Scale effects related to the rating curve of cylindrically crested Piano Key weirs

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ABSTRACT: Hydraulic scale effects on physical Piano Key weir (PKW) models were so far rarely discussed in literature, even though almost all prototypes are model-tested before construction. In parallel, physical weir models are generally known to include significant scale effects if operated under small heads, so that an up-scaling of the results derived from the latter is unreliable regarding the discharge-head function (rating curve). This comes from the fact that viscosity and surface tension of water are fluid properties which cannot be scaled simultaneously, whereas both affect the flow in models. Thus, scale effects occur particularly for small overflow heads. Literature mentions limiting heads to respect in the order of 0.03 to 0.05 m. Furthermore, a specific 'low-head behavior' regarding the transition from the clinging to the leaping nappe has been reported, which is different in a prototype and its scaled model. The latter is linked to the teapot effect and to air-water flow features. To derive general equations for the head-discharge relationship of PKWs, researchers thus have to exclude data which could be subjected to scale effects, without knowing the precise limit so far. However, the tests to be excluded are of specific interest to develop the aforementioned relationship, as PKWs are particularly efficient for these conditions. The paper discusses scale effects related to PKWs, based on an analogy considering cylindrical weirs.

1 INTRODUCTION

1.1 *Physical modeling*

Hydraulic modeling remains one of the principal engineering tools to design and optimize complex hydraulic processes, despite of the advancement in numerical simulations. One may suggest that almost all hydraulic structures and phenomena were physically modeled, given that most publications in the field refer to them. Nevertheless, physical modeling remains challenging, as model effects and scale effects have to be limited.

For free surface flows – as for instance at Piano Key weirs (PKWs) – the similitude according to Froude allows for a correct representation of the dominant forces, namely gravity and inertia. Beside these two forces, also the surface tension force and the viscosity force act on the fluid. The ratio of inertia to viscous forces gives the Reynolds number; the ratio of inertia to surface tension forces yields the Weber number. A true dynamic similarity requires achieving identical Froude, Reynolds and Weber numbers in both prototype and model. This is physically impossible when the same fluid (i.e. water) is used in both prototype and model. As a consequence, scale models based upon the Froude similitude may overestimate effects related to the fluid surface tension σ and the viscosity ν. This model-overestimation leads to retarded overflow under small weir heads, affecting the rating curve (i.e. the head-discharge relation). In order to allow for an adequate up-scaling of the model results to prototype dimensions, this overestimation of the surface tension and the viscosity has to be within acceptable limits. Such limits are typically respected when excluding model tests with heads below some 0.03 to 0.05 m.

Herein, a model family based on a numerical simulation is presented and compared with a analytical approach from literature, to validate such limiting heads for cylindrical weir crests. The latter are typical for PKWs, as they even increase their efficiency. This efficiency represents the main benefit of PKWs, spilling relatively high discharges Q under comparably small heads H. This comes from the non-linear nature of PKWs (folded back and forth in plan-view to make repeating cycles or keys) allowing for significantly longer developed weir lengths L than could be achieved using a traditional linear weir for a given spillway channel width W.

For small heads, the hydraulic effect of the edges and corners at the keys remains small, so that the flow features may be approximated with a linear weir of length L. Unfortunately, scale effects concerning the rating curve occur particularly for the "efficient" small heads, so that the accuracy of the physical model data is reduced for the most relevant operation regime of PKWs.

1.2 *Scale effects at free weir overflow*

As for reliable rating curves derived from scaled physical models, literature mentions minimal heads to respect. Otherwise, the effects of surface tension and viscosity influence the head-discharge relation, making impossible a correct up-scaling of the latter to prototype dimensions. Basically, two limitations can be identified, linked to (1) the onset of over-flow due to surface tension (Fig. 1a), and (2) flow affected by surface tension and viscosity (Fig. 1b).

The first limitation (1) may be illustrated by the following example, representing an extreme condition. It takes into account free flow over a cylindrical weir crest (discharge coefficient $C_d = 0.395$ from Castro-Orgaz (2012) for $H/R = 0.1$ and potential flow). Here, R = crest radius as shown in Fig. 2. Due to the fluid surface tension, an absolute minimal head of some $H_M = 0.004$ to 0.006 m is required for water to flow (Bollrich and Aigner 2000). In a scaled physical model, no discharge is flowing across the weir for this head (Fig. 1a). The related prototype, however, indicates a different behavior. There, the head H_P (subscript P for prototype) is equivalent to the model (subscript M) head H_M times the geometrical scale factor λ, which is $H_P = \lambda H_M = 0.300$ m, if $\lambda = 50$. With this H_P, a specific discharge of $q = 0.29$ m^2/s may be expected on prototype, if applying the Poleni equation. For a developed crest length of $L = 100$ m, an absolute discharge of almost $Q = 28.7$ m^3/s occurs, while the prediction from the model is $Q = 0.0$ m^3/s.

Once the initial effect of the surface tension is overcome and water flows, then the fluid surface tension *and* the viscosity affects the rating curve (second limitation (2)). Both, surface tension and viscosity retard the flow, at least as long as no potential flow is achieved (with $\nu = \sigma = 0$). Small discharges require thus an over-proportional head to be conveyed, as compared to potential flow (Matthew 1991). Literature quantifies the minimum head necessary to avoid "relevant" scale effects related to the rating curve of weirs. The values depend of the particular weir crest type, nevertheless, typical limit heads H are:

- around 0.03 to 0.05 m following the data of Rehbock (1909) for sharp-crested weirs with aerated nappe
- 0.07 m following Kirschmer (1928) to reproduce the free nappe (flow separates from weir) on a cylindrical weir with a crest radius $R = 0.046$ m
- around 0.05 to 0.07 m following Dillmann (1933) for sharp-crested weirs
- 0.05 m for sharp-crested weirs (Sarginson 1972)
- 0.05 m under a crest-radius of more than 0.03 m for cylindrical weirs (Sarginson 1972)
- 0.02 m for standard ogee weirs referring to the rating curve, and 0.06 m to reproduce correct jet trajectories at sharp-crested weirs (Breitschneider 1978)
- 0.05 m for broad-crested weirs regarding the rating curve (Hager and Schwalt 1994)
- Some 0.025 m for determination of the rating curve and 0.06 m to correctly model the nappe shape (Ettema 2000).

Further minimal heads to limit the effects of surface tension and viscosity on the head-discharge relation are listed by Novak et al. (2010). As for Piano Key weirs, Erpicum et al. (2013) compared a prototype rating curve with these derived from a model family, including scale factors of 1:7, 1:15, and 1:25. The side wall thicknesses were $T_s = 0.300$ m (Fig. 2) at the prototype, 0.043 m in the 1:7 scaled model, 0.020 m in the 1:15 scaled model, and 0.012 m in the 1:25 scaled model. The authors conclude that model-heads smaller than 0.03 m underestimate the discharge capacity at PKWs, and that some 0.06 m are required to correctly reproduce the flow features in terms of nappe formation and jet geometry.

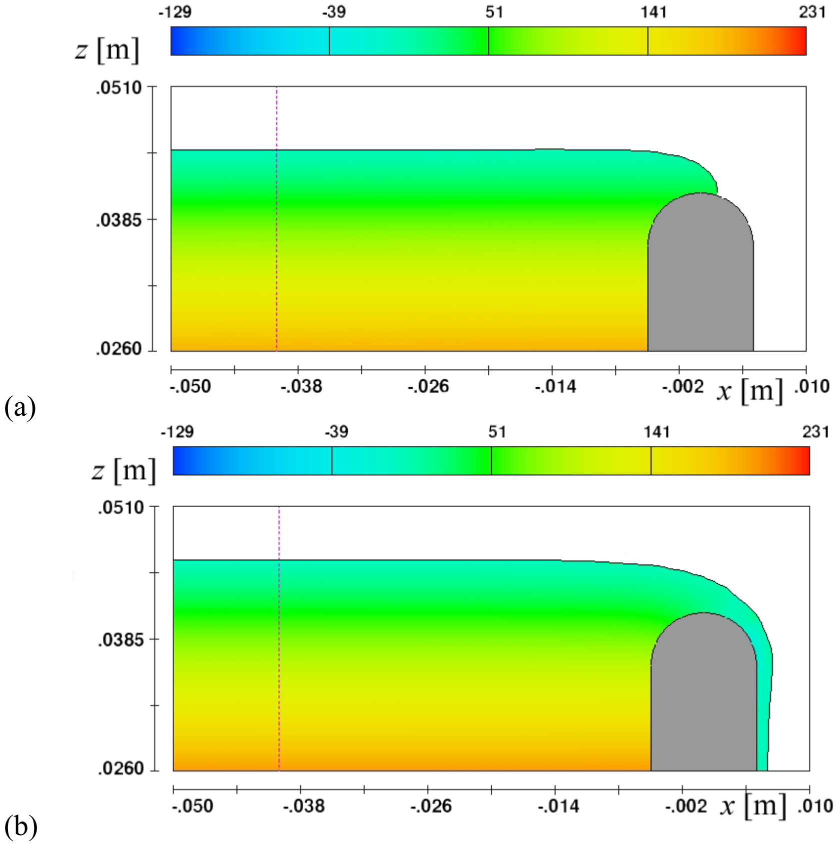

Figure 1. Water surface profiles and pressures (Pa) on a cylindrical weir of crest radius $R = 0.005$ m (from numerical simulation), for a head of $H =$ (a) 0.004 m for stagnant water (limitation 1), and (b) 0.005 m for flowing conditions with a small head (limitation 2)

2 METHOD

Several numerical simulations were conducted, investigating the rating curve of a straight cylindrical weir under typical model and prototype dimensions. These are, for PKW prototypes near the crest around $T_s = 0.20$ to 0.35 m (Vermeulen et al. 2011), and in the physical model under geometrical scale factors of $20 \leq \lambda \leq 30$ in the range of $T_s = 0.005$ to 0.02 m.

As for cylindrical weirs, the effect of the absolute weir thickness $T_s = 2R$ on the rating curve was analytically derived by Matthews (1991) and Castro-Orgaz (2012), based on higher-order curved flow theory. This provides a second tool to investigate scale effects related to the wall thickness, and to compare them with the prediction of the numerical simulation. Of course, the analytical result is only valid for (transversally) linear weirs and for small relative heads H/R. As a first approximation, they can be applied to PKWs anyway, because the hydraulic effect of the edges and corners is small for minimal heads, as e.g. Fig. 3 indicates. It may be assumed that the contraction effect at edges compensates the widening at corners. Furthermore, particularly minimal heads are affected by scale effects, so that the herein derived limits may be a priori transposed from linear cylindrical weirs to PKWs considering the developed crest length as reference to derive the rating curve. For higher heads, the rating curve is influenced by the folded PKW crest, so that the effect on the discharge coefficient C_d reduces. These cases are, however, less important regarding scale effects.

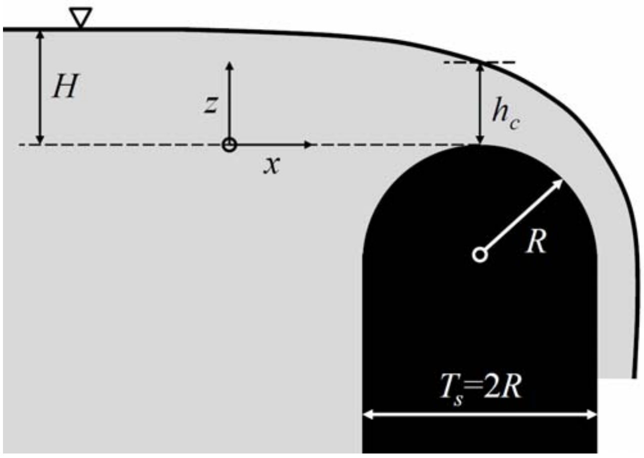

Figure 2. Definition sketch of flow over a cylindrical weir crest

Figure 3. Flow features on sectional PKW model ($T_s = 2R = 0.02$ m, cylindrical crest) under a small head (Leite Ribeiro et al. 2012)

The numerical simulations were performed using the Navier-Stokes solver *Flow-3D*, including the continuity and momentum equations based on a finite-volume approximation with a structured FAVOR grid. The flow region was subdivided into a mesh of fixed rectangular cells. The cylindrical weir was embedded by defining the fractional face areas and fractional volumes of the cells that are open to flow. The free water surface as interface between water and air was modeled with the VOF technique (Hirt and Nichols 1981).

Local average values of all dependent variables were defined within each cell, located at the cell centers, whereas the velocities were located at their faces (staggered grid arrangement). Most terms in the equations are evaluated using the current time-level values of the local variables explicitly. This produces a simple and efficient computational scheme but requires the use of a limited time-step size to maintain computationally stable and to provide accurate results. The pressure-velocity solver used the GMRES method.

The input parameters are (based on a Reynolds and Weber number investigation): gravity, viscous effects through the appropriate turbulence model, and surface tension effect. To take viscous effects into account, the two-equation *k*-*ε* model is used as the turbulence closure model. The surface tension model requires the static contact angle (chosen as 90° for water on smooth concrete wall between wetting and non-wetting condition), as well as the surface tension. Both viscosity and surface tension dependent on temperature: The simulations were performed isothermal at 20° C. For wall shear stress calculations, a surface roughness of 0.01 mm (equivalent to PVC as commonly used in physical models) was attributed to the crest for $0.005 \text{ m} \leq R \leq 0.02$ m, and 1 mm (equivalent to concrete in prototype) for $R = 0.1$ and 0.2 m. Wall shear stresses at obstacle surfaces are modeled using the usual shear stress estimations by adding (to the molecular viscosity) terms taking into account the surface roughness and the local tangential velocity distribution perpendicular to the wall, in general in a sub-grid scale.

The computation was performed in 2D with a symmetry boundary on each side. The upstream boundary condition was a fixed water level corresponding to a given head. An outflow condition was imposed at the downstream, allowing incompressible fluid flowing out with a free surface. From experience, a minimum of five cells is required to represent any physical phenomena, so that a fine rectangular mesh was defined at the crest, with 0.5 mm high cells along z (Fig. 2), and with a x/z ratio of 2. The total number of cells is around 40'000 for the smallest R, and up to 1'800'000 for the largest. The approach flow section was sufficiently large and deep to avoid effects of the velocity and to achieve a horizontal water surface at the boundary.

3 EFFECT OF VISCOSITY AND SURFACE TENSION

3.1 *Discharge coefficient*

The developed PKW crest length L may be considered as characteristic length to derive the specific discharge, as least for small heads H, as

$$q = \frac{Q}{L} \tag{1}$$

The discharge coefficient C_d for a cylindrical weir crest follows then as

$$C_d = \frac{q}{\sqrt{2gH^3}} \tag{2}$$

Accordingly, C_d may be derived from the numerical simulations, based on the related prediction of H and q. As an alternative, and to validate the latter, analytical C_d values from literature were considered. Matthew (1991) derived C_d for cylindrical weir crests based on the second- and third-order equations of the plane potential flow taking into account the streamline curvature. Based on the latter work, Castro-Orgaz (2012) derived an equation including the relative head H/R (and R/H) as well as the absolute radius R, beside the fluid parameters, as

$$C_d = 0.385\left[\begin{array}{l} 1+0.272\left(\dfrac{H}{R}\right)-0.833\left(\dfrac{\sigma}{\rho g R^2}\right)\left(\dfrac{R}{H}\right)-1.05\left(\dfrac{3}{g}\right)^{1/4}\sqrt{\nu}R^{-3/4}\left(\dfrac{R}{H}\right) \\ -0.045\left(\dfrac{H}{R}\right)^2 \end{array}\right] \tag{3}$$

There, σ = surface tension, ν = kinematic viscosity, and ρ =density. Matthew (1991) limits his Eq. (3) to roughly $0.2 \leq H/R \leq 1.0$, while the numerical simulations imply a wider application range up to approximately $H/R \leq 3.0$. Note that Eq. (3) gives explicitly the effects of R, σ, ν, and ρ, i.e. scale effects (linked to a comparison of large "prototype" R and small "model" R) can be derived from the latter for an otherwise similar H/R (and R/H). The potential flow version of Eq. (3) with $\nu = \sigma = 0$ is accordingly (Castro-Orgaz 2012)

$$C_d = 0.385\left[1+0.272\left(\frac{H}{R}\right)-0.045\left(\frac{H}{R}\right)^2\right] \tag{4}$$

Equation (4) gives the C_d values ignoring the effect of viscosity and surface tension. This is correct if describing flow with high heads with a marginal effect of the fluid parameters, thus without scale effects.

Figure 4 shows C_d values derived from the numerical simulations (Fig. 4a) and from Eq. (3) (Fig. 4b), as a function of the non-dimensional head H/R. Various crest radii are included, involving typical prototype and model values ($R = 0.005$, 0.01, 0.02, 0.1, 0.2, and 0.3 m). Note that the data for $R = 0.3$ m were provided by Castro-Orgaz (2012), measured on a physical model.

As for the limitation (1), one may see that $C_d = 0$ (no flow) occurs for small H/R combined with small R (Fig. 4a). For instance, a minimum of $H/R = 0.8$ to 0.9 is required to generate weir overflow for $R = 0.005$ m, whereas the latter occurs at $H/R < 0.1$ for $R = 0.2$ m. Expressed in absolute terms, the limitation (1) was observed at a constant head $H = 0.004$ to 0.005 m for all simulated tests, independent of R. This first limitation is less explicitly represented by Eq. (3) (Fig. 4b), which was developed for flowing water.

As for the limitation (2), it is visible in Fig. 4 that C_d is affected by R, for a constant ratio H/R. Small “model” R generate (for small H/R) C_d values which lay below those of “prototype” R (being close to potential flow according to Eq. (4)), so that the rating curve is incorrectly represented. This effect reduces, however, with increasing H/R, and is small for $H/R > 2$. Generally, the effect of viscosity and surface tension on C_d reduces with increasing relative head, with the consequence that the discharge coefficients collapse with Eq. (4) independently of R. For large relative heads, the (down-scaled) wall thickness of a cylindrically crested weir is thus not relevant, so that scale effects related to the rating curve disappear.

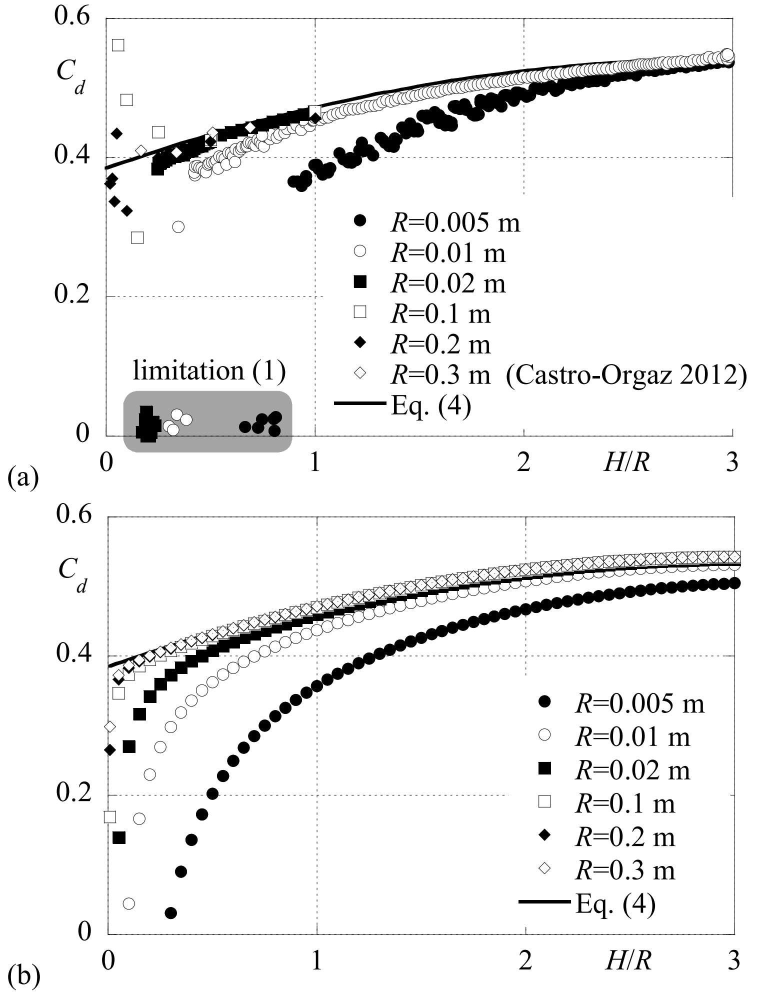

Figure 4. Discharge coefficient C_d versus H/R for different crest radii R, derived from (a) numerical simulations (0.005 m≤R≤0.2 m) and physical modeling (R = 0.3 m), and (b) an analytical derivation resulting in Eq. (3) (Matthew 1991)

The numerical simulations and the analytical solution (Eq. 3) provide basically similar results, as the comparisons of Fig. 4a with Fig. 4b shows. The analytical result is slightly more conservative, i.e. indicates a marginally higher effect of R. Additionally, limitation (1) is not included in Eq. (3), as the latter presumes flowing water. Both methods asymptotically approach potential flow according to Eq. (4) for large H/R. Given the similitude of both methods, one may accept an extension of the application range of Eq. (3) up to $H/R = 3$ (as proposed before).

Minimal heads are to respect in order to avoid significant scale effects regarding the rating curve (as listed in the literature review of chapter 1.2). These heads may be derived from the Eqs. (3) and (4), and the numerical simulations. Therefore, the C_d value of a certain parameter set (including σ, ν, ρ, and R according to Eq. 3) is compared to that resulting for potential flow (Eq. 4). Two criteria were introduced to define the minimal heads: C_d following Eq. (3) or the numerical simulation is (1) 95%, and (2) 98% of C_d following Eq. (4). This provides the values H/R with negligible scale effects for each R. Multiplying the mentioned H/R with R results in the minimal head H to respect for each value R. As visible in Fig. 5a, the effect of σ, ν, and ρ is small if $H > 0.03$ m for the 98% criterion, and $H > 0.015$ m for the 95% criterion. This limits are applicable on models ($R < 0.02$ m) as well as on prototypes ($R = 0.1$ to 0.2 m)! Note that small $R < 0.005$ m are more sensitive, and require minimal heads exceeding the aforementioned values. A model wall thickness of $R = 0.0025$ m ($T_s = 0.005$ m), for instance, achieves a maximum precision of 80% regarding C_d as compared to the prototype (potential flow) if $H/R<4$.

Figure 5b shows C_d values computed with Eqs. (1) and (2) as measured on various physical PKW models (cylindrical weir crests, $R = 0.01$ m) by Leite Ribeiro et al. (2012), zooming on small H/R. It is visible that the data tend to Eq. (3) for $H/R < 2$. The effect of the PKW cycles seems thus small for these relative heads, so that the overflow characteristic is close to that of a linear cylindrical weir. This indicates that the basic assumption, i.e. to consider a PKW under small relative heads as a linear cylindrical weir using L as reference, is appropriate.

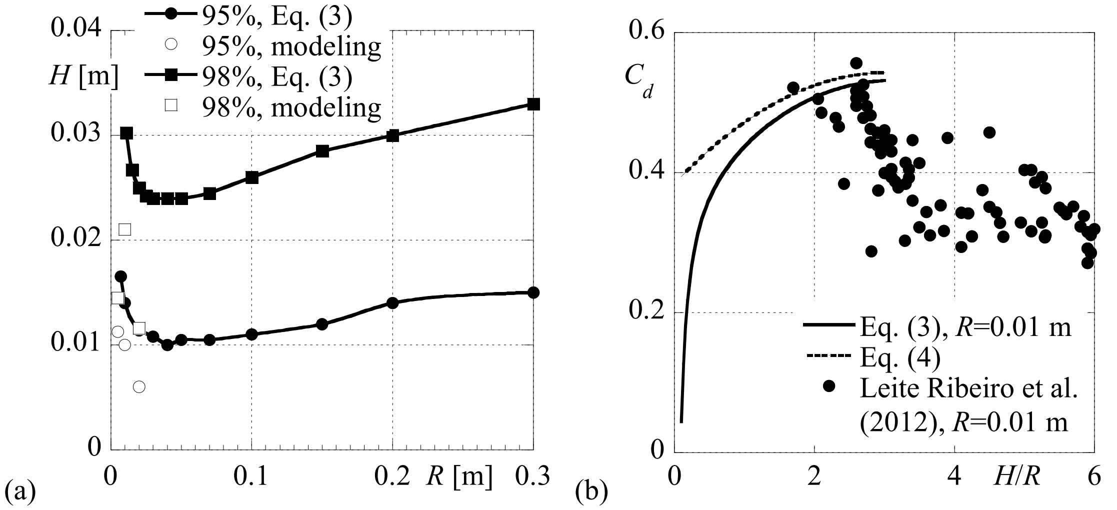

Figure 5. (a) Influence of the absolute crest radius R on head H for errors of 95% and 98% relative to the discharge coefficient C_d, and (b) comparison of C_d values derived from a PKW model and from Eq. (3), both for $R = 0.01$ m

3.2 *Crest pressure distribution*

Scale effects are not only observed related to the rating curve, but also to the behavior of the flow downstream of the crest. As shown by Erpicum et al. (2013), the prototype flow tends to separate from the crest if a certain discharge is exceeded, whereas the model flow then still clings on the structure (teapot effect). This affects the jet trajectories as well as the related disintegration and air entrainment (Pfister and Hager 2012, Pfister and Chanson 2012). The flow separation depends on the crest pressures, which are different in models as compared to prototypes.

Figure 6 shows an example of the vertical pressure profile in the flow at the crest center, derived from the aforementioned numerical simulations and under $H/R = 1$. The pressures are shown on the abscissa, normalization with the flow depth h_c at the crest (Fig. 2, subscript c). The ordinate gives the related elevation with $z/h_c = 1$ representing the water surface, and $z/h_c = 0$

the crest surface. It is visible that the pressures follow almost hydrostatic conditions near the surface, but then significantly reduce due to streamline curvature. Relevant is the difference of pressures on the crest ($z/h_c = 0$) for different R. Small values R (physical models) tend to larger pressures than large R (prototype), under otherwise identical conditions. Physical models with large scale factors ($R = 0.005$ m) tend thus to relatively higher crest pressures, retarding the separation of the flow from the weir surface. A precise limit is not derived herein, as not enough simulations were run in the request range.

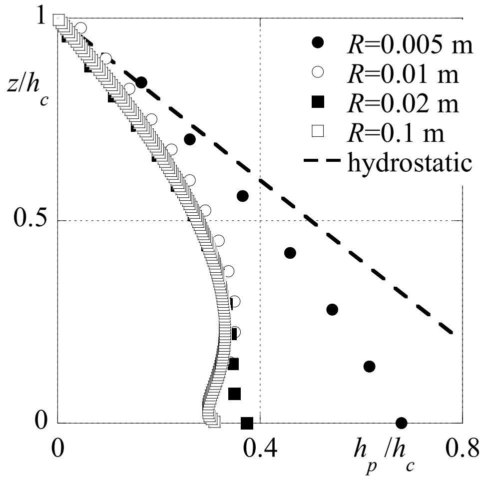

Figure 6. Vertical crest pressure h_p profiles (z/h_c versus h_p/h_c, with h_c as crest flow depth) for $H/R = 1$ under various crest radii R

4 CONCLUSIONS

Scale effects on weir flow occur in physical models for small heads, reducing the accuracy of the up-scaled head-discharge relation. As for PKWs, minimal heads to respect – in order to limit scale effects – were so far exclusively derived by Erpicum et al. (2013). They based their observation on a model family, comparing the related data with the flow features on an identical prototype. Herein, numerical simulations and an analytical approach are used to derive the minimal heads on a cylindrically crested weir as typical on PKWs. The following conclusions are drawn:

- The flow features on PKWs are dominated by the crest shape for relative heads smaller than approximately $H/R < 2$ (Fig. 2). The effect of the folded crest is then small.
- For the aforementioned small heads, the developed crest length is relevant to derive the discharge coefficient.
- Onset of flow is observed for a absolute head of 0.004 to 0.005 m (limitation 1).
- Surface tension and viscosity affect the weir flow up to potential flow conditions (limitation 2). The related limiting head is $H = 0.03$ m for $0.005 \text{ m} \leq R \leq 0.3$ m if assuming $C_{dM}/C_{dP} = 0.98$ as criterion, and $H = 0.015$ m for $C_{dM}/C_{dP} = 0.95$ (Fig. 5a). As for $R < 0.005$ m, more severe limits (i.e. larger limiting heads) have to be applied.
- Between limitation (1) and (2), physical models underestimate the discharge (i.e. the discharge coefficient) for a given head.
- Small R values (physical models) tend to generate overestimated crest pressures (as compared to large R on prototypes), so that flow separation from the profile (leaping nappe) is retarded.

Figure 7 summarizes the results by visualizing the limitations (1) and (2) for different scale factors λ, based on a typical prototype with $R = 0.15$ m ($T_s = 2R = 0.3$ m) and considering Eq. (3). If the related physical model is built for instance with $\lambda = 20$, then scale effects occur between limitations (1) equivalent to an up-scaled prototype head of $\lambda H_M = 0.08$ m and limitation (2) at $\lambda H_M = 0.31$ m. Exclusively prototype heads above 0.31 m are reliable (i.e. the model C_d values are at least 95% of those occurring under potential flow conditions) in terms of the rating curve. For an intermediate head of $\lambda H_M = 0.15$ m, the ratio $C_{dM}/C_{dP} = 0.88$, so that the model spills under these conditions only 88% of the prototype discharge.

Note that not only scale effects influence the precision of hydraulic model data. Additionally, the measurement error including head and discharge measurements is in the order of 1 to 2%, and the geometrical accuracy of the model is around 0.001 m. These errors are particularly significant, such as scale effects, under small heads. Nevertheless, one should not excessively question the accuracy of hydraulic model data, as the main input value to design a spillway, namely the discharge, results from hydrological analyses. These include typically higher uncertainties than the physical model data. A reliable hydraulic structure (e.g. a PKW) should operate as requested even if the effective boundary conditions (design discharge, concrete dimensions) are slightly different from design values.

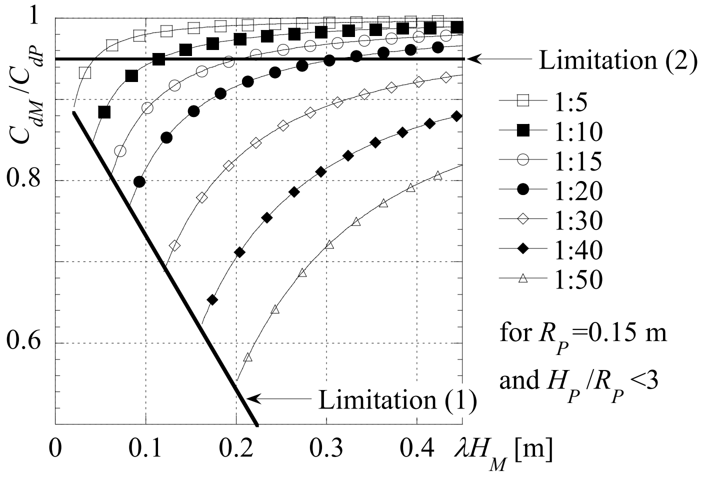

Figure 7. Relative error regarding the discharge coefficient ratio C_d prototype (subscript P) to C_d model (subscript M), for different scale factors λ

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