COMPARISON OF TWO FFT LIBRARIES ON THE AMDAHL/FUJITSU VP COMPUTER — NAG AND SIEMENS LIBRARIES

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Abstract

Fourier transforms are used by a large number of customers at the Danish Computer Centre for Research and Education (UNI•C) in Lyngby, Denmark. Two excellent FFT Libraries are offered. The NAG Library[6], which is a general purpose numerical library, has powerful FFT subroutines in the chapter C06. The Siemens VP FFT Library[8][9] has recently been installed for the UNI•C AMDAHL VP 1200 computer.

The benchmarks clearly show a significant performance increase (in some cases by a factor of 3) with the new NAG Mark 15 library as compared to the older NAG Mark 13, although this is offset by some loss of performance in other cases. The Siemens Library is shown to perform fastest for almost all cases studied.

1 NAG Library FFT routines

The NAG routines which are considered here are:

$C06FRF$: multiple 1-dimensional complex FFT's

$C06FPF$: multiple 1-dimensional real-to-hermitian FFT's

$C06FQF$: multiple 1-dimensional hermitian-to-real FFT's

These routines were introduced into Mark 12 of the NAG Fortran Library (1987), and were specifically designed to give good performance on vector-processing machines. Let m be the number of transforms and n the length of each transform.

Good vector performance was achieved principally by the following well-known techniques:

1. switching the innermost loops to ensure that the vector-length for vector operations never falls much below $\sqrt{n}$, and is usually much larger;
2. the ability to compute m transforms in a single call, allowing even greater vector lengths;
3. using a self-sorting algorithm to avoid the need for non-vectorisable re-ordering operations.

Du Croz and Mayes[2] reported speeds of between 350 and 380 Megaflops on an Amdahl VP 1200 for 128 transforms of various lengths between 180 and 256.

However, the code introduced at Mark 12 had the disadvantage that the vector operations in the innermost loop did not always work with contiguous vectors. The effect of this varied with the values of m and n, and from one machine to another. The effect was particularly noticeable on the Amdahl VP machines because of memory-bank conflicts when the offset between vector elements was divisible by a modest power of 2. The effect on performance shows up clearly when computing a single transform with n a large power of 2.

To remedy this defect, the NAG routines at Mark 15 have implemented another well-known technique, described, for example, by Swartztrauber[7]. The code switches between two forms of the algorithm, so that the code can always work with contiguous vectors in the innermost loop, while maintaining a vector-length $\geq \sqrt{n}$ (approximately). The extra cost now is that at the switchover point a matrix-transposition must be performed, and this may take an appreciable amount of time because of problems with memory-bank conflicts; however it only needs to be performed once or twice.

The performance figures being displayed below show a dramatic improvement for single transforms when n is a large power of 2. For some values of m and n, the performance at Mark 15 is a little slower than at Mark 13, but the pattern of performance (speed increasing with m and n) is more regular, and we believe that values of n that are either a power of 2, or are divisible by sizeable powers of 2 (for example, $n = 10,000$) occur most frequently in practice.

Because the NAG routines have been designed as portable code for implementation on a very wide variety of machines, they are not likely to give quite such good performance as routines which have been carefully tuned for one particular machine. But they have the advantages that they are so widely available, and that they are called in turn by other routines in the NAG Library, namely:

$C06FUF$: 2-dimensional complex FFT's

$C06HAF, -HBF, -HCF, -HDF$: multiple 1-dimensional transforms with extra symmetry (sine, cosine, quarter-wave sine, quarter-wave cosine)

$D03FAF$: solves a 3-dimensional Helmholtz equation on a rectangle

2 SIEMENS FFT VP Library

The Siemens Subroutine FFT VP Library was developed for Siemens vector computers. It covers a range of different forms of Fourier transformations and yields high performance by automatic selection of optimal vectorisation methods. The Library covers a large variety of different forms of Fourier transforms with a unifying calling interface. A particular FFT in the Library can be identified by four characteristic parameters: precision, symmetry, algorithm and dimension. The guideline for the implementation of the various transforms is their optimal performance on vector computers. With the parameter algorithm one can choose, among others, between in-place versions of the FFT[5] with minimized memory requirements and out-of-place versions[4] with optimal vectorisation properties. The Stockham autosort version for m simultaneous transforms of size n, which is best suited for vectorisation and needs no reordering were chosen for the Siemens FFT VP Library. Different symmetries of the array to be transformed, as prescribed by particular applications, are e.g. real, hermitian, even, odd. The parameter dimension distinguishes between the cases of several one-dimensional transforms and one multidimensional transform. The names for all possible FFT subroutines are constructed in the form $pFTsad$ with values for the parameter p, s, a and d corresponding to the above mentioned possibilities. For more details see[8].

3 Features of the Amdahl VP1200 computers

For a brief description see[3]; for more details, see[1].

One of the most important features of the Amdahl vector processors is the large register memory, used for vector registers. Its total capacity is 8K double precision variables on the VP1200. This vector register memory enables the vector processor to handle large amounts of vector data with great efficiency. The vector register memory can be re-configured dynamically. The extreme configurations are 8 registers of length 1024, or 256 registers of length 32. The Fortran 77/VP compiler (possibly aided by compiler directives) analyzes each DO loop and automatically inserts the required hardware instructions to re-configure the register memory at run time.

The VP-system consists of a scalar and vector unit. The scalar unit runs the IBM S/370-XA instruction set except for the addition of a few unique instructions used by VP/XA. For the vector unit there are 83 vector instructions available for six pipelines – two load/store, one add, one multiply, one divide and one mask pipeline. Five of these six pipelines may work in parallel. In addition scalar and vector instructions can be executed in parallel.

For all the measurements reported here, we used Amdahl Fortran 77/VP compiler, Version 1.30. Timing measurements were not exactly repeatable: they were subject to a variation of about 10%.

4 Remarks

The results of the benchmarks are presented in Tables 1–6.

When n is a power of 2, NAG Mark 15 is almost always faster than NAG Mark 13 — dramatically so when n is large (by a factor of up to 3 or 4). When n is a power of 3 or 5, NAG Mark 15 is usually slower than NAG Mark 13, but performs similarly if the number of transforms is large ($m = 64$).

The Siemens Library is fastest for almost all values of m and n, but is outperformed by NAG Mark 13 when n is a large power of 3 or 5 and $m = 1$. It is occasionally outperformed by NAG Mark 15. The superiority of the Siemens Library over NAG Mark 15 is greatest when $m = 4$ or 16 and can reach a factor of 2; when $m = 1$ or 64, it seldom exceeds 30%.

5 Conclusions

The results presented above illustrate some of the successes, and also some of the limitations, of efforts to provide highly efficient yet portable software for a fundamental numerical computation, the FFT.

None of the software discussed is completely portable in a purist sense, however it can certainly be described as "transportable". The Siemens VP FFT Library requires a fairly elaborate installation procedure intended to optimize the code for a particular model of Siemens VP machine (the code can also be installed on other types of machine). The NAG Library is designed to be implemented on a very wide variety of machines with — in principle — as little modification as possible, although for computationally intensive routines such as the FFT a small number of pre-selected variants may be provided, with timing programs to assist selection of the most efficient variant.

It is not surprising that the Siemens FFT routines are almost always superior to the NAG routines; however there are a few entries in the tables for radix 3 and 5 which show that there is still some scope for further improvement in the performance of the Siemens library.

For many of the values of m and n in the tables, the performance of the NAG routines comes close to that of the Siemens routines. At Mark 13 there were some cases where the NAG routines ran very much slower; at Mark 15 NAG has avoided those shortcomings, but has incurred some (less serious) loss of performance for other values of m and n. Clearly it should be possible in theory for NAG to produce code that matches the best performance of either the Mark 13 or Mark 15 code. But this would require further complication in the code and in the procedure of implementation. NAG believes that the Mark 15 code offers performance that is better balanced over different values of m and n. We recommend that users should use the Mark 15 Library without worrying whether the Mark 13 Library might give slightly better performance for particular values of m and n.

In fact, the measurements reported in this paper may give undue weight to values of n which are pure powers of 3 or 5. The main reason why the NAG Mark 13 code performs so well for such values of n when $m = 1$, is that *all* the factors of n are odd, so that no memory-bank conflicts occur. We believe that pure powers of 3 or 5 are untypical of the values of n that are used in practice; it is more likely that factors of 3 or 5 will occur in combination with powers of 2, and this is less favourable for the NAG Mark 13 code.

The ideal solution for developers of numerical subroutine libraries would be for a *de facto* standard set of FFT kernels to be defined, analogous to the BLAS in linear algebra. Optimized kernels could be provided for each important range of machines, and the FFT routines in a library could perform almost all their computation by calls to these kernels.

6 Acknowledgements

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		Mflops		
n	m	NAG Mk 13	NAG Mk 15	Siemens
128	1	31	25	27
2048	1	118	108	237
32768	1	66	274	348
131072	1	74	313	357
128	4	76	35	107
2048	4	137	157	330
32768	4	68	259	409
128	16	110	126	238
1024	16	50	171	339
8192	16	64	192	382
128	64	288	281	302
512	64	56	308	413
2048	64	67	320	390

Table 1: *Performance for Complex radix 2 FFT.*

		Mflops		
n	m	NAG Mk 13	NAG Mk 15	Siemens
81	1	26	21	27
729	1	134	107	190
6561	1	343	274	299
59049	1	429	356	392
81	4	64	49	96
729	4	184	168	274
6561	4	249	266	363
19683	4	244	306	375
27	16	81	86	108
243	16	208	163	300
2187	16	263	202	385
27	64	236	250	225
243	64	161	335	372

Table 2: *Performance for Complex radix 3 FFT.*

		Mflops		
n	m	NAG Mk 13	NAG Mk 15	Siemens
125	1	32	29	37
3125	1	254	212	285
78125	1	425	363	399
125	4	99	69	135
3125	4	275	242	345
15625	4	282	308	371
25	16	84	86	101
625	16	283	180	340
3125	16	280	202	374
25	64	240	252	251
625	64	184	356	406

Table 3: *Performance for Complex radix 5 FFT.*

		Mflops					
		Real to Hermitian			Hermitian to Real		
n	m	NAG Mk 13	NAG Mk 15	Siemens	NAG Mk 13	NAG Mk 15	Siemens
128	1	10	8	7	6	4	7
2048	1	82	46	110	77	43	100
32768	1	170	179	262	137	179	232
131072	1	98	248	288	64	250	261
128	4	34	15	29	17	11	25
2048	4	136	93	202	103	86	182
32768	4	92	203	321	58	204	285
128	16	73	64	91	58	50	91
1024	16	163	118	230	118	116	218
8192	16	87	150	299	53	155	285
128	64	192	168	180	167	147	169
512	64	243	224	285	252	226	262
2048	64	87	254	309	54	269	303

Table 4: *Performance of Real radix 2 FFT.*

		Mflops					
		Real to Hermitian			Hermitian to Real		
n	m	NAG Mk 13	NAG Mk 15	Siemens	NAG Mk 13	NAG Mk 15	Siemens
81	1	11	8	9	7	5	10
729	1	71	47	94	66	43	94
6561	1	241	170	199	248	167	203
59049	1	341	281	294	391	303	288
81	4	33	20	38	24	17	39
729	4	134	95	180	135	88	179
6561	4	212	200	294	219	208	313
19683	4	210	235	315	217	252	344
27	16	39	38	41	33	23	43
243	16	150	109	190	151	106	193
2187	16	222	163	312	228	164	338
27	64	139	123	124	101	89	121
243	64	267	254	293	290	270	300

Table 5: *Performance of Real radix 3 FFT.*

		Mflops					
		Real to Hermitian			Hermitian to Real		
n	m	NAG Mk 13	NAG Mk 15	Siemens	NAG Mk 13	NAG Mk 15	Siemens
125	1	17	11	15	11	7	15
3125	1	177	127	197	178	125	191
78125	1	358	300	319	388	319	319
125	4	53	27	58	37	21	45
3125	4	227	165	286	231	171	284
15625	4	262	233	332	275	247	341
25	16	39	35	37	36	23	37
625	16	229	138	266	231	145	268
3125	16	266	170	334	284	183	339
25	64	125	111	114	117	105	130
625	64	313	298	338	330	317	346

Table 6: *Performance of Real radix 5 FFT.*