

# Optical Nyquist-pulse generation with a power difference to the ideal sinc-shape sequence of $< 1\%$

## Erzeugung optischer sinc-förmiger Nyquist Pulssequenzen mit einer Leistungs-Abweichung zur Idealform von $< 1\%$

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### Abstract

Sinc-shaped Nyquist-pulses possess a rectangular spectrum. Thus, a sinc-pulse transmission minimizes the carrier spacing down to the baud rate, and therefore, substantially increases the transmissible data rates. These perspectives have led to a strong research activity in the field of Nyquist pulse transmission. However, all methods of Nyquist pulse generation shown up to now are rather complex, costly and none leads to ideal sinc-shaped Nyquist pulses. Thus, it has not been clear yet if Nyquist-pulse transmission can be incorporated in optical networks in an energy and cost-effective way.

Here we present a method for the generation of almost ideal sinc-shaped Nyquist pulses based on a flat and phase-locked frequency comb. The pulses can be generated with conventional modulators without any sophisticated electronics or other costly equipment. In our proof-of-concept experiment we generate sinc-shaped Nyquist-pulse sequences which show a power difference lower than 1% compared to an ideal sequence. Generated sinc pulses have a full width at half maximum (FWHM) duration of 9.8 ps, an out-of-band suppression of more than 27 dB, a signal-to-noise ratio of more than 40 dB and a jitter of 82 fs, equivalent to 0.82% of the FWHM. The pulse width and repetition rate can be changed simply by tuning the comb parameters.

### Kurzfassung

Sinc-förmige Nyquist-Pulse besitzen ein rechteckiges Spektrum. Folglich minimiert eine Datenübertragung mit sinc-Pulsen den Abstand zwischen den Trägern auf die Baud-Rate und kann daher die übertragbaren Datenraten drastisch erhöhen. Diese Aussichten führten zu einer starken Forschungsaktivität auf dem Gebiet der Nyquist-Puls-Übertragung. Alle bisher gezeigten Methoden der Erzeugung solcher Pulse sind jedoch sehr komplex, kostenaufwändig und keine führt zu ideal sinc-förmigen Nyquist Pulsen. Daher war es bisher nicht klar, ob eine Nyquist-Puls Übertragung tatsächlich energie- und kosteneffizient in optischen Netzen eingesetzt werden kann.

Wir präsentieren hier eine Methode zur Erzeugung annähernd idealer, sinc-förmiger Nyquist Pulse die auf einem flachen, phasengekoppelten Frequenzkamm basiert. Die Pulse können mit konventionellen Modulatoren, ohne den Einsatz anspruchsvoller Elektronik oder anderem kostenaufwändigem Equipment, erzeugt werden. In unserem proof-of-concept Experiment haben wir sinc-förmige Nyquist Pulssequenzen erzeugt, die eine Leistungsabweichung zur Idealform von weniger als 1 % aufweisen. Die erzeugten sinc-Pulse haben eine Halbwertsdauer (full width at half maximum, FWHM) von 9.8 ps, eine Außerbandunterdrückung von mehr als 27 dB, einen Signal-zu-Rauschabstand von mehr als 40 dB und einen Jitter von 82 fs, was 0.82% der Halbwertsdauer entspricht. Die Pulsbreite und Repetitionsrate kann einfach durch eine Abstimmung der Kammparameter verändert werden.

## 1 Introduction

To keep pace with the increasing data rates in telecommunication networks the bandwidth of the communication channels has to be exploited in the best possible way, i.e. the spectral efficiency of the channel has to be maximized. Conventional wavelength division multiplexing (WDM)-Systems work with a maximum bit-rate of 40 Gb/s and have a channel spacing of 50 GHz, resulting in a spectral efficiency of just 0.8 bit/s/Hz. A promising solution is the exploitation of multilevel modulation formats and polarization multiplexing which can increase the spectral efficiency up to several bit/s/Hz [1]. However, each doubling of the spectral efficiency with multilevel formats requires a quadrupling of the number of symbols. This is accompanied by a drastic increase in the requirements to the electronic signal processing and the energy consumption of the network. Furthermore, the higher spectral efficiency comes at the cost of a reduced tolerance to noise influences. To meet the data rate requirements over the next decade, a data aggregation of 1 Tb/s per channel with high spectral efficiency has been envisaged [2]. Such high data rates per channel exceed the limits of digital signal processing even with parallelization and the resulting baud rate is beyond the current limits of electronic circuits [3].

To solve this problem multiple lower rate channels with high spectral efficiency can be combined to a Tb/s “superchannel”. It is assumed that such superchannels can be routed through optical add-drop multiplexers (OADMs) and wavelength selective switches (WSSs) as a single entity [4]. However, the required dense packaging of the carriers is one of the main challenges in the generation of superchannels. In principle two methods can be used to minimize the guard bands between the carriers, i.e. orthogonal frequency division multiplexing (OFDM) and Nyquist wavelength division multiplexing (N-WDM). Under idealized assumptions both have the same sensitivity and spectral efficiency. However, OFDM requires a much larger receiver bandwidth and proportionally faster speed of the analog-to-digital converters [5]. Furthermore, compared to OFDM Nyquist WDM tributaries can be transmitted and processed asynchronously, the pulse shaping leads to lower peak-to-average power ratios [6] and is less sensitive to fiber nonlinearities [7].

These advantages led to a strong research activity in the field of Nyquist pulse transmission [4-13]. Recently, for instance a 32.5 Tbit/s transmission with an electronic generation of the Nyquist-pulses with an arbitrary waveform generator has been shown [9]. In optical demonstrations Gaussian pulses from a mode-locked laser have been shaped into raised-cosine Nyquist-pulses with a liquid crystal spatial modulator [10, 11], or Nyquist-pulses have been generated using a fibre optical parametric amplification pumped by parabolic pulses and a phase modulator to compensate the pump-induced chirp [13].

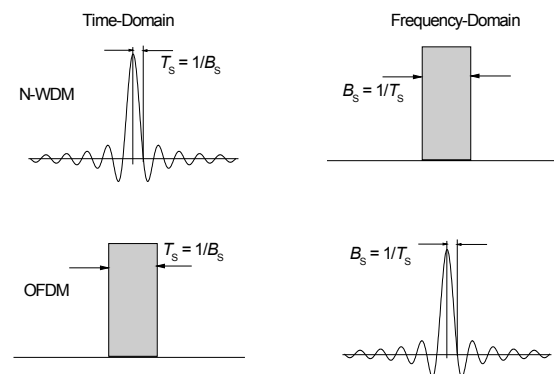
However, besides the complexity and power consumption, none of these demonstrations led to ideally sinc-

shaped Nyquist pulses. Since these pulses have no rectangular spectrum, the multiplexing requires a guard bandwidth between the channels. Here we present the generation of almost ideally sinc-shaped Nyquist pulse sequences with an almost ideally rectangular spectrum. Thus, these channels can be multiplexed with a spacing corresponding to the baud rate. For the proof-of-concept experiment state-of-the-art electro optic modulators are used so that the method could be easily integrated in existing fiber optic networks.

## 2 Theory

If several subchannels are multiplexed to a superchannel, the maximum bandwidth exploitation is given if the spectra of the subchannels are directly adjacent to each other and the bandwidth of the single spectrum is the inverse of the symbol duration  $T_s$ . The resulting rectangular spectrum leads to a sinc-shaped pulse in the time domain (N-WDM). Another possibility is a rectangular pulse which has a sinc-shaped spectrum (OFDM).

Both methods are connected via the duality principle that a rectangular function in the time or frequency domain is represented by a sinc-function in the frequency or time domain, respectively (see Fig. 1). Thus, they lead to the same transmissible baud rate, i.e. the maximum baud rate which can be transmitted over the bandwidth. However, since N-WDM has many advantages compared to OFDM, as already discussed in the introduction, here we want to concentrate on N-WDM.



**Figure 1** Time (left) and frequency (right) representations for a single sinc-shaped Nyquist pulse (top) and a spectrum for a single OFDM channel (bottom).

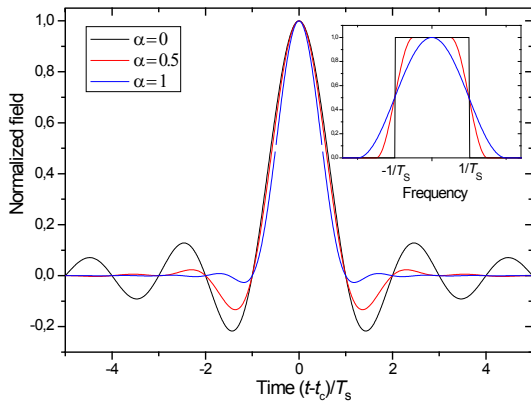
The shape of a Nyquist pulse is defined by the first Nyquist criterion, which is fulfilled by signals for which the periodic summation of the power density spectrum is constant [14].

$$\sum_{n=-\infty}^{\infty} |S(f - n/T_s)|^2 = T_s \quad (1)$$

Figure 2 shows solutions for Nyquist pulse shapes in the time and frequency domain which differ by the so called

roll-off factor  $\alpha$ . The required bandwidth (between the two zero crossings) for these pulses is [6]:

$$B_S = \frac{1}{T_S}(1 + \alpha) \quad (2)$$



**Figure 2** Nyquist pulses with different roll-off factors  $\alpha$  in the time domain. The inset shows the pulse spectra.

The generation of Nyquist pulses in the optical domain is carried out by optical filtering [4, 5, 7, 10, 11]. Since rectangular optical filters have not been shown up to now, the generated pulses are non-sinc-shaped ( $\alpha > 0$ ). However, even for non-sinc-shaped Nyquist pulses the filter must enable an accurate shaping.

According to Eq. (2), the minimum bandwidth can only be achieved for pulses with  $\alpha = 0$ , which corresponds to ideally sinc-shaped pulses with an ideal rectangular spectrum (black curve in Fig.2). For such a transmission each symbol is encoded by an ideal sinc-pulse, i.e. the pulse and therefore the duration of the transmitted symbol is unlimited. However, due to causality unlimited pulses cannot be generated. This problem can be solved by the generation of sinc-shaped pulse sequences instead of sinc-shaped pulses.

A normalized rectangular spectrum covering a bandwidth  $B_S = N\Delta f$  can be written as;

$$\text{rect}(f) = \frac{1}{N\Delta f} \text{rect}\left(\frac{f}{N\Delta f}\right) \quad (3)$$

In the dual time domain, this spectrum corresponds to a sinc function (see Fig. 1)

$$\text{sinc}(\pi N\Delta f t) = \frac{\sin(\pi N\Delta f t)}{\pi N\Delta f t} \quad (4)$$

Since all frequencies are present under the rectangular envelope and as a result of the sharp edges of the ideal function, the time domain representation is just one single, unlimited sinc-pulse. If the rectangular spectrum is multiplied by  $N$  equally spaced, narrow linewidth frequency components (a Dirac frequency comb), the result is a sampling of the rectangular function, i.e. all the comb fre-

quencies are weighted with the same amplitude and phase. The multiplication in the frequency domain corresponds to a convolution in the time domain:

$$\text{sinc}(\pi N\Delta f t) \otimes \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{n}{\Delta f}\right) = \sum_{n=-\infty}^{\infty} \text{sinc}\left(\pi N\Delta f\left(t - \frac{n}{\Delta f}\right)\right) \quad (5)$$

where  $\otimes$  represents the convolution and  $\delta$  is the Dirac delta function. Thus, the sampling in the frequency domain leads to equally spaced copies of the sinc-pulse in the time domain with a period of  $1/\Delta f$  [15-17]. However, according to the sampling theorem in frequency, only the spectrum of a time-limited signal can be sampled. In order to avoid inter-symbol interference (ISI), the maximum frequency spacing in the comb has to be  $1/\tau$ , with  $\tau$  as the duration of a single copy. Since the sinc-pulse is unlimited, the maximum frequency spacing would be necessarily zero. However, Nyquist pulses have the property to enable zero ISI. Thus, under certain conditions the periodic summation of unlimited sinc-pulses leads to a periodic sinc-pulse sequence.

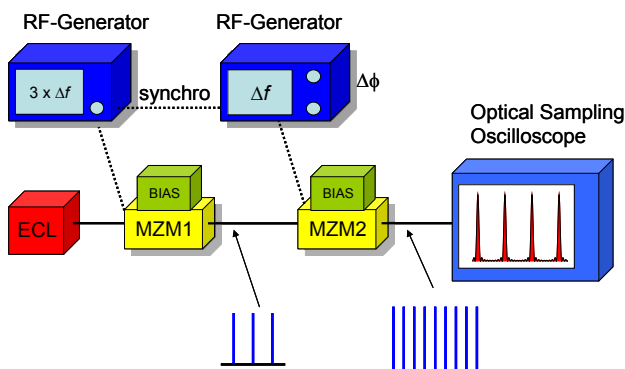
$$\sum_{n=-\infty}^{\infty} \text{sinc}\left(\pi N\Delta f\left(t - \frac{n}{\Delta f}\right)\right) = \frac{\sin(\pi N\Delta f t)}{N \sin(\pi \Delta f t)} \quad (6)$$

The right side of Eq. (6) is the Fourier transform of a frequency comb with  $N$  equally spaced frequency lines separated by  $\Delta f$ . Thus instead of scanning a rectangular spectrum, an ideal sinc pulse sequence can be obtained by a flat, phase-locked frequency comb with narrow lines. The symbol duration  $T_S$  of the sinc pulse is defined by the inverse bandwidth of the comb  $T_S = 1/(N\Delta f)$  and the repetition rate of the pulses is determined by the frequency spacing between the comb lines  $T_R = N \times T_S = 1/\Delta f$ .

### 3 Experiment

There are several possibilities to produce a flat frequency comb [18 - 21]. However, for the generation of Nyquist pulses the different frequencies in the comb must have the same or a linear varying phase. Additionally, the out-of-band suppression must be as high as possible. Therefore, methods which use a phase modulator [18, 19] or higher order sidebands of amplitude modulators [20, 21] cannot be used for this purpose. An integrated solution for a Nyquist-pulse source could be a master-slave configuration of laser diodes for instance. For the proof-of-concept experiment we choose the first order sidebands of two coupled intensity modulators. The basic experimental setup can be seen in Fig. 3. An external cavity laser (ECL) generates continuous wave (cw) light at a wavelength of 1550 nm, the first intensity modulator MZM1 modulates

the cw with  $3 \times \Delta f$ , in our case 30 GHz. The result are three laser lines, i.e. the carrier and the first two sidebands. By adjusting the RF-power and the bias of the MZM, the three laser lines have the same intensity and almost no additional sidebands are present. The second intensity modulator MZM2 modulates the three lines of the first MZM with  $\Delta f = 10$  GHz, here again the RF-power and the bias are adjusted in a way that all lines are flat and no additional sidebands are present. In the output nine laser lines can be seen. Since the two RF-Generators are locked to each other (synchro), the phases of the frequencies in the comb are locked as well. To tune the phases between the two generators the phase at the second generator can be changed ( $\Delta\Phi$ ). The result is a sequence of sinc pulses, measured with an optical sampling oscilloscope. Since we choose a  $\Delta f$  of 10 GHz, the bandwidth of our Nyquist pulses is 90 GHz and the pulses have a duration between the two zero crossings of 22.22 ps, corresponding to  $T_s = 11.11$  ps. The resulting 3 dB duration is around 10 ps.

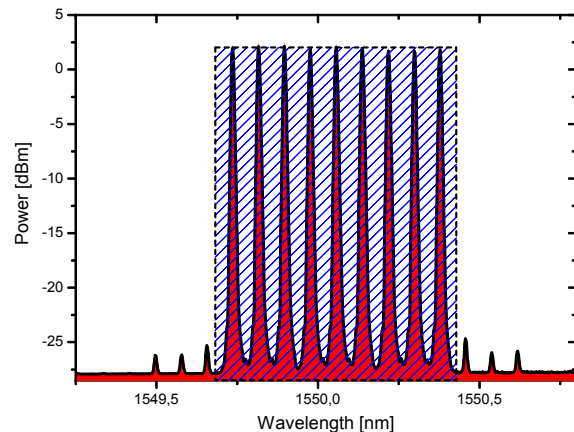


**Figure 3** Experimental setup. ECL external cavity laser, MZM Mach-Zehnder modulator, RF radio frequency.

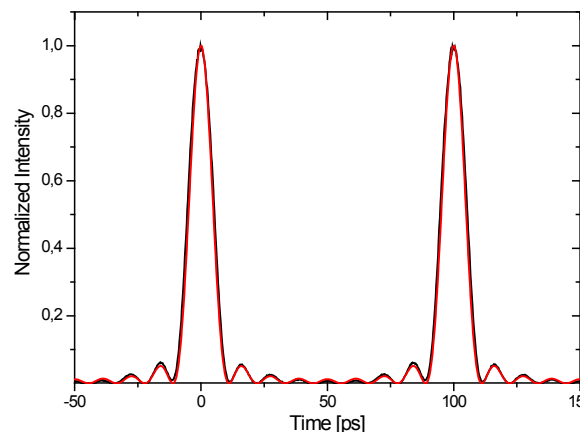
The generated 9 laser lines can be seen in Fig. 4. The apparent line broadening of the frequency lines is a result of the limited resolution of the spectrum analyzer. The real linewidth of the ECL is in the kHz-range and therefore more than 7 orders of magnitude lower than the pulse bandwidth. The measured power variation between the lines is less than 0.2 dB and the suppression of the out-of-band components higher than 27 dB. The blue box in Fig. 2 represents the spectrum of a single sinc-shaped Nyquist pulse with the same duration as that of the generated ( $T_s = 11.11$  ps).

Since the frequency components in Fig. 4 are locked with a linear phase difference, the time domain representation of the comb is a sinc-pulse sequence. Due to the high bandwidth, we measured the sequence with a 500 GHz optical sampling oscilloscope. The result (black curve) compared to an ideal sinc-pulse sequence (red) can be seen in Fig. 5. The calculation of the ideal sequence was carried out squaring Eq. (6), representing this way the power in time domain measured by the photo-detector. The generated sinc-pulse sequence is almost the same as the ideal; the power deviations are smaller than 1%. We

have measured a jitter of 82 fs, corresponding to 0.82% the FWHM and an SNR of greater than 40 dB.



**Figure 4** Frequency domain representation of the generated sinc-pulse sequence. The blue shaded box represents the spectrum of one single sinc-pulse with the same duration.



**Figure 5** Generated sinc-pulse sequence. The red trace shows the calculated ideal sequence whereas the black curve shows the measured sinc pulses.

## 4 Discussion and Conclusion

Due to the close to ideal rectangular spectrum offered by the generated sinc-pulse sequence, in the channel bandwidth of 90 GHz the maximum possible symbol rate of 90 Gbd/s can be transmitted by a multiplexing of the sequences in the time domain. However, for the modulation of each channel a modulator with a bandwidth of just 10 GHz is sufficient. Consequently, the requirements to the electronics are greatly reduced by this method. A superchannel with a data rate of 1 Tbps can be formed in the 90 GHz bandwidth if each TDM channel is modulated with a polarization multiplexed (PM) 64-QAM. For a superchannel with PM-QPSK modulation, three of the channels shown in Fig.4 can be multiplexed in the frequency domain without any guard band. Another possibility is a reduction of the pulse width by an increase of the frequency spacing, or an increasing of the number of frequency lines in the comb.

The pulse width and repetition rate can be simply changed by the comb parameters. Thus, the method offers a great flexibility to adjust the channels in the time and frequency domain to actual requirements. Additionally, it can be implemented in existing networks with cost effective, standard components of optical telecommunications.

In conclusion we have shown that by generating a flat phase locked frequency comb with a high suppression of out-of-band components, a close to perfect sinc-pulse sequence can be obtained. We address the slight deviations from the ideal trace to residual imperfections of our experimental setup. Here we presented proof-of-concept experiments, there are many possibilities to enhance or improve the setup. The frequency comb can be generated from one single modulator for instance if it is driven by the corresponding RF-signal.

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