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Wave propagation across rock joints filled with viscoelastic medium using modified recursive method

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ABSTRACT

Rock joints have significant effects on wave propagation. When the joints are filled with saturated sand or clay, the filling materials exhibit viscoelastic behavior on wave propagation. In the present study, wave propagation across single and multiple parallel joints filled with viscoelastic medium is examined. Based on a layered medium model, the recursive method is adopted and a modification is made under some special conditions for faster calculation. In the theoretical formulation, analytical solutions of wave propagation across a single viscoelastic joint as well as elastic joint are mathematically derived. Through parametric studies, it is found that the more viscous the filled medium is, the less the wave energy transmits. Meanwhile, distinct stop-pass behavior exhibits with the change of joint thickness or wave frequency for a single elastic joint. While for the wave transmission across a single viscoelastic joint, the transmission coefficient generally decreases with increasing joint thickness or wave frequency, except when the two parameters match with the pass bands of the corresponding elastic joint. When parallel joints exist, multiple wave reflections among joints influence wave transmission. The transmission coefficient decreases with increasing joint number and, the stop-pass behavior for viscoelastic joints is less significant than that for the elastic joints.

1. Introduction

When a wave propagates through rock masses, it is often attenuated (and slowed) due to the presence of rock joints (King et al., 1986; Zhao et al., 2006). For rock engineering, the prediction of wave attenuation across jointed rock masses is a great concern of rock engineers to assess the safety of underground structures built in rocks under dynamic loads.

Recently, many studies have been performed on wave propagation across non-filled joints with full consideration of different deformational behaviors by using displacement discontinuity theories (Cai and Zhao, 2000; Pyrak-Nolte et al., 1990a; Schoenberg, 1980). In these theories, stresses across joints are continuous, but displacements are not. The discontinuity in displacement is equal to the applied stress divided by the joint stiffness. These theories are utilized usually when joints generally have large persistence and small apertures relative to the wavelength (Myer, 2000; Yi et al., 1997).

Natural joints, however, are often filled with a certain amount of saturated sand, clay or gouge, where the thickness of filled material can be up to several centimeters (Barton, 1974; Sinha and Singh, 2000). For a thick filled joint, its behavior is mostly controlled by the filling material. In practice, a filled joint can be treated as a thin and weak layer sandwiched between background rock materials (layered medium model). Across the two interfaces, both displacements and stresses are continuous (Bedford and Drumheller, 1994; Brekhovskikh, 1960). The layered medium model is valid regardless of the ratio of joint thickness to wavelength. The displacement discontinuity model is also applicable for the analysis of filled joints. However, the displacement discontinuity model is an approximation of the layered medium model, and its accuracy decreases with the increasing ratio of joint thickness to wavelength (Liu et al., 1995; Rokhlin and Wang, 1991). When the joint thickness is comparable with the wavelength, e.g., ultrasonic wave (with a frequency of 10^5 Hz or more) propagation across a thick joint (with a thickness of 10^{-2} m or more), the displacement discontinuity model is no longer applicable, but the layered medium model always is.

Based on the layered medium model, the wave propagation across a single filled joint (or a sandwiched layer between two solids) has been studied for different scenarios, where the behavior of the filling material is assumed to be linear elastic (Jones and Whittier, 1967), nonlinear elastic (Li and Ma, 2009), poroelastic (Nakagawa and Schoenberg, 2007) or viscoelastic (Fehler, 1982; Rokhlin and Marom, 1986; Zhu et al., 2011). In nature, rock joints are usually in

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parallel form as joint sets. Wave propagation across multiple joints becomes complicated due to multiple wave reflections among joints (Cai and Zhao, 2000; Pyrak-Nolte et al., 1990b). Solutions of wave propagation across a layered medium have been reported in some classical books (Bedford and Drumheller, 1994; Brekhovskikh, 1960). For a periodically layered structure (a special case of layered medium), frequency spectra show large energy transmission bands (pass bands) and near-zero energy transmission bands (stop bands). The stop-pass behavior for elastic stratified system with welded interfaces (Brekhovskikh, 1960; Sve, 1971) and with non-filled joints (Nakagawa et al., 2000) has been studied and, the transmission coefficient is a function of impedance contrast, nondimensional layer thickness, and joint stiffness.

For a joint filled with saturated sand or clay, the viscoelastic behavior of the filling material is suggested by some researchers (Jaeger et al., 2007; Richer, 1977) to take account of wave propagation, and such behavior has been widely implemented in the study of soil dynamics (Das and Ramana, 2011; Verruijt, 2010). In order to consider a wide range of wave frequency and joint thickness, the layered medium model is adopted in the present study to examine wave attenuation across single and multiple parallel joints filled with viscoelastic medium. Firstly, the recursive method is briefly introduced and a modification is made under some special conditions for faster calculation. Then, analytical solutions of wave propagation across a single viscoelastic joint as well as elastic joint are mathematically derived. Subsequently, parametric studies of wave propagation across a single filled joint are conducted with respect to viscosity, joint thickness and wave frequency. Finally, the effects of multiple joints (including joint spacing and joint number) on wave transmission are examined.

2. The modified recursive method

The recursive method (or termed as propagator method or reflectivity method by some other researchers) is an efficient tool to study wave propagation across a layered medium (Brekhovskikh, 1960; Fuchs and Müller, 1971; Kennett, 1983; Sve, 1971; Treitel and Robinson, 1966). With this method, relations among different layers with respect to potential amplitude or stresses and displacements are established. Hence, the transmitted wave across N layers can be obtained through N − 1 steps. However, efforts have been persistently made to improve its efficiency since its emergence (Chen, 1993; Luco and Apsel, 1983). For example, Chen (1993) presented a systematic and efficient approach for computing the dispersion curves as well as the eigenfunctions of normal modes in a multilayered half-space medium. Compared with the previous recursive method, it is a simple and self-contained algorithm. In addition, as intrinsically excluding the growth terms, the algorithm not only exhibits the physical mechanism of the formation of the normal modes, but also is numerically more stable for high-frequency cases.

In nature, joint mechanical properties, filling condition, spatial configuration, and mechanical properties of background media vary little and can be approximated to be the same on a certain engineering scale (Jaeger et al., 2007). Therefore, a modification of the recursive method is made in this section under above assumption for faster calculation.

For a normally incident P or S wave with the form of $u_0 = A\exp(-i\omega t + ikx)$, where u_0 is the particle displacement, A is the amplitude, ω is the angular frequency, and k is the wave number, its reflection and transmission coefficients across a single joint are assumed to be R_1 and T_1, respectively.

The reflected and transmitted waves can be treated as the superposition of reflected and transmitted waves arriving at different times caused by multiple reflections between joints. Therefore, the first, second, third, fourth, …, nth reflected waves across two joints are R_1u_0, $T_1^2R_1e^{i2ks}u_0$, $T_1^2R_1e^{i2ks}R_1^2e^{i2ks}u_0$, $T_1^2R_1e^{i2ks}(R_1^2e^{i2ks})^2u_0$, …, $T_1^2R_1e^{i2ks}(R_1^2e^{i2ks})^{n-2}u_0$, respectively; and the first, second, third, …, nth transmitted waves across two joints are $T_1^2e^{iks}u_0$, $T_1^2e^{iks}R_1^2e^{i2ks}u_0$, $T_1^2e^{iks}(R_1^2e^{i2ks})^2u_0$, …, $T_1^2e^{iks}(R_1^2e^{i2ks})^{n-1}u_0$, respectively, where s is the joint spacing, and the term iks reflects the phase shift caused by wave propagation through the background medium. It is found that transmitted waves arriving at different times form a geometric sequence with a common ratio of $R_1^2e^{i2ks}$, and reflected waves arriving at different times except the first one also form a geometric sequence with a common ratio of $R_1^2e^{i2ks}$. Thus, the reflection and transmission coefficients across two joints are

$$R_2 = R_1 + \frac{{T_1}^2R_1e^{i2ks}}{1-{R_1}^2e^{i2ks}} = R_1 + \frac{{T_1}^2R_1e^{i4\pi\xi}}{1-{R_1}^2e^{i4\pi\xi}}, \tag{1}$$

$$T_2 = \frac{{T_1}^2e^{iks}}{1-{R_1}^2e^{i2ks}} = \frac{{T_1}^2e^{i2\pi\xi}}{1-{R_1}^2e^{i4\pi\xi}}, \tag{2}$$

where $ks = 2\pi s/\lambda = 2\pi\xi$, ξ is the ratio of the joint spacing to the wavelength and termed as the nondimensional joint spacing.

Similarly, the first, second, third, fourth, …, nth reflected waves across four joints are R_2u_0, $T_2^2R_2e^{i2ks}u_0$, $T_2^2R_2e^{i2ks}R_2^2e^{i2ks}u_0$, $T_2^2R_2e^{i2ks}(R_2^2e^{i2ks})^2u_0$, …, $T_2^2R_2e^{i2ks}(R_2^2e^{i2ks})^{n-2}u_0$, respectively; and the first, second, third, …, nth transmitted waves across four joints are $T_2^2e^{iks}u_0$, $T_2^2e^{iks}R_2^2e^{i2ks}u_0$, $T_2^2e^{iks}(R_2^2e^{i2ks})^2u_0$, …, $T_2^2e^{iks}(R_2^2e^{i2ks})^{n-1}u_0$, respectively. It shows that transmitted waves arriving at different times form a geometric sequence with a common ratio of $R_2^2e^{2iks}$, and reflected waves arriving at different times except the first one also form a geometric sequence with a common ratio of $R_2^2e^{2iks}$. Thus, the reflection and transmission coefficients across four joints are functions of those across two joints:

$$R_4 = R_2 + \frac{{T_2}^2R_2e^{i4\pi\xi}}{1-{R_2}^2e^{i4\pi\xi}}, \tag{3}$$

$$T_4 = \frac{{T_2}^2e^{i2\pi\xi}}{1-{R_2}^2e^{i4\pi\xi}}. \tag{4}$$

It is further found that for wave propagation across 2^n joints, the reflection and transmission coefficients can be expressed as functions of $R_{2^{n-1}}$ and $T_{2^{n-1}}$:

$$R_{2^n} = R_{2^{n-1}} + \frac{{T_{2^{n-1}}}^2R_{2^{n-1}}e^{i4\pi\xi}}{1-{R_{2^{n-1}}}^2e^{i4\pi\xi}}, \tag{5}$$

$$T_{2^n} = \frac{{T_{2^{n-1}}}^2e^{i2\pi\xi}}{1-{R_{2^{n-1}}}^2e^{i4\pi\xi}}. \tag{6}$$

These solutions (R_{2^n} and T_{2^n}) are basic solutions. However, it does not mean that reflection and transmission coefficients can only be obtained for 2^n joints. Reflection and transmission coefficients across other joint numbers can be obtained from these basic solutions through steps much less than the joint number. For example, R_3, T_3 can be obtained through one step from R_1, T_1, R_2 and T_2

$$R_3 = R_1 + \frac{{T_1}^2R_2e^{i4\pi\xi}}{1-R_1R_2e^{i4\pi\xi}} = R_2 + \frac{{T_2}^2R_1e^{i4\pi\xi}}{1-R_1R_2e^{i4\pi\xi}}, \tag{7}$$

$$T_3 = \frac{T_1T_2e^{i2\pi\xi}}{1-R_1R_2e^{i4\pi\xi}}. \tag{8}$$

R_7, T_7 can be obtained through two steps from R_1, T_1, R_2, T_2, R_4 and T_4 with the first step to obtain R_3 and T_3 as shown in Eqs. (7)–(8), and then,

$$R_7 = R_3 + \frac{T_3{}^2 R_4 e^{i4\pi\xi}}{1 - R_3 R_4 e^{i4\pi\xi}} = R_4 + \frac{T_4{}^2 R_3 e^{i4\pi\xi}}{1 - R_3 R_4 e^{i4\pi\xi}}, \tag{9}$$

$$T_7 = \frac{T_3 T_4 e^{i2\pi\xi}}{1 - R_3 R_4 e^{i4\pi\xi}}. \tag{10}$$

To be convenient, the modified recursive method is abbreviated to MRM. The MRM is superior to the previous method by the existence of basic solutions. Therefore, less steps are needed for the MRM to obtain reflection and transmission coefficients across multiple joints, especially when the joint number is large. For example, it needs only 4 steps (first R_2 and T_2, then R_4 and T_4, R_8 and T_8, finally R_{16} and T_{16}) to obtain R_{16} and T_{16} with the MRM, while 15 steps are needed to obtain R_{16} and T_{16} for the previous recursive method. In addition, once the reflection and transmission coefficients across a single joint are available, the MRM can be applied to calculate wave propagation across multiple joints with different deformational behaviors.

3. Wave propagation across a single filled joint

Different from wave propagation across the elastic medium, wave attenuation and dispersion occur when a wave propagates across the viscoelastic medium. A general solution of wave propagation across the viscoelastic medium is expressed as (Achenbach, 1973; Ewing et al., 1957; Kolsky, 2003)

$$u = A\exp(-ax)\exp[i\omega(x/C - t)], \tag{11}$$

where a is the attenuation factor and C is the phase velocity.

Many viscoelastic models, including the Maxwell model, the Kelvin model, the Kelvin–Voigt model and other models were developed to describe the viscoelastic behavior of materials for different scenarios. Among them, the Kelvin model is simple and has been widely used to describe the dynamic behavior of saturated sand or clay (Das and Ramana, 2011; Verruijt, 2010). For P wave propagation across the viscoelastic medium described by the Kelvin model with spring stiffness E and dashpot viscosity η, the attenuation factor (a) and phase velocity (C) are (Achenbach, 1973; Ewing et al., 1957; Kolsky, 2003):

$$a = \left[\frac{\rho_0 E}{2(E^2 + \eta^2\omega^2)}\left(\sqrt{1 + \frac{\eta^2\omega^2}{E^2}} - 1\right)\right]^{1/2}\omega, \tag{12}$$

$$C = \left[\frac{2\left(E^2 + \eta^2\omega^2\right)}{\rho_0 E\left(\sqrt{1 + \frac{\eta^2\omega^2}{E^2}} + 1\right)}\right]^{1/2}. \tag{13}$$

From Eqs. (12) and (13), it can be seen that both the attenuation factor and phase velocity are frequency-dependent. In addition, the attenuation factor and P wave velocity for purely elastic materials can be obtained by setting η to zero:

$$a = 0, \tag{14}$$

$$C = \sqrt{\frac{E}{\rho_0}}. \tag{15}$$

Based on the layered medium model, the reflection and transmission coefficients across a single viscoelastic joint by considering multiple wave reflections between two interfaces of the filled joint, are obtained:

$$R_1 = R_{R\to F} + \frac{T_{R\to F} T_{F\to R} R_{F\to R} e^{(-2ad + i2\omega d/C)}}{1 - R_{F\to R}{}^2 e^{(-2ad + i2\omega d/C)}}, \tag{16}$$

$$T_1 = \frac{T_{R\to F} T_{F\to R} e^{(-ad + i\omega d/C)}}{1 - R_{F\to R}{}^2 e^{(-2ad + i2\omega d/C)}}. \tag{17}$$

where $R_{R\to F} = \frac{1-r}{1+r}$, $T_{R\to F} = \frac{2}{1+r}$, $R_{F\to R} = \frac{r-1}{1+r}$, $T_{F\to R} = \frac{2r}{1+r}$, the subscripts R and F refer to rock and filled medium, respectively, r is the ratio of the impedance of rock to that of the filled medium. r is frequency-dependent because of the frequency-dependency of phase velocity in viscoelastic medium. Eqs. (16)–(17) are similar with Eqs. (1)–(2), the difference between them is that Eqs. (1)–(2) describe the dynamic response across two joints, while Eqs. (16)–(17) are used to determine the effects of two interfaces of a single filled joint on wave propagation.

Similarly, the reflection and transmission coefficients for wave propagation across a single joint filled with a purely elastic medium are obtained by introducing Eqs. (14) and (15) into Eqs. (16) and (17).

$$R_{1E} = R_{R\to F} + \frac{T_{R\to F} T_{F\to R} R_{F\to R} e^{i4\pi\zeta}}{1 - R_{F\to R}{}^2 e^{i4\pi\zeta}}, \tag{18}$$

$$T_{1E} = \frac{T_{R\to F} T_{F\to R} e^{i2\pi\zeta}}{1 - R_{F\to R}{}^2 e^{i4\pi\zeta}}, \tag{19}$$

where $\zeta = d/\lambda$ is the ratio of joint thickness to the wavelength and termed as the nondimensional joint thickness. To be convenient, joints filled with viscoelastic medium and purely elastic medium are abbreviated to be viscoelastic joint and elastic joint, respectively.

In the following calculation, it is assumed that the rock density is 2650 kg/m^3 and its P wave velocity is 5830 m/s as typical properties of the Bukit Timah granite of Singapore (Zhao, 1996), the density of the filled material is 1900 kg/m^3 and the stiffness (k) of the filled medium for both Kelvin model and elastic model is equal to 2 GPa, which are typical soil properties.

In the following section, the effects of viscosity η, joint thickness d, and wave frequency f on wave transmission across a single filled joint are studied in the frequency domain. Fig. 1 shows the magnitude of transmission coefficient across a single viscoelastic joint ($|T_1|$) as a function of η (when $\eta = 0$, the viscoelastic joint becomes an elastic one), where $d = 0.01$ m, $f = 100$ kHz. From the figure, it is found that $|T_1|$ decreases with η and, it indicates that the more viscous the filled medium is, the less the wave energy transmits. This phenomenon is due to the combined effects of wave attenuation in the

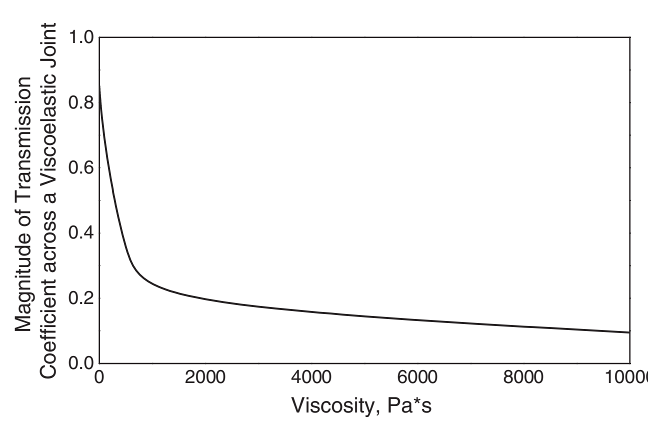

Fig. 1. The magnitude of transmission coefficient across a single viscoelastic joint as a function of viscosity, where $d = 0.01$ m and $f = 100$ kHz.

viscoelastic medium and the dependence of material impedance on viscosity.

Results of transmission coefficients across a single viscoelastic joint ($|T_1|$) and elastic joint ($|T_{1E}|$) as a function of joint thickness d are presented in Fig. 2, where $f=100$ kHz for both kinds of joints, and $\eta=1000$ Pa*s for the viscoelastic joint. It can be seen that $|T_1|$ is smaller than $|T_{1E}|$ and, distinct stop-pass behavior exhibits with the change of joint thickness for a single elastic joint. While for wave transmission across a viscoelastic joint, $|T_1|$ decreases with increasing d, except when d matches with pass bands of the elastic joint. The general tendency of $|T_1|$ decreasing with d is due to the wave attenuation in viscous medium: the larger the joint thickness is, the more the wave energy dissipates. Meanwhile, the small discrepant part is caused by the stop-pass characteristics of a layered system, which results from the change of the ratio of joint thickness to wavelength.

Wave frequency also has great effects on wave transmission across joints. Fig. 3 presents results of transmission coefficients across a single viscoelastic joint ($|T_1|$) and elastic joint ($|T_{1E}|$) as a function of wave frequency f, where $d=0.01$ m for both kinds of joints and, $\eta=1000$ Pa*s for the viscoelastic joint. Similar with Fig. 2, it is found that $|T_1|$ is smaller than $|T_{1E}|$ and, distinct stop-pass behavior exhibits with the change of wave frequency for a single elastic joint. While for wave transmission across the viscoelastic joint, $|T_1|$ decreases with increasing f, except when f matches with the pass bands of the elastic joint. The general tendency of $|T_1|$ decreasing with f is due to two factors: the dependence of attenuation factor on f (the higher the wave frequency is, the more the wave energy dissipates), and the dependence of the filling material's impedance on f. Meanwhile, the small discrepant part is due to the stop-pass characteristics of the layered system, which corresponds to the variation of the ratio of joint thickness to wavelength.

4. Wave propagation across a filled joint set

When parallel joints exist, wave propagation becomes complicated due to multiple wave reflections among joints. Based on the MRM and solutions of wave propagation across a single viscoelastic joint and elastic joint, wave attenuation across multiple filled joints is studied in this section.

Fig. 4 shows the results of transmission coefficients across two viscoelastic joints ($|T_2|$) and elastic joints ($|T_{2E}|$) as a function of nondimensional fracture spacing ξ, where $f=100$ kHz, $d=0.01$ m for both kinds of joints, and $\eta=10000$ Pa*s for the viscoelastic joints. From the figure, it is shown that the stop-pass behavior exhibits with the change of joint spacing for both scenarios of two viscoelastic and elastic joints. However, the stop-pass behavior for elastic joints is more significant than that for the viscoelastic joints: at the pass band, the transmission coefficient for the elastic scenario is much larger than that for the viscoelastic scenario; and contrarily at the stop band, the transmission coefficient for the elastic case is much smaller than that for the viscoelastic case.

The transmitted wave can be treated as the superposition of transmitted waves arriving at different times caused by multiple reflections between joints. Joint spacing causes phase shift among differently arriving waves. Therefore, for some joint spacing, transmitted waves arriving at different times enhance the superposed wave amplitude and form the pass band. While for some other joint spacing, these waves counteract one another and form the stop band. Due to the attenuation of the filling medium, amplitudes of the first arriving transmitted wave and subsequently arriving transmitted waves across the viscoelastic joints are smaller than those across the elastic joints. Thus, it is well understood that at pass bands, the transmission coefficient for the viscoelastic scenario is much smaller than that for the elastic scenario. At stop bands, for the viscoelastic joints, because later transmitted waves experience much more numbers of wave reflections and transmissions across joints, the attenuation effects (compared with the elastic scenario) on them are much greater than those on the first transmitted wave. And thus, the counteraction effects of the later transmitted waves on the first transmitted waves are not as significant as those for the elastic cases. Therefore, the transmission coefficient for the viscoelastic joints is larger than that for the elastic ones.

Results of the transmission coefficients across multiple viscoelastic joints ($|T_N|$ and $N=2, 8, 16$) as a function of nondimensional fracture spacing ξ are illustrated in Fig. 5, where $f=100$ kHz, $d=0.01$ m,

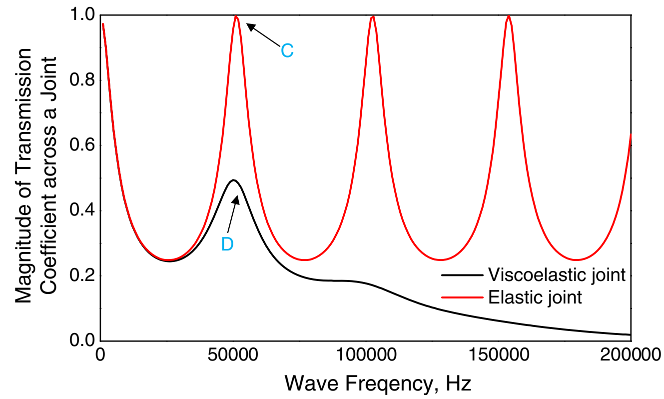

Fig. 3. The magnitude of transmission coefficient across single viscoelastic and elastic joints as a function of wave frequency, where $d=0.01$ m and, $\eta=1000$ Pa*s for the viscoelastic joint.

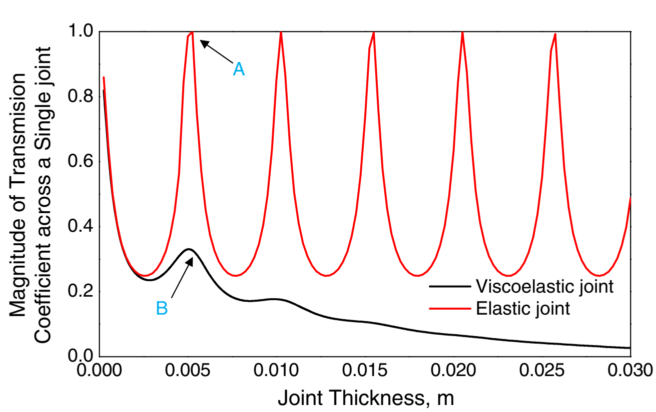

Fig. 2. The magnitude of transmission coefficient across single viscoelastic and elastic joints as a function of joint thickness, where $f=100$ kHz, and $\eta=1000$ Pa*s for the viscoelastic joint.

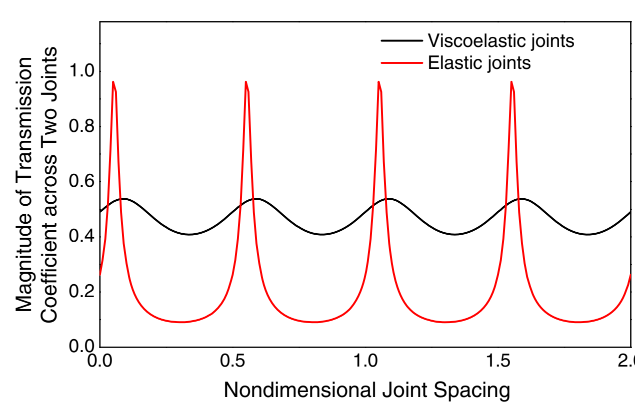

Fig. 4. The magnitude of transmission coefficient across two parallel viscoelastic and elastic joints as a function of nondimensional joint spacing, where $d=0.01$ m, $f=100$ kHz, $\eta=10000$ Pa*s for the viscoelastic joints.

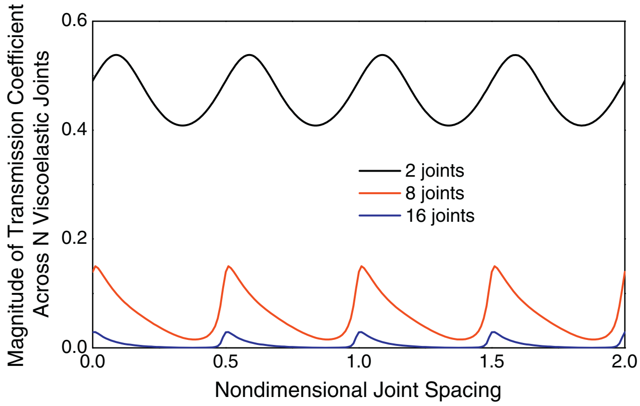

Fig. 5. The magnitude of transmission coefficient across multiple viscoelastic joints (N = 2, 8, 16) as a function of nondimensional joint spacing, where $d = 0.01$ m, $f = 100$ kHz, $\eta = 10000$ Pa*s.

and $\eta = 10000$ Pa*s. From the figure, we find that the stop-pass behavior exhibits with the change of joint spacing for the wave transmission across 2, 8 and 16 viscoelastic joints. However, the transmission coefficients at pass bands and stop bands decrease with *N*. It is due to the wave attenuation of the filling materials and wave reflection at the joint interfaces.

5. Discussion

Wave propagation across viscoelastic medium is distinguished from that across the elastic medium by frequency-dependent wave attenuation and dispersion. Four points (A–D) are shown in Figs. 2 and 3: point A corresponds to wave transmission across a single elastic joint, where $d = 0.005$ m and $f = 100$ kHz; point C refers to wave transmission across a single elastic joint, where $d = 0.01$ m and $f = 50$ kHz; point B denotes wave transmission across a single viscoelastic joint, where $d = 0.005$ m and $f = 100$ kHz; point D shows wave transmission across a single viscoelastic joint, where $d = 0.01$ m and $f = 50$ kHz. For points A and C (both cases correspond to wave transmission across a single elastic joint), the ratios of joint thickness to the wavelength for the two scenarios are the same (the phase change of differently arriving waves are the same) and correspondingly, their transmission coefficients are the same ($|T_{1E}| = 0.955$). For points B and D (both cases refer to the wave transmission across a single viscoelastic joint), because of the frequency-dependency of phase velocity in the viscoelastic medium, there exists a small difference between their ratios of joint thickness to the wavelength (the ratio is 0.483 for point B and it is 0.47 for point C). However, the difference of their transmission coefficients is relatively large: $|T_1| = 0.332$ for point B and $|T_1| = 0.494$ for point D. The above phenomena indicate that wave attenuation across an elastic joint is equally determined by thickness and wave frequency, i.e., by the ratio of joint thickness to the wavelength. However, compared with joint thickness, wave attenuation across a viscoelastic joint is more sensitive to wave frequency, i.e., for cases having the same products of *f* and *d*, the case with larger *f* has smaller $|T_1|$.

6. Conclusions

Wave propagation across viscoelastic joints is of great importance in the fundamental research and engineering application of rock dynamics and applied geophysics. Based on the layered medium model, the recursive method is adopted and modified for faster calculation, which is valid when joints and rock materials have similar mechanical properties and spatial configuration.

In the theoretical formulation, analytical solutions of wave propagation across a single viscoelastic joint as well as elastic joint are mathematically derived. Through parametric studies, it is found that the magnitude of the transmission coefficient across a viscoelastic joint is smaller than that across an elastic joint and, the transmission coefficient decreases with increasing viscosity, which indicates that the more viscous the filled medium is, the less the wave energy transmits. In addition, the magnitude of the transmission coefficient across a viscoelastic joint decreases with increasing joint thickness and wave frequency, except when the joint thickness and wave frequency match with the pass bands of the corresponding elastic joint. Meanwhile, for an elastic joint, wave attenuation is determined by the ratio of joint thickness to the wavelength, while for a viscoelastic joint, wave attenuation is more sensitive to wave frequency, compared with joint thickness.

When parallel joints exist, multiple wave reflections among joints influence wave transmission. Based on calculation results, it can be concluded that the transmission coefficient decreases with increasing joint number and, the stop-pass behavior with the change of joint spacing for the viscoelastic joints is less significant than that for the elastic joints: at the pass band, the transmission coefficient for the viscoelastic scenario is much smaller than that for the elastic scenario, and contrarily at the stop band, the transmission coefficient for the viscoelastic case is much larger than that the for elastic case.

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