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Authors:	Muttoni A., Fernández Ruiz M., Bentz E. C., Foster S. J., Sigrist V.
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Background to the Model Code 2010 Shear Provisions –

Part II Punching Shear

Aurelio Muttoni (École Polytechnique Fédérale de Lausanne, Switzerland)

Miguel Fernández Ruiz (École Polytechnique Fédérale de Lausanne, Switzerland)

Evan Bentz (University of Toronto, Canada)

Stephen Foster (University of New South Wales, Australia)

Viktor Sigrist (Hamburg University of Technology, Germany)

Abstract

This paper outlines the theoretical background of the punching shear provisions implemented in
the fib Model Code 2010 and presents a practical example of its application. It is the aim to
explain the mechanical model that forms the basis of the punching design equations, to justify
the relevance of the provisions and to show their suitability   for the design and assessment of
structures.

Key-words:

Model Code 2010, punching shear, flat slabs, critical shear crack theory, Level-of-Approximation

1. Introduction

The Model Code 2010 [1,2] constitutes a significant step forward with respect to basing design on more physical and more comprehensive models. With regards to the punching shear provisions, an in-depth review of the previous versions of the code (Model Codes 78 [3] and 90 [4]) was performed. At the time they were published, MC 78 and MC 90 constituted the state-of-the-art and, as such, these codes later inspired a generation of standards such as Eurocode 2 [5]. When MC 78 and MC 90 were published, the first mechanical models for punching shear that were based on physical behaviour were already available [6,7]. However, these models led to rather cumbersome design expressions and were difficult to use in practice. Thus, an empirical approach was preferred by national and international standards. The corresponding design formulas (derived on the basis of dimension analysis and statistical regression) accounted for the influence of a number of properties and phenomena on the shear resistance, such as concrete compressive strength, size effect, reinforcement ratio or the influence of unbalanced moments transferred by the slab.

Since the publication of MC 90, a significant amount of research has been undertaken on the

punching shear strength of slab connections and extensive reviews on the topic can be found in

[8,9,10]. Many contributions provided physical models and theoretical advances [11,12,13,14]

that lead to simple design expressions with comparable, or improved, accuracy than previous

empirical approaches. These expressions are rationally derived on the basis of the physical

models supporting the grounding theories and include some material constants fitted on the

basis of test results. These advances in mechanical modelling were acknowledged when

preparing MC 2010. The new and physically grounded design equations have the advantage

that the underlying principles can still be understood by practitioners, thus enabling

phenomenological approaches to punching shear. These models can be considered as an

evolution of the previous empirical design approaches, providing physical approaches that

explain the role of the various parameters. This is justified since the role and the influence of the

previously empirically-fitted governing parameters had been derived and incorporated on the

design expressions on the basis of mechanical analogies.

With respect to the punching shear provisions, the Critical Shear Crack Theory (CSCT), based

on a physical model, was selected as the reference model. Grounding design rules based on

physical models has a number of advantages:

- rules based on a physical model can be explained, understood and justified on physical

    principles and have the potential for further development;

- they provide a consistent platform for design in various situations (eg. with and without shear reinforcement, fibres, etc…); and

- design equations can be adapted to different cases (even if they are still not considered in current design equations) by suitably evaluating the mechanical parameters.

With respect to the CSCT, it provides simple design equations that are widely checked against experimental results and enables the use of a Level-of-Approximation (LoA) approach for design and analysis, consistent with the general principles of MC 2010 [15]. It is also worthy of note that an earlier form of the CSCT model was already adopted in Swiss concrete structures standards [16] with positive experiences.

Details on the LoA approach and on how punching shear provisions take advantage of it can be found elsewhere [15]. The idea of the approach is that the number and accuracy of the mechanical parameters used in the physical model (and thus the accuracy of the estimate of the strength) can be refined (if necessary) in a number of steps. For a preliminary estimate of the strength of a member, the mechanical parameters of the design expressions are determined in a safe and simple manner. This allows for checking the dimensions and main properties of a structure with minimum effort and sufficient accuracy and conservatism. In addition, in many cases, such safe and simplified checks are sufficient to ensure that punching is not governing and hence, no further effort is required (this is the case when column size and slab thickness are

governed by other design criteria). When a more accurate estimate of the strength is required for

critical or non-conventional elements, the calculation of the mechanical parameters of the

design expressions can be refined, which requires some additional work (successive LoA). The

LoA approach allows for a consistent use of the same theory and design equations during the

different phases of a project (preliminary design, tender or executive design, assessment of

critical details) with improving the accuracy of the strength estimates as, and when, required.

2. Code provisions

The MC 2010 establishes a consistent basis for design of beams and one-way and two-way

slabs and is based on physical models. The influence of strain and size effects on the strength of

beams in shear have long been established [17,18,19]. The same influences have been observed

for punching in two-way slabs and the physical mechanisms are similar [12,13,20]. With

respect to the punching shear provisions of MC2010, references [20] and [21] are of particular

interest, as these illustrate the basis for the design formulations.

2.1 Failure criterion

According to the CSCT the shear strength depends on the crack widths (and thus on the strains)

developing in the shear-critical region [20], see Figure 1a. For slabs failing in punching shear, a

strong gradient of bending moments and shear forces occur in the vicinity of supported areas (or

of concentrated loads) [22]. A suitable parameter for describing the strains in the shear-critical region (near the concentrated forces) was identified in [6] as the rotation of the slab (ψ), which can be considered as an integral of the curvatures for such region, see Figure 1a. According to the CSCT, the crack widths in the shear-critical region can be correlated to the product of rotation and flexural effective depth of the slab ($w \propto \psi \cdot d$ [24]). In MC2010, this dependency is incorporated in the calculation of the punching shear strength ($V_{Rd,c}$) as:

$$V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 \cdot d_v \tag{1}$$

Where b_0 refers to the length of the control perimeter (set at $d_v/2$ of the edge of the supported area), d_v to the shear-resisting effective depth of the member (accounting for penetration of the supported area in the slab), f_{ck} to the characteristic compressive strength of concrete measured in cylinder (in [MPa]), γ_c is the partial safety factor for concrete and k_ψ is the factor accounting for the opening and roughness of the cracks:

$$k_\psi = \frac{1}{1.5 + 0.9 k_{dg} \psi \cdot d} \le 0.6 \tag{2}$$

With factor k_{dg} defined as for shear ($k_{dg} = 32/(16+d_g) \ge 0.75$, where d_g refers to the maximum size of the aggregate in [mm]) and d is the effective depth to be introduced in [mm]. It can be noted that Eq. (2) is based on the criterion given in [20] where the constant terms have been adapted to the definition of term k_{dg}.

Note that this approach can be adapted to different situations. For instance, the contributions to

the punching shear strength (V_{Rd}) of any shear reinforcement can be accounted for by adding the

shear reinforcement contribution ($V_{Rd,s}$) to the concrete contribution ($V_{Rd,c}$) [21] (see Figure 2e).

That is:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \tag{3}$$

Extensive theoretical justification and experimental validation of Eq. (2) and details of the

equations defining the activation of the shear reinforcement as a function of the slab rotations

are presented elsewhere [23,25] accounting for the contributions of aggregate interlock and

tensile strength of concrete. The same approach has also been adopted in MC 2010 to account

for the contribution of steel fibres to punching shear strength [26]. Also, application to

prestressed slabs [27,28], post-installed shear reinforcement [29] or non-axis-symmetric slabs

[30,31] can be found elsewhere.

2.2 Load-rotation behaviour of the slab

Contrary to shear in beams and one-way slabs in the companion paper [32], where the

relationship between strains (crack openings) and acting bending moments can be assumed as

proportional after flexural cracking, the load-rotation behaviour of a slab is significantly

nonlinear [6,12,13,20].

A general approach for obtaining such a relationship was already investigated by Muttoni [20]

on the basis of an axis-symmetric slab accounting for equilibrium and compatibility conditions

(considering concrete tensile strength, tension-stiffening of the reinforcement and the

elastic-plastic behavior of the reinforcement and concrete). The resulting expression was

derived on the basis of a quadri-linear moment-curvature diagram (see Figs. 1b-h) and results

in:

$$V = \frac{2\pi}{r_q - r_c} \cdot \begin{pmatrix} -m_r \cdot r_0 + m_R \cdot \langle r_y - r_0 \rangle + EI_1 \cdot \psi \cdot \langle \ln(r_1) - \ln(r_y) \rangle + \\ EI_1 \cdot \chi_{TS} \cdot \langle r_1 - r_y \rangle + m_{cr} \cdot \langle r_{cr} - r_1 \rangle + EI_0 \cdot \psi \cdot \langle \ln(r_s) - \ln(r_{cr}) \rangle \end{pmatrix} \tag{4}$$

where $\langle x \rangle$ is equal to x if $x>0$ and 0 otherwise, and the various parameters are defined in

Figures 1a,b,g. This law is aimed at performing refined analyses of the behavior of slabs and is

particularly suitable for assessing the strength of existing structures (typically corresponding to

a LoA IV). For design of new structures, it can however be simplified on the basis of some

conservative assumptions leading to larger rotations for a given load level (thus associated to

larger crack widths and lower punching shear strength). For instance, by neglecting the tensile

strength of concrete and its tension-stiffening behavior (i.e. considering a bilinear

moment-curvature diagram, Fig. 1g) one can obtain the following load-rotation expression [20]

(Fig. 1h):

$$V = \frac{2\pi}{r_q - r_c} \cdot EI_1 \cdot \psi \cdot \left(1 + \ln \frac{r_s}{r_0} \right) \text{ for } r_y \leq r_0 \text{ (elastic regime)} \tag{5}$$

$$V = \frac{2\pi}{r_q - r_c} \cdot EI_1 \cdot \psi \cdot \left(1 + \ln \frac{r_s}{r_y}\right) \quad \text{for } r_0 \leq r_y \leq r_s \text{ (elastic-plastic regime)} \tag{6}$$

It can be noted that these expressions are similar to those already proposed by Kinnunen and

Nylander for characterizing the behaviour of slabs [6] and in a general manner they can be

written as:

$$V = \frac{2\pi}{r_q - r_c} \cdot EI_1 \cdot \psi \cdot f_1 \left(\frac{V_E}{V_{flex}}\right) \tag{7}$$

where $f_1(V/V_{flex})$ is a function depending on the ratio between the applied load and the bending

strength of the slab ($V_{flex} = 2\pi \cdot m_R \cdot r_s / (r_q - r_c)$). Then, the load-rotation diagram can be further

simplified by introducing the suitable values of the bending strength (V_{flex}, m_R) and the cracked

flexural stiffness (EI_1):

$$m_R = \rho \cdot d^2 \cdot f_y \cdot g_1(x_{pl} / d) \tag{8}$$

$$EI_1 = \rho \cdot \beta \cdot E_s \cdot d^3 \cdot g_2(x_{el} / d) \tag{9}$$

where ρ refers to the flexural reinforcement ratio, d to its effective depth f_y to the yield strength

of the reinforcement, β is a factor accounting for non axis-symmetric reinforcement layout (that

can be set approximately to 0.6 according to [20]), E_s is the modulus of elasticity of the

reinforcement; $g_1(x_{pl}/d)$ is a function depending on the parameter between brackets (depth of

plastic compression zone, a solution assuming a rigid-plastic behaviour for the materials can be

found in [20]) and $g_2(x_{el}/d)$ is a function depending on the parameter between brackets (depth of

elastic compression zone, a solution assuming an elastic behaviour for concrete with no tensile

strength and an elastic behaviour for the reinforcement can be found in [20]). Thus:

$$V = \frac{2\pi}{r_q - r_c} \cdot EI_1 \cdot \psi \cdot f_1\left(\frac{V}{V_{flex}}\right) = \frac{V_{flex}}{r_s \cdot m_R} \cdot EI_1 \cdot \psi \cdot f_1\left(\frac{V}{V_{flex}}\right) \tag{10}$$

That can alternatively be written as:

$$\psi = \frac{r_s \cdot m_R}{EI_1} f_2\left(\frac{V}{V_{flex}}\right) = \frac{r_s \cdot \rho \cdot d^2 \cdot f_y \cdot g_1(x_{pl}/d)}{\rho \cdot \beta \cdot E_s \cdot d^3 \cdot g_2(x_{el}/d)} f_2\left(\frac{V}{V_{flex}}\right) = \frac{r_s}{d} \frac{f_y}{E_s} f_3\left(\frac{V}{V_{flex}}\right) \tag{11}$$

The function $f_3(V/V_{flex})$ contains thus the information on the development of the rotations for the

level of applied load. According to MC 2010, such function can be estimated on the basis of the

acting bending moments and flexural strength as:

$$f_3\left(\frac{V_E}{V_{flex}}\right) = k_m \cdot \left(\frac{m_s}{m_R}\right)^{1.5} \quad \text{so that} \quad \psi = k_m \frac{r_s}{d} \frac{f_y}{E_s}\left(\frac{m_s}{m_R}\right)^{1.5} \tag{12}$$

where m_s refers to the moment used for calculation of the reinforcement in the support strip [15]

and m_R to the corresponding bending strength. The shape of the curve given by Eq. 12

reproduces with sufficient accuracy that obtained from other models(refer to Fig. 1h), such as

those found from a quad-linear or bi-linear approximation of the m-χ curve (refer to Fig. 1g).

A factor of proportionality (k_m) is also given to account if simpler (Level-of-Approximation II,

$k_m = 1.5$) or more refined (Level-of-Approximation III, $k_m = 1.2$) estimates of the acting

moment in the design strip (m_s) are considered. For design purposes, the previous formulas can

be directly used by introducing the pertinent safety format and its corresponding values (design

moment strip m_{sd}, design bending strength m_{Rd} and design yield strength of the reinforcement

f_{yd}).

For preliminary design purposes, a safer estimate of the rotation of the slab can be used by

setting $m_s = m_R$ (full reinforcement yielding at the support strip). This is typically adopted at a

LoA I, in order to check if the general dimensions of a slab are suitable.

2.3 Design and calculation of failure loads

Performing a design using this strain-based method is rather simple. It suffices to calculate the

rotation of the slab (ψ) that corresponds to the acting shear force (V_E) and, with that, to

determine the strength according to the failure criterion (V_R). In the case $V_E \leq V_R$, the strength is

sufficient (Fig. 2b). Otherwise, the punching shear strength is insufficient (Fig. 2c) and the

design of the slab has to be modified (e.g. by placing of shear reinforcement or shearheads,

enlargement of the supported area, increasing of the slab thickness or of the flexural

reinforcement).

If it is necessary to calculate the actual punching strength (which might be necessary for the

assessment of an existing structure), the intersection point of the failure criterion, which

provides the available punching shear strength for a given rotation, and the load-rotation curve

of the slab, which represents shear force for a given rotation, has to be determined (refer to

Figure 2d).

The accuracy of the MC 2010 approach is compared in Figure 3a-b to test results on slabs

without transverse reinforcement for Levels-of-Approximation II (given by Eq. 12) and using a

LoA IV approximation (Eq. 4) based on a quad-linear moment-curvature diagram. Details on

the tests compared to MC 2010 in Figure 3a-b have been presented in reference [20]. The

results compare well for both cases, with a logical improvement in accuracy for LoA IV

calculations, compared to those of LoA II. It has to be noted though that LoA II already

provides very good estimates for the measured strengths and, in these cases of punching in

symmetric slabs without moment transfer, performing a LoA III or LoA IV calculation has only

a limited influence on the results. On the other hand, the LoA III and LoA IV models provides

significant improvement in accuracy when designing slabs of irregular geometry [23].

Comparison to shear-reinforced slabs is also shown in Figure 3c for the tests presented in

references [21,25]. The results are presented for LoA II, showing how various failure modes

(punching within the shear-reinforced area, by crushing of concrete struts and outside the

shear-reinforced area) can be governing [21]. The results are again in nice agreement to the

measured failure loads.

3. Design example

In this section, the MC 2010 provisions are used for the punching shear design of a flat slab. The structure under consideration is a five-storey residential building with the geometry and main dimensions given in Figure 4. In the following, the design of the flat slab against punching of the inner column (C5) is discussed.

For the concrete, the strength class C30/37 (f_{ck} = 30 MPa, γ_c = 1.5) is assumed and for the reinforcing steel a grade B500S (f_{yk} = 500 MPa, E_s = 200 GPa, γ_s = 1.15, ductility class B) is assumed. The factored design load accounting for self-weight, dead load and live load is $q_d = 15.6 \, \text{kN/m}^2$.

3.1 First LoA: Check of the main dimensions of the structure

For preliminary design it is usually sufficient to check whether the thickness of the slab and the size of the slab connections (columns, walls) are adequate to ensure a sufficient punching shear strength. For the selected design example, and without performing a rigorous analysis, the design reaction can be estimated on the basis of contributive areas, refer to Figure 5. For the inner column, this results in R_{Ed} = 692 kN. If the loads applied on the inside the control perimeter are neglected, which is a safe and reasonable assumption with respect to the degree of accuracy, the shear force results in $V_{Ed} = R_{Ed}$ = 692 kN.

The punching shear strength for members without shear reinforcement is calculated as [2]:

$$V_{Rd} = V_{Rd,c} = k_\psi \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_0 \cdot d_v \tag{13}$$

The control perimeter b_0 can be estimated assuming a basic perimeter at a distance of $d_v/2$ from the supported area and a safe value of the eccentricity coefficient k_e of 0.90 (accounting for concentrations of loads within the shear field [15]). Thus, the control perimeter results in:

$$b_0 = k_e\left(4 \cdot b_c + \pi \cdot d_v\right) = 0.90 \cdot \left(4 \cdot 260 + \pi \cdot 200\right) = 1501\,\text{mm} \tag{14}$$

Assuming an average effective depth of $d = 200$ mm and d_v (defined as the distance from the centroid of the reinforcement layers to the supported area) equal to d, as the construction joint between the slab and the column is assumed to be at the base of the slab; that is, there is no column penetration.

For estimating the punching shear strength, the rotation of the slab is the governing parameter. A first and safe estimate of this value can be obtained assuming that failure of the slab occurs at full yielding of the flexural reinforcement in the support strip, which is a conservative assumption with respect to the crack widths [15]. This allows calculation of the the governing rotation as a function of the slenderness of the slab (expressed in terms of the ratio r_s/d, where r_s denotes the radius of contraflexure of radial bending moments) and of the yield strain of the flexural reinforcement [15]. The distance r_s can be approximated for flat slabs with regular bays

as [2]: $r_{sx} = 0.22\ell_x = 1.32$ and $r_{sy} = 0.22\ell_y = 1.23$ m (where ℓ refers to the bay span). In this case,

the rotation around the x-direction is governs (larger value of r_s) and can be calculated as [2]:

$$\psi_x = 1.5 \cdot \frac{r_{s,x}}{d} \frac{f_{yd}}{E_s} = 1.5 \cdot \frac{1.32}{0.200} \frac{435}{200000} = 0.0215 \tag{15}$$

Thus, the value of k_ψ (accounting for the opening of the shear-critical crack and for its

roughness [15]) is:

$$k_\psi = \frac{1}{1.5 + 0.9 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.9 \cdot 0.0215 \cdot 200 \cdot 0.75} = 0.227 \quad (< 0.6) \tag{16}$$

Assuming that the aggregate size is larger than 27 mm, a value $k_{dg} = 0.75$ is obtained($k_{dg} =$

$32/(16+d_g) \geq 0.75$). The punching shear strength can thus be calculated according to MC 2010

[2] as:

$$V_{Rd,c} = k_\psi \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_0 \cdot d_v = 0.227 \frac{\sqrt{30}}{1.5} 1501 \cdot 200 \cdot 10^{-3} = 249 \text{ kN} \leq V_{Ed} \tag{17}$$

In this case, the punching shear resistance is lower than the design shear force. In order to check

whether the placement of shear reinforcement will suffice to strengthen the slab or other

alternatives have to be provided (shearheads, changes of thickness or of supported area

dimensions), the maximum punching shear strength of a shear-reinforced slab has to be

determined. In the MC 2010 [2] this can be done with help of the coefficient k_{sys}. This

coefficient is equal to the ratio between design shear force and the shear strength due to the

concrete contribution. A similar approach has also been adopted by other authors [33] on the

basis of the Eurocode 2 empirical formulas [5]. The coefficient k_{sys} varies between 2.0, for lower performing systems, and 2.8, for higher performing systems. In the present case there is a demand of:

$$k_{sys} = \frac{V_{Ed}}{V_{Rd,c}} = \frac{692}{249} = 2.78 \tag{18}$$

This value implies that it is possible to choose shear reinforcement that will provide sufficient strength and no increases of the column size or of the thickness of the slab are necessary.

3.2 Second LoA: Design for punching shear

For preliminary design purposes and in order to check the dimensions of a structure, it is assumed that the flexural reinforcement yields at failure of the slab. However, in a tender or executive design this (safe) assumption can be improved, if necessary. To do so, the amount of flexural reinforcement over the column needs to be known.

The design of the flexural reinforcement can be carried out for example with help of the "advanced strip method" [34], the "direct design method", the "equivalent frame method" [35] or by using the finite element method (FEM). For the following, the latter option was used resulting in the reinforcement layout shown in Figure 6. The resistance in bending is calculated according to the rigid-plastic theory as:

$$m_{Rd} = \rho \cdot d^2 \cdot f_{yd}(1\text{-}0.5\rho \cdot f_{yd}/f_{cd}) \tag{19}$$

which, for the investigated region, leads to a value of $m_{Rd} = 115$ kNm/m (calculated with $d = 204$ mm). Based on the FEM analysis a more refined value of the design reaction of $R_d = 664$ kN is obtained, which is in close agreement with the previous estimate and the corresponding bending moments acting on top of the column are found as $M_{Ed,x} = 8$ kNm and $M_{Ed,y} = 1$ kNm. The total bending moment $M_{Ed} \approx 8$ kNm.

According to MC 2010 [2], the design shear force can be reduced by subtracting the loads applied within the control perimeter from the column reaction (Figure 7):

$$A_c = b_c^2 + 2b_c \cdot d_v + \frac{\pi}{4}d_v^2 = 0.206 \text{ m}^2, \quad \text{thus} \quad V_{Ed} = R_d - q_d A_c = 661 \text{ kN} \tag{20}$$

In this case the design shear force is only slightly reduced; however, in other cases (such as foundations or post-tensioned slabs) this reduction might be significant. The calculation of the eccentricity coefficient k_e can also be performed in a more accurate manner. Instead of the safe estimate adopted in LoA I ($k_e = 0.90$) the the general expression provided by MC 2010 for this parameter [2] can be used:

$$k_e = \frac{1}{1 + e_u/b_u} \tag{21}$$

The load eccentricity e_u is calculated to $e_u = |M_{Ed}/V_{Ed}| = 8 \cdot 10^3/661 = 12$ mm, accounting for the coincident position of the centroids of the column and of the control perimeter. The diameter of

a circle with the same surface as the region A_c inside the basic control perimeter is $b_u = (4A_c/\pi)^{0.5} = 513$ mm. This yields $k_e = 0.977$ and $b_0 = 1642$ mm.

The determination of the design rotations, accounting for the flexural reinforcement, starts with the definition of the width of the bending moment support strip. This parameter can be calculated as [15] $b_s = 1.5(r_{sx} \cdot r_{sy})^{0.5} = 1.91$ m. The bending moments acting in the support strip result in:

$$m_{sd,x} = \frac{V_{Ed}}{8} + \left| \frac{M_{Ed,x} - V_{Ed} \cdot \Delta e_x}{2b_s} \right| = \frac{661}{8} + \left| \frac{8}{2 \cdot 1.91} \right| = 84.7 \ \text{kNm/m} \tag{22}$$

Finally, at LoA II, the estimate of the design rotations can be improved with respect to LoA I [2]:

$$\psi_x = 1.5 \frac{r_{s,x}}{d} \frac{f_{yd}}{E_s} \left(\frac{m_{sd}}{m_{Rd,x}} \right)^{1.5} = 1.5 \frac{1.32}{0.204} \frac{435}{200000} \left(\frac{84.7}{115} \right)^{1.5} = 0.0133 \tag{23}$$

where the rotation around the x-axis is governing (rotation in the y-axis is smaller with $m_{sd,y} = 82.9$ kNm/m and $\psi_y = 0.0121$). Thus:

$$k_\psi = \frac{1}{1.5 + 0.9 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.9 \cdot 0.0133 \cdot 204 \cdot 0.75} = 0.30 \ (\le 0.6) \tag{24}$$

$$V_{Rd,c} = k_\psi \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot b_0 \cdot d_v = 0.300 \frac{\sqrt{30}}{1.5} 1642 \cdot 204 \cdot 10^{-3} = 367 \ \text{kN} \le V_{Ed} \tag{25}$$

The punching shear strength of the slab without shear reinforcement is insufficient and

reinforcement is required. The shear carried by the concrete, however, is assessed to be higher

from that of the LoA I model and, thus, the total steel reinforcement required is reduced.

This result confirms that of the LoA I analysis. Taking advantage of the improved values k_ψ and

ψ, a detailed design of the required shear reinforcement can be performed. To do so, the three

potential failure modes of shear-reinforced slabs (i.e., crushing of concrete struts, punching

within the shear-reinforced zone and punching outside the shear-reinforced zone [21]) have to

be checked.

a) Maximum punching shear strength (crushing of concrete struts)

The required value of k_{sys} results from the previous value of $V_{Rd,c}$:

$$k_{sys} = \frac{V_{Ed}}{V_{Rd,c}} = \frac{661}{367} = 1.80 \tag{26}$$

This implies that the slab can be shear-reinforced using any available shear reinforcement

system. In this example vertical stirrups or links suitably detailed according to MC 2010 [2] are

used.

b) Design of punching shear reinforcement

At failure, V_{Ed} is equal to V_{Rd} and the punching shear strength is calculated as $V_{Rd} = V_{Rd,c} + V_{Rd,s}$, where the contribution of the shear reinforcement $V_{Rd,s} = V_{Ed} - V_{Rd,c}$ has to satisfy $V_{Rd,s} \geq 0.5 V_{Ed}$ (minimum shear reinforcement); in the present case this condition is governing: $V_{Rd,s} \geq 331$ kN.

The required shear reinforcement can thus be determined as [2]:

$$V_{Rd,s} = A_{sw} \cdot k_e \cdot \sigma_{swd} \tag{27}$$

where A_{sw} denotes the cross-sectional area of the shear reinforcement located between $0.35 \cdot d_v$ and d_v from the edge of the supported area (Figure 8) and σ_{swd} is the average stress activated in the shear reinforcement due to the opening of the critical shear crack. This latter parameter can be calculated as:

$$\sigma_{swd}(\leq f_{ywd}) = \frac{E_s \cdot \psi}{6}\left(1 + \frac{f_{bd}}{f_{ywd}}\frac{d}{\phi_w}\right) = \frac{200000 \cdot 0.0133}{6}\left(1 + \frac{3}{435}\frac{204}{8}\right)$$
$$= 521\,\text{MPa} \rightarrow \sigma_{swd} = f_{ywd} \tag{28}$$

The values f_{bd}, f_{ywd} and ϕ_w refer to the bond strength, yield strength and diameter of the shear reinforcement, respectively. Thus

$$A_{sw} = \frac{V_{Rd,s}}{k_e \cdot \sigma_{swd}} = \frac{331}{0.977 \cdot 435} = 779 \ \text{mm}^2 \tag{29}$$

Choosing, for instance, 8 mm-diameter links at a spacing of 100 mm in both x- and y-directions ($\rho_w = 0.5\%$) results in $A_{sw} = \rho_w \cdot A_{cw} = 1263$ mm^2, where A_{cw} is the concrete area within d_v and $0.35 \cdot d_v$ from the supported area (0.253 m^2), refer to Figure 8.

For shear studs or other types of shear reinforcement arranged either radially or in a cruciform

shape (see Figure 9), A_{sw} may be determined from:

$$A_{sw} = n_r \cdot A_\phi \cdot d / s \tag{30}$$

where n_r is the number of lines of stud, or shear, reinforcement radiating from the column, s is

the nominal stud spacing, $s = \max(s_1; s_0 + 0.5 \cdot s_1)$, where s_0 is the distance of the first shear

reinforcement unit to the supported area and s_1 is the spacing of the studs in the radial direction,

and A_ϕ is the cross-sectional area of one stud.

c) Extent of the shear-reinforced area

The extent of the area where shear reinforcement has to be provided can be determined by

calculating the punching shear strength outside the shear-reinforced area (accounting for the

concrete contribution $V_{Rd,c}$ only). The shear-resisting effective depth ($d_{v,out}$) has to be reduced in

this case because of the concrete cover of the shear reinforcement in the compression side

(soffit) of the slab ($d_{v,out} = d - c = 174$ mm). The required perimeter to ensure sufficient

punching shear strength results in

$$b_0 = \frac{V_{Ed}}{k_\psi \cdot \frac{\sqrt{f_{ck}}}{\gamma_c} \cdot d_v} = \frac{661 \cdot 10^3}{0.300 \frac{\sqrt{30}}{1.5} \cdot 174} = 3468 \ \text{mm} \tag{31}$$

which is smaller than that available: $b_{0,out} = 0.99\cdot (4\cdot800+\pi\cdot174) = 3708$ mm, see Figure 8, where the factor 0.99 is the coefficient k_e for the outer perimeter. It can be noted that the reduction of the shear forces acting inside the control perimeter have been neglected as a safe assumption.

4. Conclusions

Punching shear design procedures - especially those of the previous Model Codes (MC 1978 and MC 1990) - have been thoroughly reviewed during the preparation of the Model Code 2010. The new Model Code provisions provide a consistent, physical, approach to shear design, including that of punching, with design equations on the mechanical model provided by the Critical Shear Crack Theory. It is to be recognised that the CSCT approach for two-way shear and the SMCFT approach for one-way shear, presented in the Part I paper, are coherent and, for the first time, a consistent physical modelling philosophy is established that links these two situations.

The main advantages of having a physically based model are:

1. A set of clear and understandable design equations are established that directly incorporate size and strain effects and enables a consistent treatment of members with transverse reinforcement and/or fibres.

2. The accuracy of the strength estimate can be progressively improved, if necessary, by following a Level-of-Approximation approach. Simple, safe and low effort expressions are provided for preliminary design. If more accurate estimates of the shear strength are required (for instance for tender or executive designs or for the assessment of existing structures), the accuracy of the design expressions can easily be improved by performing some additional work better reproducing the actual load-rotation behaviour of the member.

3. As the design expressions are based on physical models both for determining the strength and load-rotation behaviour, they are open for incorporation future developments and construction technologies (new punching shear reinforcing systems, new grades or types of steel and/or concrete) as the mechanical parameters and the underlying model(s) for determining them can be adjusted, or extended, simply.

Finally, an example is presented that demonstrates how the LoA methodology may be utilised effectively depending on the accuracy needed and/or the level of design required, preliminary or detailed, and on the determination of shear reinforcement, using different arrangements, to achieve a required performance.

Acknowledgements

Appendix 1: Notation

The following symbols are used within this paper:

A_c	=	cross-sectional area of concrete
A_{cw}	=	cross-sectional area of concrete where shear reinforcement is activated
$A_{c\phi}$	=	cross-sectional area of one shear reinforcement
A_{sw}	=	cross-sectional area of a shear reinforcement
E_s	=	modulus of elasticity of the reinforcement
EI_0	=	uncracked flexural stiffness
EI_1	=	cracked flexural stiffness
M_{Ed}	=	transfer moment (design value; subscripts $_x$, $_y$ referring to the considered directions)
R_{Ed}	=	reaction of supported area (design value)
V	=	shear force
V_E	=	acting shear force
V_{Ed}	=	design value of acting shear force
V_{flex}	=	level of shear force leading to failure in bending
V_R	=	punching shear strength
V_{Rd}	=	design punching shear strength
$V_{Rd,c}$	=	design concrete contribution to punching shear strength
$V_{Rd,s}$	=	design shear reinforcement contribution to punching shear strength
b_0	=	shear-resisting control perimeter
b_c	=	size of square column
b_s	=	strip width
b_u	=	Diameter of a circle with the same surface as the region inside the basic control perimeter
d_v	=	shear-resisting effective depth
d	=	effective depth
d_g	=	maximum diameter of the aggregate

e_u = load eccentricity with respect to the centroid of the basic control perimeter

f_i, g_i = functions

f_{bd} = design bond strength

f_{ck} = characteristic compressive strength of concrete (measured on cylinder)

f_{cd} = design value of the compressive strength of concrete (measured on cylinder)

f_y = yield strength of flexural reinforcement

f_{yd} = design yield strength of flexural reinforcement

f_{yk} = characteristic value of the yield strength of the flexural reinforcement

f_{ywd} = design yield strength of the shear reinforcement

k_{dg} = coefficient for aggregate size (= 32 /(16 mm + d_g))

k_e = coefficient of eccentricity

k_m = factor of proportionality

k_{sys} = efficiency factor of a punching shear reinforcing system

k_ψ = factor accounting for crack widths and roughness of cracks on shear strength

ℓ = span length (subscripts $_x$, $_y$ referring to the considered directions)

m_{cr} = cracking moment

m_r = radial moment

m_s = average moment per unit length (design of flex. reinforcement) in the strip

m_{sd} = average design moment per unit length (design of flex. reinforcement) in the strip (subscripts $_x$, $_y$ referring to the considered directions)

m_R = average flexural strength per unit length in the support strip

m_{Rd} = average design flexural strength per unit length in the support strip (subscripts $_x$, $_y$ referring to the considered directions)

n_r = number of lines of studs

q_d = applied load (design value)

r_0 = radius of the critical shear crack

r_1 = radius of the zone where cracking is stabilized

r_q = distance between the column the line of contraflexure of bending moments

r_s	=	distance between the column and the line of contraflexure of moments (subscripts x, y referring to the considered directions)
r_c	=	column radius
r_y	=	radius of yielded zone
s	=	reference stud spacing
s_0	=	distance of the first shear reinforcement unit to the supported area
s_1	=	spacing of the studs in the radial direction
w	=	critical shear crack opening
x_{el}	=	depth of uncracked concrete
x_{pl}	=	depth of plastic zone of concrete
β	=	efficiency factor of the bending reinforcement for stiffness calculation
χ	=	curvature
χ_{TS}	=	reduction of curvature due to tension-stiffening
γ_c	=	partial safety factor of concrete
γ_s	=	partial safety factor of steel
ϕ_w	=	diameter of shear reinforcement
ρ	=	flexural reinforcement ratio
ρ_w	=	transverse reinforcement ratio
σ_{swd}	=	design stress in shear reinforcement
ψ	=	rotation of the slab outside the supported area region (subscripts x, y referring to the considered directions)

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Figures
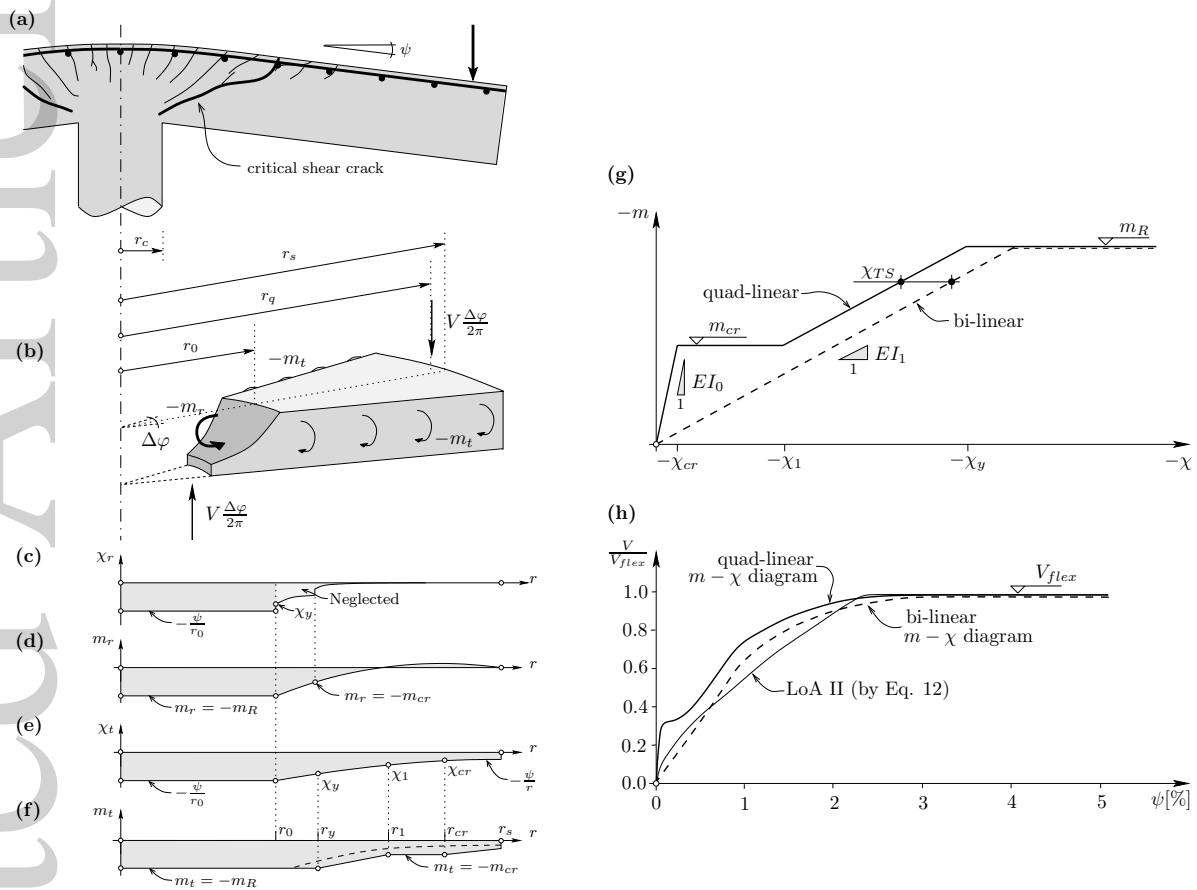
Figure 1:   Physical model for obtaining suitable load-roation relationships in flat slabs: (a) investigated region and critical shear crack; (b) acting moments, forces and dimensions; (c-d) acting radial curvature and moments; (e-f) acting tangential curvature and moments; (g) quadrilinear moment-curvature diagram; and (h) corresponding load-rotation relationships (results calculated for an axis-symmetric slab $\rho = 0.48\%$, $h = 155$ mm, $r_c = 150$ mm, $r_s = r_q = 856$ mm)
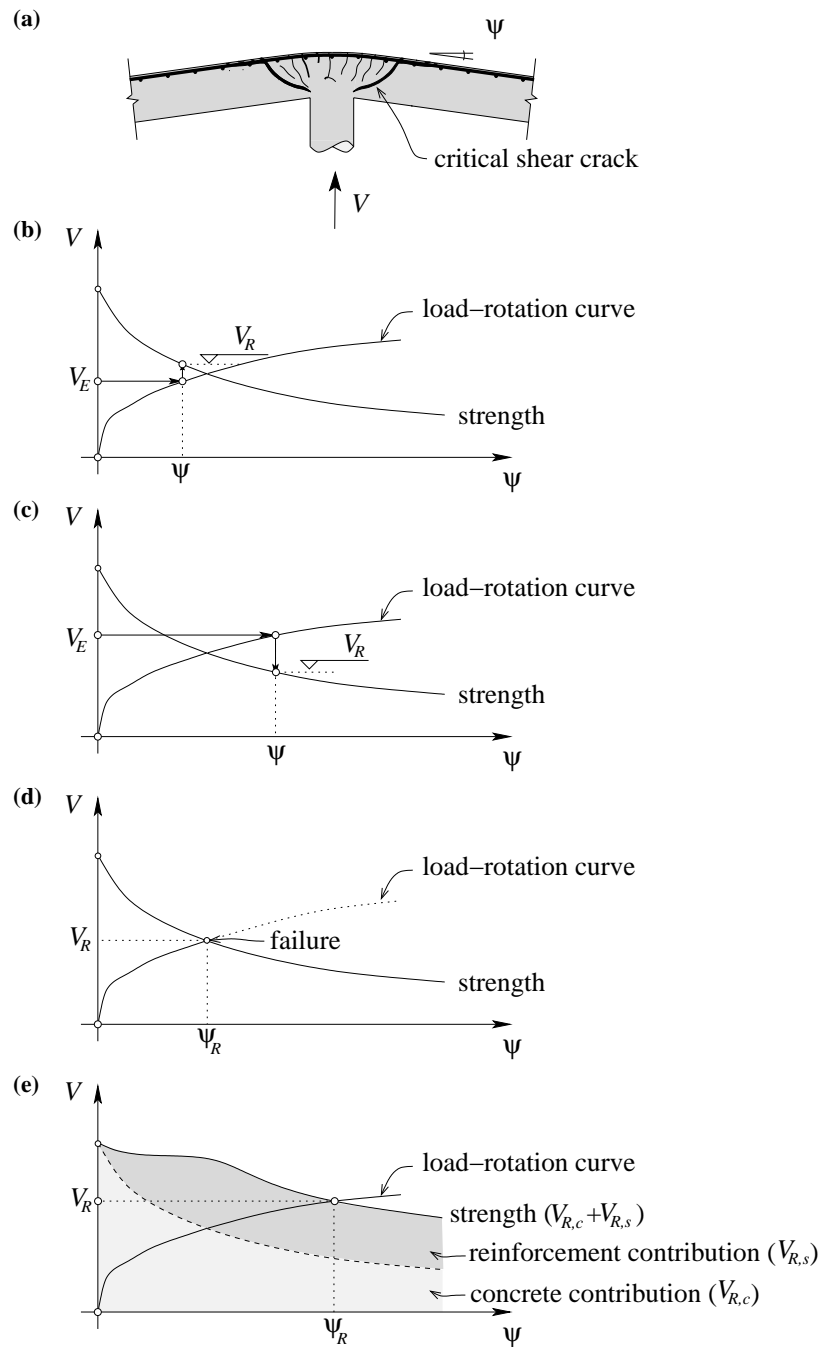
Figure 2: Calculation of punching shear strength according to the CSCT: (a) rotation of the slab near the supported area region; (b,c) calculation of the punching shear strength (V_R) for the rotation developed for a given applied load (V_E); (d) intersection between failure criterion and load-rotation curve for calculation of punching shear strength (V_R); and (e) failure criterion accounting for concrete and shear reinforcement contribution
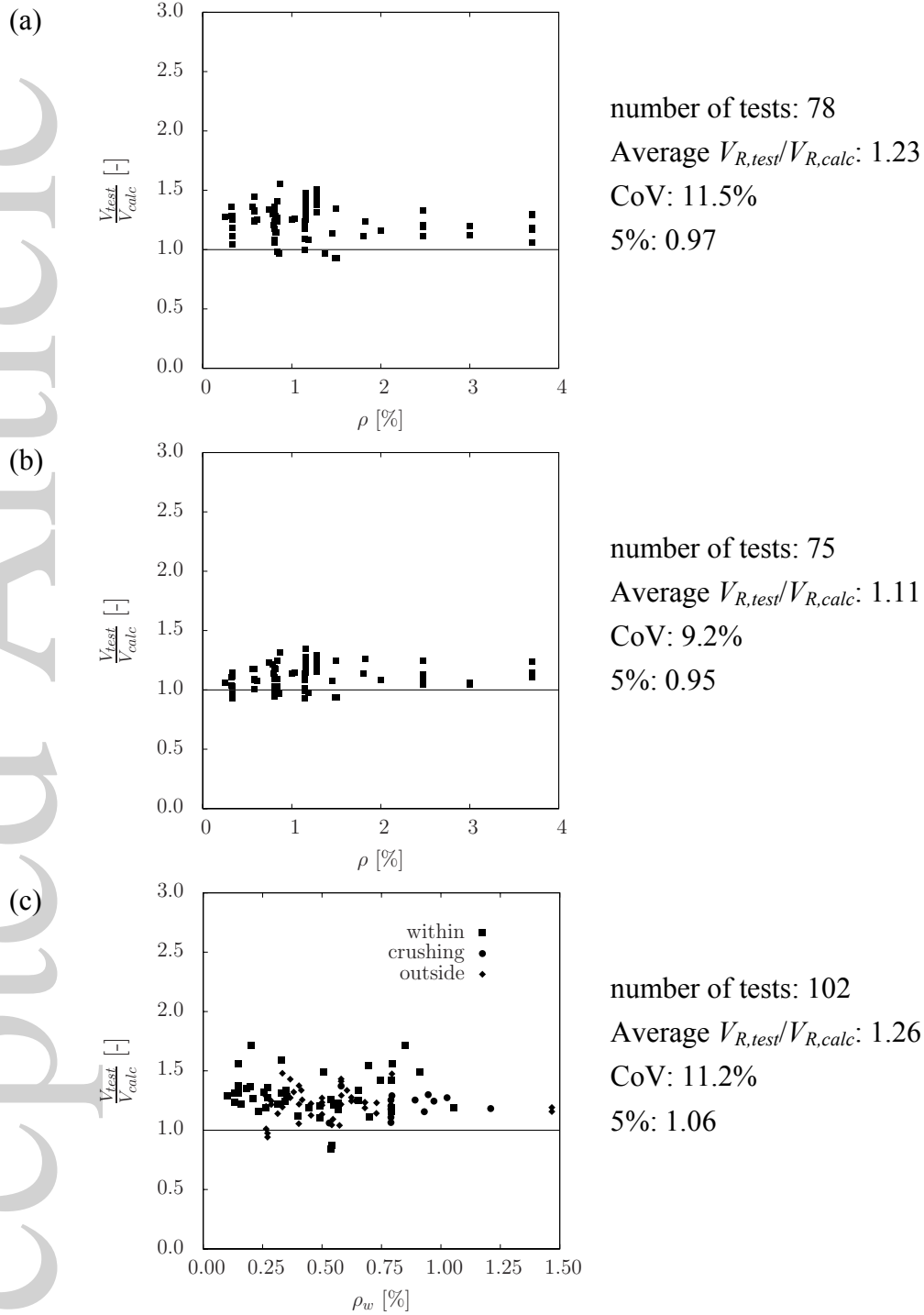
Figure 3:   Comparison of Model Code 2010 provisions with test results: (a) and (b) specimens without transverse reinforcement for LoAs II and IV, respectively; and (c) specimens with transverse reinforcement for LoA II.

(a)

(c)

(b)
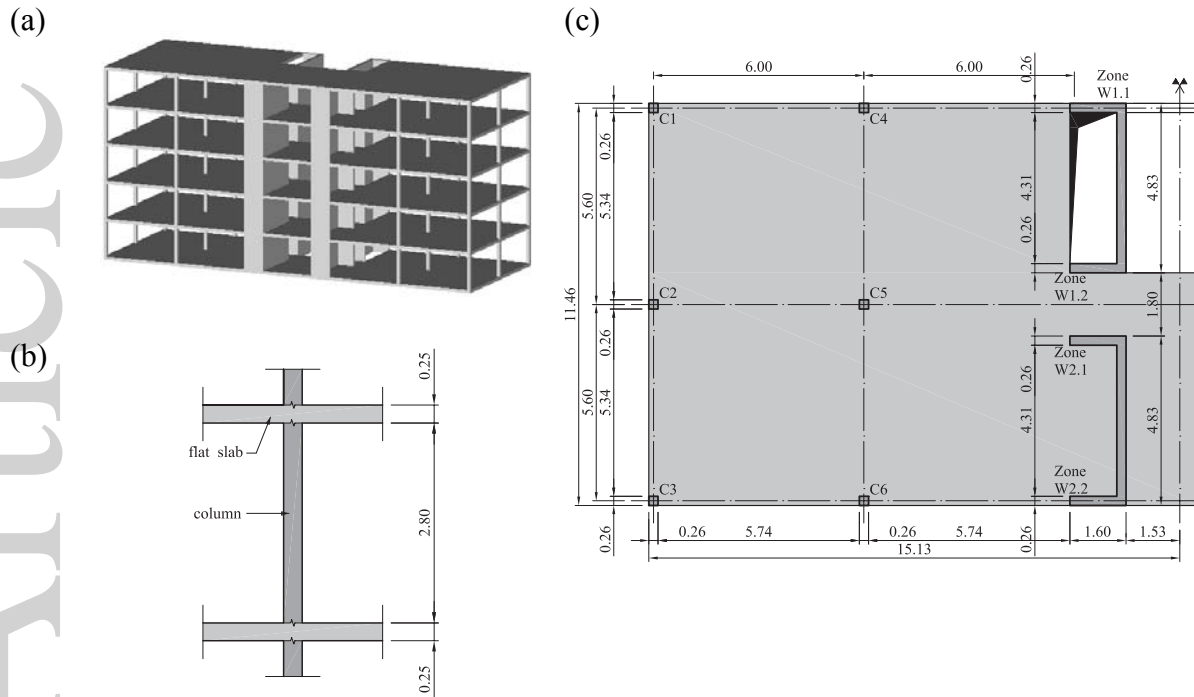
Figure 4:   Design example: (a) view of building; (b) cross section of flat slab and

supporting columns; and (c) main dimensions (in [m], $\ell_x = 6.00$ m, $\ell_y = 5.60$ m)
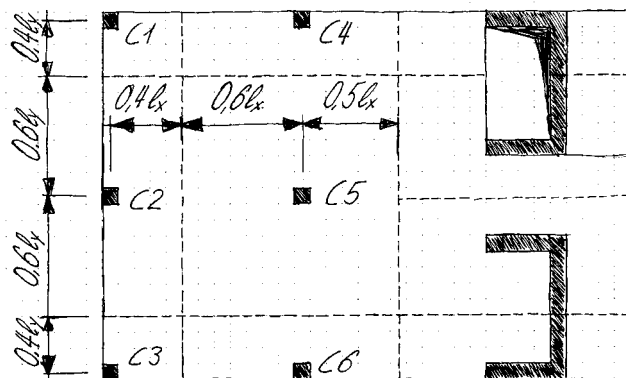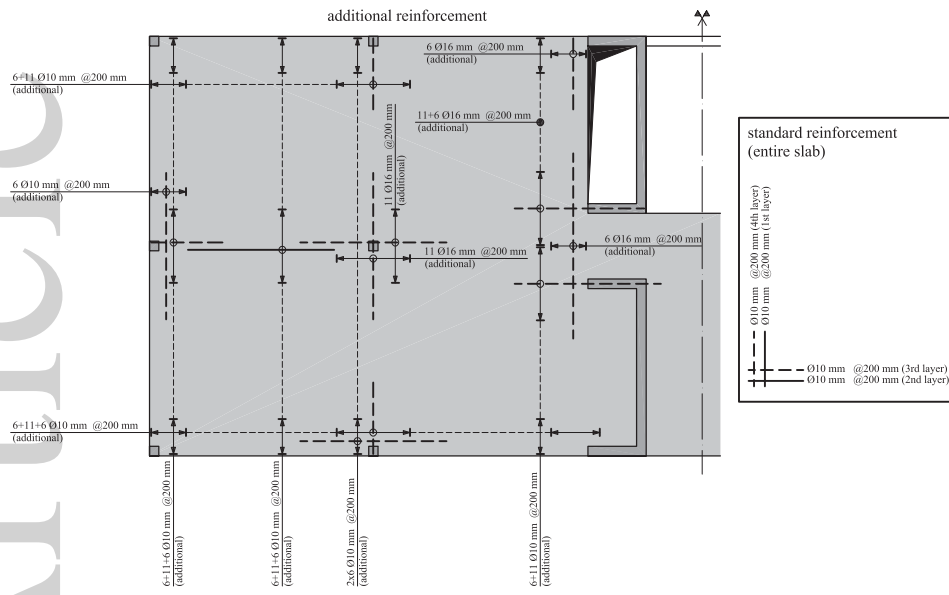

Figure 5:   Approximated contributive areas for each column

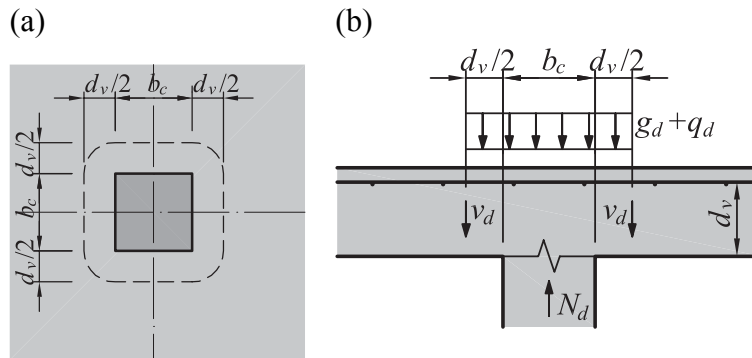Figure 6:　　　　　Sketch of flexural reinforcement.

Figure 7:　　　　　Loads applied within the control perimeter: (a) plan view; and (b) cross-section
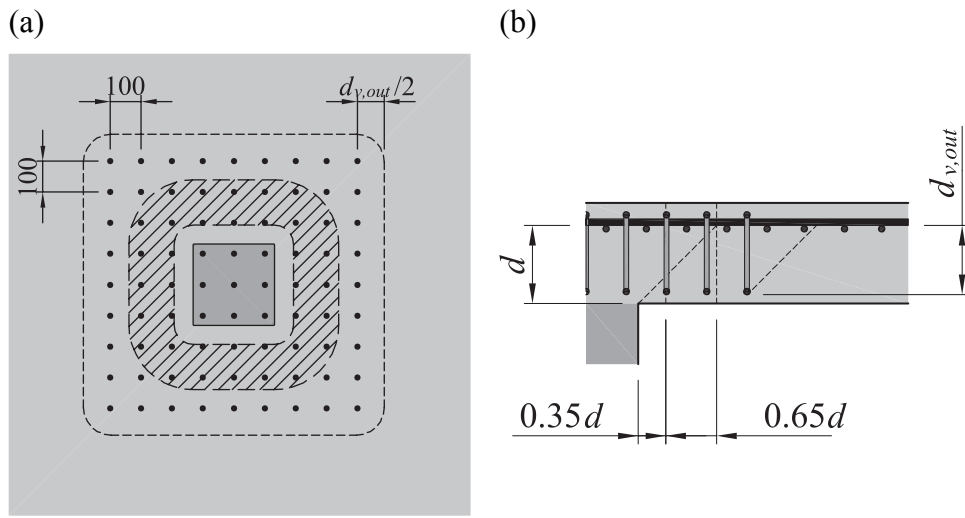
Figure 8:   Shear reinforcement arrangement: (a) plan view; and (b) cross-section
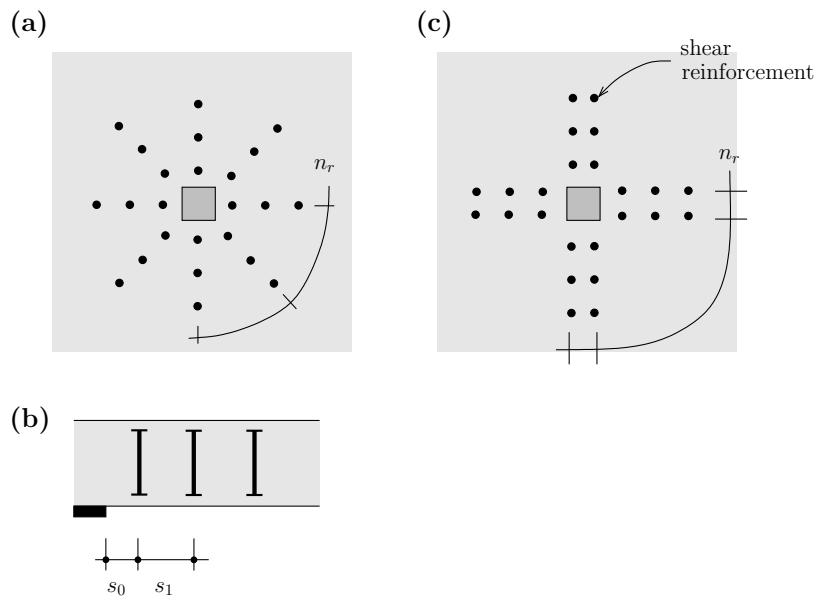

Figure 9:   Arrangement of shear reinforcement: (a) radial arrangement ($n_r = 8$ in this case);

(b) detail of distances of first and second shear reinforcement; and (c) cruciform

shape ($n_r = 8$ in this case)