Supplementary information for: All-optical signal processing using dynamic Brillouin gratings

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March 14, 2013

1 Introduction

In this document, more details are given to explain, to support and to extend the results presented in the main manuscript. In particular, the details refer to the definitions, the methods and the parameters of both the theoretical calculations. Moreover, in this document higher order differentiation is presented and the effects of self-phase modulation (SPM) considered. A comment on the role of spontaneous Brillouin noise (SpBN)is also provided.

2 Theoretical Model

The equations describing the slowly varying wave envelopes of the optical waves DW, RP, WP, RW (respectively $A_d(z,t)$, $A_w(z,t)$, $A_r(z,t)$, $A_{ret}(z,t)$) and of the AW, Q, are given by:

$$\begin{aligned}
\partial_z A_d + \frac{1}{v_s}\partial_t A_d &= -\frac{\eta g_B}{2} Q A_w + j\zeta_K(|A_d|^2)A_d, &(1)\\
-\partial_z A_w + \frac{1}{v_s}\partial_t A_w &= \frac{\eta g_B}{2} Q^* A_d + j\zeta_K(|A_w|^2)A_w, &(2)\\
\partial_z A_{ret} + \frac{1}{v_f}\partial_t A_{ret} &= -\frac{\eta g_B}{2} Q A_r + j\zeta_K(|A_{ret}|^2)A_{ret}, &(3)\\
-\partial_z A_r + \frac{1}{v_f}\partial_t A_r &= \frac{\eta g_B}{2} Q^* A_{ret} + j\zeta_K(|A_r|^2)A_r, &(4)\\
2\tau_B \partial_t Q + Q &= A_d A_w^* + A_{ret}^* A_r + \xi. &(5)
\end{aligned}$$

Besides the reference cited in the main manuscript [1] these equations can be also found in [2]. The symbols ∂_z, ∂_t indicate respectively space and time derivatives, v_s, v_f are the group velocities for the slow and fast axis of the PMF, $\eta = c_0 n \epsilon_0/2$ is a normalization factor (c_0 is the light speed in vacuum, n the refractive index, g_B the SBS gain coefficient) such that wave intensities $|A|^2$ are measured in V^2/m^2. The equations also take into consideration the SPM through the term proportional to $\zeta_K = \zeta\gamma$, where γ is the Kerr coefficient, $\zeta = \eta A_{eff}$, where A_{eff} is the fiber effective area. The AW is defined by $Q = 2v_a^2\rho/(\imath\gamma_e\epsilon_0\Omega_B\tau_B)$, where v_a is the sound speed, γ_e the electrostriction coefficient, ϵ_0 the vacuum permittivity, τ_B the phonon lifetime. The SpBN is also taken into account through the random variable ξ, as defined in [3]. For the sake of simplicity it will be set $v_s = v_f = c = c_0/n$.

2.1 Information storage

Neglecting the SPM and the SpBN, for the writing process (no RP and RW), in a reference frame moving with the AW (z, i.e. practically fixed with the fiber) the equations are reduced to:

$$\begin{aligned} \partial_t A_d + c\partial_z A_d &= -\eta_B A_w q \exp(-\Gamma_B t) & (6)\\ \partial_t A_w - \partial_z A_w &= \eta_B A_d q^* \exp(-\Gamma_B t) & (7)\\ \partial_t q &= \Gamma_B A_d A_w^* \exp(\Gamma_B t), & (8) \end{aligned}$$

where $\eta_B = c\eta g_B/2$. By formally integrate Eq. (8) in time, at a fixed position $z \in \{0, L\}$ the result is:

$$q(z,t) = \Gamma_B \int_0^t A_{d0}\left(\frac{z}{c} - t'\right) A_{wL}^*\left(\frac{z-L}{c} + t'\right) \exp(\Gamma_B t')dt' \tag{9}$$

In the ideal case, the AW is calculated straightforwardly:

$$\begin{aligned} q(z,t) &= \Gamma_B \int_0^t A_{d0}\left(\frac{z}{c} - t'\right) \varepsilon_w \exp(-\imath\theta_w)\delta\left(\frac{z-L}{c} + t'\right) \exp(\Gamma_B t')dt' & (10)\\ &= \Gamma_B \varepsilon_w \exp(-\imath\theta_w) A_{d0}\left(2\frac{z-z_c}{c}\right) \exp\left(-\Gamma_B \frac{z-L}{c}\right) \end{aligned}$$

where z_c is the point of the collision between the WP and the DW.

2.2 Information retrieval

The stored AW moves at the sound speed v_a and decays with the acoustic lifetime τ_B (see Table 1 for practical values). If the reading process is carried out within a few ns from the storage time, the AW motion can be completely neglected and the decay is expected to cause a decrease of the wave amplitude but no distortion. When the interaction between the DW and the WP is over, equations (3)-(4)-(5) govern the reading process on the PMF fast axis. Through a formal change of variables $z' = z - ct$ (reference frame moving with the RW $A_{ret}(z', t)$) the governing equations are:

$$\begin{aligned} \partial_t A_r - 2c\partial_{z'} A_r &= \eta_B A_{ret} q^* \exp(-\Gamma_B t), & (11)\\ \partial_t A_{ret} &= -\eta_B A_r q \exp(-\Gamma_B t), & (12)\\ \partial_t q - v_{fa}\partial_{z'} q &= \Gamma_B A_{ret} A_r^* \exp(\Gamma_B t), & (13) \end{aligned}$$

Let us consider an ideal Dirac function RP, injected from the same fiber side from which the WP was previously injected ($z = L$) with a delay t_0; the RP propagates without distortion till it interacts with the stored AW:

$$A_r(z', t) = A_{rL}\left(\frac{z'-L}{c} + 2t\right) = \varepsilon_r \exp(\imath\theta_r)\ \delta\left(\frac{z'-L}{c} + 2t - t_0\right), \tag{14}$$

while in the new reference frame the stored AW (Eq. (10)) reads:

$$q(z', t) = \Gamma_B \varepsilon_w \exp(-\imath\theta_w) A_{d0}\left(2\frac{z'-z_c}{c} + 2t\right) \exp\left[-\Gamma_B\left(\frac{z'-L}{c} + t\right)\right]. \tag{15}$$

While the RP walks off the AW, a part of it is backscattered into the RW that, similarly to what was done for the writing process, can be calculated by integrating Eq. (12) also substituting Eqs. (14),(15):

$$\begin{aligned} A_{ret}(z', t) &= -\eta_B \Gamma_B \varepsilon_w \varepsilon_r \exp[\imath(\theta_r - \theta_w)] \times & (16)\\ &\times \int_0^t \delta\left(\frac{z'-L}{c} + 2t' - t_0\right) A_{d0}\left(2\frac{z'-z_l}{c} + 2t'\right) \exp\left[-\Gamma_B\left(\frac{z'-L}{c} + 2t'\right)\right] dt' \\ &= A_{out} A_{d0}\left(\frac{z'}{c} + t_0\right), \end{aligned}$$

where $A_{out} = -\eta_B \Gamma_B \varepsilon_w \varepsilon_r \exp[-\Gamma_B t_0 + \imath(\theta_r - \theta_w)]$ is a constant.

Therefore, in the reference frame fixed with the fiber the RW is:

$$A_{ret}(z, t) = A_{out} A_{d0}\left(\frac{z}{c} - t + t_0\right). \tag{17}$$

2.3 Arbitrary waveform time differentiation

The time differentiation of the DW can be achieved when the RP, injected from $z = L$ and propagating without distortion till it interacts with the stored AW, is the time derivative an ideal Dirac distribution:

$$A_r(z', t) = A_{rL}\left(\frac{z' - L}{c} + 2t\right) = \varepsilon_r^{(1)} \exp(\imath\theta_r^{(1)})\, \delta^{(1)}\left(\frac{z' - L}{c} + 2t - t_0\right). \tag{18}$$

The stored AW is still given by Eq. (15). Substituting Eqs. (15) and (18) into Eq. (12) and integrating one gets:

$$\begin{aligned} A_{ret}(z', t) &= \eta_B \Gamma_B \varepsilon_w \varepsilon_r^{(1)} \exp[\imath(\theta_r^{(1)} - \theta_w)] \times \\ &\times \int_0^t \delta^{(1)}\left(\frac{z' - L}{c} + 2t' - t_0\right) A_{d0}\left(2\frac{z' - z_c}{c} + 2t'\right) \exp\left[-\Gamma_B\left(\frac{z' - L}{c} + 2t'\right)\right] dt' = \\ &= A_{out}^{(1)}\left[\frac{dA_{d0}}{dt}\left(\frac{z'}{c} + t_1\right) - \Gamma_B A_{d0}\left(\frac{z'}{c} + t_0\right)\right], \end{aligned} \tag{19}$$

where $A_{out}^{(1)} = -\eta_B \Gamma_B \varepsilon_w \varepsilon_r^{(1)} \exp[-\Gamma_B t_0 + \imath(\theta_r^{(1)} - \theta_w)]/2$. In the reference frame fixed with the fiber:

$$A_{ret}(z, t) = A_{out}^{(1)}\left[\frac{dA_{d0}}{dt}\left(\frac{z}{c} + t_0 - t\right) - \Gamma_B A_{d0}\left(\frac{z}{c} + t_0 - t\right)\right]. \tag{20}$$

2.4 True time reversal

In order to obtain the DW time reversal, it is necessary to exchange the roles of the waves A_r, A_{ret} in Eqs. 3-4-5. Therefore, a forward-propagating RP is injected from $z = 0$ and its interaction with the stored AW generates a backward propagating RW at a frequency $\omega_{ret} = \omega_r - \Omega_B$. The phase matching condition between slow and fast axis becomes $\omega_r = \omega_d(1 + \Delta n/n)$. Through a change of variables $z'' = z + ct$ (reference frame is moving with A_{ret}) the governing equations become:

$$\partial_t A_r + 2c\partial_{z''} A_r = -\eta_B A_{ret} q \exp(-\Gamma_B t), \tag{21}$$

$$\partial_t A_{ret} = \eta_B A_r q^* \exp(-\Gamma_B t), \tag{22}$$

$$\partial_t q + c\partial_{z''} q = \Gamma_B A_{ret}^* A_r \exp(\Gamma_B t). \tag{23}$$

Let the RP at the input side ($z = 0$) be a Dirac function, propagating without distortion till it interacts with the stored AW:

$$A_r(z'', t) = A_{r0}\left(\frac{z'' - L}{c} - 2t\right) = \varepsilon_r \exp(\imath\theta_r)\, \delta\left(\frac{z'' - L}{c} - 2t - t_0\right). \tag{24}$$

The stored AW (Eq. (10)) in the new reference frame reads:

$$q(z'', t) = \Gamma_B \varepsilon_w \exp(-\imath\theta_w) A_{d0}\left(2\frac{z'' - z_c}{c} - 2t\right) \exp\left[-\Gamma_B\left(\frac{z'' - L}{c} - t\right)\right]. \tag{25}$$

By formally integrating Eq. (22) and substituting Eqs. (24) and (25):

$$\begin{aligned} A_{ret}(z'', t) &= \eta_B \Gamma_B \varepsilon_w \varepsilon_r \exp[\imath(\theta_r + \theta_w)] \times \\ &\times \int_0^t \delta\left(\frac{z'' - L}{c} - 2t' - t_0\right) A_{d0}^*\left(2\frac{z'' - z_c}{c} - 2t'\right) \exp\left[-\Gamma_B\left(\frac{z'' - L}{c}\right)\right] dt' \\ &= A_{ttr} A_{d0}^*\left(\frac{z''}{c} + t_0\right) \exp\left[-\Gamma_B\left(\frac{z'' - L}{c}\right)\right], \end{aligned} \tag{26}$$

where $A_{ttr} = \eta_B \Gamma_B \varepsilon_w \varepsilon_r \exp[\imath(\theta_r + \theta_w)]$.

Finally, in the reference frame fixed with the fiber, one gets:

$$A_{ret}(z, t) = A_{ttr} A_{d0}^*\left(\frac{z}{c} + t + t_0\right) \exp\left[-\Gamma_B\left(\frac{z - L}{c} + t\right)\right]. \tag{27}$$

3 The effect of the Self Phase Modulation

The WP and RP powers in experiments are usually large (about 100W peak power) so one might ask what the effects of the SPM could be. In fact, though the fiber used is short (of the order of a few meters), the accumulated nonlinear phase shift can be large at the position where the pulses collide. As for the cross-phase modulation (XPM) its effects are certainly negligible, because the overlap time between the WP and the DW is very small due to the counter-propagation of the waves.

Here we show that the SPM contribution can be analytically predicted, with a very good degree of approximation, and its effects are not detrimental for the signal processing functions that have been demonstrated. The theory is corroborated by a numerical analysis. In fact, it is legitimate to assume that the SPM affects the propagation only till the point of collision in Eqs. (1)-(5) and therefore to assume that each pulse is affected by a predictable, constant, phase shift $\theta = \gamma P_0 d$, where d is the distance between the input (or output) and the collision point ($d = z_c$ or $d = L - z_c$). The model is tested in the particular case of a second order time differentiation.

3.1 Second order time differentiation

In the simulation, we consider a PMF of length $L = 5\ m$. The other fiber parameters are defined in Table 1. The DW is a superposition of two real Gaussian pulses, each of them with a FWHM of 2 ns (see Fig. 1a). The WP is the first derivative of a 500 ps Gaussian pulse (see Fig. 1a). The stored AW profile actually reproduces the first derivative of the DW, spatially compressed by a factor of 2, as it has been explained in the main manuscript. In Fig. 1b the modulus of the ideal first order derivative of the DW (dashed blue curve) is compared to the numerical (red curve) and the theoretical (with approximated SPM phase shift - dark green curve) store wave modulus. Similarly, in Fig. 1c, the ideal first derivative of the DW (dashed blue line) is compared with the numerical (red curves) and the theoretical (with approximated SPM phase shift - dark green curves) stored acoustic wave. The continuous curves refer to the real part, while the dashed curves to the imaginary part. This figure shows that the SPM effect is well described by the theory proposed in this section.

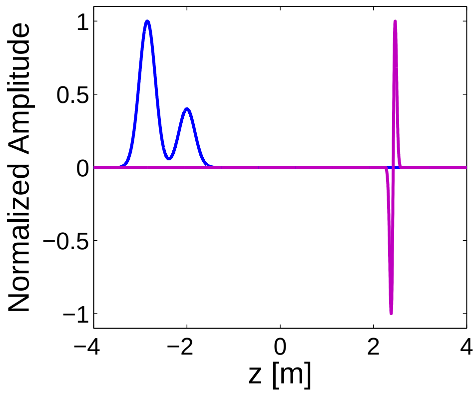
In order to obtain the second order derivative of the DW, let us consider a RP which is also the first order derivative of a Gaussian pulse (i.e. the same as WP in the simulations), as shown in Fig. 2a. The RW reproduces the second order time differentiation of the input data pulse, with a very good agreement. In fact, in Fig. 2b the modulus of the ideal second order derivative of the DW (dashed blue curve) is compared to the numerical (black curve) and the theoretical (approximated SPM phase shift - dark red curve) RW modulus. Similarly, in Fig. 2c, the ideal second order derivative of the DW (dashed blue curve) is compared with the numerical (black curves) and the theoretical (approximated SPM phase shift - dark red curves) RW. The continuous curves represent the real part and the dashed curves the imaginary part.

4 Experimental sources of distortion

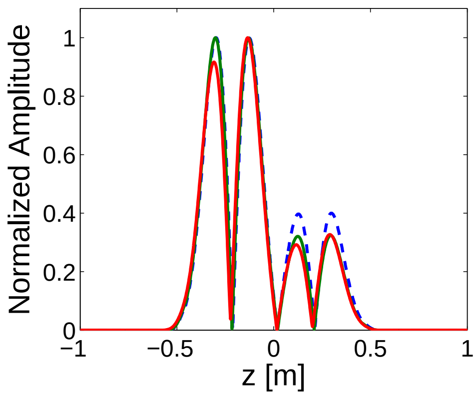
In the experiments, due to the lack of a suitable modulator, the write and read pulsewidth were 400 ps FWHM pulses, so their duration is comparable to the single pulse of the sequence. Therefore, they are not close to the ideal delta Dirac functions. This introduces some distortion in the output pulse, as shown in Fig. 3, where a numerical simulation using the experimental input sequence but with shorter (50ps, FWHM) write and read pulses is presented. Note in particular that both the extinction ratio and the rise and decay times are affected by the finite pulsewidth. Additional source of distortion (like that appearing in the pulse leading edge) arises from temperature and birefringence gradients, which causes a shift of the Brilluoin gain and of the peak reflectivity wavelength with substantial reflected amplitude change.

5 The effects of the spontaneous Brillouin noise

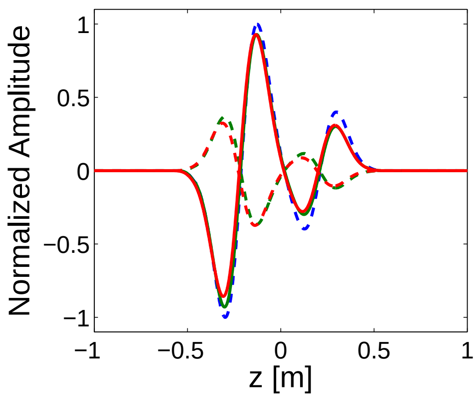
The contribution of SpBN is shown by ξ term in eq. 5, which actually represents a Langevin noise source describing the thermal excitation of AW. The statistical properties of ξ, described by a white Gaussian random variable, are investigated in [3], in which the AW is expressed in terms of density ρ in the dynamical equations, so the noise variance is calculated obtaining eq. 16 in [3]. Let us remark that here we defined the AW in terms



(a)



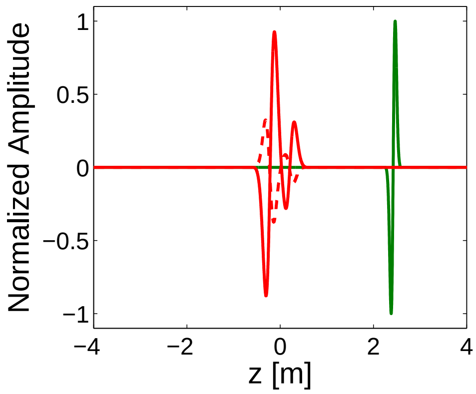
(b)



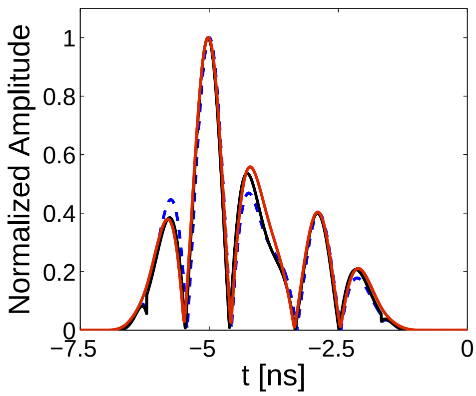
(c)

Figure 1: (a): Spatial profile of the DW (blue) and the WP (magenta). (b): Spatial profile of stored AW modulus; comparison between the ideal first order derivative of the DW (dashed blue), the numerical obtained (red) and the theoretical (approximated SPM phase shift - dark green) stored wave. (c): Same as (b); the continuous curves refer to the real part, the dashed curves to the imaginary part.

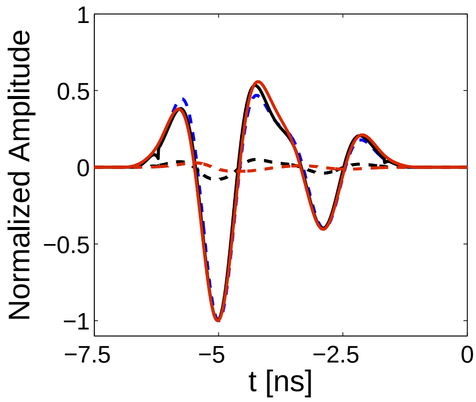
of Q rather than ρ ($Q = 2v_a^2\rho/\left(\imath\gamma_e\epsilon_0\Omega_B\tau_B\right)$) so the noise variance can be straightforwardly recalculated in terms of Q.



(a)



(b)



(c)

Figure 2: (a): Spatial profile of the DW (blue) and the WP (magenta). (b): the comparison between the ideal second order derivative of the DW (dashed blue), the numerical (black) and the theoretical (approximated SPM phase shift - dark red) RW. (c): Same as in (b); the continuous curves represent the real part, the dashed curves the imaginary part.

We considered the SpBN in the numerical simulations for completeness. However, it has been verified that the SpBN induced fluctuations do not affect the SBS signals: noise contribution is few orders of magnitude

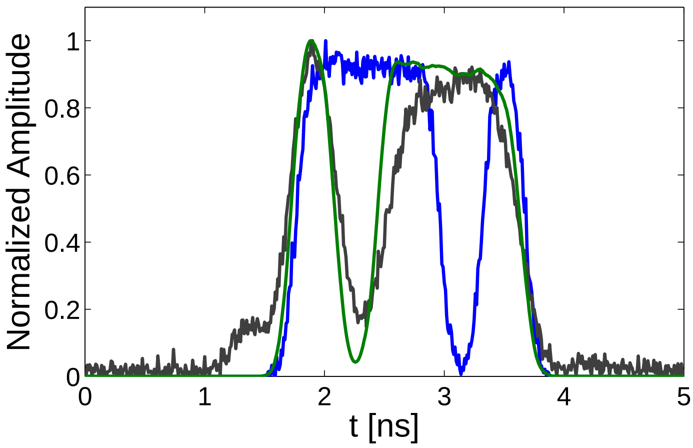


Figure 3: True time reversal: the experimental input data waveform (blue curve), the experimental retrieved waveform (black curve), the numerically obtained retrieved waveform (green curve) when the data waveform is the experimental but using shorter ($50ps$, FWHM) write and read pulses.

lower than SBS waveforms. This fact has been also corroborated by experiments, which results are not affected by SpBN.

6 PM fiber parameters definition

In Tab. 1, we give the parameters used in the main manuscript and also in this document.

PMF birefringence	$\Delta n = 5 \cdot 10^{-4}$
Acoustic wave velocity	$v_a = 5970.7\ m/s$
Electrostriction coefficient	$\gamma_e = 1.8$
Refractive index	$n = 1.5$
Light group velocity	$c = 2 \cdot 10^8\ m/s$
Permittivity	$\epsilon_0 = 1/36\pi \cdot 10^{-9}\ A^2 s W^{-1} m^{-1}$
Brillouin frequency shift	$\Omega_B = 2\pi(10.93\ GHz)\ rad/s$
Phonon lifetime	$\tau_B = 5\ ns$
SBS gain coefficient	$g_B = 5 \cdot 10^{-11}\ m/W$
Effective area	$A_{eff} = 40\ \mu m^2$
Kerr's coefficient	$\gamma = 0.0032\ W^{-1} m^{-1}$

Table 1: PM fiber parameters.

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