Generation of tripolar vortical structures on the beta plane

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A new feature of the long-time evolution of a strong vortex with initially monotonic potential vorticity is found by direct numerical solution of the quasigeostrophic equivalent barotropic equation. Two satellites, which emerge after splitting of an annulus, appear at the vortex periphery. Rotation and oscillation of the tripolar structure may lead to increased mixing near the boundary of the vortex core. The translation of strong monopoles is found to be well described, even for times longer than the linear Rossby wave period, by a recent approximate theory for the evolution of an azimuthal perturbation with mode number $l=1$.

I. INTRODUCTION

Generation of large-scale vortical structures, remaining coherent during many turnaround times, has been recognized to be typical in quasi-two-dimensional flows. Coherent vortices of various forms are often encountered in planetary atmospheres and oceans, as well as in laboratory experiments. Such propagating structures may trap particles and convect them over distances much larger than their scale size. The understanding of their dynamics is therefore of great importance for the description of transport mechanisms.

Geophysical flows are anisotropic due to the variation of the Coriolis parameter with the latitude—the β effect—which allows for Rossby wave propagation. In a magnetized plasma the inhomogeneity of the plasma density is a source of free energy, allowing for the evolution of drift waves with properties similar to Rossby waves. Coherent vortex structures in plasmas are believed to be important for understanding of anomalous transport across the magnetic flux surfaces.

The interaction between coherent vortices and Rossby waves has been intensively studied during the last decades, both analytically, numerically, and in laboratory experiments. On the β plane, vortexlike solutions were proved to exist only with zonal direction of propagation. In the traditional quasigeostrophic approximation such steadily propagating solutions have a dipolar structure with zero net angular momentum.[1] Within a more general shallow water model a wide class of westward propagating anticyclonic monopoles of size larger than the deformation radius can also steadily persist.[2] Thus meridional transport on the β plane cannot be described in terms of steadily propagating structures.

An approximate theory for the initial evolution of an azimuthal $l=1$ (l is the azimuthal mode number) perturbation, setting up a secondary flow of dipolar nature, giving rise to an acceleration of the monopolar vortex both westward and meridionally, has been proposed by Sutyrin.[3] This approach was successfully used for describing initial behavior of a geostrophic point vortex and the influence of the vortex radius on the propagation of strong Gaussian vortices on the β plane.[4] Detailed analysis of the evolution of singular vortices on the β plane was recently performed by Reznik.[5,6] For a piecewise constant potential vorticity distribution in the vortex core the effects of the distortion in the vortex shape on the evolution of the $l=1$ perturbation has been analyzed recently.[7] To consider the long time evolution of a monopolar vortex it is necessary to take into account the appearance of higher azimuthal modes, as well as the change in the internal vortex structure due to the meridional drift and Rossby wave radiation.[8]

In the present work, the nonstationary evolution of monopolar vortices on the β plane is studied numerically. In particular, we emphasize the emerging of tripolar structures due to meriodional displacement of the vortex center.

II. THEORETICAL FORMULATION

We consider the quasigeostrophic equivalent barotropic approximation describing conservation of the potential vorticity

$$\frac{D\Gamma}{Dt}=\frac{\partial\Gamma}{\partial t}+J\{\phi,\Gamma\}=0, \tag{1}$$

where J is the Jacobian operator and ϕ is the streamfunction. We adopt local Cartesian coordinates (x,y), with positive x eastward and positive y northward. In terms of the streamfunction the velocity (u,v) and the potential vorticity Γ are

$$u=-\frac{\partial\phi}{\partial y},\quad v=\frac{\partial\phi}{\partial x}, \tag{2}$$

$$\Gamma=q+\beta y=\nabla^2\phi-\phi+\beta y. \tag{3}$$

Nondimensional variables are used with the deformation radius (Rossby radius) as the spatial scale, and β, being proportional to the gradient of the Coriolis parameter, is the maximum velocity of the Rossby waves.

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Equation (1) obeys the symmetry relation $\phi(x,y,t) = -\phi(x,-y,t)$. Thus, we present only the evolution of cyclones, i.e., structures with negative maximum of the streamfunction.

On the f-plane ($\beta=0$) cyclonic vortices with an initial Gaussian streamfunction have an annulus of positive radial gradient of the potential vorticity around the core negative radial gradient. Such vortices may be termed shielded vortices and they are prone to an $l=2$ instability.[9] As this instability evolves it deforms the core into an ellipse and causes the annulus of negative potential vorticity around the core of the initial vortex to collect into two satellites, forming a tripole; intensively studied both numerically and experimentally during the last years.[10–14]

The β effect causes monopolar vortices to propagate, by introducing an azimuthal perturbation with mode number $l=1$, giving rise to a secondary flow of dipolar nature. Advection by the planetary vorticity, βy, and the nonlinear self-interaction of the $l=1$ perturbation produce a $l=2$ disturbance. Thus on the β plane we can expect the appearance of the $l=2$ instability and ultimately the generation of a tripolar vortex. This behavior was recently observed by Hesthaven *et al.*[15] using the quasigeostropic equivalent barotropic model [Eq. (1)], and an initial Gaussian streamfunction. A similar behavior was also demonstrated in laboratory and numerical experiments by Carnevale *et al.*[16] employing a barotropic model with a topographic β effect and essentially infinite deformation radius.

Here, we focus our attention on the evolution of vortices which have an initial Gaussian potential vorticity distribution (for $\beta=0$);

$$q_{\text{init}}=q_{\text{m}}\exp(-r^2/2a^2), \tag{4}$$

ensuring stability of the vortex to small perturbations in the absence of the β effect. q_m is the amplitude of the vortex, measured relative to β. We refer to this structure as a nonshielded vortex.

A strong monopole on the β plane will have closed isolines of potential vorticity; a criteria for this is that $d\Gamma/dy|_{x=\text{const}} < 0$ somewhere along y. Inside the vortex core, having a radius r_c, fluid particles are trapped and will be convected with the propagating vortex, indicating the transport property of the vortex. The radius of the vortex core is found as

$$r_c^2=-2a^2\ln\{(\beta/q_m)\,[y_c+(a^2/y_c)]\}, \tag{5}$$

where y_c is the location of the local minimum of Γ along $x=0$;

$$y_c^2=-2a^2\ln(\beta a^2/q_m y_c).$$

On the β plane the azimuthal $l=1$ perturbation is introduced by the β effect. During a time which is less than the characteristic Rossby wave period $T_{\text{R}}=2\pi/\beta$, the evolution of the secondary flow due to this perturbation around a strong vortex can be described by considering the $l=1$ perturbation[3] in the form

$$\phi=\phi_{\text{init}}+\beta r\,\text{Re}[\psi(r,t)\exp(-i\theta)],$$

$$\Gamma=q_{\text{init}}+\beta r\,\text{Re}[\xi(r,t)\exp(-i\theta)], \tag{6}$$

where $\psi(r,t)$ and $\xi(r,t)$ are complex functions describing the radial structure of the $l=1$ perturbation of the streamfunction and potential vorticity, respectively. Here, (r,θ) are the polar coordinates relative to the center of the vortex. From Eq. (3) we obtain the relation between ψ and ξ,

$$r\psi=I_1(r)\int_r^\infty K_1(\rho)[i-\xi(\rho,t)]\rho^2\,d\rho$$
$$+K_1(r)\int_0^r I_1(\rho)[i-\xi(\rho,t)]\rho^2\,d\rho. \tag{7}$$

The evolution of ξ is, in the lowest-order approximation, described by

$$\frac{\partial\xi}{\partial t}=i\Omega\xi+b(u+iv-i\psi), \tag{8}$$

$$\Omega(r)=r^{-1}\frac{d\phi_{\text{init}}}{dr},\quad b(r)=r^{-1}\frac{dq_{\text{init}}}{dr}, \tag{9}$$

$$u+iv=-0.5\int_0^\infty K_1(\rho)\left(\frac{b}{b(0)}+1+i\xi\right)\rho^2\,d\rho. \tag{10}$$

Here, Ω is the angular velocity of the vortex, b describes the radial gradient of the potential vorticity in the vortex, βu and βv are the zonal and meridional components, respectively, of the vortex propagation velocity.

Initially $\xi=i$ and $\psi=0$, implying that the vortex starts by drifting westward with a speed $\beta u=\beta\Omega(0)/b(0)$, according to Eq. (10). Then the differential rotation of the $l=1$ potential vorticity [the term $i\Omega\xi$ in Eq. (8)] and the advection of the potential vorticity of the vortex by the asymmetric flow relative to the vortex center [the last term in Eq. (8)] produce a west-meridional acceleration of the vortex center. The trajectory of the nonshielded vortex was calculated according to Eqs. (7)–(10).[3] We compare this trajectory with the results of our numerical solution of Eq. (1) (see Fig. 3).

Because of the material conservation of the potential vorticity, Γ, fluid is trapped in the core of a strong vortex and transported with the vortex center. Thus inside the vortex core the azimuthally averaged $\langle q\rangle$ decreases with meridional displacement y_0 of the vortex center as follows:[4]

$$\langle q\rangle\approx q_{\text{init}}(r)-\beta y_0(t). \tag{11}$$

When the averaged enstrophy inside the vortex core reaches a minimum, the meridional displacement of the center may be obtained as

$$y_m=2\,\frac{\int_0^{r_c}q_{\text{init}}r\,dr}{\beta r_c^2}. \tag{12}$$

The value of y_m gives an estimate of the maximum meridional displacement of the vortex, assuming that the radius of the core does not decrease.

From Eq. (11) we see, that even for an initially monotonic radial distribution of the potential vorticity in the vortex core, an annulus of opposite radial gradient of $\langle q\rangle$ may develop around the core due to the meridional drift.

Thus, also for nonshielded monopolar vortices, we may expect the excitation of the $l=2$ instability and the resulting generation of a tripolar structure on the β plane.

III. NUMERICAL RESULTS

For the numerical investigations we have employed a spectral collocation scheme in a double periodic domain of size $L_x \times L_y$ and with a resolution given by $M \times N$. An explicit third-order Adam–Bashforth predictor–corrector method is used for time integration and a zero-padding scheme is implemented to remove aliasing errors. A hyperviscosity term proportional to $\nabla^6 \Gamma$ was added implicitly to Eq. (1) to ensure numerical stability. This viscous term has negligible influence for the gross properties of the vortex evolution, but has a large influence on the small scale behavior, allowing for reconnection of isolines of potential vorticity. The code was implemented on an Amdahl VP1100 with a typical executing time for each time unit, T, in Eq. (1) of 14 CPU sec for 172^2 active modes, and 100 time steps per T.

We have studied the vortex dynamics for different values of the initial amplitude, q_m, in Eq. (4). In all cases we have chosen $a=1$ and $\beta=0.1$.

A localized, nearly linear perturbation ($q_m < \beta$) decays due to Rossby wave dispersion in agreement with linear calculations.[17] The flow field remains symmetric relative to the x axis with long waves propagating westward and short waves appearing eastward. In this low amplitude case there is no core in the potential vorticity and, consequently, there is no transport of trapped particles.

By increasing the vortex intensity, a core consisting of closed isolines of Γ appears and the vortex evolution becomes essentially nonlinear. For $q_m = 6\beta$ the maximum meridional drift is $y_m \approx 4.2$, according to Eq. (12) and the radius of the vortex core is found to $r_c \approx 1.3$, using Eq. (5). The numerical calculations show a decay of the vortex with near linear rate in agreement with previous simulations.[18] The flow field becomes slightly asymmetric due to nonlinear effects. A tripolar structure does not arise in this case. In the development of the potential vorticity field we observed that the vortex core maintained its identity essentially longer than T_R, although gradually shrinking. The vortex core drifts west poleward, providing transport of fluid particles up to $y_0 \approx 2$ at $T \approx 4T_R$, where the core almost disappeared. Thus, even though the decay rate is near linear, the monopole transports trapped particles at a distance of the order of the initial vortex.

For $q_m \gg \beta$ an estimation of the meridional drift for a nonshielded cyclone gives $y_m \simeq q_m \beta^{-1}$ according to Eq. (12). Thus we expect that such a strong vortex is able to provide long-distance meridional transport.

Calculations for $q_m = 30\beta$ show that the decay rate of the streamfunction amplitude is essentially smaller than for the low amplitude case in agreement with previous calculations.[18] We observe that the core of the vortex becomes elliptic and finally a tripolar structure appears in the potential vorticity field, as seen in Fig. 1. The satellites of the tripole rotate nonstationary, in contrast to the f-plane tripole,[10] and perform some complicated oscillatory motion around the vortex core. The satellites of the tripole are formed by reconnection of the isolines of potential vorticity in the annulus surrounding the core. This reconnection is possible due to the numerical hyperviscosity.

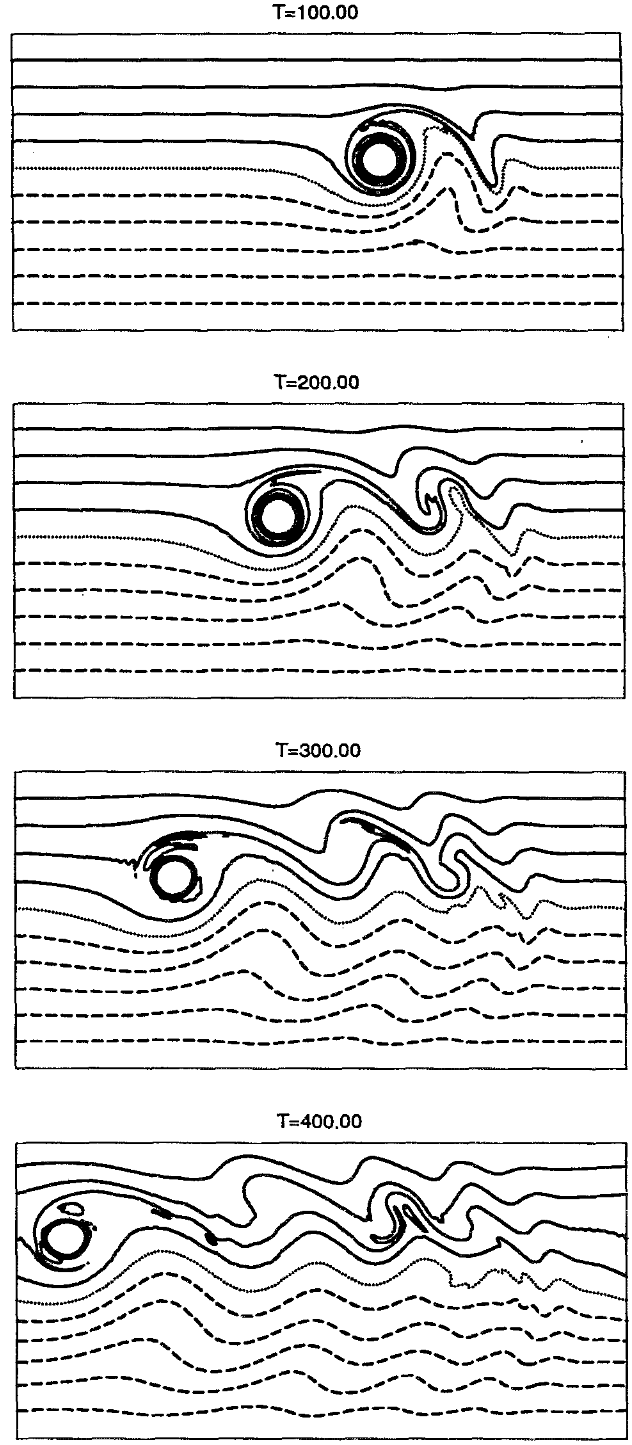

FIG. 1. The evolution of the potential vorticity, Γ, for a strong nonshielded monopolar vortex with $q_m = 30\beta$. Shown are isolines of Γ. Dashed lines represent negative values, solid lines positive values. The dotted line represents the zero line of Γ. $L_x=50$, $L_y=25$, $M=512$, $N=256$.

In Fig. 2 we have plotted the temporal evolution of the

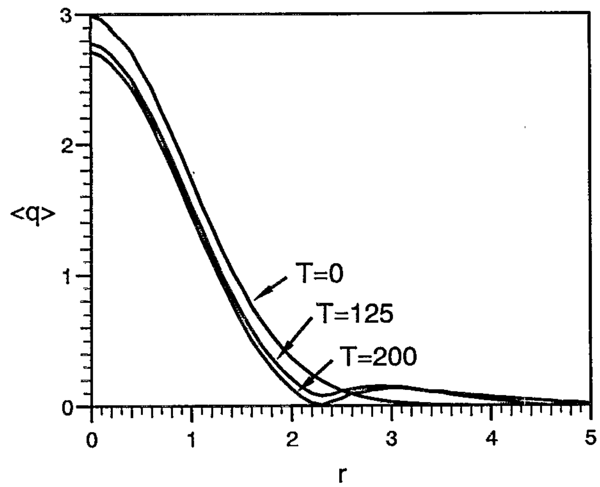

FIG. 2. Azimuthally averaged $\langle q \rangle$ at the times indicated.

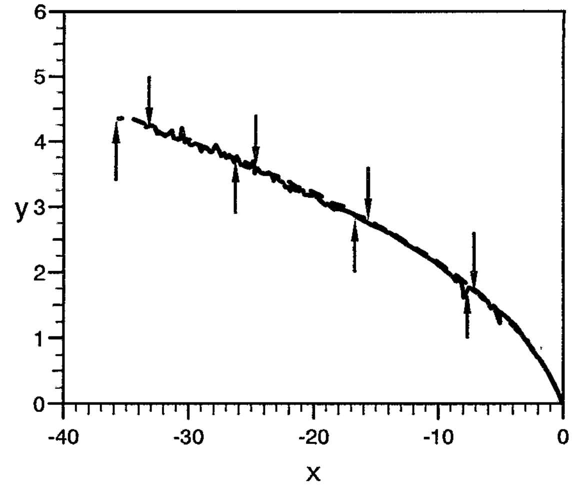

FIG. 3. Trajectory of the center of the strong nonshielded monopole (solid line) in Fig. 1 compared with the prediction (dashed line) based on the azimuthal $l=1$ perturbation theory. The markers indicate the position of the vortex center, according to the numerical solution (downward pointing) and the predicted trajectory (upward pointing). The markers are shown with a time interval of 100.

azimuthally averaged $\langle q \rangle$. Inside the core, $r < r_c = 2.1$, we clearly see that the evolution of $\langle q \rangle$ follows Eq. (11), i.e., the radial gradient is unaltered. Outside the core we observe that the radial gradient of $\langle q \rangle$ changes sign, and the local minimum tends to zero. The asymptotic value of $\langle q \rangle$ at large distances is unchanged and the bump observed on Fig. 2 is possibly caused by Rossby wave effects. The tripole starts to emerge at $T \approx 250$, which is slightly after the local minimum of $\langle q \rangle$ reaches zero at $T \approx 200$, as seen in Fig. 2. We suggest that the appearance of the observed tripole may have the same physical reason as that for f-plane tripoles, i.e., an instability of the axisymmetric vortex with nonmonotonic profile of the potential vorticity. However, further investigations are necessary in order to qualify this.

Comparison of the vortex trajectory with the one calculated, according to Eqs. (7)–(10), shows remarkably good agreement for a period of time, which is significantly longer than the linear Rossby wave dispersion time, $T_R \simeq 60$, and even after the emergence of the tripolar structure as shown in Fig. 3. We have also indicated the position of the vortex at identical moments and we do observe in Fig. 3 that the vortex velocity seems to be slightly smaller than according to the approximate theory for evolution of the azimuthal $l=1$ mode. Behind the vortex core we observe a strongly nonlinear wave pattern, indicating strong interaction between the vortex and the surrounding flow. This effect is not included in the theory, Eqs. (7)–(10), and may account for the observed discrepancy.

At $T=400$ we see that the meridional displacement, $y_0 \simeq 4.5$, of the vortex center is larger than the initial vortex size. However, y_0, is still quite far away from the maximum meridional displacement, $y_m \approx 12$, as obtained from Eq. (12), but from Fig. 3 it is clear that the vortex continues to drift meridionally, maintaining the identity of the core.

IV. CONCLUSION

Our study shows that the evolution of monopolar vortices on the β plane strongly depends on their intensity. Vortices, decaying like a linear Rossby wave, provide, however, some west-meridional fluid transport of fluid trapped inside the closed isolines of the potential vorticity. Strong monopolar vortices provide an efficient meridional transport mechanism. The trajectory is found to be well described by considering the development of the azimuthal perturbation with $l=1$[3,4] for a time period longer than the characteristic period of Rossby waves.

An important new feature of the evolution of a strong unshielded monopolar vortex on the β plane is the appearance of a tripolar structure during the meridional displacement of the vortex center. Rotation and oscillation of the tripole may lead to increased mixing near the boundary of the vortex core. This physical mechanism will play an important role in the evolution of coherent vortices by providing an exchange of material between the vortex core and the surrounding flow.

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