Journal of Applied Geophysics 91 (2013) 31–38

Contents lists available at SciVerse ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

A thin-layer interface model for wave propagation through filled rock joints

J.C. Li [a,*], W. Wu [b], H.B. Li [a], J.B. Zhu [c], J. Zhao [b]

[a] State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan 430071, China
[b] Ecole Polytechnique Fédérale de Lausanne (EPFL), Laboratory for Rock Mechanics (LMR), CH-1015 Lausanne, Switzerland
[c] Graduate Aeronautical Laboratories and Department of Mechanical and Civil Engineering, Division of Engineering and Applied Science, California Institute of Technology, Pasadena, CA 91125, USA

ARTICLE INFO

Article history:
Received 17 November 2012
Accepted 11 February 2013
Available online 18 February 2013

Keywords:
Wave propagation
Filled rock joint
Thin-layer interface model
Displacement discontinuity method
Stress-closure relation

ABSTRACT

The present study essentially employs a thin-layer interface model for filled rock joints to analyze wave propagation across the jointed rock masses. The thin-layer interface model treats the rough-surfaced joint and the filling material as a continuum medium with a finite thickness. The filling medium is sandwiched between the adjacent rock materials. By back analysis, the relation between the normal stress and the closure of the filled joint are derived, where the effect of joint deformation process on the wave propagation through the joint is analyzed. Analytical solutions and laboratory tests are compared to evaluate the validity of the thin-layer interface model for filled rock joints with linear and nonlinear mechanical properties. The advantages and the disadvantages of the present approach are also discussed.

1. Introduction

Joints significantly affect the physical and mechanical behaviors of the rock masses. Besides unfilled joints, filled joints are also widely existed in rock masses in nature. Filled joints are typically joints with apertures filled with soft and loose materials, such as sand and clay. Under the effect of a stress wave, the deformation behavior of a filled joint is complicated. Meanwhile, the stress wave propagation across the filled joint is strongly influenced by the presence of the filling material.

For a rock joint filled with a specific filling material, it is generally to develop and utilize appropriate laboratory tests to evaluate the realistic physical and mechanical behavior of the joint. For example, by using a triaxial apparatus Sinha and Singh (2000) carried out the test for rock joints filled with gouge and found that the filling material significantly affect the stiffness and the strength of the filled joint. Based on the modified SHPB test, Li and Ma (2009) studied the dynamic property of the filled joints when the filling materials are sand and clay with different thickness and water contents. The test results show that under normal dynamic loads, the relation between the pressure and the closure of the joint is nonlinear. From the test results, Ma et al. (2011) proposed a three-phase medium model for the filled rock joints. Later, Wu et al. (2012) extended the SHPB test to study the loading rate dependency of filled rock joints.

The mechanical property of a joint is related to its relative deformation modes (Bandis et al., 1983; Sharma and Desai, 1992). Under dynamic or static loads, the deformation mode of a joint or an interface between two structures may be various, such as stick, slip, debonding and rebonding (Desai et al., 1984), for welded and non-welded interfaces. The stick mode belongs to the welded case. To investigate wave propagation, the interfaces in a layered media are often modeled to be welded (Bedford and Drumheller, 1994; Brekhovskikh, 1980; Ewing et al., 1957; Kennett, 1983; Miklowitz, 1978). The stress and displacement at the welded interface are both continuous.

Rock joints are usually considered as non-welded interfaces in a rock mass. Recently, wave propagation across a filled or unfilled rock joint has been addressed systematically using theoretical methods. The displacement discontinuity method (DDM) (Miller, 1977; Schoenberg, 1980) is one typical method, in which the joint is modeled as a non-welded interface with linear or nonlinear property. In the DDM, the stresses across a joint are continuous, whereas the displacements across it are discontinuous. Pyrak-Nolte et al. (1990a, b) adopted DDM to derive the closed-form solution for a harmonic incidence across a rock joint. Coupled with the method of characteristic (Bedford and Drumheller, 1994; Ewing et al., 1957), the DDM was also used for analyzing normal longitudinal (P) wave propagation across a single unfilled rock joint with nonlinear property (Zhao and Cai, 2001). Based on the method of characteristic and the DDM, Li et al. (2010) analyzed wave propagation across a filled joint which was modeled as a non-welded interface with exponential behavior. Perino et al. (2012) used the scattering matrix method to analyze wave propagation across elastic and viscoelastic joints. To simplify the problem, the aperture of each joint was considered to be zero

* Corresponding author.
E-mail address: jcli@whrsm.ac.cn (J.C. Li).

0926-9851/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jappgeo.2013.02.003

in the foregoing analytical studies. This assumption is valid only when the joints are planar, large in extent and small in thickness compared with the wavelength of an incident wave. In another word, the joint in the analytical methods was modeled as a zero-thickness interface.

Different from the assumption of the zero-thickness interfaces, a joint or an interface between two solids can be represented as a thin-layer interface, which was proposed by Desai et al. (1984). The thin-layer interface concept was that the joint should be replaced by an equivalent solid or continuum medium with a finite and small thickness. Sharma and Desai (1992) thought that a thin-layer interface or a zero-thickness interface for a joint should be essentially the same from the physical point of view. By modeling the interface between two solids as a thin viscoelastic layer with stiffness and inertia term, wave propagation was addressed by Rokhlin and Wang (1991). Later, the thin viscoelastic layer interface concept was extended by Zhu et al. (2011) to study wave propagation across filled joints. The results from Rokhlin and Wang (1991), Li et al. (2010) and Zhu et al. (2011) showed that the thickness of a filled joint influences wave propagation in a rock mass. In practical situation, when a stress wave propagates across a filled joint with one thin thickness, i.e. a thin-layer interface, the displacement $u(x,t)$ at each side of the joint is continuous, as shown in Fig. 1(a). If the joint is assumed as a non-welded interface with zero-thickness, there is a distinct jump in the displacement at the zero-thickness interface, which is modeled as the displacement discontinuity boundary condition and shown in Fig. 1(b). Meanwhile, there appears a time delay in Fig. 1(a), which may not happen in Fig. 1(b). Therefore, the calculation performance may be different from two treatments, i.e. the thin-layer and zero-thickness interfaces.

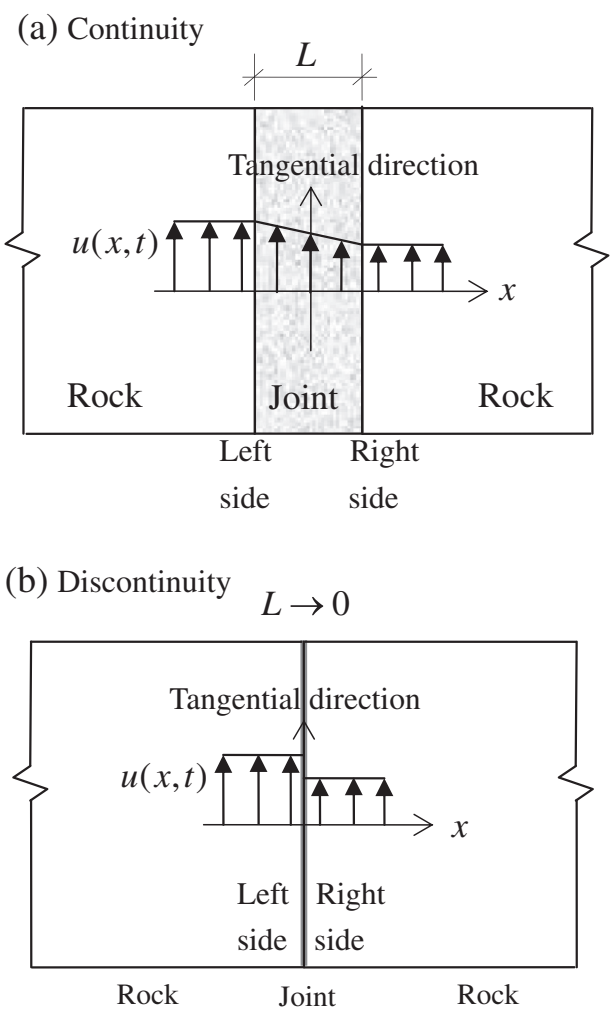

Fig. 1. Schematic view of the displacement at the two sides of a filled joint.

This study is motivated by the need to better understand the role of filled joints on the P-wave propagation. In the study, the filling material in a rock joint is supposed as a thin-layer elastic medium and the joint is equivalent to be a thin-layer interface with one thickness. The two sides of the filling medium are welded to the adjacent rocks. Based on the method of characteristic (MC), the interaction between a stress wave and the joint is analyzed. The wave propagation equation is established for the filled joint with the thin-layer interface model (TLIM). The normal stress and the closure for the joint are derived herein. Two verifications are then carried out, one is to compare the analytical results with those from the existing methods based on zero-thickness interface model (ZTIM), the other is to compare with the test results. Finally, the causes of the discrepancy between two interface models, the potential application and limitations of the present approach are discussed.

2. Theoretical formulations

2.1. Problem description

Assume there is a joint in a linear elastic, homogeneous and isotropic rock. The joint is filled with one geological material, such as soil or sand. Here the filling material is equivalent as an elastic and homogeneous medium different from the adjacent rock. The filled joint is considered as a thin-layer interface between two intact rocks. The thickness of the thin-layer interface is denoted as L, as shown in Fig. 1(a). When a plane P wave impinges on one side of the filling medium, reflection and transmission take place from the two sides of the medium, respectively. In the paper, the two sides of the filling medium are welded to adjacent rocks. During wave propagation, the stress and the displacements at the both sides of the filled joint are continuous. In this section, analytical study for wave propagation across the filled joint modeled in Fig. 1(a) will be conducted.

2.2. Basic equations for stresses and particle velocities

Based on one-dimensional wave propagation theory, two waves propagate in two opposite directions in one continuous medium. Bedford and Drumheller (1994) derived the relations between the particle velocity v and the stress σ. The relation shows that $zv(x,t)+\sigma(x,t)=const$ along any straight right-running (R–R) characteristic line with slope c and $zv(x,t)-\sigma(x,t)=const$ along any straight left-running (L–R) characteristic line with slope $-c$ in the $x-t$ plane, where z is the P wave impendence and $z=\rho c$, c is the P wave propagation velocity in the medium and $c=\sqrt{E/\rho}$, ρ is the density and E is the Young's modulus of the medium. For convenience, the compressive stress is defined to be positive and the tensile stress to be negative.

Fig. 2 schematically shows the characteristic lines at the interface of two media (Bedford and Drumheller, 1994). The position for the interface of two media is at x_i. The wave propagation velocities of the two media are denoted as c_1 and c_2, respectively, and the densities are ρ_1 and ρ_2, respectively. The two media can also be identical. Along the R-R characteristic line ab, there is

$$z^-v^-\left(x_i,t_{j+1}\right)+\sigma^-\left(x_i,t_{j+1}\right)=z^-v^+\left(x_{i-1},t_j\right)+\sigma^+\left(x_{i-1},t_j\right) \tag{1}$$

And along the L-R characteristic line ad, there is

$$z^+v^+\left(x_i,t_{j+1}\right)-\sigma^+\left(x_i,t_{j+1}\right)=z^+v^-\left(x_{i+1},t_j\right)-\sigma^-\left(x_{i+1},t_j\right) \tag{2}$$

where $v^-(x_i,t_{j+1})$ and $v^+(x_i,t_{j+1})$ are the particle velocities at time t_{j+1} before and after the interface at position x_i, respectively; $\sigma^-(x_i,t_{j+1})$ and

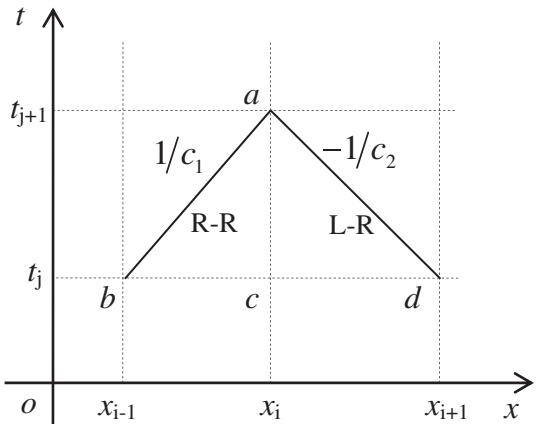

Fig. 2. Schematic view of the characteristic line for right-running (R-R) and left-running (L-R) waves.

$\sigma^+(x_i,t_{j+1})$ are the normal stresses at time t_{j+1} before and after the interface at x_i, respectively; z^- and z^+ are the wave impendence of the two media, respectively, and $z^-=\rho_1 c_1$ and $z^+=\rho_2 c_2$; $v^+(x_{i-1},t_j)$ and $\sigma^+(x_{i-1},t_j)$ are the particle velocity and the normal stress, respectively, at time t_j after the interface at x_{i-1}; $v^-(x_{i+1},t_j)$ and $\sigma^-(x_{i+1},t_j)$ are the particle velocity and the normal stress, respectively, at time t_j before the interface at x_{i+1}.

If the media are welded, the particle velocity and the stress before and after the interface at x_i are continuous, that is

$$v^-\left(x_i,t_j\right)=v^+\left(x_i,t_j\right) \tag{3}$$

$$\sigma^-\left(x_i,t_j\right)=\sigma^+\left(x_i,t_j\right) \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (1) and considering Eq. (2), there is

$$v^+\left(x_i,t_{j+1}\right)=v^-\left(x_i,t_{j+1}\right)=Av^+\left(x_{i-1},t_j\right)+Bv^-\left(x_{i+1},t_j\right)+C\left[\sigma^+\left(x_{i-1},t_j\right)-\sigma^-\left(x_{i+1},t_j\right)\right] \tag{5}$$

where $A=z^-/(z^-+z^+)$, $B=z^+/(z^-+z^+)$ and $C=1/(z^-+z^+)$. The stresses at x_i can be obtained from Eq. (2) and written as,

$$\sigma^+\left(x_i,t_{j+1}\right)=\sigma^-\left(x_i,t_{j+1}\right)=z^+v^+\left(x_i,t_{j+1}\right)-z^+v^-\left(x_{i+1},t_j\right)+\sigma^+\left(x_{i-1},t_j\right)+\sigma^-\left(x_{i+1},t_j\right) \tag{6}$$

where $v^+(x_i,t_{j+1})$ can be determined from Eq. (5). Eqs. (5) and (6) show that the particle velocity and the stress at x_i can be expresses as functions of the particle velocities and the stresses at x_{i-1} and x_{i+1}.

2.3. Analysis of normal stress and closure of a filled joint

Assume the filling medium in a joint is divided into N sub-layers along the wave propagation direction, and the position for the left side of the filled joint is at x_2. If $v^+(x_1,t)$ equals to the incident wave $v_I(t)$, the reflected wave from the left side of the joint is

$$v_R(t)=v^-(x_2,t)-v_I(t-\Delta t) \tag{7}$$

where Δt is the time interval between the two adjacent sub-layers in the rock or the filling medium, that is, $\Delta t=(x_2-x_1)/c_r$ or $\Delta t=(x_{i+1}-x_i)/c_f$ $(i=2\sim N)$, c_r and c_f are the wave propagation velocities in the rock and the filling media, respectively. At the right side of the filled joint shown in Fig. 1(a), the transmitted wave is

$$v_T(t)=v^+(x_{N+2},t) \tag{8}$$

At the left side of the joint, the strain of the rock $\varepsilon_{left}(t)$ is

$$\varepsilon_{left}(t)=\frac{v_I(t-\Delta t)}{c_r}+\frac{v_R(t)}{c_r} \tag{9}$$

and the particle velocity for the rock $v_{left}(t)$ is

$$v_{left}(t)=v_I(t-\Delta t)-v_R(t) \tag{10}$$

The strain and the particle velocity of the rock, $\varepsilon_{right}(t)$ and $v_{right}(t)$, at the right side of the joint can respectively be given by

$$\varepsilon_{right}(t)=\frac{v_T(t)}{c_r} \tag{11}$$

$$v_{right}(t)=v_T(t) \tag{12}$$

From the Hooke's law, the average normal stress on the rock joint is

$$\sigma(t)=\frac{E_r}{2}\left[\varepsilon_{left}+\varepsilon_{right}\right]=\frac{E_r}{2c_r}[v_I(t-\Delta t)+v_R(t)+v_T(t)] \tag{13}$$

When the strain rate of the joint $\dot{\varepsilon}(t)$ is obtained from

$$\dot{\varepsilon}(t)=\frac{1}{L}\left[v_{left}-v_{right}\right]=\frac{1}{L}[v_I(t-\Delta t)-v_R(t)-v_T(t)], \tag{14}$$

we can calculate the normal closure of the joint from the initial time to time t_j, that is

$$\Delta L\left(t_j\right)=\varepsilon L=L\int_0^{t_j}\dot{\varepsilon}dt=\int_0^{t_j}[v_I(t-\Delta t)-v_R(t)-v_T(t)]dt \tag{15}$$

ρ_r and ρ_f are the densities of the rock and the filling material, respectively. With the boundary conditions $v^+(x_1,t)=v_I(t)$ and $\sigma^+(x_1,t)=\rho_r c_r v_I(t)$, and the initial conditions $v^m(x_i,t_1)=0$ and $\sigma^m(x_i,t_1)=0$ ($m=-,+$, and $i=2$ to $N+2$), the particle velocities $v^-(x_2,t)$ and $v^+(x_{N+2},t)$ at the left and right sides, respectively, of the filled joint can be obtained from wave propagation Eqs. (5) and (6). From Eqs. (7) to (8), we can calculate the reflected and transmitted waves, respectively. A combination of Eqs. (13) and (15) yields the relation between the normal stress and the closure of the filled joint.

3. Verification of the approach

In this section, the validity of the approach for wave propagation across a filled joint with thin-layer interface model (TLIM) is evaluated by using three results from existing methods and experimental tests. The first two results are from analytical studies based on the displacement discontinuity method (DDM) with zero-thickness interface model (ZTIM) for unfilled rock joints. One is an exact solution derived by Pyrak-Nolte et al. (1990b) for harmonic wave propagation across a linear joint. The other is the solution derived by Zhao and Cai (2001), who coupled the DDM and the MC for wave propagation across a nonlinear joint. The third set of results is from Split Hopkinson Pressure Bar (SHPB) tests through which the strain wave propagation across artificial rock joints filled with sand was measured.

During the SHPB tests, the sand was used as the filling material with porosity 40%. Parameters for the sand and the rock measured from the test are adopted in the following analysis, that is, the wave propagation velocities in the rock and the filling medium are $c_r=5600$ m/s and $c_f=210$ m/s, respectively, and the densities of

the rock and the sand are $\rho_r = 2800$ kg/m^3 and $\rho_f = 1700$ kg/m^3, respectively.

3.1. Comparison with analytical results for a linear joint

In this section, the incident wave is assumed to be a harmonic wave with the amplitude $A_I = 1$ m/s and the frequency $f = 100$ Hz. When the time interval is assumed to be $\Delta t = 1/(2100f)$ s, the thickness for one sub-layer equals to $c_f\Delta t$ or $c_f/(2100f)$. For example, if the thickness of one filled joint is 2 mm and the frequency f is 100 Hz, the thin-layer of the filling medium is divided into two sub-layers. From Eqs. (13) and (15), the normal stress and closure of the filled joint can be calculated, respectively. The normal stiffness of the joint k_n is defined as the ratio of the normal stress to the joint closure, i.e. $k_n = \sigma/\Delta L$. The calculation results reveal that k_n keeps constant for different frequencies if the joint thickness is a given value, but changes with the joint thickness for a given frequency. From the calculated normal stress and the closure of the joints, the stiffness k_n for joint thickness 2, 4 and 8 mm can be found in Fig. 3 for the harmonic incident wave with $f = 100$ Hz. It is observed from the figure that the stiffness k_n decreases with increasing joint thickness.

The transmission coefficient T_{p-p} is defined to be the ratio of the magnitudes between the transmitted and the incident waves. For the harmonic incident wave, the transmitted wave can be calculated from Eqs. (5), (6) and (8). The relationship between T_{p-p} and the filled joint thickness L is shown in Fig. 4(a) when the frequency of the incident wave f is 100 Hz. If the joint thickness L is 2 mm, the variation of T_{p-p} with the change of frequency f is shown in Fig. 4(b).

Based on the DDM for joints with zero-thickness interface model (ZTIM), the transmission coefficient T_{p-p} was expressed as (Pyrak-Nolte et al., 1990b)

$$T_{p-p} = \frac{2k_n/(\rho_r c_r)}{\sqrt{(2\pi f)^2 + 4{k_n}^2/(\rho_r c_r)^2}} \tag{16}$$

If the joint stiffness k_n shown in Fig. 3 is adopted in Eq. (16), the variations of T_{p-p} with the frequency of incident waves and the thickness of filled joints can be obtained, which are also drawn in Fig. 4(a) and (b), respectively. It can be seen from Fig. 4 that for the two interface models the tendencies of T_{p-p} with the variation of the joint thickness or the frequency are very similar.

The relation between T_{p-p} and the joint thickness shown in Fig. 4(a) can be understood from Fig. 4 and Eq. (16). As shown in Fig. 4, the larger thickness of a filled joint leads to a weaker joint stiffness. Since T_{p-p} in

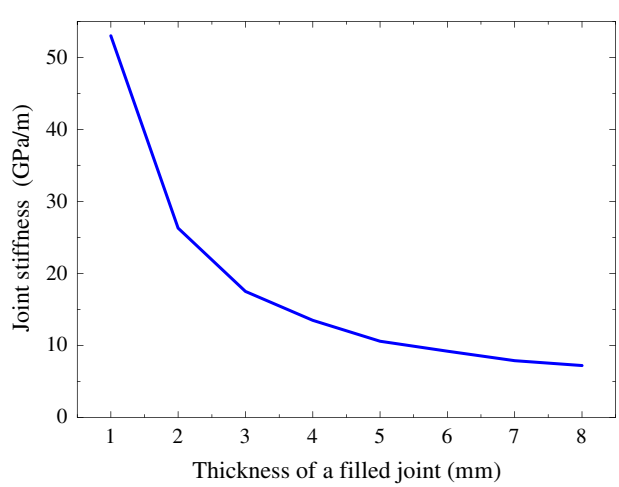

Fig. 3. Variation of the stiffness of filled joints with the joint thickness (from the thin-layer interface model).

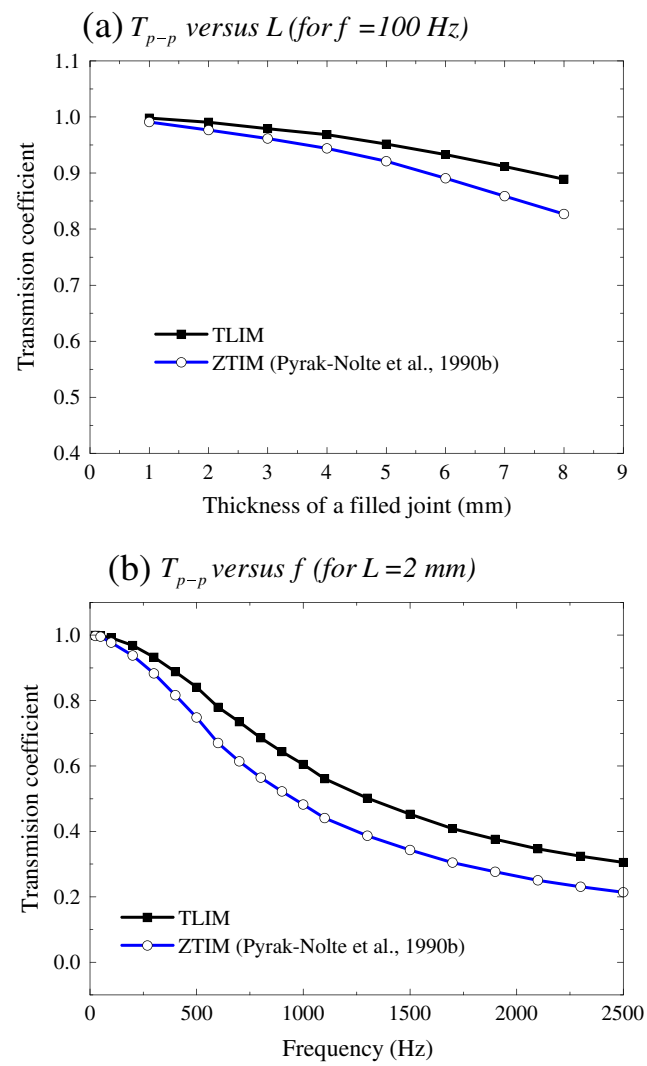

Fig. 4. Variation of the transmission coefficient with the joint thickness and the frequency of incidences based on two interface models.

Eq. (16) is related to the joint stiffness, T_{p-p} decreases with increasing joint thickness. It can also be observed from Fig. 4(a) that the discrepancy of T_{p-p} for the two interface models, i.e. TLIM and ZTIM, increases with larger thickness of the filled joint. When the joint thickness L is 2 mm, T_{p-p} from the two methods are very close, while the discrepancy becomes much bigger and around 1.5% for $L = 8$ mm. For a given thickness, the T_{p-p} calculated from TLIM is always larger than that from ZTIM.

In Fig. 4(b), T_{p-p} from two interface models for filled joints both decrease with increasing frequency. This phenomenon can be understood as a rock joint always acts as a wave filter to filter out high frequency waves instead of the low frequency waves. Fig. 4(b) also shows that when the frequency is lower, e.g., f less than 200 Hz, T_{p-p} from the thin-layer interface model are close to those from the zero-thickness interface model. When f becomes bigger, the discrepancy between T_{p-p} from two methods turns to obvious.

Fig. 4(a) and (b) indict that the transmission coefficient is related to not only the thickness of the joint but also the frequency of the incident wave. It demonstrates that wave propagation depends on the ratio of the joint thickness L to the incident wavelength λ_0, i.e. L/λ_0, where $\lambda_0 = c_r/f$. The value of T_{p-p} reduces with the increasing L/λ_0. Meanwhile, Fig. 4(a) and (b) illustrate that the discrepancy of T_{p-p} between two interface models becomes obvious, when the ratio L/λ_0 is bigger. Otherwise, for a smaller L/λ_0, the calculation results from the two

interface models are almost the same for wave propagation across a filled joint.

3.2. Comparison with analytical results for a nonlinear joint

In Section 3.1, the joint thickness in process of wave propagation is not considered for calculating the strain rate $\dot{\varepsilon}$. Compared to the adjacent intact rock, the filling material is softer. In another word, the relative motion of the filled joint is obviously larger than that of the rock. If the effect of the joint thickness process on the strain rate is considered, Eq. (14) should be rewritten as

$$\dot{\varepsilon} = \frac{1}{l(t)}\left[v_{left} - v_{right}\right] = \frac{1}{l(t)}[v_I(t-\Delta t) - v_R(t) - v_T(t)] \tag{17}$$

where $l(t)$ is the joint thickness in process of wave propagation, i.e. $l(t) = L - \Delta L(t-\Delta t)$, and $\Delta L(t-\Delta t)$ is the joint closure which can be obtained from the joint closure at the previous time step $t-\Delta t$. Eq. (15) for the normal closure of the joint at time t_j is expressed as

$$\Delta L\left(t_j\right) = \varepsilon L = L\int_0^{t_j}\dot{\varepsilon}dt = \int_0^{t_j}\frac{L}{l(t)}[v_I(t-\Delta t) - v_R(t) - v_T(t)]dt \tag{18}$$

The incident wave adopted in this section is in half-cycle sinusoidal waveform, i.e. $v_I(t) = A_I\sin(2\pi ft)$, where $t = 0 \sim 1/(2f)$, $A_I = 1$ m/s and $f = 100$ Hz. When the incident wave impinges the left side of the filled joint, as shown in Fig. 1(a), Eqs. (5) and (6) are still used for the stresses and the particle velocities at the two sides of the joint, and the reflected and transmitted waves caused from the left and right sides of the joint can be obtained from Eqs. (7) and (8). The transmitted waves calculated from the thin-layer interface model for filled joints with thickness 2 and 4 mm are shown in Fig. 5(a) and (b), respectively.

Fig. 5 also shows the transmitted waves from the analytical method by Zhao and Cai (2001), in whose study the zero-thickness interface model was used for nonlinear joints. In the analysis by Zhao and Cai (2001), the mechanical property of the nonlinear joint was expressed as a hyperbolic curve function, that is

$$\sigma = \frac{k_{ni}\Delta L}{1 - \Delta L/L_{\max}} \tag{19}$$

where k_{ni} is the initial stiffness of the joint and equals to the values shown in Fig. 3, that is k_{ni} are 26.3 and 13.5 GPa/m for the filled joints with thickness 2 and 4 mm, respectively; and L_{max} is the maximum closure of the joint, here L_{max} is considered to be the joint thickness. The transmitted waves shown in Fig. 5(a) and (b) from the two interface models of the filled joints are very similar to each other.

Fig. 6 shows the relation between the transmission coefficient T_{p-p} and the joint thickness when the thin-layer interface model (TLIM) and the zero-thickness interface model (ZTIM) for filled joints are taken into account. When the joint thickness is 2 mm, T_{p-p} for the two interface models are 0.994 and 0.993, respectively; when the joint thickness is 4 mm, T_{p-p} for the two interface models are 0.978 and 0.973, respectively. In Fig. 4(a) for linear joints, T_{p-p} are 0.992 and 0.977 from two interface models, respectively, for the 2 mm-thick joints, and T_{p-p} are 0.968 and 0.944 for the two interface models, respectively, for the 4 mm-thick joint. It can be concluded that for a given joint thickness and one interface model, T_{p-p} shown in Fig. 5 is larger than that shown in Fig. 4(a). That is, for the TLIM, the analytical results considering the effect of the joint closure process $l(t)$ are greater than those not including the effect of $l(t)$.

On the other hand, the discrepancies of T_{p-p} shown in Fig. 6 between the two models are 0.1% and 0.5% for the cases of 2 and 4 mm, respectively, which are smaller than the discrepancies of T_{p-p} shown in Fig. 4(a), i.e. 1.5% and 2.5% for the two thicknesses, respectively. Moreover, the discrepancy of T_{p-p} between the two models increases with increasing joint thickness.

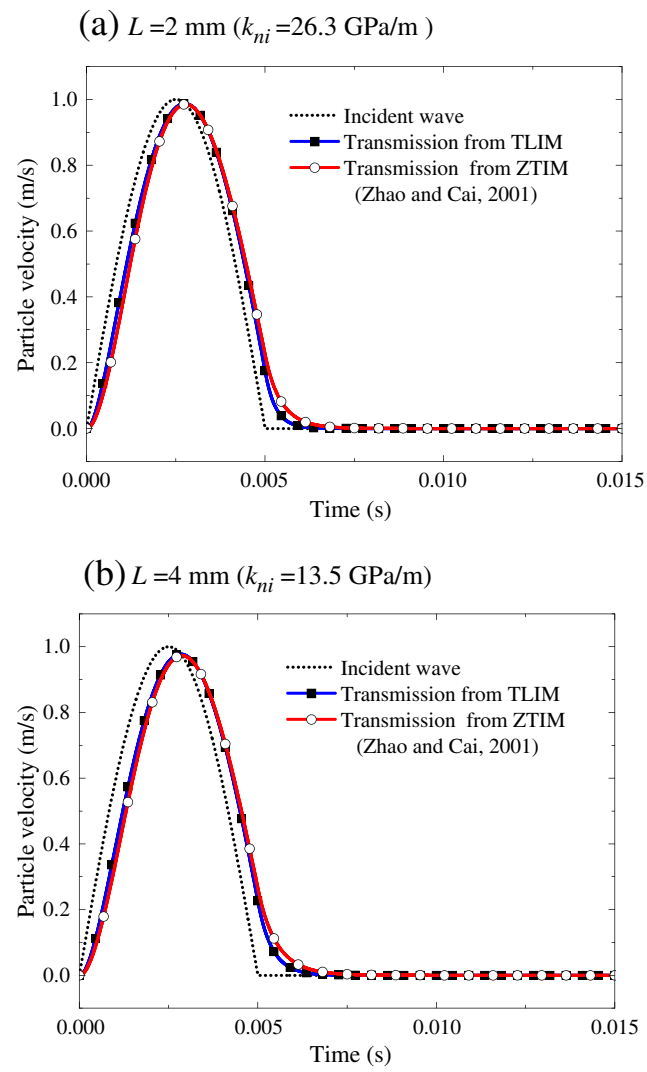

Fig. 5. Comparison for the transmitted waves from two interface models.

When the transmitted and reflected waves are calculated, the normal stress and the closure of the filled joints can be obtained from Eqs. (13) and (18), respectively. Fig. 7 shows the relation between the joint closure

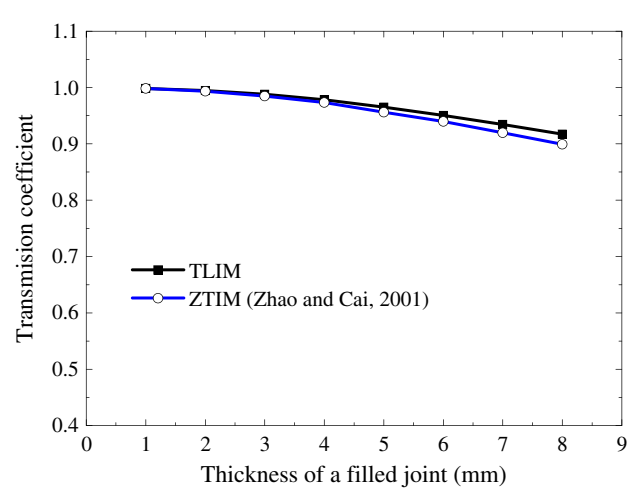

Fig. 6. Comparison for the transmission coefficients from two interface models.

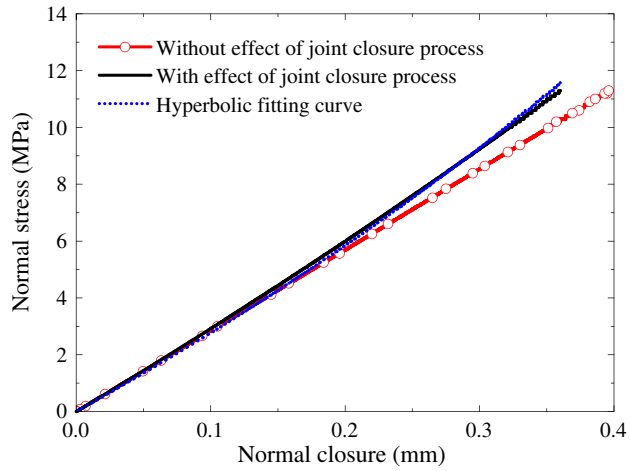

Fig. 7. Relationship between the normal stress and the joint closure.

ΔL and normal stress σ on the 2 mm-thick filled joint with and without considering the effect of the joint closure process $l(t)$ on the strain rate. It can be seen from Fig. 7 that if the effect of $l(t)$ is considered, σ is expressed as a curve function for ΔL, that is, the property of the joint appears nonlinear. Otherwise, the property of the joint is linear without considering the effect of $l(t)$. By comparison, it is observed from Fig. 7 that the slop of the straight-line is the same with the slope of the curve at the initial point (0, 0). In another word, the stiffness of the linear joint equals to the initial stiffness of the nonlinear joint, which indicates that the value of k_{ni} is determined reasonably to calculate the transmitted waves in Fig. 5 and the transmission coefficient in Fig. 6 from ZTIM. When $k_n = 26.3$ GPa/m from Fig. 4 for the 2 mm-thick joint is adopted as k_{ni}, the relation between σ and ΔL is then calculated from Eq. (19), which is shown as the dot curve in Fig. 7. It can be seen from the figure that the dot curve is very similar to the solid curve with the effect of joint closure process.

The comparisons shown in Figs. 5, 6 and 7 indicate that it is more reasonable to take into account the effect of joint closure process on the analysis for wave propagation across a filled joint with TLIM.

3.3. Comparison with experimental test results

The experimental test was conducted using a Split Hopkinson Pressure Bar (SHPB) apparatus (shown in Fig. 8) (Wu et al., 2012), which consists of an artificial filled joint between a pair of square bars, a loading system with a striker bar, and a LabVIEW data acquisition unit. The norite square bars have a cross section of 40×40 mm and 1500 mm in length. The norite striker bar has the same cross section and 200 mm in length, and instantaneously launched by a compressed spring. The data acquisition unit is used for signal triggering, recording and storage. A rubber pulse shaper (10 mm in diameter and 1 mm thickness) is stuck at the impact end of the incident bar to generate a non-dispersive low-rate incident wave and protect the contacting ends of the striker and incident bars. Two groups of strain gauges connected into the full Wheatstone bridge mounted on each bar record the incident and reflected waves. The waves are superposed due to the short length of two bars. With the application of the wave separation technique, the stress time response of each joint interface can be analyzed in order to calculate the wave transmission coefficient. The quartz sands are used as the filling material, which have a density of 2620 kg/m^3, a porosity of 40% and particle size 1–2 mm. The sands are filled in a pre-set gap between two bars and held by a confining box during tests. A total of 9 groups of SHPB tests were carried out at three thicknesses of the filled joints, i.e. 2, 4 and 8 mm. For each joint thickness, there were three tests with the same input energy.

The incident strain waves measured from the test are shown in Fig. 9 for joints with thickness 2, 4 and 8 mm. When the fast Fourier and inverse Fourier transforms are used, the incident strain wave can be expressed as the sum of a series of harmonic waves with different frequencies. From Eqs. (5) to (6), the stresses and the particle velocities at the two sides of the joint can be obtained, which are the combination result for each harmonic incident strain wave. For each test, the transmitted and reflected waves can be calculated from Eqs. (7) to (8). The results are illustrated in Fig. 9. Fig. 10 shows the transmission coefficients from the tests and from the present approach using the TLIM for filled joints with thicknesses 2, 4 and 8 mm. The discrepancy between the test and analytical results ranges from 0.7%~17.0%. It can also be found from the fitting curves in Fig. 10 that the transmission coefficient decreases with increasing thickness of the joints, for a given incidence.

According to the transmitted and reflect strain waves, the average normal stress and the joint closure are calculated from Eqs. (13) and (18), respectively, where the effect of the joint closure process on the joint strain is taken into account. The relation between the normal stress and the joint closure for the filled joints is shown in Fig. 11 for three joint thicknesses. It can be seen from the figure that the normal stress on the joint can be expressed as a curve function of the joint closure. When the normal stresses σ are the same, the joint closure ΔL for $L = 8$ mm is the greatest while ΔL for $L = 2$ mm is the smallest in the three cases.

By comparison, Fig. 10 shows that the strain rate dependence on the joint deformation in process provides highly satisfactory correlation with laboratory test results. The reasonably good agreement verifies

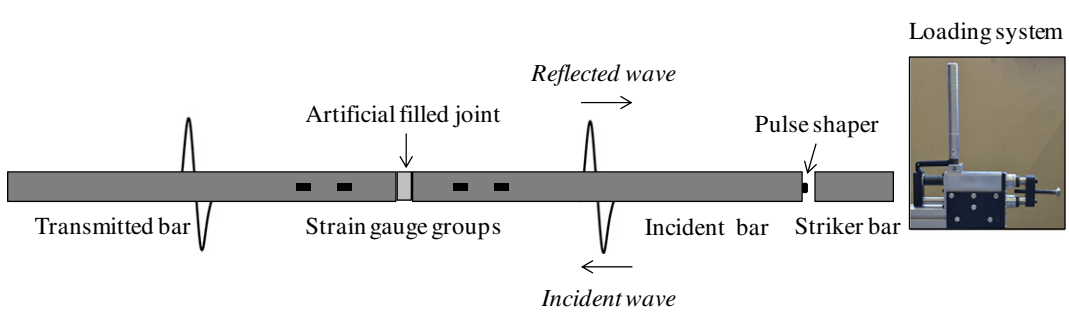

Fig. 8. Schematic view of the split Hopkinson rock bar (SHPB) system.

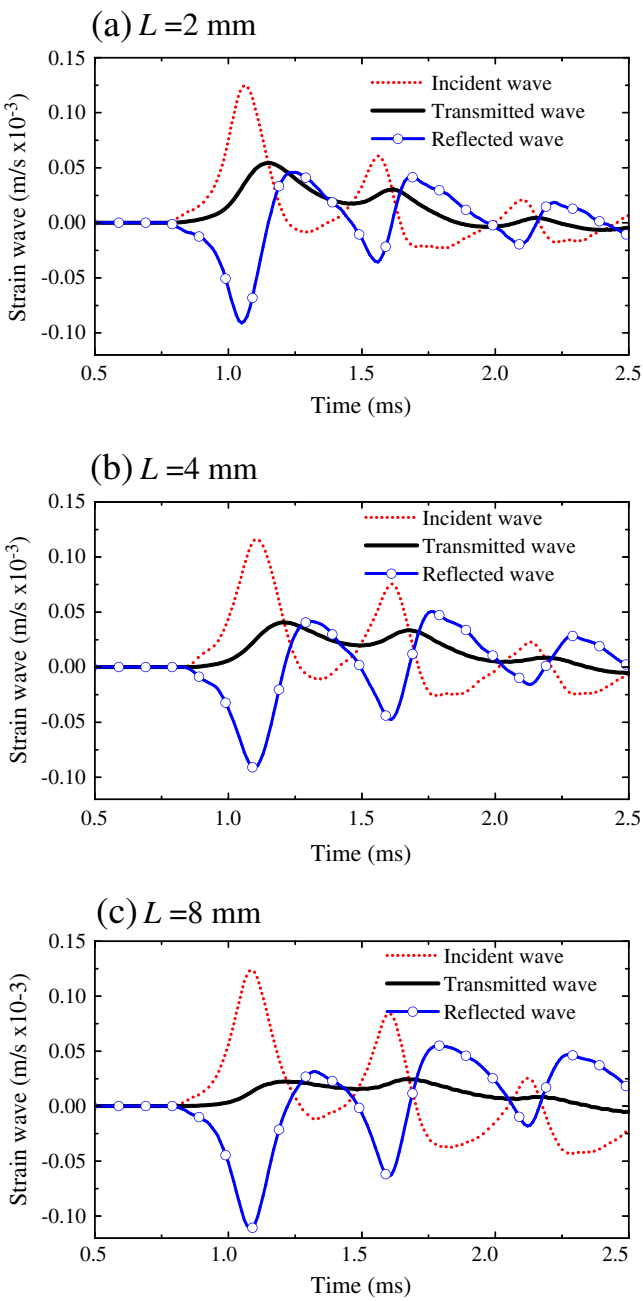

Fig. 9. Reflected and transmitted waves from TLIM for filled joints.

that the present approach can predict the behavior of filled joints subjected to longitudinal stress waves.

4. Discussions

During wave propagation across a thin-layer medium, multiple reflections between two sides of the joint appear. However, the phenomena for multiple reflections in a joint disappear for wave propagation across the joint with zero thickness. For a given incidence, a wider layer gives rise to more time delay for wave propagation and more obvious effect of multiple reflections on the transmitted wave. Hence, for the present study, the discrepancy for T_{p-p} between two interface models is the smallest for the joint with thickness 1 mm and the biggest for the joint with thickness 8 mm, as shown in Figs. 4(a) and 6.

Fig. 4(b) shows that for the joint thickness 2 mm, the discrepancy for T_{p-p} between two interface models is about 0.2% for f 25 Hz, 1.5% for f 100 Hz, 14% for f 500 Hz, 20% for f 1000 Hz and 29.8% for f 2500 Hz. Therefore, for a given joint thickness, if the frequency of an incident wave is low, or the ratio of the incident wavelength to the joint thickness λ_0/L is big, the effect of multiple reflections on the transmitted wave is insignificant. Fig. 4(b) also indicts that the equivalence of the two interface models is satisfied when the ratio of the incident wavelength to the joint thickness λ_0/L is big enough, such as the value of λ_0/L for $f = 50$ Hz and $L = 2$ mm in Fig. 4(b).

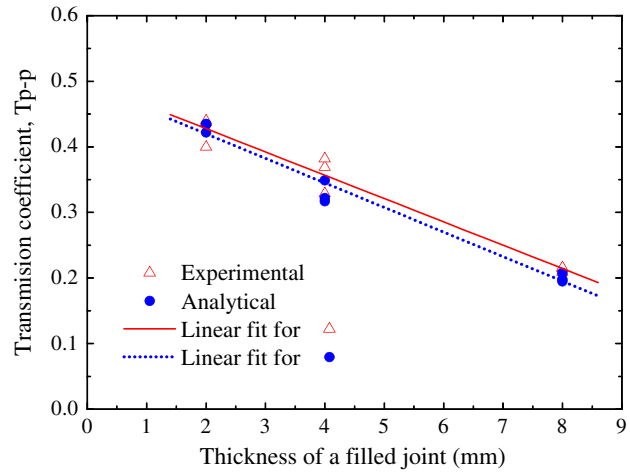

Fig. 10. Transmission coefficients for filled joints with different thicknesses.

Fig. 6 shows that the analytical results based on TLIM and ZTIM are similar when the joint thickness is smaller than 4 mm and the incident frequency is 100 Hz. When the joint thickness is bigger than 4 mm, the discrepancy of the transmission coefficient T_{p-p} between two methods appears and increases with increasing joint thickness. Hence, if the effect of a filled joint thickness on wave propagation cannot be omitted, the TLIM should be adopted.

In reality, the filling materials are usually composed of two or more phase media, such as solid particles, water and air. The filling material in the present study is assumed as a single phase, homogeneous and elastic medium. In addition there exist multiple filled joints in a rock mass. Extension of the thin-layer interface model to study wave propagation across generalized and multiple filled joints needs further investigation.

The thin-layer interface model can directly be used for P wave propagation across filled joints even the joint normal stiffness is not known. This is different from the previous method, in which the joint normal stiffness must be obtained in advance to analyze wave propagation.

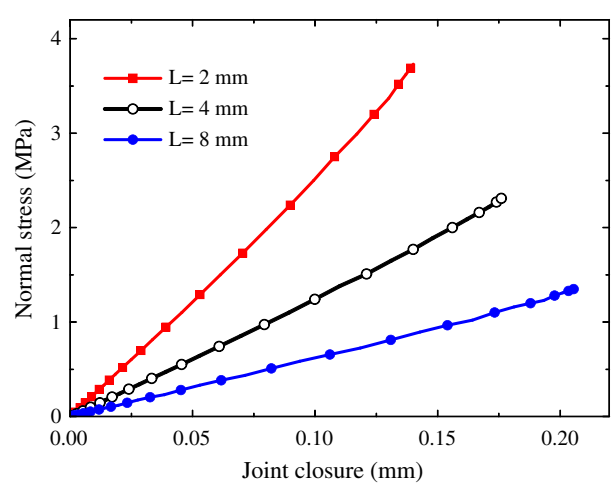

Fig. 11. The relationship between the normal stress and the closure for filled joints with TLIM.

Moreover, according to the interaction between the thin-layer filling medium and the adjacent rocks, the relation between the normal stress and the joint closure is deduced. Therefore, the mechanical property of the filled joints can be estimated theoretically using the present approach.

5. Conclusions

In this paper, a thin-layer interface model is developed for wave propagation across filled rock joints. The study suggests that the effect of joint deformation process should be considered to improve the calculation. Meanwhile, transmission coefficients from the present approach generally agree well with the measured data from the SHPB test, which verifies the thin-layer interface model is effective to analyze the interaction between the stress waves and the filled joints.

The present study also shows that for a given incidence, the transmission coefficient decreases with increasing thickness of the joints. The transmission coefficient decreases also with increasing frequency of the incident wave, for a given joint thickness.

The comparison between the thin-layer interface model and the zero-thickness interface model shows that the discrepancy from the two interface models is caused by the ratio of the incident wavelength to the thickness of the filled joint. If the ratio is large, the analytical results from the two interface models approach to the same.

The intension of the study is to find a new method to analyze wave propagation across filled joints even if the dynamic properties of the joints are unknown. In traditional methods, analytical study for wave propagation across jointed rock masses is not available until the dynamic property of filled joints is obtained from tests. In addition, the relation between the normal closure and the pressure on a filled joint can be estimated by back analysis from the present approach, which provides reference to tests to some extent.

Acknowledgements

The author acknowledges the support of the Chinese National Science Research Fund (41272348, 11072257, 51025935, 51174190), the Major State Basic Research Project of China (2010CB732001), and the Swiss National Science Foundation (200021_124846).

References

Bandis, S.C., Barton, N.R., Lumsden, A.C., 1983. Fundamental of rock joint deformation. International Journal of Rock Mechanics and Mining Science and Geomechanics Abstracts 20, 249–268.

Bedford, A., Drumheller, D.S., 1994. Introduction to Elastic Wave Propagation. Wiley, Chichester.

Brekhovskikh, L.M., 1980. Waves in Layered Media. Academic, San Diego.

Desai, C.S., Zaman, M.M., Lightner, J.G., Siriwardance, H.J., 1984. Thin-layer element for interfaces and joints. International Journal for Numerical and Analytical Methods in Geomechanics 8 (1), 19–43.

Ewing, W.M., Jardetzky, W.S., Press, F., 1957. Elastic Waves in Layered Media. McGraw-Hill, New York.

Kennett, B.L.N., 1983. Seismic Wave Propagation in Stratified Media. Cambridge University Press, London.

Li, J.C., Ma, G.W., 2009. Experimental study of stress wave propagation across a filled rock joint. International Journal of Rock Mechanics and Mining Sciences 46 (3), 471–478.

Li, J.C., Ma, G.W., Huang, X., 2010. Analysis of wave propagation through filled rock joint. Rock Mechanics and Rock Engineering 43 (6), 789–798.

Ma, G.W., Li, J.C., Zhao, J., 2011. Three-phase medium model for filled rock joint and interaction with stress waves. International Journal for Numerical and Analytical Methods in Geomechanics 35 (1), 97–110.

Miklowitz, J., 1978. Elastic Wave and Waveguides. North-Holland, Amsterdam.

Miller, R.K., 1977. An approximate method of analysis of the transmission of elastic waves through a frictional boundary. Journal of Applied Mathematics 44 (4), 652–656.

Perino, A., Orta, R., Barla, G., 2012. Wave propagation in discontinuous media by the scattering matrix method. Rock Mechanics and Rock Engineering 45 (5), 901–918.

Pyrak-Nolte, L.J., Myer, L.R., Cook, N.G.W., 1990a. Transmission of seismic-waves across single natural fractures. Journal of Geophysical Research 95 (B6), 8617–8638.

Pyrak-Nolte, L.J., Myer, L.R., Cook, N.G.W., 1990b. Anisotropy in seismic velocities and amplitudes from multiple parallel fractures. Journal of Geophysical Research 95 (B7), 11345–11358.

Rokhlin, S.I., Wang, Y.J., 1991. Analysis of boundary conditions for elastic wave interaction with an interface between two solids. Journal of the Acoustical Society of America 89 (2), 503–515.

Schoenberg, M., 1980. Elastic wave behavior across linear slip interfaces. Journal of the Acoustical Society of America 68 (5), 1516–1521.

Sharma, K.G., Desai, C.S., 1992. Analysis and implementation of thin-layer element for interfaces and joints. Journal of Engineering Mechanics 118 (12), 2442–2462.

Sinha, U.N., Singh, B., 2000. Testing of rock joints filled with gouge using a triaxial apparatus. International Journal of Rock Mechanics and Mining Sciences 37, 963–981.

Wu, W., Li, J.C., Zhao, J., 2012. Loading rate dependency of dynamic responses of rock joints at low loading rate. Rock Mechanics and Rock Engineering 45 (3), 421–426.

Zhao, J., Cai, J.G., 2001. Transmission of elastic P-waves across single fractures with a nonlinear normal deformational behavior. Rock Mechanics and Rock Engineering 34, 3–22.

Zhu, J.B., Perino, A., Zhao, G.F., Barla, G., Li, J.C., Ma, G.W., Zhao, J., 2011. Seismic response of a single and a set of filled joints of viscoelastic deformational behaviour. Geophysical Journal International 186 (3), 1315–1330.