

Contributed paper

Flexures: simply subtle

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(Received 30 June 2010; accepted 27 July 2010)

Flexures are enjoying a new boom in numerous high-precision and extreme-environment applications. This paper presents some general aspects of flexure design, showing simple principles, and also some subtler issues concerning kinematic design, stiffness compensation, large reduction ratios and rectilinear as well as circular movements.

1. Introduction

Although the basic principles of flexible bearings have been known for several decades, the design methods that could be found in the literature have long remained fragmented. Only recently, the interest for flexures and their applications has grown, leading to a more systematic treatise of the respective design methodologies (Smith 2000; Henein 2001). This development has been driven by the increasing need for motion accuracies in the nanometer ranges, in extreme environments like vacuum, cryogenic or high temperatures, radiations or outer space. In the particular field of instrumentation for accelerator facilities, active optics and precision mechanisms of all kinds (Henein, Kjelberg & Zelenika 2002) are more often relying on flexures.

While in conventional mechanical bearings motion is obtained by sliding or rolling between solid bodies, flexible bearings rely on the elastic properties of matter allowing several advantages to be obtained: no friction and associated hysteresis, no wear, no need for lubrication, no risk of jamming, no backlash and possibility of monolithic manufacturing; the main sources of errors are systematic and therefore simple control laws can be used.

This short article illustrates some simple, yet subtle, aspects of the design of flexure-based mechanisms.

2. Flexure design based on a kinematic approach

One of the methods to design flexure-based mechanisms is to consider flexures as idealized joints with finite stiffnesses about their various degrees of freedom (DOFs).

A leaf spring, for example, which is one of the simplest building blocks for flexures, has three natural DOFs, as shown in figure 1(a). Rotations of a few degrees of

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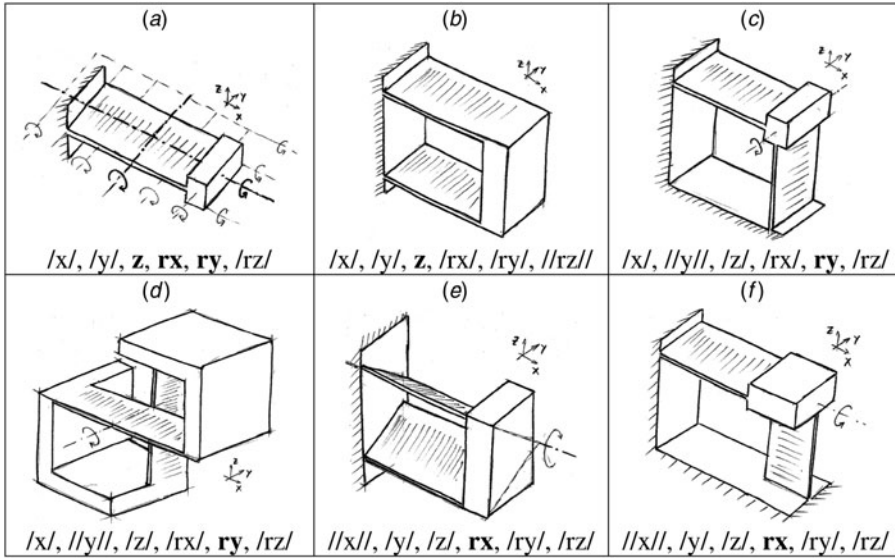


FIGURE 1. Simple flexures and their DOFs.

the mobile block are possible about all the axes that run through the plane of the blade and are parallel to either x or y . The two axes that are drawn with a thick line are those for which the stress in the blade is the lowest for a given rotation angle, i.e. the preferred rotation axes.

The most common one-DOF flexures (figure 1b–f) are produced by connecting a mobile block to a fixed base by two such leaf springs. According to Grüblers' criterion (Grübler 1927), these structures have 0 DOF (a kinematical structure composed of two three-DOFs links and one closed loop has: $2 \cdot 3 - 1 \cdot 6 = 0$ DOF). This demonstrates that those classical flexures have an internal over-constraint, i.e. one of their DOFs is determined twice.

In figure 1, a bold letter indicates a DOF about that axis (e.g. ' z ' means that the translation of the mobile block is possible along the z axis, ' rz ' means that the rotation of the mobile block is possible about the z axis). Two slashes (e.g. ' $/z/$ ') indicate that the corresponding DOF is blocked by one of the blades. Four slashes (e.g. ' $//z//$ ') indicate that the corresponding DOF is blocked by two blades, i.e. this DOF is over-constrained once.

The flexures work correctly only if the dimensional errors associated with those over-constraints are small enough to be absorbed elastically by the flexible elements themselves or by the bodies they are connecting. This aspect has to be taken into account in their design, machining and assembly. Some design principles have been proposed to cope with these peculiarities (Schellenkens *et al.* 1998).

3. Stiffness compensation and bistability

Ideal bearings are movable by infinitesimal forces or torques. This is not the case of simple flexures which present a finite spring constant to resist motion intended away from their neutral position. Nevertheless, if a preloaded spring is used in combination with a flexure in an arrangement where the spring loses elastic energy when the flexure is moved away from its neutral position, then the overall stiffness

of the structure will be reduced. This phenomenon can be explained by the fact that a significant part of the energy required to move the flexure is then provided by the preload springs instead of the externally applied driving force. Special designs allow to practically reduce the natural stiffness of flexures by several orders of magnitude (figure 2), approaching a zero-stiffness ideal behaviour. Other designs allow achieving negative stiffness: in the latter case, the flexures presents a bistable behaviour.

4. Achieving large reduction ratios

Striving for nanometric motion accuracies often leads to the use of mechanical transmissions with very large reduction ratios in order to attenuate the motion of available actuators having micrometric accuracies. It is well known in the state of the art that flexures can be used as reduction mechanisms, but the classical solutions generally have non-linear characteristics (i.e. the reduction factor is not constant over the motion range). In comparison, a non-conventional design such as the nanoconverter (figure 3) (Henein 2006) presents the following key advantages: it exhibits a constant reduction factor; it can achieve very high reduction factors (typically up to 1000); it can be designed to be tunable using a simple tuning screw or shim to select the reduction factor over a wide range (typically 20–1000); its simple planar structure can be manufactured monolithically (no need for assembly) using a wide variety of techniques (e.g. wire-EDM (electrodischarge machining), laser cutting, silicon etching, LIGA (lithography, electroplating, and molding)).

A commercial linear actuator with micrometric motion accuracy drives horizontally the point A (figure 3) of the input stage to A' (rectilinear displacement). This motion is transmitted to the intermediate stage that is guided by a classical parallel spring stage (with blade length L): point B moves to B'. Due to the shortening of the blade projection, the motion of this stage is a well-known parabolic translation (Henein 2001): $y_1 = -3x_1^2/(5L)$, where $x_1 \cong x$. A third blade of length L (called 'converting blade') that has an offset deformation x_0 links the intermediate stage to the output stage. The output stage is guided vertically by a classical parallel spring stage. The motion x_1 causes the converting blade to shorten, following the same

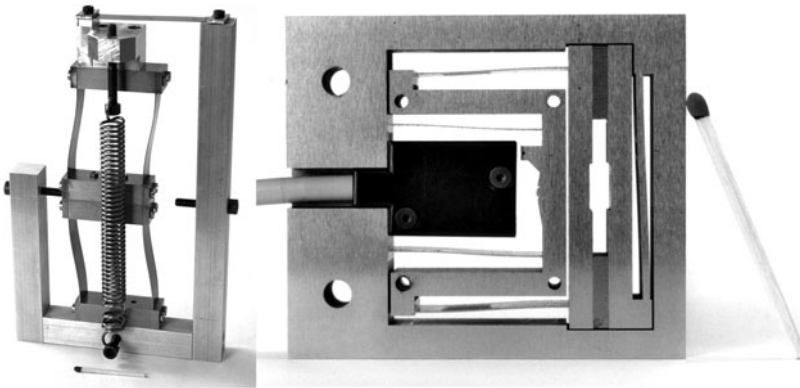


FIGURE 2. Parallel spring stage equipped with a stiffness compensation mechanism. Left: Mock-up model showing the architecture. Right: Same arrangement machined monolithically by wire-EDM. The two dark shims have been inserted to preload the vertical blade that is used as a spring.

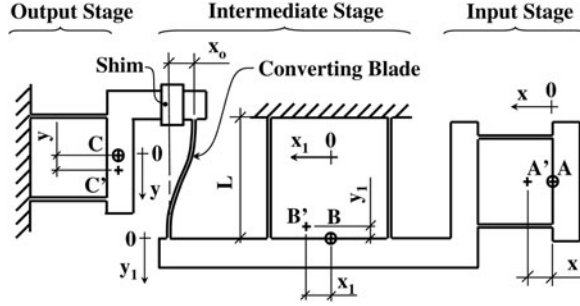


FIGURE 3. Working principle of the nanoconverter.

parabolic law as the two blades of the intermediate stage, but with an offset x_0 . The resulting motion y of the output stage (motion from C to \dot{C}) is equal to the differential shortening of the blades (subtraction of two parabolas with an offset):

$$y = \frac{3(x + x_0)^2}{5L} - \frac{3x^2}{5L} = \frac{6x_0}{5L}x + \frac{3x_0^2}{5L}; \quad i = \frac{x}{y} = \frac{5L}{6x_0}.$$

Therefore, if the origin of the y -axis is adequately chosen, the displacement y of the output stage is simply proportional to the displacement x of the actuator, with a reduction ratio i that is constant over the whole displacement range and is inversely proportional to the offset x_0 . Choosing an offset x_0 that is small compared to the blade length L leads to very large demagnification ratios. This is mechanically very easy to carry out by using a shim as illustrated in figure 2 or by monolithical manufacturing.

5. Rectilinear and circular flexures

Producing purely rectilinear movements can be achieved with the well-known compound parallel spring stage (Henein 2001) (figure 4, right), (some other less common structures exist like the five- or six-folded leaf springs (Schellenkens *et al.* 1998) or the flexure-based version of the ‘Sarrus mechanism’ (Henein 2001).

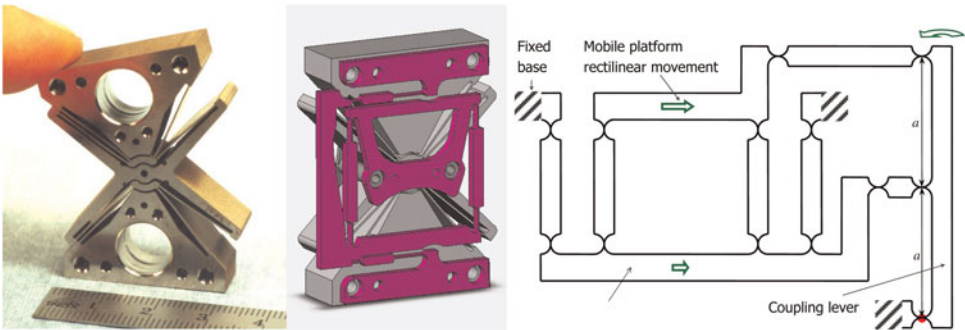


FIGURE 4. Left: Picture of the monolithic ‘butterfly’ flexure pivot. Angular stroke up to $\pm 15^\circ$; parasitic translation (centre shift) below $2 \mu\text{m}$ for $\pm 10^\circ$ strokes; machined monolithically in titanium. Center: Pivot equipped with a coupling plate suppressing the internal DOF of the butterfly. Right: Compound parallel spring stage with its coupling lever.

A similar arrangement can be used to produce almost pure rotations like in the butterfly pivot (figure 4) (Henein *et al.* 2003). But in the latter case, the compensation is not perfect due to a rotation of the parasitic shift vectors. Moreover, all those arrangements introduce additional internal DOFs to the structures, which affect their transverse stiffnesses (quasi-static effect) and introduce undesired low eigenmodes (dynamic effect). The drawbacks can be compensated by using internal coupling chains suppressing those undesired DOFs. The slaving chains can be implemented in translation as well as in rotation. The coupling plate (figure 4, centre) fulfils the same role as the coupling lever of the classical compound rectilinear stage (right).

6. Conclusion

During the last decades, the old and simple idea of using the elastic deformation of solid bodies to produce well-defined movements has been used in many application fields. The simplest original components (e.g. cross spring-pivots and leaf spring stages) are today replaced by sophisticated mechanical structures with complex kinematics. In future, flexure-based mechanisms will play an increasingly important role in the technological system upon which our society relies for living.

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