Capillary effect on watertable fluctuations in unconfined aquifers

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Water Resources Research, accepted for publication on 3 April 2013

Abstract

Parlange and Brutsaert [1987] derived a modified Boussinesq equation to account for the capillary effect on watertable dynamics in unconfined aquifers. *Barry et al*. [1996] solved this equation subject to a periodic boundary condition. Their solution shows significant influence of capillarity on watertable fluctuations, which evolve to finite-amplitude standing waves at the high frequency limit. Here, we propose a new governing equation for the watertable, which considers both horizontal and vertical flows in an unsaturated zone of finite thickness. An approximate analytical solution for periodic watertable fluctuations based on the new equation was derived. In agreement with previous results, the analytical solution shows that the unsaturated zone's storage capacity permits watertable fluctuations to propagate more readily than predicted by the Boussinesq equation. Furthermore, the new solution reveals a capping effect of the unsaturated zone on both the amplitude and phase of the watertable fluctuations as well as the watertable overheight. Due to the finite thickness of the unsaturated zone, the capillary effect on watertable fluctuations is modified mainly with reduced amplitude damping and phase shift.

Keywords: tide; wave; coastal aquifer; Boussinesq equation; groundwater wave

1. Introduction

Oceanic oscillations produce watertable fluctuations in coastal unconfined aquifers. As they propagate inland, the watertable fluctuations are attenuated with increasing time lags. These fluctuations, representing basic characteristics of coastal groundwater, provide important information for understanding the properties and behavior of coastal aquifers, and have been subjected to numerous investigations [e.g., *Parlange et al*., 1984; *Nielsen et al.*, 1990; *Jiao and Tang*, 1999; *Li et al.*, 2000a; *Li and Jiao*, 2003; *Jeng et al*., 2005]. Although most previous research has been focused on tide-induced low-frequency watertable dynamics [e.g., *Nielsen*, 1990; *Li et al*., 2000b,c; *Jeng et al.*, 2002], high-frequency watertable fluctuations due to waves have also been studied [*Waddell*, 1976; *Li et al*., 1997].

Traditionally, models of watertable fluctuations are based on the Boussinesq equation, which predicts increasing rates of amplitude damping with the frequency of the oceanic oscillations [e.g., *Parlange et al*., 1984; *Nielsen*, 1990]. According to these models, high frequency waves would not induce watertable fluctuations in coastal unconfined aquifers to any considerable distance inland, a result inconsistent with field observations. *Li et al*. [1997] found that consideration of capillarity explains the transmission of high frequency watertable fluctuations in coastal aquifers.

Parlange and Brutsaert [1987] examined the capillary effect on watertable dynamics. As the watertable fluctuates, the pressure distribution above the watertable varies, resulting in local water exchange across the watertable. *Parlange and Brutsaert* [1987] modified the Boussinesq equation with an additional term to account for this mass transfer process. *Barry et al*. [1996] combined the approaches of *Parlange et al.* [1984] and *Parlange and Brutsaert* [1987]. They obtained and applied a depth-integrated model with capillarity incorporated to study the propagation of small-amplitude oscillations in an unconfined aquifer and derived an approximate analytical solution. Their results showed that the damping rate of the watertable fluctuations reaches an asymptotic finite value as the forcing frequency on the boundary increases. In other words, damping effects on high frequency watertable fluctuations are bounded. Under the influence of capillarity, high-frequency waves can be transmitted into the aquifer over a considerable distance, as observed in the field. Moreover, the analytical solution of *Barry et al*. [1996] predicts that at the high frequency limit, watertable fluctuations become standing waves, also consistent with field observations [*Li et al*., 1997]. This solution was further extended to a higher order by *Jeng et al*. [2005]. It should be noted that the capillary effect on watertable dynamics has implications for a range of processes and phenomena in unconfined aquifers, for example, pumping tests [e.g., *Moench*, 2008]

Despite the progress in the theoretical development, laboratory experiments have shown that the modified Boussinesq equation with the capillarity correction and other approximations for vertical flow effects still cannot describe fully the watertable behavior under the influence of boundary oscillations [e.g., *Cartwright et al.,* 2003]. Our goal here is to extend the previous work to improve the watertable dynamics model. The modified Boussinesq equation of *Parlange and Brutsaert* [1987] considered local water exchange across the watertable assuming only vertical flow in the unsaturated zone. This same assumption was used by *Barry et al.* [1996]. In this work, both horizontal and vertical flows are incorporated. The new approach also takes into account the (finite) thickness of the unsaturated zone.

2. Theory

As shown in Fig. 1, we consider the watertable fluctuations in a rectangular unconfined aquifer subjected to the influence of head oscillation at a side boundary (x = 0). The coordinate system and various physical quantities (parameters) are defined in the figure. The flow in the saturated and unsaturated zones underlying the watertable behavior is described by Richards' equation [*Richards*, 1931],

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[K(\psi)\frac{\partial \Phi}{\partial x}\right] + \frac{\partial}{\partial z}\left[K(\psi)\frac{\partial \Phi}{\partial z}\right], \tag{1}$$

where θ [L] is the soil water content, $\Phi = \psi + z$ [L] is the hydraulic head, ψ is the pressure head, z [L] is the elevation and $K(\psi)$ [LT^{-1}] is the hydraulic conductivity. The model of *Gardner* [1958] is used to describe θ and K as functions of ψ, i.e.,

$$\theta = (\theta_s - \theta_r)\exp(\alpha\psi) + \theta_r \quad \text{for} \quad \psi < 0, \tag{2a}$$

$$\theta = \theta_s \quad \text{for} \quad \psi \ge 0, \tag{2b}$$

and

$$K(\psi) = K_s \exp(\alpha\psi) \quad \text{for} \quad \psi < 0, \tag{3a}$$

$$K(\psi) = K_s \quad \text{for} \quad \psi \ge 0, \tag{3b}$$

Where K_s [LT^{-1}] is the saturated hydraulic conductivity (assumed to be uniform and isotropic); α [L^{-1}] is related to the capillary rise length scale inversely; θ_s and θ_r [-] are the saturated and residual water content, respectively; and $n_e = \theta_s - \theta_r$ is the effective porosity [-].

2.1 Approximation under the hydrostatic pressure assumption

Under the assumption of hydrostatic pressure, the hydraulic head (Φ) is constant in the vertical direction and

$$\Phi = h \, , \tag{4}$$

where h is the watertable elevation [L]. The pressure head in the unsaturated zone is given by

$$\psi = h - z \, . \tag{5}$$

Integrating equation (1) with respect to z from the impermeable base ($z=0$) to the surface ($z=Z_0$) gives,

$$\int_0^{Z_0} \frac{\partial \theta}{\partial t} \mathrm{d}\, z = \int_0^{Z_0} \frac{\partial}{\partial x}\left[K(\psi) \frac{\partial \Phi}{\partial x} \right] \mathrm{d}\, z \, , \tag{6}$$

where the no (vertical) flow boundary condition has been applied at both the base and surface. The left hand side of equation (6) can be evaluated as follows,

$$\begin{aligned} \int_0^{Z_0} \frac{\partial \theta}{\partial t} \mathrm{d}\, z &= -n_e \exp\left[\alpha\left(h - Z_0\right)\right] \frac{\partial\left(h - Z_0\right)}{\partial t} + n_e \frac{\partial h}{\partial t} - n_e \exp\left[\alpha\left(h - Z_0\right)\right] \frac{\partial Z_0}{\partial t} \\ &= n_e \left\{1 - \exp\left[\alpha\left(h - Z_0\right)\right]\right\} \frac{\partial h}{\partial t} . \end{aligned} \tag{7}$$

Upon evaluation, the right hand side of equation (6) becomes,

$$\int_0^{Z_0} \frac{\partial}{\partial x}\left[K(\psi) \frac{\partial \Phi}{\partial x} \right] \mathrm{d}\, z = K_s \frac{\partial}{\partial x}\left\{ h \frac{\partial h}{\partial x} + \frac{1}{\alpha}\frac{\partial h}{\partial x} - \frac{1}{\alpha}\exp\left[\alpha\left(h - Z_0\right)\right] \frac{\partial h}{\partial x} \right\} . \tag{8}$$

Substituting equations (7) and (8) into equation (6) yields a new governing equation for the watertable dynamics,

$$n_e \left\{1 - \exp\left[\alpha\left(h - Z_0\right)\right]\right\} \frac{\partial h}{\partial t} = K_s \frac{\partial}{\partial x}\left\{ h \frac{\partial h}{\partial x} + \frac{1}{\alpha}\frac{\partial h}{\partial x} - \frac{1}{\alpha}\exp\left[\alpha\left(h - Z_0\right)\right] \frac{\partial h}{\partial x} \right\} . \tag{9}$$

Further details of the derivation can be found in the supplementary material. It can be shown that equation (9) reduces to the standard Boussinesq model [e.g., *Bear*, 1972] as $\alpha \to \infty$ (the case where the unsaturated zone is neglected).

2.2 Non-hydrostatic pressure correction

A non-hydrostatic pressure correction can be made to equation (5), i.e.,

$$\psi = h - z + P, \tag{10}$$

where P is the dynamic pressure head [L] and depends on the vertical (Darcy) flow velocity (w) in the unsaturated zone, i.e.,

$$\frac{\partial P}{\partial z} = -\frac{w}{K}. \tag{11}$$

Mass conservation requires (assuming no recharge)

$$\frac{\partial \theta}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{12}$$

where u is the horizontal (Darcy) flow velocity [L/T].

Integrating equation (12) in the vertical direction from a location (z) within the unsaturated zone to the surface (Z_0) gives

$$\int_z^{Z_0} \frac{\partial \theta}{\partial t} \mathrm{d}z + \int_z^{Z_0} \frac{\partial u}{\partial x} \mathrm{d}z + \int_z^{Z_0} \frac{\partial w}{\partial z} \mathrm{d}z = 0. \tag{13}$$

With no (vertical) flow at Z_0, equation (13) leads to

$$w = \int_z^{Z_0} \frac{\partial \theta}{\partial t} \mathrm{d}z + \int_z^{Z_0} \frac{\partial u}{\partial x} \mathrm{d}z, \tag{14a}$$

with

$$\int_z^{Z_0} \frac{\partial \theta}{\partial t} \mathrm{d}z = n_e \frac{\partial h}{\partial t} \left\{ \exp\left[\alpha(h - z)\right] - \exp\left[\alpha(h - Z_0)\right] \right\}, \tag{14b}$$

$$\int_z^{Z_0} \frac{\partial u}{\partial x} \mathrm{d}z = K_s \left[\left(\frac{\partial h}{\partial x} \right)^2 + \frac{1}{\alpha} \frac{\partial^2 h}{\partial x^2} \right] \left\{ \exp\left[\alpha(h - Z_0)\right] - \exp\left[\alpha(h - z)\right] \right\}. \tag{14c}$$

Thus,

$$\frac{\partial P}{\partial z}=-\frac{w}{K}=\left\{\exp\left[\alpha\left(z-Z_0\right)\right]-1\right\}\left[\frac{n_e}{K_s}\frac{\partial h}{\partial t}-\left(\frac{\partial h}{\partial x}\right)^2-\frac{1}{\alpha}\frac{\partial^2 h}{\partial x^2}\right], \tag{15}$$

where it has been assumed that $K\approx K_s\exp\left[\alpha\left(h-z\right)\right]$ since the magnitude of P is generally small compared with that of (h - z) and hence $\exp\left[\alpha\left(h-z+P\right)\right]\approx\exp\left[\alpha\left(h-z\right)\right]$ (i.e., negligible effect of P on K); and

$$P=-\int_z^{Z_0}\frac{\partial P}{\partial z}\mathrm{dz}=\left[\frac{1}{\alpha}\frac{\partial^2 h}{\partial x^2}-\frac{n_e}{K_s}\frac{\partial h}{\partial t}+\left(\frac{\partial h}{\partial x}\right)^2\right]\left\{\frac{1}{\alpha}-\frac{1}{\alpha}\exp\left[\alpha\left(z-Z_0\right)\right]-Z_0+z\right\}. \tag{16}$$

P also varies with x, which leads to an additional term in equation (8):

$$\begin{aligned}\int_h^{Z_0}K_s\exp\left[\alpha\left(h-z\right)\right]\frac{\partial P}{\partial x}\mathrm{dz}\approx\frac{n_e}{\alpha^2}\{\ & 2\exp\left[\alpha\left(h-Z_0\right)\right]-2\\ &+\alpha\left(Z_0-h\right)\exp\left[\alpha\left(h-Z_0\right)\right]\\ &+\alpha\left(Z_0-h\right)\ \}\frac{\partial^2 h}{\partial x\partial t}.\end{aligned} \tag{17}$$

With this term added to the right hand side of equation (9), the new governing equation for the watertable with a non-hydrostatic pressure correction is derived,

$$\mathrm{F}n_e\frac{\partial h}{\partial t}=K_s\frac{\partial}{\partial x}\left(\mathrm{M}h\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial x}\left(\mathrm{N}\frac{\partial^2 h}{\partial x\partial t}\right), \tag{18a}$$

with

$$\mathrm{F}=1-\exp\left[\alpha\left(h-Z_0\right)\right], \tag{18b}$$

$$\mathrm{M}=1+\frac{1}{\alpha h}\left\{1-\exp\left[\alpha\left(h-Z_0\right)\right]\right\}, \tag{18c}$$

$$\mathrm{N}=\frac{n_e}{\alpha^2}\left\{2\exp\left[\alpha\left(h-Z_0\right)\right]-2+\alpha\left(Z_0-h\right)\exp\left[\alpha\left(h-Z_0\right)\right]+\alpha\left(Z_0-h\right)\right\}. \tag{18d}$$

F is positive and smaller than unity, reflecting a reduction in the effective void space for local water storage and leading to enhancement of watertable fluctuations. M is larger than unity due to the horizontal flux in the unsaturated zone. N is the non-hydrostatic pressure correction term, which accounts for the effect of vertical flow in the unsaturated zone. To make a non-hydrostatic pressure correction for the saturated zone, the work of *Parlange et al.* [1984] and *Liu and Wen* [1997] can be adopted to include an additional term in N, i.e.,

$$\mathrm{N}=\frac{n_e}{\alpha^2}\left\{2\exp\left[\alpha\left(h-Z_0\right)\right]-2+\alpha\left(Z_0-h\right)\exp\left[\alpha\left(h-Z_0\right)\right]+\alpha\left(Z_0-h\right)\right\}+\frac{D^2 n_e}{3}, \quad (19)$$

where D is the average aquifer thickness.

3. Analytical solution

Using perturbation, we solved equation (18) subject to a periodic boundary condition for a semi-infinite aquifer domain (details in the supplementary material). The first-order approximation gives the following solution for the primary frequency (ω),

$$h=D+A\exp\left(-x\mathrm{k}_{\mathrm{US}}\mathrm{F}_1\right)\cos\left(\omega t-x\mathrm{k}_{\mathrm{US}}\mathrm{F}_2\right), \quad (20a)$$

with

$$\mathrm{k}_{\mathrm{US}}=\sqrt{\frac{\mathrm{R}_1\omega}{2\mathrm{R}_2}}, \quad (20b)$$

$$\mathrm{N}_{\mathrm{US}}=\frac{\mathrm{R}_2}{\mathrm{R}_3\omega}, \quad (20c)$$

$$\mathrm{F}_1=\sqrt{\frac{\mathrm{N}_{\mathrm{US}}}{\sqrt{1+\mathrm{N}_{\mathrm{US}}{}^2}}+\frac{\mathrm{N}_{\mathrm{US}}}{1+\mathrm{N}_{\mathrm{US}}{}^2}}, \quad (20d)$$

$$\mathrm{F}_2=\sqrt{\frac{\mathrm{N}_{\mathrm{US}}}{\sqrt{1+\mathrm{N}_{\mathrm{US}}{}^2}}-\frac{\mathrm{N}_{\mathrm{US}}}{1+\mathrm{N}_{\mathrm{US}}{}^2}}, \quad (20e)$$

$$\mathrm{R}_1=n_e\left\{1-\exp\left[\alpha\left(D-Z_0\right)\right]\right\}, \quad (20f)$$

$$\mathrm{R}_2=K_sD+K_s\frac{1}{\alpha}\left\{1-\exp\left[\alpha\left(D-Z_0\right)\right]\right\}, \quad (20g)$$

$$\mathrm{R}_3=\frac{n_e}{\alpha^2}\left\{2\exp\left[\alpha\left(D-Z_0\right)\right]-2+\alpha\left(Z_0-D\right)\exp\left[\alpha\left(D-Z_0\right)\right]+\alpha\left(Z_0-D\right)+\frac{D^2\alpha^2}{3}\right\}, \quad (20h)$$

where A is the amplitude of the hydraulic head oscillations at the boundary. When ω approaches infinity, $\mathrm{F}_1=1$ and $\mathrm{F}_2=0$, and watertable fluctuations become standing waves. The second-order solution was also derived (supplementary material).

To validate the approximate analytical solution, equation (18) was solved numerically. A harmonic analysis on the predicted watertable fluctuations given by the numerical solution was conducted to determine the damping rate and wave number (phase shift) associated with the oscillations at the primary frequency (ω). Comparison of these results with the analytical predictions shows reasonably good agreement between the two (Fig. 2). The results also display considerable variations in the damping rate and wave number with the thickness of the unsaturated zone (with Z_0 varying for fixed D). The variations are not monotonic – both parameters increase with Z_0 to a maximum followed by a gradual decline as described by equation (20).

Dispersion relation

The dispersion relation given by the new analytical solution was compared with those of previous solutions, based on the experimental case by *Cartwright et al*. [2003]. This relation characterizes the watertable fluctuations and can be expressed by a complex number (k) combining the damping rate (k_r) and wave number (k_i), i.e., $k = k_r + ik_i$.

The solution for the watertable fluctuations based on the traditional Boussinesq equation gives the following dispersion relation [*Parlange et al*., 1984],

$$k = (1+i)\sqrt{\frac{n_e \omega}{2K_s D}}, \tag{21}$$

where $k_r = k_i$. *Cartwright et al*. [2003] found $k_r / k_i \approx 2.3$ based on the results from their sand flume experiment. The condition $k_r / k_i > 1$ can be explained by the capillary effect and/or the vertical flow effect. For the former effect, *Barry et al.*'s [1996] solution gives,

$$k = \sqrt{\frac{n_e\omega}{2DK_s}\left[\frac{N_{CAR}}{\sqrt{1+N_{CAR}{}^2}}+\frac{N_{CAR}}{1+N_{CAR}{}^2}\right]} + \sqrt{\frac{n_e\omega}{2DK_s}\left[\frac{N_{CAR}}{\sqrt{1+N_{CAR}{}^2}}-\frac{N_{CAR}}{1+N_{CAR}{}^2}\right]}i\,, \tag{22a}$$

with

$$N_{CAR} = \frac{K_s}{B\omega}\,, \tag{22b}$$

where B is an equivalent capillary fringe thickness [*Parlange and Brutsaert*, 1987]. For the latter effect, *Nielsen et al.* [1997] suggested the following dispersion relation for medium-depth aquifers,

$$kD\tan kD = i\frac{n_e\omega D}{K_s}\,. \tag{23}$$

A complex porosity (n_c) instead of n_e can be used in equation (23) to further incorporate the capillary effect [*Cartwright et al.*, 2003],

$$n_{\mathrm{c}} = \frac{n_e}{1+2.5\left(i\dfrac{n_e\omega H_\psi}{K_s}\right)^{2/3}}\,. \tag{24}$$

Cartwright et al. [2003] found that neither equation (22) nor equation (23) described well the dispersion relation observed in data from their laboratory experiment (Fig. 3). Direct application of the present solution with measured parameter values also failed to predict the observation; however, the data fell on the dispersion relation curve. With the saturated hydraulic conductivity adjusted from the measured value (0.00047 m/s) to 0.0008 m/s, the present solution predicted the experimental results. Considering the possible uncertainty associated with the K_s measurement, this adjustment by a factor less than two was relatively small, indicating the applicability of the dispersion relation given by the present solution.

Mean watertable elevation (overheight)

Oceanic oscillations induce not only watertable fluctuations but also an overheight – i.e., the mean watertable elevation far inland is higher than the mean water (head) level at the boundary [*Knight*, 1981]. The overheight increment (Δ), due to non-linearity, is given by [e.g., *Parlange et al.*, 1984],

$$\Delta = D\left[\sqrt{1+\frac{1}{2}\left(\frac{A}{D}\right)^2}\right] \approx D\left[1+\frac{1}{4}\left(\frac{A}{D}\right)^2\right]. \tag{25}$$

Barry et al. [1996] found that the capillarity affects the time-averaged mean-square height of the watertable. However, as the landward distance approaches infinity, their result reduces to equation (25). The present solution to 2nd order as shown in the supplementary material also predicts the mean watertable with an overheight,

$$H_{\mathrm{OVER}} = D\left[1+\frac{1}{4}\mathrm{N}_{\mathrm{OVER}}\left(\frac{A}{D}\right)^2\right], \tag{26a}$$

with

$$\mathrm{N}_{\mathrm{OVER}} = \frac{\mathrm{R}_4\,\mathrm{F}_1 - \mathrm{R}_5\,\mathrm{F}_2\,\omega}{\mathrm{R}_2\,\mathrm{F}_1}, \tag{26b}$$

$$\mathrm{R}_4 = K_s D\left\{1-\exp\left[\alpha\left(D-Z_0\right)\right]\right\}, \tag{26c}$$

$$\mathrm{R}_5 = \frac{n_e}{\alpha^2}\left\{\exp\left[\alpha\left(D-Z_0\right)\right]\alpha D + \alpha^2 D\left(Z_0 - D\right)\exp\left[\alpha\left(D-Z_0\right)\right] - \alpha D\right\}, \tag{26d}$$

where the dimensionless number $\mathrm{N}_{\mathrm{OVER}}$ is positive and mostly less than unity for the physical conditions considered, indicating the existence of a watertable overheight but less than that predicted by equation (25). As ω increases, F_2 decreases and approaches zero and the overheight approaches $D\left[1+\frac{1}{4}\left(A/D\right)^2 \mathrm{R}_4/\mathrm{R}_2\right]$ which is still lower than the traditional overheight value for $0 < \mathrm{R}_4/\mathrm{R}_2 < 1$. As shown in Fig. 4,

N_{OVER} is affected by and increases with Z_0 until an asymptotic value is reached. This effect of finite unsaturated zone thickness is particularly strong for small α (strong capillarity).

4. Concluding remarks

Parlange and Brutsaert's [1987] work enabled investigation into the capillary effect on watertable fluctuations in coastal unconfined aquifers induced by oceanic oscillations and has stimulated further studies in the area. In particular, their work provided an explanation for high-frequency watertable fluctuations caused by waves.

Here, we have extended their approach to derive a new governing equation and analytical solution that incorporate both horizontal and vertical flows in governing the watertable dynamics as well as the effect of the finite size of the unsaturated zone. These effects are shown to influence the dispersion relation of the watertable fluctuations and the mean watertable height. While the comparison with experimental data indicates improved predictions by the present solution compared with those given previously, further validation is required, particularly in relation to the effect of the finite unsaturated zone thickness. New experiments must be carried out under controlled conditions to provide comprehensive datasets for the validation. These experiments should include measurements of hydraulic heads (capillary pressure) in the unsaturated zone and tracer tests to track the watertable dynamics and unsaturated flow near the watertable.

Acknowledgments

This work was supported by the National Basic Research Program of China (973 program, 2012CB417005), National Natural Science Foundation of China (51009059) and National Science-Technology Support Program of China (2012BAB03B01).

References

Barry, D.A., S.J. Barry and J.-Y. Parlange (1996), Capillarity correction to periodic solution of the shallow flow approximation. In mixing in Estuaries and Coastal seas, Coastal Estuarine Stud., vol. 50, edited by Pattiaratchi, C.B., pp. 496-510, AGU, Washington, D.C., USA.

Bear, J. (1972). Dynamics of Fluids in Porous Media. American Elsevier Publ. Co., New York, USA.

Cartwright, N., P. Nielsen and S. Dunn (2003), Water table waves in an unconfined aquifer: Experiments and modeling. *Water Resources Research*, *39*(12), 1330, DOI: 10.1029/2003WR002185.

Gardner, W.R. (1958), Some steady-state solutions of the unsaturated moisture flow equations with application to evaporation from a water table. *Soil Science*, *85*(4), 228-232.

Jeng, D.-S., L. Li, and D.A. Barry (2002), Analytical solution for tidal propagation in a coupled semi-confined/phreatic coastal aquifer. *Advances in Water Resources*, *25*(5), 577-584, DOI: 10.1016/S0309-1708(02)00016-7.

Jeng, D.-S., B.R. Seymour, D.A. Barry, L. Li and J.-Y. Parlange (2005), New approximation for free surface flow of groundwater: Capillary correction. *Advances in Water Resources*, *28*(10), 1032-1039, DOI: 10.1016/j.advwatres.2004.05.012

Jiao, J. J. and Z. H. Tang (1999), An analytical solution of groundwater response to tidal fluctuation in a leaky confined aquifer. *Water Resources Research*, *35*(3), 747-751, DOI: 10.1029/1998WR900075.

Knight, J. H. (1981), Steady periodic-flow through a rectangular dam. *Water resources Research*, 17(4), 1222-1224, DOI: 10.1029/WR017i004p01222.

Li, H. L. and J. J. Jiao (2003), Tide-induced seawater-groundwater circulation in a multi-layered coastal leaky aquifer system. *Journal of Hydrology*, *274*(1-4), 211-224 DOI: 10.1016/S0022-1694(02)00413-4.

Li, L., D.A. Barry, J.-Y. Parlange and C.B. Pattiaratchi (1997), Beach water table fluctuation due to wave run-up: Capillarity effects. *Water Resources Research*, *33,* 935-945, DOI: 10.1029/96WR03946.

Li, L., D.A. Barry, C. Cunningham, F. Stagnitti and J.-Y. Parlange (2000a), A two-dimensional analytical solution of groundwater responses to tidal loading in an estuary and ocean. *Advances in Water Resources*, *23*(8), 825-833, DOI: 10.1016/S0309-1708(00)00016-6.

Li, L., D.A. Barry, F. Stagnitti and J.-Y. Parlange (2000b), Groundwater waves in a coastal aquifer: A new governing equation including vertical effects and capillarity. *Water Resources Research*, *36*(2), 411-420, DOI: 10.1029/ 1999WR900307.

Li, L., D.A. Barry, F. Stagnitti, J.-Y. Parlange and D.-S. Jeng (2000c), Beach water table fluctuations due to spring-neap tides: Moving boundary effects. *Advances in Water Resources*, *23*(8), 817-824, DOI: 10.1016/S0309-1708(00)00017-8.

Liu, P. L.-F. and J. Wen (1997), Nonlinear diffusive surface waves in porous media. *Journal of Fluid Mechanics*, *347*, 119-139, DOI: 10.1017/S0022112097006472.

Moench, AF (2008), Analytical and numerical analyses of an unconfined aquifer test considering unsaturated zone characteristics, *Water Resources Research*, 44(6), Article Number: W06409, DOI: 10.1029/2006WR005736.

Nielsen, P. (1990), Tidal dynamics of the water table in beaches. *Water Resources Research*, *26*(9), 2127-2134, DOI: 10.1029/WR026i009p02127.

Nielsen, P., A.M. Aseervatham, J.D. Fenton P. and Perrochet (1997), Groundwater waves in aquifers of intermediate depths. *Advances in Water Resources*, *20*(1), 37-43, DOI: 10.1016/S0309-1708(96)00015-2.

Parlange, J.-Y., F. Stagnitti, J.L. Starr and R.D. Braddock (1984), Free-surface flow in porous media and periodic solution of the shallow-flow approximation. *Journal of Hydrology 70*(1-4), 251-263, DOI: 10.1016/0022-1694(84)90125-2.

Parlange, J.-Y. and W. Brutsaert (1987), A capillarity correction for free surface flow of groundwater. *Water Resources Research*, *23*(5), 805-808, DOI: 10.1029/WR 023i005p00805.

Richards, L.A. (1931), Capillary conduction of liquids in porous mediums. *Physics 1*, 318-333, DOI: 10.1063/1.1745010.

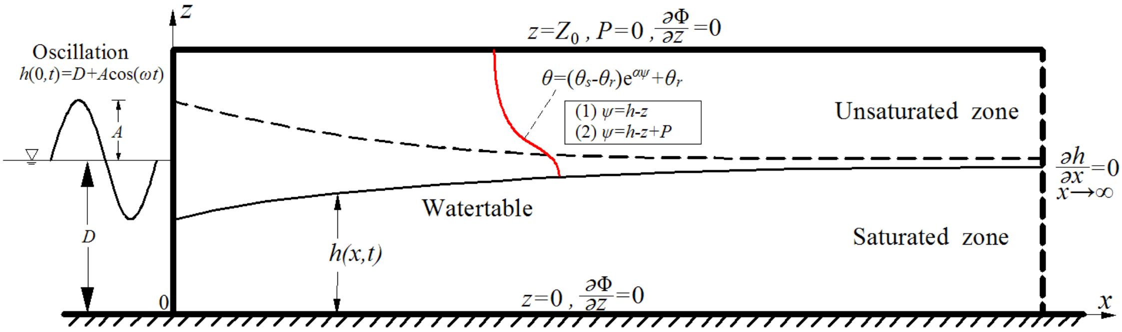

Figure 1. Schematic diagram of watertable fluctuations in a rectangular unconfined aquifer under the influence of a periodic boundary condition.

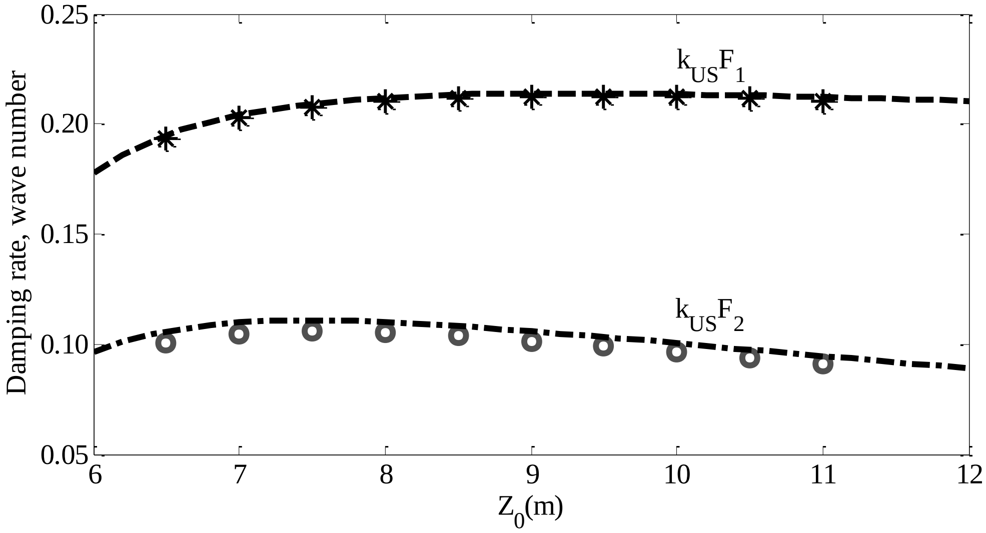

Figure 2. Comparison between the damping rate and wave number of the primary (ω) watertable fluctuations given by the analytical and numerical solutions. Stars and circles are the numerical results of the damping rate and wave number, respectively. Dashed and dash-dotted lines are the analytical predictions of the damping rate and wave number (equation (20a)), respectively. Parameters values used were A = 1 m, D = 5 m, α = 1 m^{-1}, K_s = 0.00047 m/s and n_e = 0.3.

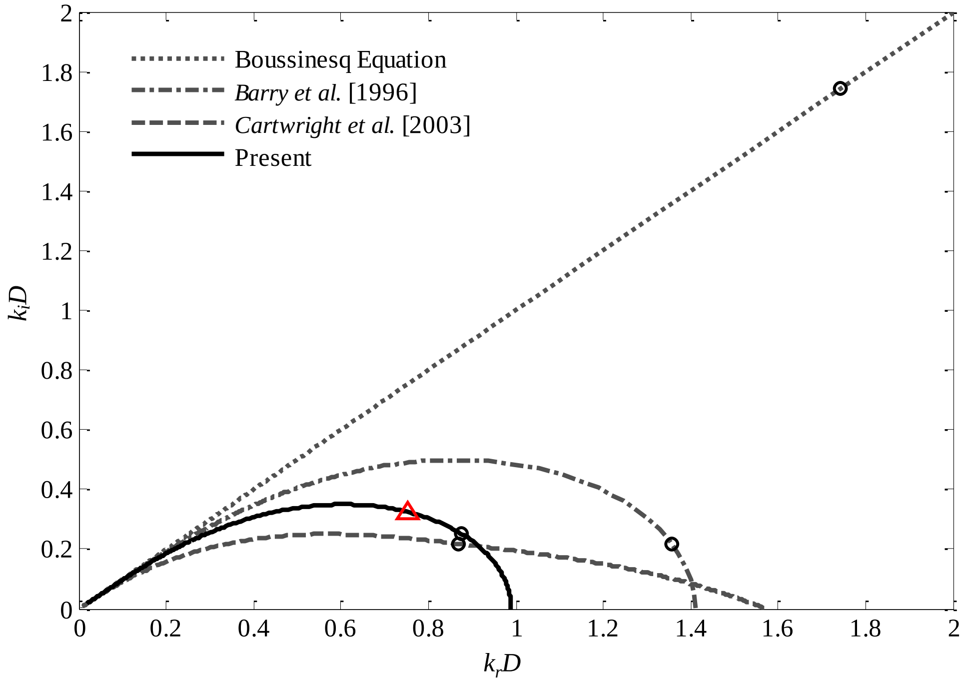

Figure 3. Comparison of amplitude damping rates k_rD (real part of KD) and wave numbers k_iD (for the primary frequency ω) given by different dispersion relation equations. The triangle denotes the measured value based on the experimental result. The circle on each curve indicates the calculated values based on measured parameter values and using the corresponding dispersion relation equation. The parameter values used were T = 772 s, D = 1.094 m, n_e = 0.32, K_s = 0.00047 m/s, H_ψ = 0.55 m and α = 1 m^{-1}.

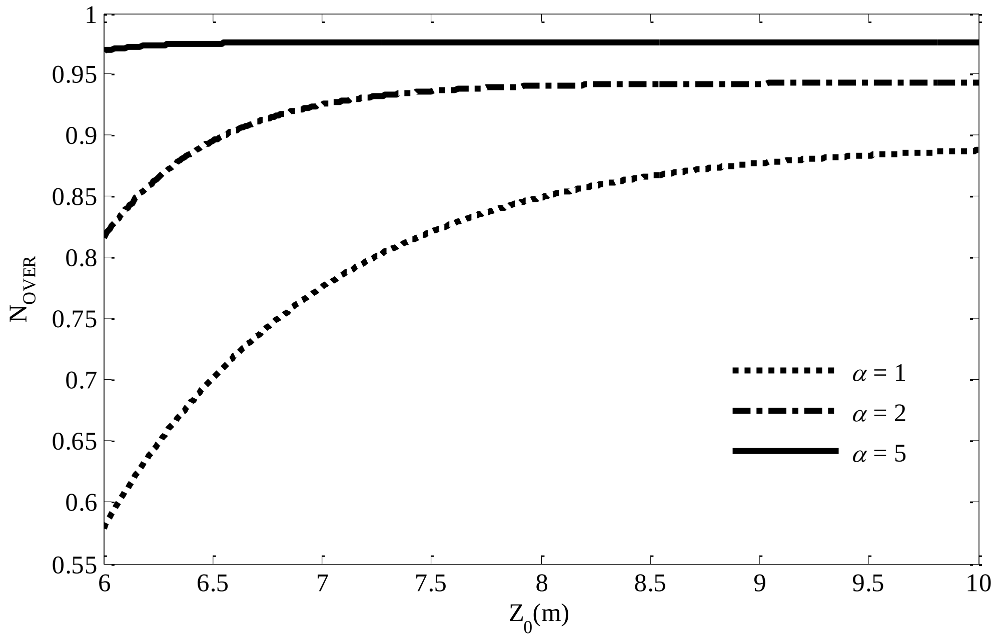

Figure 4. Variation of overheight index (N_{OVER}) with Z_0 for different α values (K_S = 0.0005 m/s, D = 5 m, n_e = 0.3 and T = 12 h).