A simple formula for the trapped fraction in tokamaks including the effect of triangularity

O. Sauter

Centre de Recherches en Physique des plasmas, Ecole Polytechnique Fédérale de Lausanne, CRPP-EPFL, CH-1015 Lausanne, Switzerland

E-mail: `Olivier.Sauter@epfl.ch`

Abstract. In 2002, a simple formula for the trapped fraction in terms of inverse aspect ratio was published in Ref. [1]. An improved formula has been derived valid for a wide variety of triangularity, between -0.5 and +0.5, elongation, up to 2, and aspect ratio, down to 1.5.

It is shown that the elongation does not play a role and the effect of negative triangularity essentially leads to an increase of the effective inverse aspect ratio ϵ and therefore of the trapped fraction.

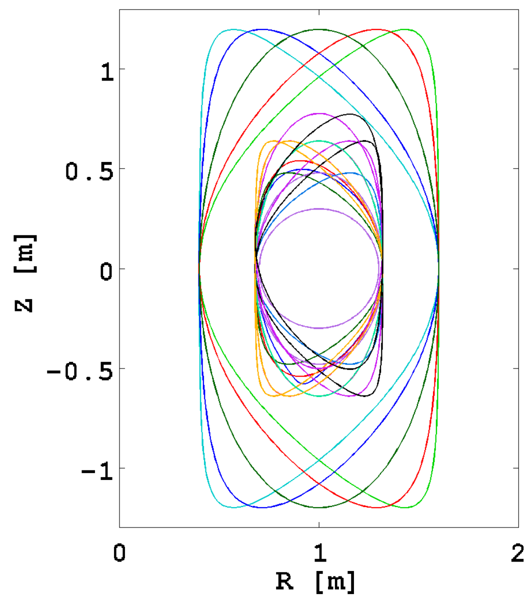

Figure 1. Plasma boundaries used to compute the trapped fraction with the CHEASE equilibrium code.

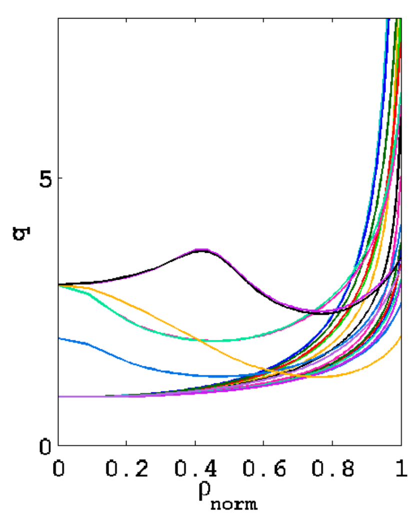

Figure 2. q profiles of the various equilibria used to determine the new fit for f_t.

1. Trapped fraction fit

The trapped fraction was given in terms of the inverse aspect ratio ϵ in Ref. [1] (Eq. (4)):

$$f_t = 1 - \frac{(1-\epsilon)^2}{(1+1.46\sqrt{\epsilon})\,\sqrt{1-\epsilon^2}} \tag{1}$$

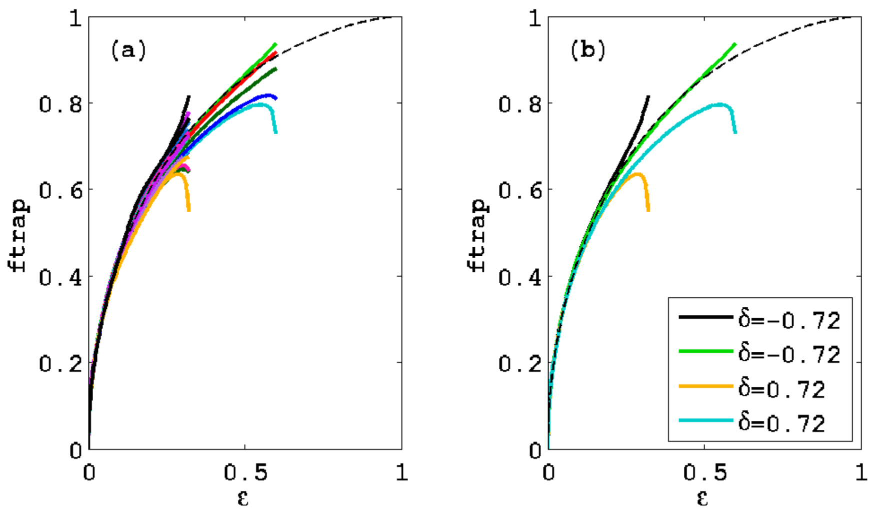

Figure 3. (a) f_t versus inverse aspect ratio using the various equilibria shown in Fig. 1. In dashed line, the formula given in Eq. (1) is plotted. (b) same but for the four triangularity mentioned in the legend with $\kappa = 2$ and $\epsilon_{edge} = 0.3$ and 0.6.

Of course it can be calculated with the full formula, as performed in CHEASE [2], or using the approximation determined in Ref. [3] to avoid double integrals. However, f_t does not depend only on ϵ, even though it is the main dependence. We have run CHEASE with various shapes as shown in Fig. 1, normalised to have $R_{geom} = 1$, with $-0.8 \le \delta \le +0.8$, $1 \le \kappa \le 2$, $\epsilon \le 0.6$ and monotonic and reverse shear q profiles. The q profiles are shown in Fig. 2.

The resulting trapped fraction for all these equilibria are shown in Fig. 3a versus ϵ. The original fit, Eq. (1), is also plotted as a black dashed line. In Fig. 3b, we plot four cases with extreme triangularities, high elongation and for two edge aspect ratio. One sees that the negative triangularity cases have a higher trapped fraction. In order to change as little as possible the original formula, which is simple and works well, we note that the trapped fraction at negative delta is similar to the one obtained at higher ϵ, and conversely for positive δ. Therefore we use an effective ϵ of the form $\epsilon - \delta$. The final formula we propose is:

$$\epsilon_{eff} = 0.67\,(1. - 1.4\,\delta\,|\delta|)\,\epsilon, \tag{2}$$

$$f_t = 1. - \sqrt{\frac{1-\epsilon}{1+\epsilon}}\;\frac{1-\epsilon_{eff}}{1+2\sqrt{\epsilon_{eff}}} \tag{3}$$

Eq. (3) provides a simple formula for f_t which is nevertheless quite general and keeps the main limits, namely that $f_t = 0$ at $\epsilon = 0$ and $f_t = 1$ at $\epsilon = 1$. At extremely large ϵ and negative δ, $f_t(Eq.(3))$ can be larger than 1. Therefore Eq. (3) should be used with: $f_t = min(1., f_t(Eq.(3)))$. However these parameters, region marked “X” in Fig. 4, are not reachable in present days machine therefore does not limit the validity

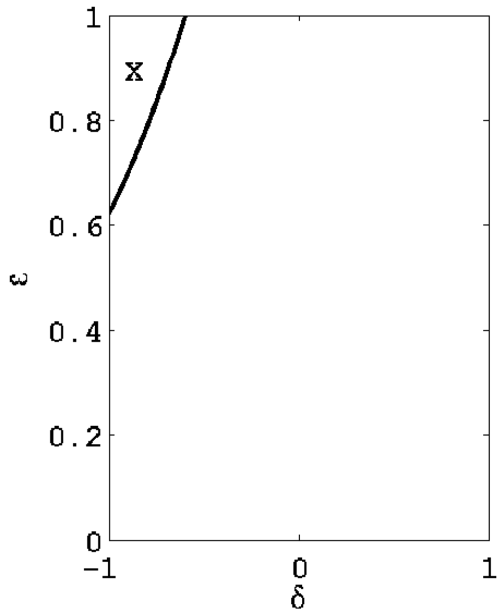

Figure 4. In the region marked by an "X", Eq. (3) yields $f_t > 1$, so use $min(1., f_t)$. However this region of parameters is very hard to obtain experimentally.

of Eq. (3).

We show in Fig. (5) the trapped fraction versus the original fit, Eq. (1) (Fig. 5a), and versus the new fit, Eq. (3) (Fig. 5(b)). The new fit does encapsulate all the dependencies and provides a value of the trapped fraction within 3-4%. Note that it takes into account very large changes in elongation, without an explicit dependence on elongation, this is because elongation is symmetric with respect to low and high field side, thus does not change much the trapped fraction. On the contrary, $\delta = -0.5$ can give a value of f_t up to 20% higher than with $\delta = 0.5$, near the plasma edge, as shown in Fig. 6. As seen in Fig. (6a) and (6c), the fit is such as to underestimate slightly the effect of δ on f_t, to be on the safe side. A factor of 1.5 instead of 1.4 could be used in Eq. (3). In this case, an increase of 17% would be obtained from the fit in these two triangularity cases, as compared to 15% shown in Fig. (6c) and 22% in reality (Fig. (6b)).

2. Conclusion

The trapped fraction has been calculated in axisymmetric tokamak cases for equilibria with various elongation, up to 2, triangularity, between -0.8 and +0.8, edge aspect ratio and q profiles. The previous formula proposed in Ref. [1], depending only on the inverse aspect ratio ϵ, is not sufficient in particular near the plasma edge. This is due to the significant effect of triangularity. A new formula is proposed, Eq. (3), which yields reasonable values of the trapped fraction, within 4%, using only the triangularity as

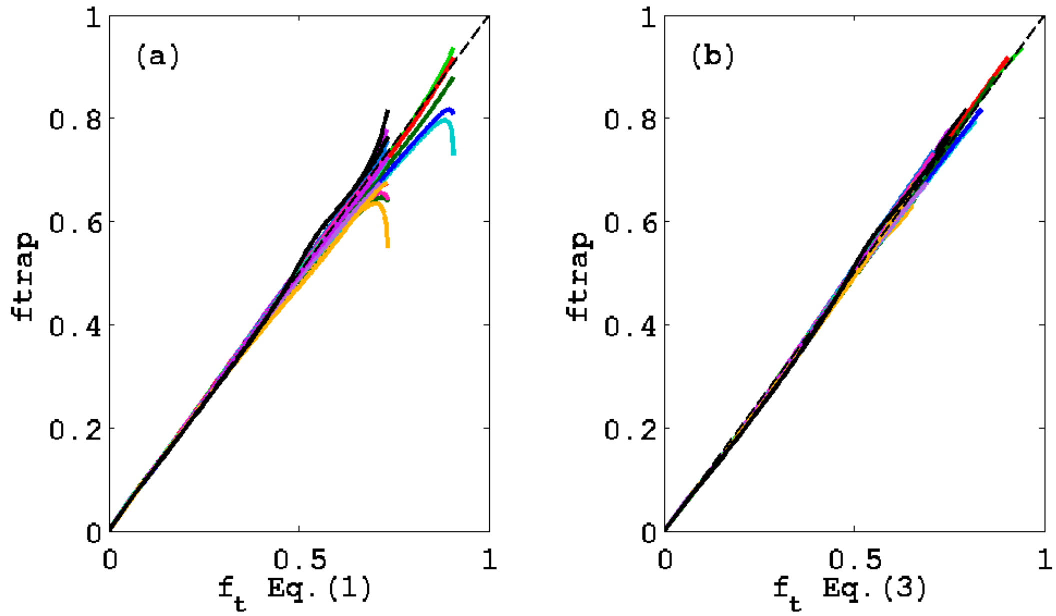

Figure 5. (a) f_t versus the fit given in Eq. (1). (b) f_t versus the new fit given in Eq. (3).

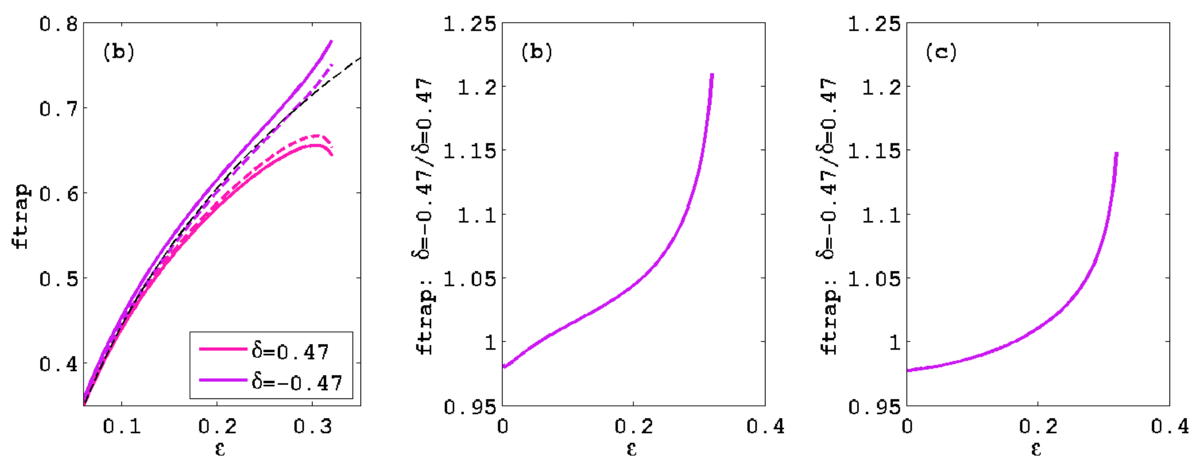

Figure 6. (a) f_t (solid lines) for a case with $\delta = 0.5$ and $\delta = -0.5$, each with $\epsilon_{edge} = 0.32$ and $\kappa = 2$. The black dashed line corresponds to the fit in Eq. (1) and the other dashed lines to the new fit Eq. (3). (b) Ratio of $f_t(\delta = -0.5)/f_t(\delta = 0.5)$. (c) same as (b) but using the fit, Eq. (3), to calculate the ratio, that is ratio of the dashed color lines in (a).

extra parameter. This study also shows that the trapped fraction near the edge can be 20% higher with negative triangularity as compared to positive triangularity equilibria.

3. Acknowledgement

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