Global electromagnetic simulations of tokamak scrape-off layer turbulence

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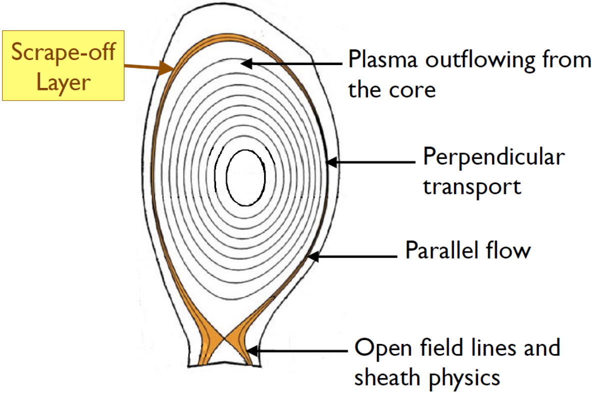
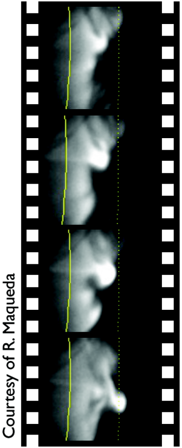
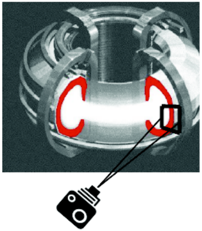
Scrape-off layer physics crucial for magnetic fusion



Heat load to PFCs, rotation, impurities, L-H transition...

Properties of the SOL





- $L_{fluc} \sim \langle L \rangle_t$
- $n_{fluc} \sim \langle n \rangle_t$
- Collisional magnetized plasma
- Low frequency modes $\omega \ll \omega_{ci}$
- Open field lines

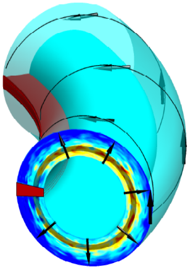
Questions we need to answer

- What instabilities are present and which one is dominant ?
- What is the mechanism setting the turbulence levels ?
- How does the SOL width change with plasma parameters ?
- What is the role of electromagnetic effects ?
- How is toroidal rotation generated in the SOL ?
- How are impurities transported ?
- Is SOL transport related to the density limit ?
- How is the SOL coupled with the closed flux surface region ?

A tool to simulate SOL turbulence

Global Braginskii Solver (GBS) [Ricci *et al.*, PPCF **54**, 124047, 2012]

- Drift-reduced Braginskii equations
  $d/dt \ll \omega_{ci}$, $k_\perp^2 \gg k_\parallel^2$
- Evolves 3D fields : n, T_e, ϕ, $V_{||e}$, $V_{||i}$
- Annular region of full torus, full flux-surface
- Flux-driven, no separation between equilibrium and fluctuations
- Global balance between plasma outflow from the core, turbulent transport, and parallel losses



Equations will be given in normalized units...

- Coordinate system : $(y, x, z) \rightarrow (\textit{poloidal length}, \textit{radial}, \textit{toroidal})$
- Equations expressed in normalized units :
  - $L_\perp \rightarrow \rho_s$
  - $L_\parallel \rightarrow R$
  - $v \rightarrow c_s$
  - $t \sim \gamma^{-1} \rightarrow R/c_s$
- Simplified notation :
  - $p_0 = \langle p \rangle_t$ with $t \gg \gamma^{-1}$
  - $L_p = -\langle p/\partial_x p \rangle_t$

Drift-reduced Braginskii equations to describe the SOL

$$\partial_t n = -\frac{R}{B}[\phi, n] + \frac{2}{B}\left[\hat{C}(p_e) - n\hat{C}(\phi)\right] - \nabla_{\parallel}\left(n v_{\parallel e}\right) + S_n$$

$$\partial_t \nabla_{\perp}^2 \phi = -\frac{R}{B}\left[\phi, \nabla_{\perp}^2 \phi\right] + \frac{2B}{n}\hat{C}(p_e) - v_{\parallel i}\nabla_{\parallel}\nabla_{\perp}^2\phi + \frac{B^2}{n}\nabla_{\parallel} j_{\parallel}$$

$$\partial_t\left(v_{\parallel e} + \frac{m_i \beta_e}{m_e 2}\psi\right) = -\frac{R}{B}\left[\phi, v_{\parallel e}\right] - v_{\parallel e}\nabla_{\parallel} v_{\parallel e} + \frac{m_i}{m_e}\left\{-\nu\frac{j_{\parallel}}{n} + \nabla_{\parallel}\phi - \frac{1}{n}\nabla_{\parallel} p_e - 0.71\nabla_{\parallel} T_e - \frac{2}{3n}\nabla_{\parallel} G_e\right\}$$

$$\partial_t v_{\parallel i} = -\frac{R}{B}\left[\phi, v_{\parallel i}\right] - v_{\parallel i}\nabla_{\parallel} v_{\parallel i} - \frac{1}{n}\nabla_{\parallel} p_e$$

$$\partial_t T_e = -\frac{R}{B}[\phi, T_e] - v_{\parallel e}\nabla_{\parallel} T_e + \frac{4}{3}\frac{T_e}{B}\left[\frac{7}{2}\hat{C}(T_e) + \frac{T_e}{n}\hat{C}(n) - \hat{C}(\phi)\right] + S_{T_e} + \frac{2}{3}T_e\left[0.71\nabla_{\parallel} v_{\parallel i} - 1.71\nabla_{\parallel} v_{\parallel e} + 0.71\left(\frac{v_{\parallel i} - v_{\parallel e}}{n}\right)\nabla_{\parallel} n\right]$$

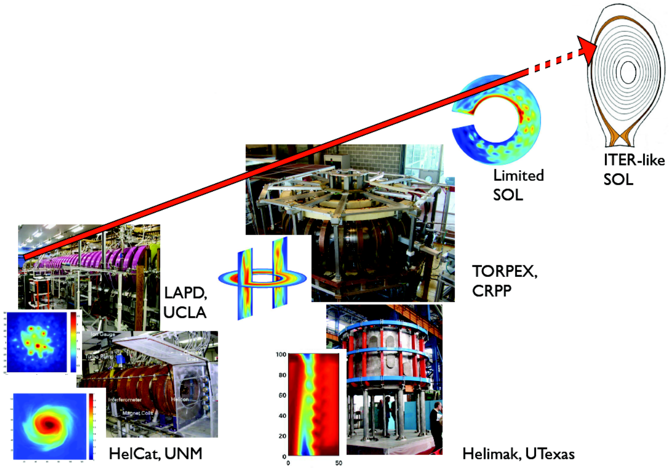
Need BC for n, $v_{\parallel e}$, $v_{\parallel i}$, T_e, $\nabla_{\perp}^2\phi$, ψ, and ϕ

BCs at the Magnetic Pre-Sheath entrance (MPS)

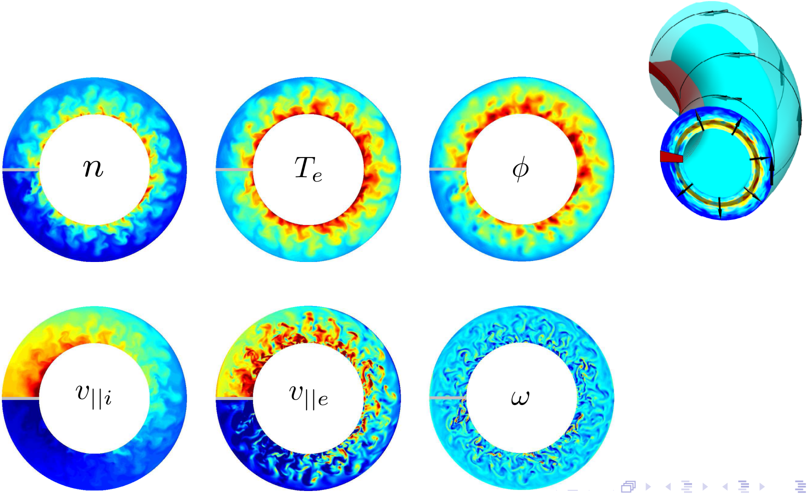
- SOL interfaces with the limiter at the MPS
  - Ions accelerated towards wall with $v = c_s$
  - Large electric field $\partial_y \phi \sim \phi/\rho_s$
- Drift-Braginskii eqs. invalid inside the Magnetic Pre-Sheath
- Derived model describing SOL–MPS *entrance* interface
- Generalized version of Bohm-Chodura BCs for ALL fluid fields

[Loizu *et al*, Phys. Plasmas **19**, 122307, 2012]

Understanding developed by increasing complexity



Examples of 3D simulations (poloidal cross-sections)



Topics under investigation

- **Turbulent saturation mechanisms**
- **Identification of the main instabilities**
- **Electromagnetic effects**
- **Size scaling**
- **Intrinsic rotation**
- Toroidicity effects (finite aspect ratio, Shafranov shift...)
- Impurity transport

We will discuss topics in **bold** face

Modes saturate due to pressure non-linearity

We observe in simulations [Ricci *et al.* Phys. Plasmas **20**, 010702 (2013)] :

- Perturbation removes background pressure gradient

$$\partial_r p_1 \sim \partial_r p_0 \rightarrow \frac{p_1}{p_0} \sim \frac{\sigma_x}{L_p}$$

- Radial eddy length described by linear non-local theory

  [Ricci *et al.*, PRL **100**, 225002 (2008)]

$$\sigma_x \approx \sqrt{L_p/k_y}$$

- Turbulent flux dominated by radial $\mathbf{E} \times \mathbf{B}$ convection

$$\Gamma_1 = R \left\langle p_1 \frac{\partial \phi_1}{\partial y} \right\rangle$$

Saturation model yields $\mathbf{E} \times \mathbf{B}$ turbulent flux

Gradient removal hypothesis

$$\frac{p_1}{p_0} \approx \frac{\sigma_x}{L_p}$$

$$\partial_t p = -R\left[\phi, p\right]$$

$$\partial_y \phi = \gamma(p_1/p_0)(L_p/R)$$

$$\Gamma_1 \approx R\left\langle p_1 \partial_y \phi_1 \right\rangle$$

$$\boxed{\Gamma_1 \sim p_0 \left(\frac{\gamma}{k_y}\right)_{\max}}$$

Self-consistent prediction of pressure gradient length

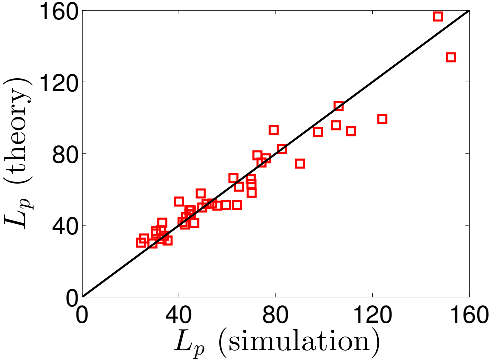
In steady state, $\nabla \cdot \Gamma_1$ balances parallel losses $\sim \nabla_\parallel \cdot (p v_{\parallel e})$, hence

$$\boxed{L_p \approx q \left(\frac{\gamma}{k_y} \right)_{\max}}$$

- Results in iterative scheme to *predict* L_p self-consistently :
  - Obtain $\gamma = f(\underbrace{L_p}_{vary}, \underbrace{k_y}_{scan}, \underbrace{R, q, \nu, \hat{s}, m_i/m_e}_{fixed})$ from linear code
  - Compare $q\,(\gamma/k_y)_{\max}$ with input L_p
  - Vary L_p until LHS = RHS (bisection, secant method, etc..)

Good agreement between theory and simulations

L_p predicted using self-consistent procedure



GBS simulations : $R = 500$–2000, $q = 3$–6, $\nu = 0.01$–1, $\beta = 0$–3×10^{-3}

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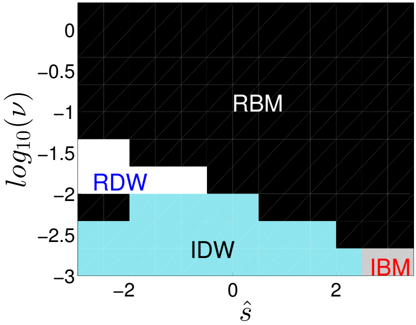
Dominant instability depends principally on q, ν, $\hat{s}$

- Which instability dominates in the non-linear stage ?
  - Resistive/inertial ballooning modes/drift waves ?

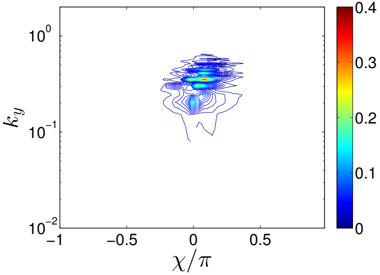


Circular, limited circular plasmas → $\hat{s}_a \approx 2$ → RBM

Dominant instability confirmed using GBS

Compute phase between potential and density fluctuations :

(as indicated in [B.Scott, Phys. Plasmas **12**, 062314, 2005])



$\chi \approx 0$
Drift wave
$\hat{s} < 0$

$\chi \approx \pi/2$
Resistive ballooning mode
$\hat{s} > 0$

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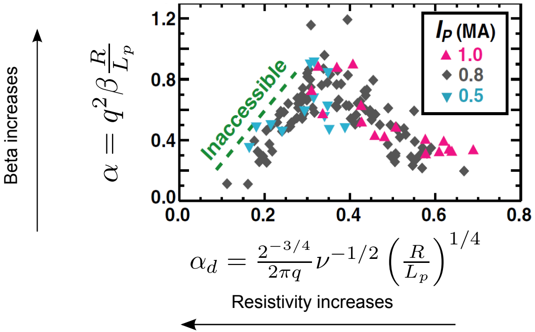
SOL turbulence : interplay between β, ν, and ω_*



[LaBombard *et al.*, *Nucl Fusion* (2005), lower-null L-mode discharges]

Important to understand resistive → ideal ballooning mode transition

Resistive ballooning modes destabilized by EM effects

- Starting from reduced MHD, obtain simple dispersion relation

$$\gamma^2\left(\nu+\frac{\beta_{e0}}{2}\frac{\gamma}{k_\perp^2}\right)=2\frac{R}{L_p}\left(\nu+\frac{\beta_{e0}}{2}\frac{\gamma}{k_\perp^2}\right)-\frac{k_\parallel^2}{k_\perp^2}\gamma$$

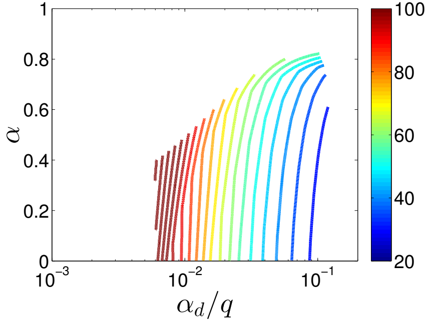
- Neglecting ideal ballooning mode, the resistive branch gives

$$\left(\gamma^2-\gamma_b^2\right)k_\perp^2=-\gamma\left(\frac{1-\alpha}{q^2\nu}\right)$$

and we identify $\gamma\sim\gamma_b=\sqrt{2R/L_p}$ and $k_b\sim\sqrt{(1-\alpha)/(\nu\gamma_b)}/q$

Electromagnetic phase space

- Build a dimensionless phase space...
- Combine simple dispersion relation with $L_p \approx q\,(\gamma/k_y)_{\mathrm{max}}$

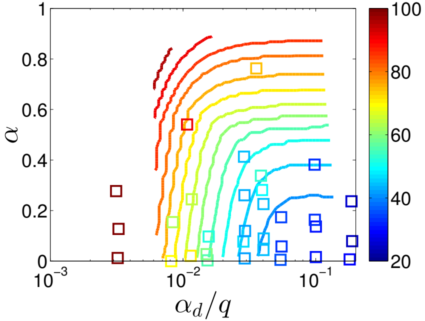


(Color gives L_p for each contour)

- Enhanced transport regime found at high ν, high β

Electromagnetic phase space

- Build dimensionless phase space with full linear system...
- Verify turbulent saturation theory with GBS simulations
  - $R = 500$, $\beta_e = 0$ to 3×10^{-3}, $\nu = 0.01, 0.1, 1$, $q = 3, 4, 6$



(Contours of L_p given by theory, squares are GBS simulations)

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SOL length scales with R, q, β, ν

SOL $\perp$ transport driven by *gradient removal* saturated RBMs

- Combine saturation theory with typical linear growth rate and wavelength

$$L_p = q\frac{\gamma}{k_y}$$

$$\gamma_b = \sqrt{2R/L_p}$$

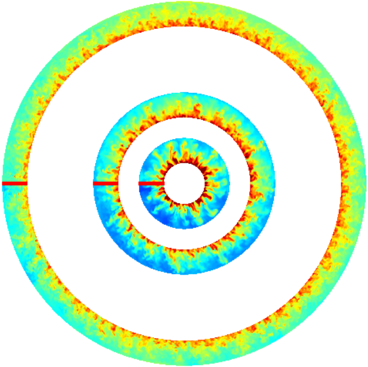
$$k_b = \sqrt{(1-\alpha)/(\nu\gamma_b)}/q$$

- Our simple model leads to a dimensionless scaling :

$$\boxed{L_p/R^{1/2} \approx \left[2\pi\alpha_d(1-\alpha)^{1/2}/q\right]^{-1/2}}$$

Effect of increasing plasma size favorable

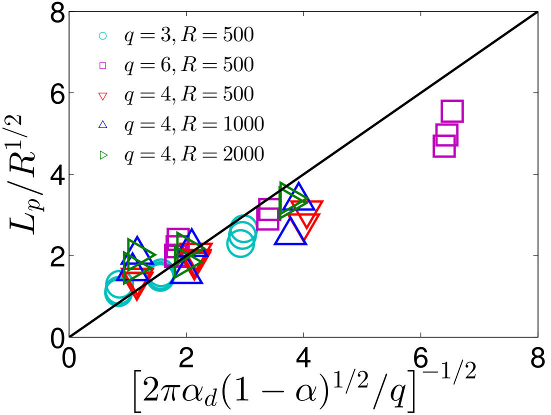
GBS simulations with $R = 500, 1000, 2000$ (TCV size)



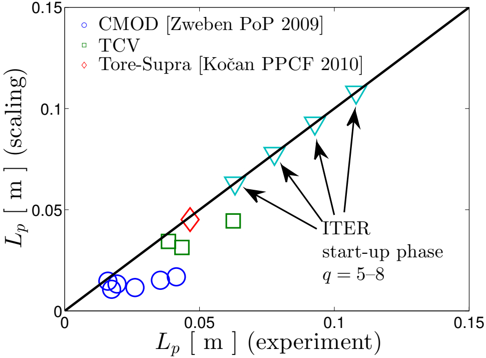
Poloidal cross sections of density

Scaling follows GBS simulation data

Comparison carried out over wide range of parameters (R, q, β, ν)



Comparison with L-mode limited discharges (preliminary !)



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B.LaBombard (MIT)
S.Zweben (PPPL)

$$L_p \approx 7.97 \times 10^{-8} q_a^{8/7} R^{5/7} B^{-4/7} T_e^{-2/7} n_e^{2/7}$$ [m,T, eV, m^{-3}]

Topics under investigation

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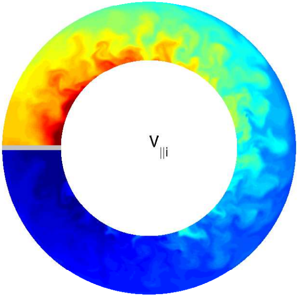
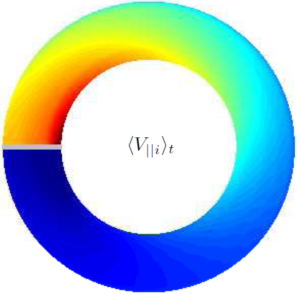
Intrinsic toroidal rotation

- Tokamak plasmas have been observed to rotate toroidally in the absence of momentum injection.
- Effects on MHD stability and turbulent transport
- Important effect for ITER where torque/particle is small
- Experimental evidence for the role of SOL flows in determining core rotation profiles in L-mode [LaBombard NF 2004]
- SOL flows set boundary conditions on the confined plasma and can determine the L-H power threshold [LaBombard PoP 2008]

GBS simulations show intrinsic toroidal rotation

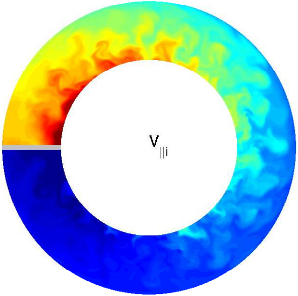


Snapshot

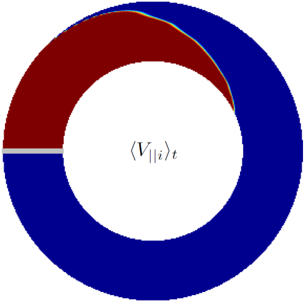


Time-average

GBS simulations show intrinsic toroidal rotation



Snapshot



Time-average +/-

- There is a finite volume-averaged toroidal rotation ($\sim 0.3 c_s$)

2D equation for the equilibrium flow

- Time averaged momentum balance equation coupled to BCs

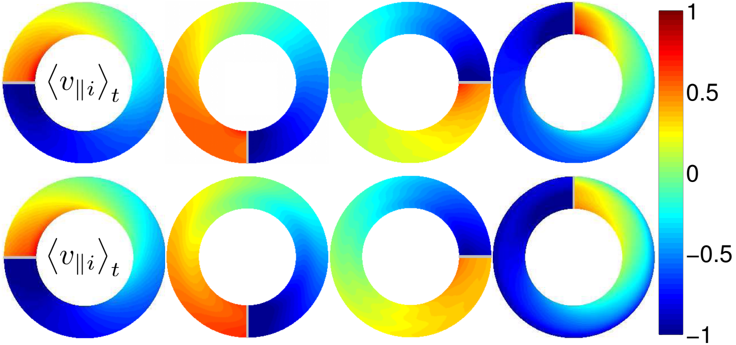
$$\overbrace{-D_I \frac{\partial^2 \bar{v}_{\|i}}{\partial x^2} + v_I \frac{\partial \bar{v}_{\|i}}{\partial x}}^{turbulent\ contribution\ (radial)} + \overbrace{\frac{\partial \bar{\phi}}{\partial x}\frac{\partial \bar{v}_{\|i}}{\partial y}}^{poloidal} + \overbrace{\epsilon \frac{\bar{v}_{\|i}}{q}\frac{\partial \bar{v}_{\|i}}{\partial y}}^{parallel} + \overbrace{\frac{\epsilon}{\bar{n}q}\frac{\partial \bar{p}}{\partial y}}^{generation} = 0$$

$$\underbrace{v_{\|i}^{\pm} = c_s^{\pm} - \frac{q}{\epsilon}\frac{\partial \bar{\phi}}{\partial x}^{\pm}}_{boundary\ condition}$$

- Role of the sheath driving toroidal rotation
  - Source term through boundary condition
  - Asymmetry of pressure profile

GBS simulations agree with the theory

$\langle v_{\parallel i} \rangle_t$ from GBS simulations



$\langle v_{\parallel i} \rangle_t$ from Theory

(limiter position → HFS, down, LFS, up)

Summary and conclusions

- Developed and verified model for turbulent saturation
  - Pressure non-linearity flats background pressure profile
- Identified dominant instability in non-linear steady state
  - Resistive ballooning modes relevant for SOL in limited plasmas
- Derived a simple scaling for SOL width
  - Agrees with simulation results over wide parameter range
  - Will be compared with experiment (in progress)
- Sheath BC drives significant toroidal rotation in SOL

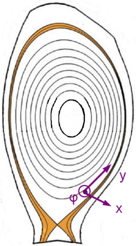
A simple theory of SOL intrinsic rotation

- Within the drift-reduced Braginskii model :

$$\frac{\partial v_{||i}}{\partial t} + v_{||i}\nabla_{||}v_{||i} + (\mathbf{v}_E \cdot \nabla)v_{||i} + \frac{1}{m_i n}\nabla_{||}p = 0$$

- Time-averaging :

$$\bar{v}_{||i}\nabla_{||}\bar{v}_{||i} + \frac{1}{B_\varphi}\langle\nabla \cdot \Gamma_v\rangle_t + \frac{1}{m_i\bar{n}}\nabla_{||}\bar{p} = 0$$



- $\langle\Gamma_{v,y}\rangle_t \simeq \Gamma^{EQ}_{v,y} = \bar{v}_{||i}\frac{\partial\bar{\phi}}{\partial x}$
- $\langle\Gamma_{v,x}\rangle_t \simeq \langle\Gamma^{TURB}_{v,x}\rangle_t = -\langle\tilde{v}_{||i}\frac{\partial\tilde{\phi}}{\partial y}\rangle_t$

Estimate of $\tilde{v}_{||i}$

- Linearising the parallel ion momentum equation :

$$\gamma \tilde{v}_{||i} \simeq \frac{1}{B_\varphi} \frac{\partial \bar{v}_{||i}}{\partial x} \frac{\partial \tilde{\phi}}{\partial y}$$

- Thus we have

$$\langle \Gamma_{v,x}^{TURB} \rangle_t \sim \langle \tilde{v}_{||i} \frac{\partial \tilde{\phi}}{\partial y} \rangle_t \sim \left\langle \left(\frac{\partial \tilde{\phi}}{\partial y} \right)^2 \right\rangle_t$$

Estimate of $\frac{\partial\tilde{\phi}}{\partial y}$

- Using the pressure continuity equation :

$$\frac{\partial \tilde{p}}{\partial t} \sim \frac{1}{B_\varphi}\frac{\partial \tilde{\phi}}{\partial y}\frac{\partial \bar{p}}{\partial x} \underbrace{\Longrightarrow}_{\partial_x \tilde{p} \sim \partial_x \bar{p}} \frac{1}{B_\varphi}\frac{\partial \tilde{\phi}}{\partial y} \sim \frac{\gamma}{k_x}$$

where $k_x = \sqrt{k_y / L_p}$ and $\gamma = c_s\sqrt{2/RL_p}$. [Ricci PRL 2007]

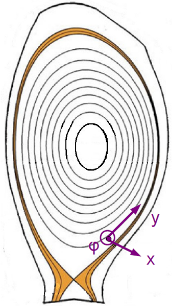
- The turbulent radial momentum flux is then

$$\Gamma_x^{TURB} \simeq -B_\varphi \sqrt{\frac{2L_p}{R}}\frac{c_s}{k_y}\frac{\partial \bar{v}_{||i}}{\partial x}$$

2D equation for the equilibrium flow

$$\underbrace{-D_I \frac{\partial^2 \bar{v}_{||i}}{\partial x^2} + v_I \frac{\partial \bar{v}_{||i}}{\partial x}}_{radial} + \underbrace{\frac{\sigma_\varphi}{|B_\varphi|} \frac{\partial \bar{\phi}}{\partial x} \frac{\partial \bar{v}_{||i}}{\partial y}}_{poloidal} + \underbrace{\alpha \sigma_y \bar{v}_{||i} \frac{\partial \bar{v}_{||i}}{\partial y}}_{parallel} + \underbrace{\frac{\alpha \sigma_y}{m_i \bar{n}} \frac{\partial \bar{p}}{\partial y}}_{generation} = 0$$

- $D_I = \sqrt{\frac{2L_p}{R} \frac{c_s}{k_y}}$ has units of a diffusion coefficient
- $v_I = D_I / 2L_T$ has units of speed
- $B_\varphi = \sigma_\varphi |B_\varphi|$, $B_p = \sigma_y |B_p|$
- The solution of this equation requires boundary conditions



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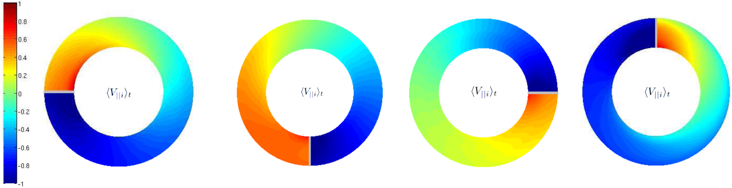
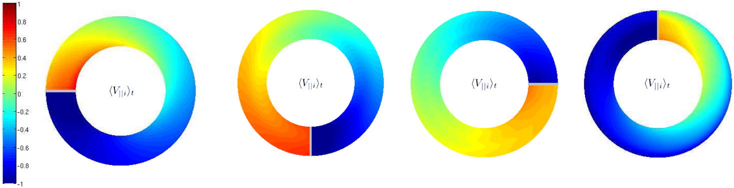


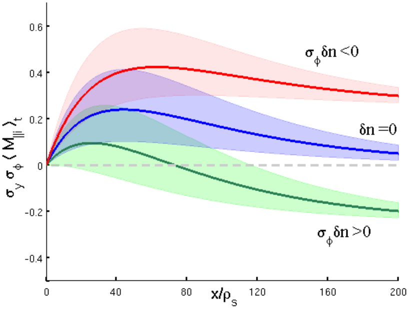
Simulation

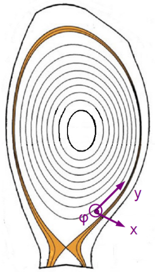


Theory

Experimental trends are reproduced

$$M_{||}(x,0) = \left(\sigma_\varphi \sigma_y \underbrace{\frac{\Lambda}{2\alpha} \frac{\rho_s}{L_T} e^{-x/L_T}}_{sheath} - \frac{\sigma_y}{2} \underbrace{\left(\frac{\delta n}{n} + \frac{\delta T}{T} \right)}_{asymmetry} \right) \left(1 - e^{-x/\lambda}\right)$$





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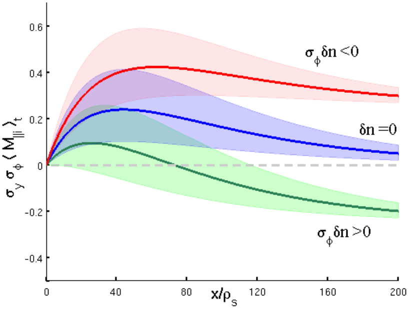
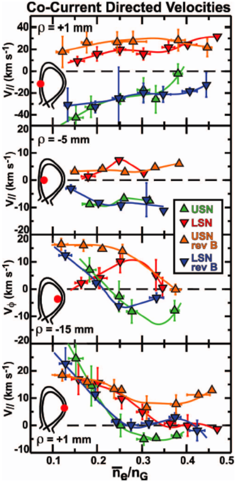


- $M_{||} \lesssim 1$
- Typically co-current
- Rice scaling $V_\varphi \sim T_e / I_p$
- Can become counter-current by reversing **B** (σ_φ) or divertor position (δn)

Experimental trends are reproduced



[LaBombard PoP 2008]

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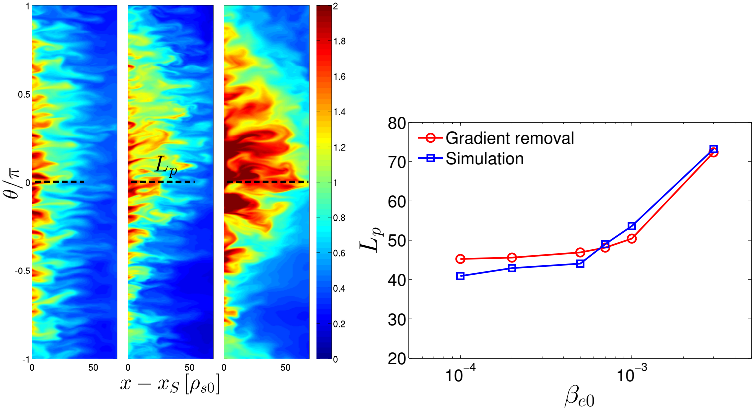
- Rice scaling $V_\varphi \sim T_e/I_p$
- Can become counter-current by reversing $\mathbf{B}$ (σ_φ) or divertor position (δn)

Dynamics of long wavelength SOL modes crucial

- Parallel dynamics and EM effects important



Extra slides : Why global ? why full-n ?

- Global vs Local ?
  - Flux-tube only valid if $k_x L_{eq} \gg 1$ but $k_x L_{eq} \sim \sqrt{k_y L_{eq}} \gtrsim 1$
- Full-n vs Delta-n ?
  - In the SOL $\delta n/n \sim 1$ so cannot separate $\bar{n}$ and $\tilde{n}$
- Flux-driven vs Gradient-driven ?
  - Need to evolve the equilibrium profile (e.g. mode saturation)

Extra slides : Effect of the source details ?

- Details of the radial shape of the source not important
- Poloidal shape of the source may be important (asymmetries, recycling) - to be studied
- Effect of source strength being explored : what do we expect ?
  - If $\gamma_{lin} > V'_{ExB}$: no difference i.e. $L_p \sim \rho_s$
  - If source strong to make $\gamma_{lin} \sim V'_{ExB}$: turbulence suppression ?

[Ricci et al PRL 2007]

Extra slides : How about kinetic effects ?

- SOL is fairly collisional :
  - $\lambda_{ei} \ll L_{||}$
  - $\nu^* > 1$
  - $\nu_{ei} > \gamma_L$
- Kinetic effects may be considered as a higher order correction
  - e.g. Landau damping in Ohm's law

Extra slides : Importance of neutrals ?

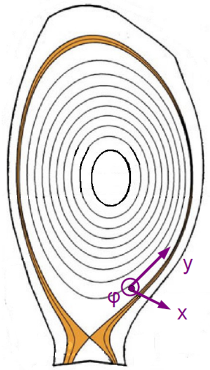
- For the magnetic presheath BC : inertia $\gg$ i-n collisions ?
  - Yes, as long as : $\omega_{ci} \sin \alpha \gg \nu_{in}$
- For the SOL equilibrium : ionization ? recombination ?
  - High recycling can affect the $V_{||i}$ profile - to be studied
  - Intrinsic rotation theory may breakdown in detached regime
- For the SOL fluctuations : effect on the turbulence ? blobs ?
  - Nature of turbulence unchanged, but can add some damping
  - Cross-field currents due to i-n collisions can affect blobs

Extra slides : Is the sheath resistive ? Ryutov's model ?

- Misconception about the concept of "sheath resistivity" :
  - The sheath is essentially collisionless, $\lambda_D \ll \rho_s \ll \lambda_{ie}$
  - How to define an effective resistivity if $j_{||} \neq j_{||}(E_{||})$?
- Ryutov model for sheath resistivity :
  - Linearized Ohm's law written as $\nabla_{||}\tilde{\phi} = \nu\tilde{j}_{||} \sim \nu\tilde{\phi}$

Extra slides : Parallel vs Toroidal rotation ?

- $V_\varphi = V_{||} \cos\alpha + V_d \sin\alpha$
- $V_d = \frac{\mathbf{E}_x \times \mathbf{B}}{B^2} - \frac{(\nabla p_i)_x \times \mathbf{B}}{enB^2}$
- $V_d / c_s \sim \rho_s / L_\phi \ll 1$



Extra slides : Ion temperature effects ?

- For the magnetic presheath :
  - FLR effects on wall absorption can affect BC - to be studied
- For the SOL equilibrium :
  - Finite T_i introduces Pfirsch-Schluter flows
- For the SOL fluctuations : effect on turbulence ?
  - RBM physics similar with ion temperature
  - ITG physics appears, but not critical for SOL

Extra slides : Electromagnetic effects ?

- GBS has EM effects - ideal ballooning modes present
- GBS could be used to get a "wall BC" for MHD codes
- Magnetic presheath BC are electrostatic - to be extended

[Ricci et al PPCF 2012, Halpern et al PoP 2013]

Extra slides : Summary of the BC

$$v_{||i} = c_s \left(1 + \theta_n - \frac{1}{2}\theta_{T_e} - \frac{2\phi}{T_e}\theta_\phi\right)$$

$$v_{||e} = c_s \left(\exp\left(\Lambda - \eta_m\right) - \frac{2\phi}{T_e}\theta_\phi + 2(\theta_n + \theta_{T_e})\right)$$

$$\frac{\partial\phi}{\partial s} = -c_s \left(1 + \theta_n + \frac{1}{2}\theta_{T_e}\right) \frac{\partial v_{||i}}{\partial s}$$

$$\frac{\partial n}{\partial s} = -\frac{n}{c_s} \left(1 + \theta_n + \frac{1}{2}\theta_{T_e}\right) \frac{\partial v_{||i}}{\partial s}$$

$$\frac{\partial T_e}{\partial s} \simeq 0$$

$$\omega = -\cos^2\alpha\left[(1 + \theta_{T_e})\left(\frac{\partial v_{||i}}{\partial s}\right)^2 + c_s\left(1 + \theta_n + \theta_{T_e}/2\right)\frac{\partial^2 v_{||i}}{\partial s^2}\right]$$

where $\theta_A = \frac{\rho_s}{2\tan\alpha}\frac{\partial_x A}{A}$, and $\eta_m = e(\phi_{mpe} - \phi_{wall})/T_e$. [Loizu et al PoP 2012]