Sufficient Conditions for Feasibility and Optimality of Real-Time Optimization Schemes - I. Theoretical Foundations (Supplementary Material)

Gene A. Bunin, Grégory François, Dominique Bonvin

Simulated Results for the Example of Section 4

In addition to the algorithms reported in the main text, the following three algorithms were also examined:

Algorithm A1 – Ideal Target

Here, $\mathbf{u}_{k+1}^*$ is taken as the plant optimum at all iterations.

Algorithm A2 – Projected Gradient Descent with Diminishing Step

This is the same as Algorithm 2 in the example of Section 3, with the gradient of the cost used to determine the optimal target at each iteration.

Algorithm A3 – Modifier Adaptation with Affine Correction

A model of the plant is available:

$$\begin{array}{l} \phi(\mathbf{u}) = (u_1 - 0.3)^2 + (u_2 - 0.5)^2 \\ g_1(\mathbf{u}) = -4u_1^2 - 4u_1 + 2u_2 - 0.5 \leq 0 \\ g_2(\mathbf{u}) = 3u_1^2 + u_1 + 0.5u_2 - 0.5 \leq 0 \\ g_3(\mathbf{u}) = -2u_1^2 - 0.5(u_2 - 0.15)^2 + 0.01 \leq 0 \end{array} . \tag{1}$$

Modifier terms are then calculated at every iteration:

$$\begin{array}{l} \varepsilon_{k,j} = g_{p,j}(\mathbf{u}_k) - g_j(\mathbf{u}_k) \\ \boldsymbol{\lambda}_{k,j} = \nabla g_{p,j}(\mathbf{u}_k) - \nabla g_j(\mathbf{u}_k) \\ \boldsymbol{\lambda}_{k,\phi} = \nabla \phi_p(\mathbf{u}_k) - \nabla \phi(\mathbf{u}_k) \end{array} , \tag{2}$$

and the model with an affine correction is then optimized to compute $\mathbf{u}_{k+1}^*$:

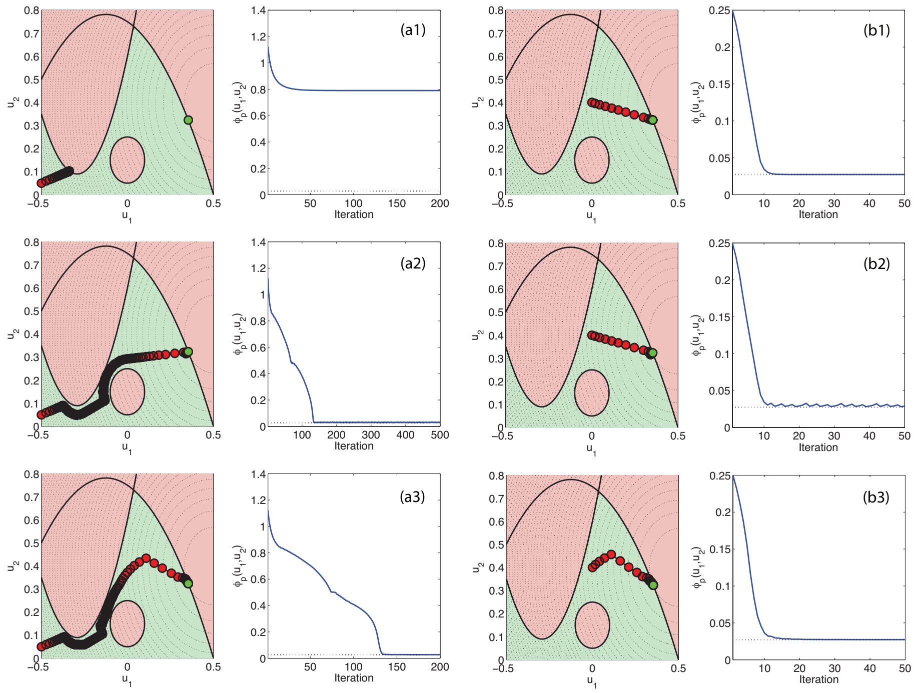

Figure 1: Performance of Algorithm A1 (ideal target). Only in (a1) is there a major issue with convergence to the optimum, as the algorithm runs into the concave constraint and is unable to progress any further.

$$
\begin{aligned}
\mathbf{u}_{k+1}^* = \arg \underset{u_1, u_2}{\text{minimize}} \quad & \phi(\mathbf{u}) + \boldsymbol{\lambda}_{k,\phi}^T \mathbf{u} \\
\text{subject to} \quad & g_1(\mathbf{u}) + \varepsilon_{k,1} + \boldsymbol{\lambda}_{k,1}^T (\mathbf{u} - \mathbf{u}_k) \leq 0 \\
& g_2(\mathbf{u}) + \varepsilon_{k,2} + \boldsymbol{\lambda}_{k,2}^T (\mathbf{u} - \mathbf{u}_k) \leq 0 \\
& g_3(\mathbf{u}) + \varepsilon_{k,3} + \boldsymbol{\lambda}_{k,3}^T (\mathbf{u} - \mathbf{u}_k) \leq 0 \\
& u_1 \in [-0.5, 0.5], u_2 \in [0, 0.8]
\end{aligned} \quad . \tag{3}
$$

Figures 1-3 present the results.

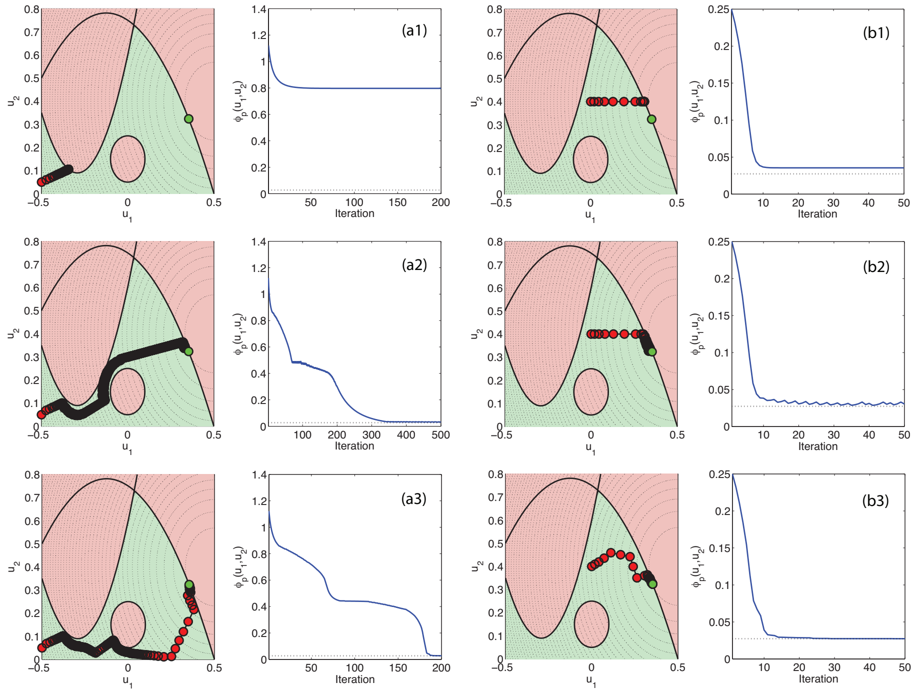

Figure 2: Performance of Algorithm A2 (gradient descent). Notice that the algorithm does not reach the optimum unless at least the condition of Theorem 3 is enforced. We also note that the algorithm requires an excessive number of iterations in (a2), which is due to the diminishing step size of the gradient descent law. While this may be seen as a poorly designed algorithm, we point out that applying the full SCFO avoids this issue (Case (a3)) due to the repeated use of a relatively large δ_ϕ.

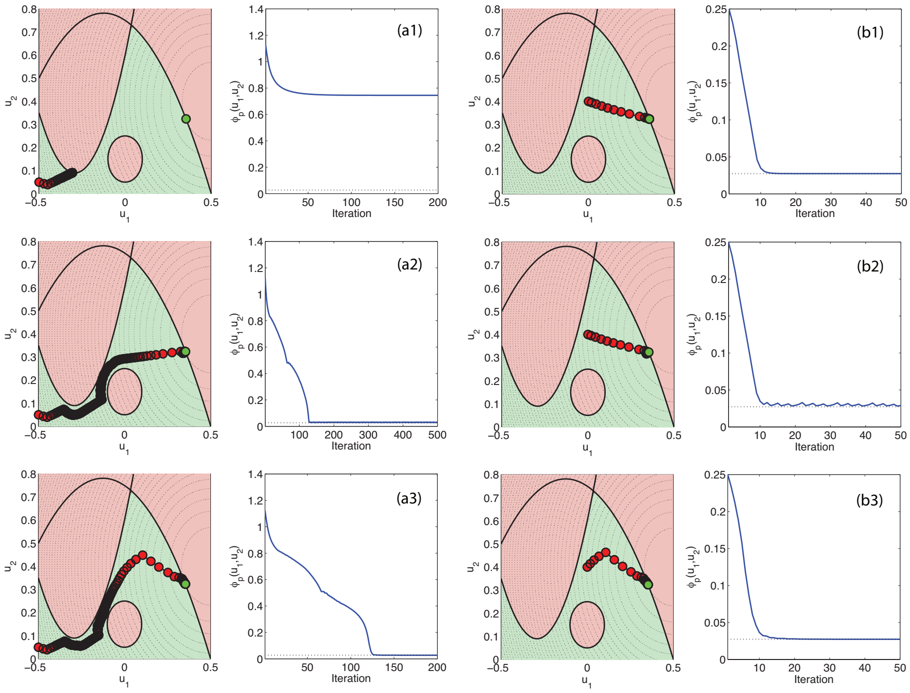

Figure 3: Performance of Algorithm A3 (modifier adaptation). Convergence (or approximate convergence) to the optimum is achieved in all cases except (a1). Here, enforcing the SCFO slows down convergence slightly for the second initial point (compare (b1) and (b3)), but the loss in optimality is quite small.