2.31
$\beta^{E}_{\text{morning}}$	0.0283	0.122	1.21*
$\beta^{B}_{\text{morning}}$	0.0790	0.341	1.86

(* Statistical significance < 90%)

Table A.3: Estimated parameters for the model with joint RP (24 OD pairs) and SP data

Parameters	Estimated value	scaled value for SP	t-test
$\beta_{\text{price}}^{E,NS}$	-1.28	-9.57	-24.73
$\beta_{\text{price}}^{B,NS}$	-1.16	-8.64	-12.46
$\beta_{\text{price}}^{E,S}$	-1.25	-9.32	-25.66
$\beta_{\text{price}}^{B,S}$	-1.17	-8.71	-13.17
$\beta_{\text{price}}^{B-OP}$	0.493	3.68	4.00
$\beta_{\text{time}}^{E,NS}$	-0.060	-0.445	-5.07
$\beta_{\text{time}}^{B,NS}$	-0.072	-0.534	-3.72
$\beta_{\text{time}}^{E,S}$	-0.045	-0.333	-4.39
$\beta_{\text{time}}^{B,S}$	-0.0058	-0.429	-3.350
$\beta_{\text{morning}}^{E}$	0.0154	0.115	1.21*
$\beta_{\text{morning}}^{B}$	0.0414	0.309	2.02

(* Statistical significance < 90%)

Table A.4: Demand indicators for OD1 when estimated with 24 ODs

	alt.	stops	class	VOT($\frac{€}{h}$)	price elas.	time elas.
OD1	1	one-stop	E	20.16	-1.23	-0.19
	2	one-stop	E	11.17	-1.22	-0.19
	3	one-stop	E	9.38	-1.13	-0.25
	4	non-stop	E	8.15	-1.06	-0.06
	5	non-stop	E	8.15	-1.03	-0.06
	6	non-stop	E	8.15	-1.11	-0.06
	7	non-stop	B	25.33	-1.12	-0.08
	8	non-stop	E	8.15	-1.15	-0.06
	9	non-stop	B	25.33	-1.10	-0.08
	10	non-stop	E	8.15	-1.16	-0.06
	11	non-stop	B	25.33	-1.15	-0.08
	12	non-stop	E	8.15	-1.16	-0.06

A.2 Fleet assignment sub-problems

A.2.1 IFAM

$$z^*_{\text{IFAM}} = \max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} t_{j,i}\bar{b}_{j,i})\bar{p}_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f}x_{k,f} \tag{A.1}$$

$$s.t. \sum_{k\in K} x_{k,f} = 1 \qquad \forall f\in F^M \tag{A.2}$$

$$\sum_{k\in K} x_{k,f} \le 1 \qquad \forall f\in F^O \tag{A.3}$$

$$y_{k,a,t^-} + \sum_{f\in In(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in Out(k,a,t)} x_{k,f} \qquad \forall [k,a,t]\in N \tag{A.4}$$

$$\sum_{a\in A} y_{k,a,t_n} + \sum_{f\in CT} x_{k,f} \le R_k \qquad \forall k\in K \tag{A.5}$$

$$y_{k,a,minE_a^-} = y_{k,a,maxE_a^+} \qquad \forall k\in K, a\in A \tag{A.6}$$

$$\sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)} \delta_{i,f}d_i - \sum_{j\in I_s}\delta_{i,f}t_{i,j} + \sum_{j\in(I_s\setminus I'_s)}\delta_{i,f}t_{j,i}\bar{b}_{j,i} \le \sum_{k\in K}\pi^h_{k,f} \qquad \forall h\in H, f\in F \tag{A.7}$$

$$\sum_{h\in H}\pi^h_{k,f} \le Q_k x_{k,f} \qquad \forall f\in F, k\in K \tag{A.8}$$

$$\sum_{j\in I_s} t_{i,j} \le d_i \qquad \forall h\in H, s\in S^h, i\in I_s \tag{A.9}$$

$$x_{k,f}\in\{0,1\} \qquad \forall k\in K, f\in F \tag{A.10}$$

$$y_{k,a,t}\ge 0 \qquad \forall [k,a,t]\in N \tag{A.11}$$

$$\pi^h_{k,f}\ge 0 \qquad \forall h\in H, k\in K, f\in F \tag{A.12}$$

$$0\le d_i\le \bar{d}_i \qquad \forall i\in I \tag{A.13}$$

$$t_{i,j}\ge 0 \qquad \forall i\in I, j\in I \tag{A.14}$$

This model uses the given price values ($\bar{p}_i$) in the dataset. It assumes that the recapture ratios, $b_{i,j}$ are given by a logit formula. With the given itinerary attributes in the dataset, recapture ratios ($\bar{b}_{i,j}$) and forecasted demand ($\bar{d}_i$) are calculated and taken as an input. The model optimizes the revenue minus operating costs (A.1). The revenue function explicitly includes the spill and recapture effects. In addition to the schedule planning decisions, seat allocation to each class of passengers is optimized. Constraints (A.7) maintain the balance between demand and allocated seats. Constraints (A.8) ensure that actual capacity of aircraft is respected. The number of spilled passengers from an itinerary cannot be more than the expected demand of that itinerary which is given by constraints (A.9).

A.2.2 IFAM$^{'}$

$$
\begin{aligned}
& z^*_{\text{IFAM}'} = \\
& \max \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I_s^{'})} u_i \bar{p}_i - \sum_{\substack{k\in K\\ f\in F}} C_{k,f} x_{k,f} && \text{(A.15)}\\
& \text{s.t.} \sum_{k\in K} x_{k,f} = 1 && \forall f\in F^M \quad \text{(A.16)}\\
& \sum_{k\in K} x_{k,f} \le 1 && \forall f\in F^O \quad \text{(A.17)}\\
& y_{k,a,t^-} + \sum_{f\in \text{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in\text{Out}(k,a,t)} x_{k,f} && \forall [k,a,t]\in N \quad \text{(A.18)}\\
& \sum_{a\in A} y_{k,a,\text{minE}_a^-} + \sum_{f\in CT} x_{k,f} \le R_k && \forall k\in K \quad \text{(A.19)}\\
& y_{k,a,\text{minE}_a^-} = y_{k,a,\text{maxE}_a^+} && \forall k\in K, a\in A \quad \text{(A.20)}\\
& \sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I_s^{'})} \delta_{i,f} u_i \le \sum_{k\in K} \pi^h_{k,f} && \forall h\in H, f\in F \quad \text{(A.21)}\\
& \sum_{h\in H} \pi^h_{k,f} \le Q_k x_{k,f} && \forall f\in F, k\in K \quad \text{(A.22)}\\
& \sum_{i\in I_s} u_i = 1 && \forall h\in H, s\in S^h \quad \text{(A.23)}\\
& u_i \le v_s \exp\left(V_i(\bar{p}_i, z_i; \beta)\right) && \forall h\in H, s\in S^h, i\in (I_s\setminus I_s^{'}) \quad \text{(A.24)}\\
& u_j = v_s \exp\left(V_j(\bar{p}_j, z_j; \beta)\right) && \forall h\in H, s\in S^h, j\in I_s^{'} \quad \text{(A.25)}\\
& x_{k,f}\in\{0,1\} && \forall k\in K, f\in F \quad \text{(A.26)}\\
& y_{k,a,t} \ge 0 && \forall [k,a,t]\in N \quad \text{(A.27)}\\
& \pi^h_{k,f} \ge 0 && \forall h\in H, k\in K, f\in F \quad \text{(A.28)}\\
& u_i \ge 0 && \forall h\in H, s\in S^h, i\in I_s \quad \text{(A.29)}\\
& v_s \ge 0 && \forall h\in H, s\in S^h \quad \text{(A.30)}
\end{aligned}
$$

This model does not have the pricing decision and therefore prices are represented by $\bar{p}_i$ for the ease of explanation.

A.3 Revenue sub-problems

The revenue sub-problems optimize the revenue with a fixed capacity, which is provided by the solution of the fleet assignment models. Therefore the fleet assignment solutions are input parameters for revenue models and represented by $\bar{x}_{k,f}$.

A.3.1 RMM

$$z^*_{\text{RMM}} = \max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} t_{j,i}\bar{b}_{j,i})\bar{p}_i \tag{A.31}$$

$$\text{s.t.} \sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)}\delta_{i,f}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} t_{j,i}\bar{b}_{j,i})$$

$$\leq \sum_{k\in K}\pi^h_{k,f} \qquad \forall h\in H, f\in F \tag{A.32}$$

$$\sum_{h\in H}\pi^h_{k,f} \leq Q_k\bar{x}_{k,f} \qquad \forall f\in F, k\in K \tag{A.33}$$

$$\sum_{j\in I_s} t_{i,j} \leq d_i \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) \tag{A.34}$$

$$\pi^h_{k,f} \geq 0 \qquad \forall h\in H, k\in K, f\in F \tag{A.35}$$

$$0 \leq d_i \leq \bar{d}_i \qquad \forall h\in H, s\in S^h, i\in I_s \tag{A.36}$$

$$t_{i,j} \geq 0 \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s), j\in I_s \tag{A.37}$$

Note that this RMM does not have the pricing decision and therefore price ($\bar{p}_i$), recapture ratios ($\bar{b}_{i,j}$), and the forecasted demand values ($\bar{d}_i$) are input parameters to the model.

A.3.2 RMM-PR

$$z^*_{\text{RMM-PR}} = \max \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} t_{j,i}b_{j,i})p_i \tag{A.38}$$

$$\text{s.t.} \sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)}\delta_{i,f}(d_i - \sum_{j\in I_s} t_{i,j} + \sum_{j\in(I_s\setminus I'_s)} t_{j,i}b_{j,i})$$

$$\leq \sum_{k\in K}\pi^h_{k,f} \qquad \forall h\in H, f\in F \tag{A.39}$$

$$\sum_{h\in H}\pi^h_{k,f} \leq Q_k\bar{x}_{k,f} \qquad \forall f\in F, k\in K \tag{A.40}$$

$$\sum_{j\in I_s} t_{i,j} \leq d_i \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) \tag{A.41}$$

$$\tilde{d}_i = D_s\frac{\exp(V_i(p_i, z_i;\beta))}{\sum_{j\in I_s}\exp(V_j(p_j, z_j;\beta))} \qquad \forall h\in H, s\in S^h, i\in I_s \tag{A.42}$$

$$b_{i,j} = \frac{\exp(V_j(p_j, z_j;\beta))}{\sum_{k\in I_s\setminus\{i\}}\exp(V_k(p_k, z_k;\beta))} \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s), j\in I_s \tag{A.43}$$

$$\pi^h_{k,f} \geq 0 \qquad \forall h\in H, k\in K, f\in F \tag{A.44}$$

$$0 \leq d_i \leq \tilde{d}_i \qquad \forall h\in H, s\in S^h, i\in I_s \tag{A.45}$$

$$LB_i \leq p_i \leq UB_i \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) \tag{A.46}$$

$$t_{i,j} \geq 0 \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s), j\in I_s \tag{A.47}$$

$$b_{i,j} \geq 0 \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s), j\in I_s \tag{A.48}$$

A.3.3 RMM-PR$'$

$$z^*_{\text{RMM-PR}'} = \max \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} u_i p_i) \tag{A.49}$$

$$\sum_{i\in I_s} u_i = 1 \qquad \forall h\in H, s\in S^h \tag{A.50}$$

$$u_i \le v_s \exp(V_i(p_i, z_i;\beta)) \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) \tag{A.51}$$

$$u_j = v_s \exp(V_j(p_j, z_j;\beta)) \qquad \forall h\in H, s\in S^h, j\in I'_s \tag{A.52}$$

$$\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} \delta_{i,f} u_i \le \sum_{k\in K} \pi^h_{k,f} \qquad \forall h\in H, f\in F \tag{A.53}$$

$$\sum_{h\in H} \pi^h_{k,f} \le Q_k \bar{x}_{k,f} \qquad \forall f\in F, k\in K \tag{A.54}$$

$$\pi^h_{k,f} \ge 0 \qquad \forall h\in H, k\in K, f\in F \tag{A.55}$$

$$LB_i \le p_i \le UB_i \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) \tag{A.56}$$

$$u_i \ge 0 \qquad \forall h\in H, s\in S^h, i\in I_s \tag{A.57}$$

$$v_s \ge 0 \qquad \forall h\in H, s\in S^h \tag{A.58}$$

A.4 Illustration for the market shares with the reformulated model

We illustrate how the logit model behaves in the reformulated integrated model given in Chapter 6. For that purpose we pick an OD-pair, i.e. a market segment, provided in experiment 2. The selected market segment consists of the itineraries 3, 5, 7, 9, and 11 that are presented in Table 6.1. Itinerary 9 is not operated since the corresponding flight is canceled as a decision of the model.

The detailed results are provided in Table A.5. Note that the last row represents the no-revenue option, i.e. an itinerary offered by a competing airline, which have a fixed price that cannot be controlled by the airline. In the second column, we present the assigned capacity for the corresponding flights of the itineraries. In this simple example, all the itineraries are non-stop. Therefore, the capacity is used by a single itinerary which makes the analysis easier. Third column is the resulting price for the itineraries and the fourth column provides the corresponding utilities. The sixth column gives the upper bound on price which is indeed the right hand side of the constraints (6.18). To remind that, this upper bound is determined by the relative attractiveness of each itinerary with respect to the no-revenue option. The seventh column on the other hand provides the resulting market shares for all the itineraries in the market segments. As discussed in section 7.5.2, the market share constraints (6.18) are introduced as inequality constraints due to the bounds on the price values. It is observed that the only itinerary which has a realized price that is different than the upper bound is itinerary 7. Therefore, the realized market share for itinerary 7 is exactly equal to $v\exp(V_i)$, i.e. the market share constraint is active.

Finally, in the last column we present a set of hypothetical market share values for the

Table A.5: An illustration of the market shares for the reformulated model - experiment 2

v for the OD-pair:			2.401				
UB on price for the OD-pair:			250				
itineraries	cap. (seats)	price	utility V_i	$\exp(V_i)$	UB on market share $v\exp(V_i)$	resulting market share u_i	hypothetical market share
3	50	250	-2.196	0.111	0.267	0.130 (50 pax)	0.189 (73 pax)
5	50	250	-2.196	0.111	0.267	0.130 (50 pax)	0.189 (73 pax)
7	117	**238.9**	-2.067	0.127	**0.304**	**0.304** (117 pax)	0.215 (83 pax)
11	50	250	-2.196	0.111	0.267	0.130 (50 pax)	0.189 (73 pax)
no-revenue		235	-2.058	0.128		0.306 (118 pax)	0.217 (83 pax)

itineraries. We take the resulting utilities for the itineraries (excluding 9) and compute the market shares based on the logit formula (6.2). It is done in order to have a feeling of the realized spill and recapture effects. When the resulting market shares are compared to these hypothetical values, it is seen that itineraries 3, 5, and 11 lose passengers and result with a lower market share. On the other hand, itinerary 7 and the no-revenue option attract passengers with a lower price. We see that in total 69 passengers are spilled from itineraries 3, 5 and 11. The remaining itineraries in the market, which are itinerary 7 and the competing alternative, attract a number of the spilled passengers proportional to their expected utility. The recapture ratio for itinerary 7 is 0.127/(0.127+0.128) = 0.498 which corresponds to 34 of 69 passengers. As a result, the realized demand for itinerary 7 becomes 83 + 34 = 117 with the spill and recapture effects.

This analysis shows that, in the presence of price bounds, it is necessary to manage the market share variables, u_i, using inequality constraints as in the model (6.2). Indeed the same decision on the pricing of the itineraries would imply unfeasible number of passengers, i.e. the assigned capacity would not be sufficient the resulting demand. Therefore, the inequality constraints on the market shares give flexibility to the model and the resulting profit is increased.

A.5 Integrated model for the case of Clip-Air

In this section we provide the integrated airline scheduling, fleeting and pricing model presented in chapter 6 for the case of Clip-Air. We mention the main differences with respect to the model for standard aircraft. First of all, in addition to the fleet assignment variable $x_{k,f}$, that represents the assignment of capsules, variable x_f^w is introduced for the assignment of wings to the flights. The operating cost in the objective is written as the sum of the costs for wings and capsules. The relation between the wings and capsules are maintained by the set of constraints given in (A.61). When there is no wing assigned for the flight, capsules cannot be assigned neither. The constraints for the fleet assignment process is included for both wings and capsules as given in (A.62)-(A.67). Supply-demand balance constraints (A.68) and (A.69) are presented according to Clip-Air fleet as explained in section 8.2.1. The remaining constraints are similar to previous formulations.

$$z^*_{\text{IFAM-PR}^{\text{Clip-Air}'}} = \max \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I_s')} u_i p_i - \sum_{f\in F}(C_f^w x_f^w + \sum_{k\in K} C_{k,f} x_{k,f}) \tag{A.59}$$

$$\text{s.t.} \sum_{k\in K} x_{k,f} = 1 \qquad \forall f\in F^M \tag{A.60}$$

$$\sum_{k\in K} x_{k,f} \le x_f^w \qquad \forall f\in F \tag{A.61}$$

$$y^w_{a,t^-} + \sum_{f\in In(a,t)} x_f^w = y^w_{a,t^+} + \sum_{f\in Out(a,t)} x_f^w \qquad \forall [a,t]\in N \tag{A.62}$$

$$\sum_{a\in A} y^w_{a,\text{minE}_a^-} + \sum_{f\in CT} x_f^w \le R_w \tag{A.63}$$

$$y^w_{a,minE_a^-} = y^w_{a,maxE_a^+} \qquad \forall a\in A \tag{A.64}$$

$$y^k_{a,t^-} + \sum_{\substack{f\in In(a,t)\\ k\in K}} k\, x_{k,f} = y^k_{a,t^+} + \sum_{\substack{f\in Out(a,t)\\ k\in K}} k\, x_{k,f} \qquad \forall [a,t]\in N \tag{A.65}$$

$$\sum_{a\in A} y^k_{a,\text{minE}_a^-} + \sum_{\substack{f\in CT\\ k\in K}} k\, x_{k,f} \le R_k \tag{A.66}$$

$$y^k_{a,minE_a^-} = y^k_{a,maxE_a^+} \qquad \forall a\in A \tag{A.67}$$

$$\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I_s')} \delta_{i,f} u_i \le \pi_{f,h} \qquad \forall h\in H, f\in F \tag{A.68}$$

$$\sum_{h\in H} \pi_{f,h} \le \sum_{k\in K} Q\, k\, x_{k,f} \qquad \forall f\in F \tag{A.69}$$

$$\sum_{i\in I_s} u_i = 1 \qquad \forall h\in H, s\in S^h \tag{A.70}$$

$$u_i \le v_s \exp(V_i(p_i, z_i;\beta)) \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s') \tag{A.71}$$

$$u_j = v_s \exp(V_j(\bar{p}_j, z_j;\beta)) \qquad \forall h\in H, s\in S^h, j\in I_s' \tag{A.72}$$

$$x_f^w \in \{0,1\} \qquad \forall f\in F \tag{A.73}$$

$$x_{k,f} \in \{0,1\} \qquad \forall k\in K, f\in F \tag{A.74}$$

$$y^w_{a,t} \ge 0 \qquad \forall [a,t]\in N \tag{A.75}$$

$$y^k_{a,t} \ge 0 \qquad \forall [a,t]\in N \tag{A.76}$$

$$\pi_{f,h} \ge 0 \qquad \forall f\in F, h\in H \tag{A.77}$$

$$\text{LB}_i \le p_i \le \text{UB}_i \qquad \forall h\in H, s\in S^h, i\in(I_s\setminus I_s') \tag{A.78}$$

$$u_i \ge 0 \qquad \forall h\in H, s\in S^h, i\in I_s \tag{A.79}$$

$$v_s \ge 0 \qquad \forall h\in H, s\in S^h \tag{A.80}$$

A.6 Inverse demand function

Market share, u_i, is given by:

$$u_i = v_s \exp(\beta p_i + z_i), \tag{A.81}$$

which is similar to (6.5). If we have the inverse function we can write the price as a function of the choice probability:

$$p_i = \frac{1}{\beta}(\ln(\frac{u_i}{v_s}) - z_i) \tag{A.82}$$

The revenue for each itinerary, R_i, is given by $u_i p_i D_s$ which can be written as:

$$R_i = \frac{1}{\beta} D_s u_i (\ln(\frac{u_i}{v_s}) - z_i) \tag{A.83}$$

The Hessian for R_i is therefore given by:

$$H = \begin{pmatrix} \frac{\partial^2 R_i}{\partial u_i^2} = D_s \frac{1}{\beta}\frac{1}{u_i} & \frac{\partial^2 R_i}{\partial u_i \partial p_i} = -D_s \frac{1}{\beta}\frac{1}{v_s} \\ \frac{\partial^2 R_i}{\partial p_i \partial u_i} = -D_s \frac{1}{\beta}\frac{1}{v_s} & \frac{\partial^2 R_i}{\partial p_i^2} = D_s u_i \frac{1}{\beta}\frac{1}{v_s^2} \end{pmatrix}$$

where u_i, D_s, v_s are ≥ 0 by definition. $\beta \leq 0$ since it gives the effect of price on the utility. Therefore $\frac{\partial^2 R_i}{\partial u_i^2}$ and $\frac{\partial^2 R_i}{\partial p_i^2} \leq 0$ The determinant of the Hessian is given by:

$$\begin{aligned} &\frac{\partial^2 R_i}{\partial u_i^2}\frac{\partial^2 R_i}{\partial p_i^2} - \frac{\partial^2 R_i}{\partial u_i \partial p_i}\frac{\partial^2 R_i}{\partial p_i \partial u_i}, \\ &\quad = D_s^2 \frac{1}{\beta^2}\frac{1}{v_s^2} - D_s^2 \frac{1}{\beta^2}\frac{1}{v_s^2} = 0, \end{aligned}$$

which shows that the revenue function is concave (not strictly concave) $\forall u \geq 0, v_s \geq 0$.

A.7 The integrated model with the logarithmic transformation

In this section, we provide the integrated model with the logarithmic transformation proposed in chapter 7. We refer to this transformed integrated scheduling, fleeting and pricing model as IFAM-PR$^{\ln}$. The presented formulation is based on the integrated model IFAM-PR$'$ that is given in chapter 6. Remember that, the most important advantage of the log transformation is the reduced complexity of the problem which allows obtaining solutions without the need for bounds on the price. The integrated models given in chapters 4 and 6 on the other hand, does not result with any feasible solution without the bounds due to complexity.

$$
\begin{aligned}
& z^*_{IFAM-PR^{\ln}} = \\
\max & \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in(I_s\setminus I'_s)} R_i - M1_i(R_i - \exp(R'_i))^2 \\
& - \sum_{h\in H}\sum_{s\in S^h}\sum_{i\in I_s} M2_i(u_i - \exp(u'_i))^2 \\
& - \sum_{k\in K}\sum_{f\in F} C_{k,f}x_{k,f} && && \text{(A.84)} \\
\text{s.t.} & \sum_{k\in K} x_{k,f} = 1 && \forall f\in F^M && \text{(A.85)} \\
& \sum_{k\in K} x_{k,f} \le 1 && \forall f\in F^O && \text{(A.86)} \\
& y_{k,a,t^-} + \sum_{f\in \mathrm{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in \mathrm{Out}(k,a,t)} x_{k,f} && \forall [k,a,t]\in N && \text{(A.87)} \\
& \sum_{a\in A} y_{k,a,\mathrm{minE}^-_a} + \sum_{f\in CT} x_{k,f} \le R_k && \forall k\in K && \text{(A.88)} \\
& y_{k,a,\mathrm{minE}^-_a} = y_{k,a,\mathrm{maxE}^+_a} && \forall k\in K, a\in A && \text{(A.89)} \\
& \sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} \delta_{i,f}u_i \le \sum_{k\in K}\pi^h_{k,f} && \forall h\in H, f\in F && \text{(A.90)} \\
& \sum_{h\in H}\pi^h_{k,f} \le Q_k x_{k,f} && \forall f\in F, k\in K && \text{(A.91)} \\
& \sum_{i\in I_s} u_i = 1 && \forall h\in H, s\in S^h && \text{(A.92)} \\
& u'_i = v'_s + \beta_i p'_i + z_i && \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) && \text{(A.93)} \\
& u'_j = v'_s + \beta_j P'_j + z_j && \forall h\in H, s\in S^h, j\in I'_s && \text{(A.94)} \\
& u_i \le \exp(u'_i) && \forall h\in H, s\in S^h, i\in I_s && \text{(A.95)} \\
& R'_i = \ln(D_s) + u'_i + p'_i && \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) && \text{(A.96)} \\
& R_i \le \exp(R'_i) && \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) && \text{(A.97)} \\
& R_i \ge 0 && \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) && \text{(A.98)} \\
& R'_i \in \mathbb{R} && \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) && \text{(A.99)} \\
& u_i \ge 0 && \forall h\in H, s\in S^h, i\in I_s && \text{(A.100)} \\
& u'_i \in \mathbb{R} && \forall h\in H, s\in S^h, i\in I_s && \text{(A.101)} \\
& v'_s \in \mathbb{R} && \forall h\in H, s\in S^h && \text{(A.102)} \\
& p'_i \in \mathbb{R} && \forall h\in H, s\in S^h, i\in(I_s\setminus I'_s) && \text{(A.103)} \\
& x_{k,f} \in \{0,1\} && \forall k\in K, f\in F && \text{(A.104)} \\
& y_{k,a,t} \ge 0 && \forall [k,a,t]\in N && \text{(A.105)} \\
& \pi^h_{k,f} \ge 0 && \forall h\in H, k\in K, f\in F && \text{(A.106)}
\end{aligned}
$$

We illustrate the different decisions of the transformed model and the previous formulations. For that purpose we select instance 1 (see Table 5.1), which is used in several sections of the thesis. For this data instance, the solution of models IFAM-PR given in chapter 4 and IFAM-PR$'$

Table A.6: An illustrative example for the transformed model - experiment 1

	Previous models IFAM-PR/IFAM-PR$'$		**Transformed model IFAM-PR$^{\ln}$**	
Revenue		65,717		70,482
Operating costs		50,626		46,751
Profit		15,091		23,731
Number of flights		8		8
Transported passengers		284		256
Allocated seats		124		74
Itineraries	demand	price	demand	price
1	*canceled*		*canceled*	
2	9	365.61	9	365.61
3	12	141.26	12	141.26
4	8	**375**	8	**380.57**
5	11	143.06	11	143.06
6	2	**525**	2	**528.10**
7	33	**225**	33	**275.34**
8	37	**250**	37	**318.98**
9	**49**	**203.06**	**34**	**275.34**
10	**50**	**250**	**37**	**314.97**
11	5	**449.28**	5	**463.71**
12	31	**170.75**	31	**174.89**
13	4	**450**	4	**530.88**
14	33	**200**	33	**255.65**
15	*canceled*		*canceled*	
16	*canceled*		*canceled*	

Boldface values show the differences between the results of the models

given in chapter 6 are the same. In Table A.6 we compare the results for IFAM-PR$^{\ln}$ and the previous models. It is observed that, the canceled flights are the same flights for the models. However with the transformed model, higher prices are obtained for most of the itineraries since there are no bounds on the price and since the price elasticity allows for it. As the prices are increased, relatively less number of passengers are attracted and the allocated capacity is decreased. Therefore, the resulting profit is higher.

Note that constraints (A.93) are kept as equality constraints since there are no bounds on the price. In order to see if the transformed integrated model is a valid transformation of IFAM-PR$'$, we impose the same bounds as used in chapters 4 and 6. Moreover, we change the constraints (A.93) back to $\leq$ constraints in order to allow spill due to the bounds on price. It is observed that the same solution as IFAM-PR$'$ is obtained given that a high enough penalty is imposed.

A.8 Lagrangian relaxation

In this section, we demonstrate the application of Lagrangian relaxation method over the IFAM-PR$^{\ln}$ model that is given in Appendix A.7. In order to obtain valid bounds we need convexity, which is addressed by section 7.6. Since the presented piecewise linear approximation of the revenue maximization problem is not tractable in terms of computational time, we present the methodology over the integrated model without the piecewise linear approximation.

First of all, for a more compact representation, we merge the constraints (A.90) and (A.91) by:

$$\sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} \delta_{i,f} u_i \le \sum_{k\in K} Q_k x_{k,f} \quad \forall f \in F, \tag{A.107}$$

where we remove the decision on the seat allocation, namely the decision variable $\pi^h_{k,f}$.

For Lagrangian relaxation we relax the constraints (A.107) and introduce the Lagrangian multipliers λ_f for each flight f. Therefore, the objective function (A.84) is re-written as:

$$\begin{aligned} z(\lambda) = \max \sum_{h\in H}\sum_{s\in S^h} & \left(\sum_{i\in(I_s\setminus I'_s)} R_i - M1_i(R_i - \exp(R'_i))^2 - \sum_{i\in I_s} M2_i(u_i - \exp(u'_i))^2 \right) \\ & - \sum_{k\in K}\sum_{f\in F} C_{k,f} x_{k,f} \\ & + \sum_{f\in F} \lambda_f \left(\sum_{k\in K} Q_k x_{k,f} - \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} \delta_{i,f} u_i \right), \end{aligned} \tag{A.108}$$

which is subject to constraints (A.85)-(A.89) and (A.92)-(A.106). By re-arranging the terms we can write the objective function as:

$$\begin{aligned} z(\lambda) = \max \sum_{h\in H}\sum_{s\in S^h} & \left(\sum_{i\in(I_s\setminus I'_s)} R_i - M1_i(R_i - \exp(R'_i))^2 - \sum_{i\in I_s} M2_i(u_i - \exp(u'_i))^2 \right) \\ & - \sum_{f\in F} \lambda_f \left(\sum_{h\in H}\sum_{s\in S^h} D_s \sum_{\in(I_s\setminus I'_s)} \delta_{i,f} u_i \right) \\ & - \sum_{f\in F}\sum_{k\in K} \left(C_{k,f} - \lambda_f Q_k \right) x_{k,f}. \end{aligned} \tag{A.109}$$

The model now can be decomposed into two subproblems. The first is a revenue maximization model where the Lagrangian multipliers modify the revenue depending on the demand-capacity balance. The objective function is given by:

$$z_{\text{RMM}}(\lambda) = \max \sum_{h\in H}\sum_{s\in S^h}\left(\sum_{i\in(I_s\setminus I'_s)} R_i - M1_i(R_i - \exp(R_i^{'}))^2 - \sum_{i\in I_s} M2_i(u_i - \exp(u_i^{'}))^2\right)$$
$$- \sum_{f\in F}\lambda_f\left(\sum_{h\in H}\sum_{s\in S^h} D_s \sum_{\in(I_s\setminus I'_s)} \delta_{i,f}u_i\right) \tag{A.110}$$

which is subject to constraints (A.92)-(A.103).

The second subproblem is a fleet assignment model where seat prices are assigned through Lagrangian multipliers. The objective function is given by:

$$z_{\text{FAM}}(\lambda) = \min \sum_{f\in F}\sum_{k\in K}\left(C_{k,f} - \lambda_f Q_k\right)x_{k,f}, \tag{A.111}$$

which is subject to constraints (A.85)-(A.89), (4.10) and (A.104)-(A.105).

A.8.1 Solving the Lagrangian dual via sub-gradient optimization

We apply sub-gradient optimization to solve the Lagrangian dual $z_D = \min_{\lambda\geq 0}\max z(\lambda)$. The gradient for flight f is defined as:

$$G_f = \sum_{k\in K} Q_k x_{k,f} - \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} \delta_{i,f}u_i \tag{A.112}$$

The step size for flight f is defined as:

$$T_f = \frac{\eta(z_{UB} - z_{LB})}{\sum_{f\in F} G_f^2}, \tag{A.113}$$

where η is a scale parameter, typically initialized at 2; z_{UB} and z_{LB} are upper and lower bounds, respectively. We update the Lagrangian multipliers using the gradient and the step size by:

$$\lambda_f = \max(0, \lambda_f - T_f G_f). \tag{A.114}$$

A.8.2 Lagrangian heuristic

At each iteration of the solution of the Lagrangian dual z_D, the optimal solution of $z(\lambda)$ may violate the capacity constraints (A.107) for some $f \in F$. Therefore we need to obtain a primal feasible solution which serves as a lower bound. As an heuristic way to achieve such a lower bound we use the optimal solution to $z_{FAM}(\lambda) = \{\bar{x}, \bar{y}\}$ and fix this fleet assignment solution and provide as an input to $\widetilde{\text{RMM}}^{\ln}$ given in chapter 7. Remember that this model has capacity constraints and with the optimized fleet assignment solution we will obtain a primal feasible solution.

A.8.3 Overall algorithm

Having provided the necessary steps, we can give the pseudo-code of the Lagrangian relaxation procedure. *no improvement()* function checks if the upper bound is improved in the last 4

Algorithm 2 Subgradient procedure

Require: z_{LB}, $k_{\max}$, ϵ
  $\lambda^0 := 0$, $k := 0$, $z_{UB} := \infty$, $\eta := 2$
  repeat
    $\{\bar{u}, \bar{p}'\} :=$ solve $z_{\text{RMM}}(\lambda^k)$
    $\{\bar{x}, \bar{y}\} :=$ solve $z_{\text{FAM}}(\lambda^k)$
    $z_{UB}(\lambda^k) := z_{\text{RMM}}(\lambda^k) - z_{\text{FAM}}(\lambda^k)$
    update the upper bound $z_{UB} := \min(z_{UB}, z_{UB}(\lambda^k))$
    if no improvement(z_{UB}) **then**
      $\eta := \eta/2$
    end if
    obtain a primal feasible solution $lb := z_{\widetilde{\text{RMM}}^{\ln}}(\bar{x})$
    update the lower bound $z_{LB} := \max(z_{LB}, lb)$
    $G :=$ compute sub-gradient$(z_{UB}, z_{LB}, \{\bar{u}, \bar{x}\})$
    $T :=$ compute step$(z_{UB}, z_{LB}, \bar{u}, \bar{x}\})$
    update multipliers $\lambda^{k+1} := \max(0, \lambda^k - TG)$
    $k := k+1$
  until $||TG||^2 \le \epsilon$ **or** $k \ge k_{\max}$

iterations in order to reduce the scale if there is no improvement. As the stopping criteria, the maximum number iterations, $k_{\max}$, can be selected or the algorithm can be kept running until the update on the Lagrangian multipliers becomes less than the threshold value ϵ.

A.9 A generalized Benders' decomposition framework

A Generalized Benders' Decomposition framework can be designed for the reformulated model with logarithmic transformation, IFAM-PR$^{\ln}$, based on the Mixed Integer Nonlinear Programming chapter of Li and Sun (2006). Similar to the Lagrangian relaxation procedure explained in section A.8, we need convexity in order to have valid bounds. However, in order to illustrate the idea, we present it over IFAM-PR$^{\ln}$. Given that, an efficient algorithm is designed for the solution of the piecewise linear approximation of the model given in section 7.6, the revenue part should be modified accordingly. The resulting convexity will then enable to obtain valid bounds.

In order to apply the idea to our case, we fix the fleet assignment variables $x_{k,f}$ and obtain the sub-problem that is similar to the revenue maximization problem, $\widetilde{\text{RMM}}^{\ln}$ given in chapter 7, section 7.5.2. The sub-problem optimizes the price and market share in order to maximize the revenue with a fixed set of fleet assignment decisions, $x_{k,f}$. The solution of the sub-problem provides Benders' cuts to the master problem through the supply-demand balance constraints (A.107). We define λ_f's as the dual variables associated with these constraints. The master problem is a fleet assignment model, where the revenue related decisions are fixed. The master problem is provided as follows:

$$
\begin{aligned}
z_{\text{master}} = \max \quad & \alpha && && \text{(A.115)}\\
\text{s.t. } \alpha \le & \sum_{h\in H}\sum_{s\in S^h} D_s \sum_{i\in(I_s\setminus I'_s)} \exp(\bar{p}_i'^{c})\bar{u}_i^c - \sum_{k\in K}\sum_{f\in F} C_{k,f}\bar{x}_{k,f}^c \\
& + \sum_{k\in K}\sum_{f\in F}(Q_k\lambda_f^c - C_{k,f})[x_{k,f} - \bar{x}_{k,f}^c] && \forall c \in \text{CUTS} && \text{(A.116)}\\
& \sum_{k\in K} x_{k,f} = 1 && \forall f \in F && \text{(A.117)}\\
& y_{k,a,t^-} + \sum_{f\in \text{In}(k,a,t)} x_{k,f} = y_{k,a,t^+} + \sum_{f\in \text{Out}(k,a,t)} x_{k,f} && \forall [k,a,t] \in N && \text{(A.118)}\\
& \sum_{a\in A} y_{k,a,\text{minE}_a^-} + \sum_{f\in CT} x_{k,f} \le R_k && \forall k \in K && \text{(A.119)}\\
& y_{k,a,\text{minE}_a^-} = y_{k,a,\text{maxE}_a^+} && \forall k \in K, a \in A && \text{(A.120)}\\
& x_{k,f} \in \{0,1\} && \forall k \in K, f \in F && \text{(A.121)}\\
& y_{k,a,t} \ge 0 && \forall [k,a,t] \in N && \text{(A.122)}
\end{aligned}
$$

The objective value is bounded by the potential profit that would be obtained with a change on the fleet assignment solution, as given by (A.116). The first part of this bound is the actual profit given by the fixed decision variables $\bar{p}'$, $\bar{u}$ and $\bar{x}$. The actual revenue is modified by the Lagrangian multipliers λ_f. These multipliers, that are introduced in the Benders' cuts, represent the main idea of the framework. The information on the potential revenue change, by a modification on the fleet assignment, is carried with λ_f^c's at each iteration c. In the framework presented by Li and Sun (2006) there are also feasibility cuts, which are added when the sub-problem is infeasible. In our case, the sub-problem is always feasible since with any fleet assignment solution, a feasible revenue solution can be obtained. The supply-demand balance is represented by inequality constraints (7.86) which allows the feasibility with any supply capacity. So we only have the cuts given in (A.116).

The λ_f multipliers should be obtained through the optimality conditions of the sub-problem and transfered to the master problem. Then the capacity provided by the master problem is transfered to the sub-problem. This iterative framework is provided in Algorithm 3.

As common with Benders' Decomposition, the convexity of the master problem increases with the number of cuts. Algorithms can be developed in order to accelerate the process (e.g. Rei et al., 2009).

Algorithm 3 Generalized Benders' Framework

Require: Choose the initial fleet assignment solution $\bar{x}^1$, and ϵ

$\lambda^0 := 0$, $k := 1$, $z_{LB} := -\infty$, $z_{UB} := \infty$, $k := 1$

repeat

Step 1. Solve the subproblem with fixed fleet assignment $\widetilde{\text{RMM}}^{\ln}(\bar{x}^k)$. We obtain the price, $\bar{p}'^k$, the market share $\bar{u}^k$ and the Lagrangian multipliers λ^k. We update the lower bound, $z^k_{LB} := \max(z^{k-1}_{LB}, z^*_{\text{IFAM-PR}^{\ln}}(\bar{x}^k, \bar{u}^k, \bar{p}'^k))$. For the lower bound the objective function of the integrated model (IFAM-PR$^{\ln}$) with fixed decision variables is calculated.

Step 2. Solve the master problem with λ^k, $\bar{u}^k$ and $\bar{p}'^k$, obtain $\bar{x}^{k+1}$ with the objective function value α^k. Set $z_{UB} = \alpha^k$.

$k := k + 1$

until $z_{LB} \geq z_{UB} + \epsilon$

B Glossary

B.1 Definitions

airline schedule planning: the set of processes for designing future airline schedules to maximize airline profitability. It includes the decisions on the markets to serve, the frequency, the time schedule for flights, the aircraft type, aircraft routing and the crew.

aggregate demand model: a demand model where the behavior of individuals are modeled with the attributes of the alternatives rather than the characteristics of individuals.

available seat kilometers (ASK): the number of seats available multiplied by the number of kilometers flown.

Clip-Air capsules: the cabin part of CliP-Air that is designed to carry the passengers/freight and that is separated from the wing.

Clip-Air wing: the flying unit of Clip-Air that is designed to carry the engines and the flight (cockpit) crew.

day-to-day variation of demand: the variation of the number of passengers that are willing to travel on an itinerary/flight from day to day; e.g. on certain market segments demand may be higher on Mondays compared to other days of the week.

demand responsiveness: the ability of a system/model to react to the changes in the demand patterns.

disaggregate demand model: a demand model where the behavior of individuals are modeled taking into account the different characteristics of individuals in addition to the attributes of the alternatives. Therefore, the estimations can be obtained for different groups of individuals in the population.

fleet assignment: the process of assigning aircraft types to the flights.

fleeting: the output of a fleet assignment model which gives the assignment of aircraft types to the flights.

flight density: the average number of flights per OD pair for a given data instance.

flight leg: operation which starts with exactly one take-off and ends with exactly one landing.

hub and spoke: an airline network type where the flights connect only through the main airports that are called hubs.

itinerary: the product offered for passengers between an origin and destination which may consist of multiple flight legs.

itinerary-based fleet assignment: a fleet assignment model where the demand information is considered at the itinerary level rather than the flight level.

itinerary choice model: discrete choice model for the behavior of passengers for their decision on which itinerary to fly.

latent variable: a variable that cannot be directly observed but rather inferred from other observed variables; e.g. attitudes and perceptions of individuals that have an impact on their behavior.

latent class: a segment of population which is identified based on the latent characteristics of individuals.

load factor of a flight: the ratio of the number of passengers to the total number of seats assigned to the flight.

local search: the method for solving an optimization problem through local changes in the solution in a given neighborhood.

market segment: the passengers that desire to travel between an origin airport and a destination airport for a given class; e.g. economy class passengers who want to travel from Geneva to Boston.

mandatory flights: the set of flights which needs to be served by the airline.

mixed passenger and freight transportation: the transportation of passenger and freight on the same transportation unit. It is typical for airlines to carry mixed load on the same aircraft.

mode choice model: discrete choice model for the behavior of passengers for their decision on which transportation mode to perform their trips.

modularity (in transportation): the concept for the design of a transportation system where a set of functional units can be composed into a transportation mode. In the context of Clip-Air, the configuration can be altered with different number of capsules attached to the wing.

multi-modal transportation: the transportation of passengers/freight through multiple means of transportation modes.

no-revenue options: alternative itineraries in a market segment which represent the alternatives provided by the competing airlines. These alternatives are included in order to have a reference value for the market price and the airline does not have a control on the attributes of these alternatives.

optional flights: the set of flights which might be served based on their profitability, i.e. they can be canceled if they are not expected to be profitable.

peer-to-peer network (airlines): an airline network type where the airports in the network are connected to each other without the need for a main airport (hub). The flights therefore are operated between each airport pair.

perturbation: a change in the expected value of a parameter in the system.

price sampling: the process of randomly selecting the price of the itineraries in a given range of prices.

quality service index (QSI): a method of assessing the relative quality of different services in order to forecast the market share of these services in a market.

recapture: the process of accommodating passengers on other itineraries in the market segment in case their originally desired itinerary has capacity restrictions.

recapture ratio: the ratio of passengers who can be accommodated on a specific itinerary when spilled from their desired itinerary. This ratio typically depends on a demand model.

revealed preferences: the preferences of individuals which can be revealed from their actual choices; e.g. preference of an individual towards the use of public transportation can be revealed from the actual trips he/she performs.

revenue management: the process of understanding, anticipating and influencing consumer behavior in order to maximize the revenue. For airlines, this process is based on demand forecasts for flights/itineraries and is performed with the decisions on capacity allocation for different cabin classes and the pricing of alternatives for the considered market segments.

robustness: the ability of a system to resist change without adapting its initial configuration. For a model it can be considered as the stability of the decisions with changing input parameters.

sensitivity analysis: the analysis of a system/model in order to assess the sensitivity of the outputs/results to the uncertainties or fluctuations in the inputs/parameters.

spill: the process of rejecting passengers due to capacity limitations so that passengers are not able to fly on their desired itinerary.

spill cost: the lost revenue due to spilled passengers.

spill rate: the average number of spilled passengers divided by the total demand for the flight/itinerary.

standard fleet: the fleet composed of standard aircraft that are already used in airline industry.

stated preferences: the preferences which are stated by the individuals but not confirmed by their actual choice; e.g. preference of an individual towards the use of a future transportation alternative can only be based on his/her statement.

time-of-day preferences: the preferences of travelers for the departure time of their flights which is commonly used in demand modeling.

time-space network: the graphical representation of a network where a node has information about both time and space and an arc represents the movements of the aircraft.

transported passengers per available seat kilometers (TPASK): the total number of transported passengers divided by the available seat kilometers.

turnaround time of an aircraft: the time needed between the unloading of the aircraft at the end of a flight and the loading for the next flight.

unconstrained demand: the number of passengers that would travel on an itinerary/ a flight if there was no capacity limitations.

value of time (VOT): the amount that a traveler would be willing to pay in order to reduce the traveling time.

variable neighborhood search (VNS): a metaheuristic method for solving combinatorial optimization problems where the size of the neighborhood varies according to the quality of the solution.

B.2 Notation

Sets related to the optimization models

A : the set of airports indexed by a
$B1$, $B2$: sets of breakpoints for the piecewise linear approximation of the choice variable, indexed by b
C : the choice set for a general setting of a logit model
C^o : the set of alternatives provided by the considered supplier, $C^o \in C$
C^c : the set of alternatives provided by the competitors, $C^c \in C$
CT : the set of flights flying at count time
F: the set of flight legs indexed by f

F_M : the set of mandatory flights
F_O : the set of optional flights
H : set of cabin classes indexed by h
I_s : the set of itineraries in segment s, indexed by i
$I_s^{'}$: the set of no-revenue itineraries, $I_s^{'} \in I_s$
K : the set of fleet types indexed by k
Ł : the set of fixed assignments in the local search heuristic indexed by ℓ
N : the set of individuals for a general setting of a logit model indexed by n
$N(k,a,t)$: the set of the nodes in the time-line network, for fleet type k, airport a and time t
$\text{In}(k,a,t)$: the set of inbound flight legs for node (k,a,t)
$\text{Out}(k,a,t)$: the set of outbound flight legs for node (k,a,t)
S^h : the set of market segments indexed by s, for cabin class h
T : the set of time of the events in the network indexed by t

Parameters of the optimization models

$\bar{b}_{i,j}$: the fixed recapture ratio for the passengers spilled from itinerary i, and redirected to itinerary j - used in IFAM and RMM
$C_{k,f}$: operating cost for flight f when operated by fleet type k, also used for Clip-Air capsules
C_f^w : operating cost of Clip-Air wing for flight f
Cap_f : the capacity assigned to flight f
$\bar{d}_i$: the fixed demand of itinerary i based on the logit model - used in IFAM and RMM
D_i : unconstrained demand for itinerary i
D_s : unconstrained demand for market segment s
LB_i : the lower bound on the price of the itinerary i
M, $M1$, $M2$, M_1, M_2 : arbitrary large numbers used in optimization problems
$max\text{E}_a^+$: the time just after the last event at airport a
$min\text{E}_a^-$: the time just before the first event at airport a
$\bar{p}_i$: the fixed price of itinerary i - used in IFAM, IFAM$^{'}$, and RMM, also represents the price of no-revenue options
Q_k : the capacity of fleet type k in number of seats, used as Q for the case of Clip-Air in order to represent the number of seats per capsule
R_k : available number of aircraft of type k, also used for Clip-Air capsules
R_w : available number of Clip-Air wings
UB_i : the upper bound on the price of the itinerary i
$\bar{x}_{k,f}$: fixed parameter used as the fixed fleet assignment solution in RMM-PR and RMM-PR$^{'}$
$x_{k_\ell^{\text{fixed}},f_\ell^{\text{fixed}}}$: a fixed fleet assignment solution used in the heuristic, where flight f_ℓ^{fixed} is fixed to aircraft k_ℓ^{fixed}
z_i : airline case - the vector of explanatory variables for itinerary i
$z_{i,n}$: general case - the vector of explanatory variables for alternative i and individual n

β, β_n : the vector of coefficients to be estimated for the logit model
$\delta_{i,f}$: 1 if itinerary i uses flight leg f, 0 otherwise

Variables of the optimization models

$b_{i,j}$: recapture ratio for the passengers spilled from itinerary i, and redirected to itinerary j
d_i : realized demand of itinerary i
$\tilde{d_i}$: expected demand of itinerary i based on the logit model
p_i : price of itinerary i
$p_i^{'}$: the logarithm of the price variable
$R_{i,n}$: the revenue obtained from individual n for the purchase of product i
$R_{i,n}^{'}$: the logarithm of the revenue variable $R_{i,n}$
$t_{i,j}$: redirected passengers from itinerary i to itinerary j
u_i : market share of itinerary i
V_i : airline case - the utility of itinerary i
$V_{i,n}$: the general case - the utility for alternative i and individual n
$x_{k,f}$: binary variable, 1 if fleet type k is assigned to flight f, 0 otherwise, also used for Clip-Air capsules
x_f^w : binary variable, 1 if a Clip-Air wing is assigned to flight f, 0 otherwise
y_{k,a,t^-} : the number of type k planes at airport a just before time t
y_{k,a,t^+} : the number of type k planes at airport a just after time t
y_{a,t^-}^k : the number of Clip-Air capsules at airport a just before time t
y_{a,t^+}^k : the number of Clip-Air capsules at airport a just after time t
y_{a,t^-}^w : the number of Clip-Air wings at airport a just before time t
y_{a,t^+}^w : the number of Clip-Air wings at airport a just after time t
$y_{i,n}$: a general representation for the choice probability for alternative i and individual n based on the logit model
$y_{i,n}^{'}$: the logarithm of the variable $y_{i,n}$

$\pi_{k,f}^h$: assigned seats for flight f on a type k aircraft for cabin class h, used as $\pi_{f,h}$ for the case of Clip-Air in order to represent the total seats assigned to class h passengers on flight f
υ_n : a variable defined to represent $\frac{1}{\sum_{j\in C}\exp(V_j)}$ for individual n
$\upsilon_n^{'}$: the logarithm of variable υ_n
υ_s : a variable defined to represent $\frac{1}{\sum_{j\in I_s}\exp(V_j)}$ for market segment s
$\upsilon_s^{'}$: the logarithm of variable υ_s
$\omega1_{b,i,n}, \omega2_{b,i,n}$: binary variables defined for the piecewise linear approximation of the choice probability variable $y_{i,n}$

Other notations

$f()$: the function that represents the penalty term for the logarithmic transformation
F^g_{flown} : the set of flights which are flown at iteration g of the local search heuristic
$g()$: a general convex function in order to represent the constraints of the revenue maximization problem
n_{fixed} : the number of fixed fleet assignment solutions
n_{inc} : the increment in the number of fixed fleet assignment solutions in the VNS procedure
n_{max} : the maximum number of fixed fleet assignment solutions in the VNS procedure
n_{min} : the minimum number of fixed fleet assignment solutions in the VNS procedure
$notImpr$: the number of subsequent iterations of the local search heuristic without any improvement in the objective function
$\bar{p}^g_i$ for the value in iteration g of the local search heuristic
prob^g_f : the probability of fixing the assignment of flight f at iteration g of the local search heuristic
SR^g_f : the spill rate for flight f in iteration g of the local search heuristic
SR^g_i : the spill rate for itinerary i in iteration g of the local search heuristic
SR^g_{max} : the maximum spill rate in iteration g of the local search heuristic
SR^g_{mean} : the average spill rate per itinerary in iteration g of the local search heuristic
time : the elapsed time since the start of the local search heuristic
time_{max} : the maximum running time for the local search heuristic

λ_f : Lagrangian multipliers for flight f in the Lagrangian relaxation and GBD procedures

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Research

Research projects

Inferring transport model preferences from attitudes - 2009-2011

Development of discrete choice models with latent variables

Clip-Air concept: Integrated schedule planning for a new generation of aircraft - Since 2010

Development of airline schedule planning and fleet assignment models

MyTosa: Simulation tool for the dimensioning, commercial promotion and case study set-up for ABB's revolutionary "Catenary-Free" 100% electric urban public mass-transportation system: Tosa 2013 - Since May 2012

Development of optimization models for the location of charging stations

Publications in International Journals

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Atasoy, B., Salani, M., and Bierlaire, M. (2011). Integrated schedule planning with supply-demand interactions. Proceedings of the 11^{th} Swiss Transport Research Conference (STRC) May 11-13, 2011.

Atasoy, B., Glerum, A., Hurtubia, R., and Bierlaire, M. (2010). Demand for public transport services: Integrating qualitative and quantitative methods. Proceedings of the 10^{th} Swiss Transport Research Conference (STRC) September 1 - 3, 2010.

Hurtubia, R., **Atasoy, B.**, Glerum, A., Curchod, A., and Bierlaire, M. (2010). Considering latent attitudes in mode choice: The case of Switzerland. Proceedings of the World Conference on Transport Research (WCTR) July 11-15, 2010.

Küçük, B., Güler, N., and Eskici, B. (2008). A dynamic simulation model of academic publications and citations. Proceedings of the 26^{th} International System Dynamics Conference July 20-24, 2008.

External Seminars

Gave 20 talks at OR and transportation conferences/seminars/workshops.
Details: `http://transp-or.epfl.ch/personnal-seminars.php?Person=ATASOY`

Teaching activities

EPFL (2009-)

- Introduction to differentiable optimization - Fall 2010, 2011, 2012
- Operations Research - Spring 2010
- Master thesis supervision for 2 mathematics and 1 civil engineering students
- Semester project supervision for 4 mathematics and 5 civil engineering students
- Helped for the one week Discrete Choice Analysis Course - 2011, 2012

Bogazici University (2007-2009)

- IE 423 Quality Engineering - Fall 2007
- IE 304 Operations Research III: Stochastic Models - Spring 2008
- IE 306 Systems Simulation - Spring 2009

Reviewing

4OR: A Quarterly Journal of Operations Research

Annals of Operations Research

EURO Journal on Transportation and Logistics

Decision Support Systems

Journal of Airline and Airport Management

Computer-Aided Civil and Infrastructure Engineering

Skills

Computer: C, C++, C♯, Matlab, Python, R, AMPL, GLPK, GAMS, Biogeme

Languages: Turkish (mother tongue), English (advanced), French (intermediate)

Honors, Awards, & Fellowships

Best Poster Runner Up Award at the 26^{th} International Conference of the System Dynamics Society held at Athens, Greece on 20 - 24 July, 2008.

Graduated with high honor M.Sc. (3.92/4.00) and B.Sc. (3.71/4.00) degrees.

Scholarship from TUBITAK (The scientific and technological research council of Turkey) during Master studies.

Scholarship from Bogazici University during bachelor studies.

Ranked 5^{th} in the national university entrance exam in Turkey among 1.5 million participants.

References

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