# On the Selection of the Most Appropriate MPC Problem Formulation for Buildings

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**Abstract**

*Model Predictive Control (MPC) for buildings has gained a lot of attention recently. It has been shown that MPC can achieve significant energy savings in the range between 15-30% compared to a conventional control strategy, e.g., to a rule-based controller. However, there exist several reports showing that the performance of MPC can be inferior to that of a well-tuned conventional controller. Possible reasons are at hand: i) minimization is typically not performed over energy but instead over some input quantity that has a different meaning ii) a model mismatch and inaccuracies in weather predictions can cause wrong predictions of future behavior which can result in undesirable behavior of the control signal (e.g. oscillations) and, as a consequence, in increase in energy consumption. This behavior has been observed when applying one of the widely used economic MPC formulation to the building of Czech Technical University in Prague. These oscillations are not an issue for buildings only, but also for every economic MPC that minimizes the absolute value of the control action. In this paper, we discuss all the these aspects of the implementation of MPC on a real building, show and analyze data from MPC operation on the university building and finally propose and validate an MPC formulation that alleviates the sensitivity to model mismatch and inaccuracies in weather predictions.*

***Keywords – Energy Savings; Model Predictive Control; Optimization***

## 1. Motivation

It is a well known fact that in developed countries the energy consumption in buildings accounts for around 40 % of the total final energy and more than half of this amount is consumed in HVAC (Heating, Ventilation and Air Conditioning) systems [1]. Therefore, improvements in algorithms for Building Automation Systems (BAS) can significantly contribute to desired energy savings.

In recent years, there have appeared a lot of simulation or real case studies evaluating advanced control algorithms applied to BAS showing savings potential of these strategies ranging up to 40 %. One of the intensively studied control techniques for BAS is the Model Predictive Control (MPC) [2, 3, 4, 5]. The objective of the MPC algorithm is to optimally select control inputs in such a way that the energy consumption is minimized and, at the same time, comfort requirements are met. In the following, we assume basic familiarity with the MPC control technique (i.e., the notions of objective function, constraints, decision and slack variables, etc.; for details please refers to, e.g., [6]).

Recently, there has been presented a wide variety of papers dealing with MPC applied to control of BAS with the following properties: *i)* The MPC controller takes disturbance predictions (occupancy, weather etc.) into account, adjusting control actions appropriately [2, 3]. *ii)* The thermal mass of the building can be utilized in a better way compared to conventional control strategies [7]. *iii)* Thermal comfort indices can be easily included into the formulation of MPC problem and therefore the performance of MPC can result in a better subjective thermal comfort [8, 9, 10]. *iv)* Variable energy prices can easily be included into the formulation of the optimization problem [11, 12]. *v)* Minimization of the energy peaks can be handled by MPC and thus energy loads can be shifted within certain time frame [3, 13, 14] (beneficial because of both the possibility of tariff selection and lowering operational costs). In the above-mentioned papers, the conclusions are usually drawn from numerical simulations on detailed building models, e.g., EnergyPlus, Trnsys, etc.; however, experimental setups of MPC have also been reported, showing energy savings potential of up to 30 % compared to conventional control strategies [3, 15, 16].

The objective of this paper is, however, slightly different from the objective of the aforementioned ones. Based on the experience from four heating-seasons of MPC deployment on a real pilot building [15], we point out main challenges that mar the idealistic world of MPC encountered in most academic studies. In addition, we propose a new MPC formulation that tries to circumvent these problems.

## 2. Problem Description

From the analysis of a long-term behavior, we can point out the following three main issues that need to be tackled in order to obtain a robust and reliable control strategy:

***Oscillatory behavior:*** The objective of MPC for buildings is to minimize energy consumption and thus reduce the energy bill. As the energy cost is an affine function of energy consumption[1], the MPC problem cost function is typically of the form

$$J = \sum_{k=0}^{N_u-1} |R_k u_k|_1 + \sum_{k=0}^{N_y-1} |Q_k(y_k - y_k^r)|_2^2, \tag{1}$$

where $k$ is the discrete time, $N_u$ and $N_y$ are the control and prediction horizon respectively, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are the vectors of system inputs and outputs respectively, $y^r$ is the vector of the reference trajectories for the output signals and finally, $R_k$ and $Q_k$ are (possibly time-varying) weighting matrices. In this cost function, 1-norm (i.e., the sum of absolute values) of the input energy is to be minimized. However, 1-norm MPC, which can be cast as a Linear Program (LP), always activates some of the constraints as the solution lies on one of the vertices of the constraint polytope and hence such an optimization problem results in a bang-bang-type deadbeat or idle control that is undesirable

[1]The constant term in the affine function represents especially maintenance costs

for buildings [17, 18]. In addition, MPC works in a receding horizon fashion when at every time-step, a finite-horizon optimal control problem (FHOCP) is solved and only the first control move is applied to the system. In the following time step, the next FHOCP is solved with updated measurements, disturbance predictions and comfort requirements. A small change in these parameters may cause an abrupt change in the optimal solution. Sensitivity of the optimal solution to the LP to a parameter change is case-dependent and difficult to assess a priory. Although in general, this sensitivity is higher for 1-norm control problems (leading to LPs) than for problems with a quadratic cost function leading to quadratic programs (QPs). Note that the quadratic norm for the comfort only does not significantly change the 1-norm-like behavior especially when there are few comfort violations (the slack variables are not active and the 1-norm-like behavior dominates). On the contrary, weighting of energy using a quadratic norm leads to a smooth input profile; the problem, of course, is that the energy bill is not proportional to the square of energy.

***Robustness to model inaccuracy and disturbance prediction errors:*** Buildings are complex systems, each is unique and therefore a detailed modeling of every building where MPC shall be applied is economically unjustifiable. Hence one has to expect that the model will always be inaccurate. Disturbance predictions are also subject to (sometimes significant) errors. These facts increase the importance of the two aforementioned issues.

Fig. 1 shows an example of the undesirable behavior recorded during ten days of a normal operation of MPC on our pilot building. Besides disturbances and room temperature that is to be kept at a certain comfort level, we can observe progress of supply water temperature, which is the only manipulated variable that is being computed by MPC. We can observe undesirable oscillatory behavior causing higher energy consumption towards the end of the data series. This behavior happens when a standard 1-norm-like MPC formulation considered in the majority of academic papers is used.

***Recursive feasibility:*** In the literature, various MPC problem formulations for buildings have been proposed (a review will be given in Section 3.). Some of the problem formulations, however, do not guarantee recursive feasibility and therefore cannot be used as a long-term, reliable control strategy.

***Small and high comfort violation:*** In practice, it is acceptable that BAS can cause small violation of comfort but major and/or persistent violations are unacceptable. Freezing occupants are not willing to hear anything about "inaccurate model" or "infeasible optimization problem".

During normal building operation, a reasonable tradeoff between energy consumption and comfort can be found using cost function weighting factors. However, during some special events, these settings can be inappropriate. An example of such event is the Christmas holiday that allows for a long-term setback in the case of university building. At the end of the setback there is a need for enormous amount of energy that has to be delivered into the building

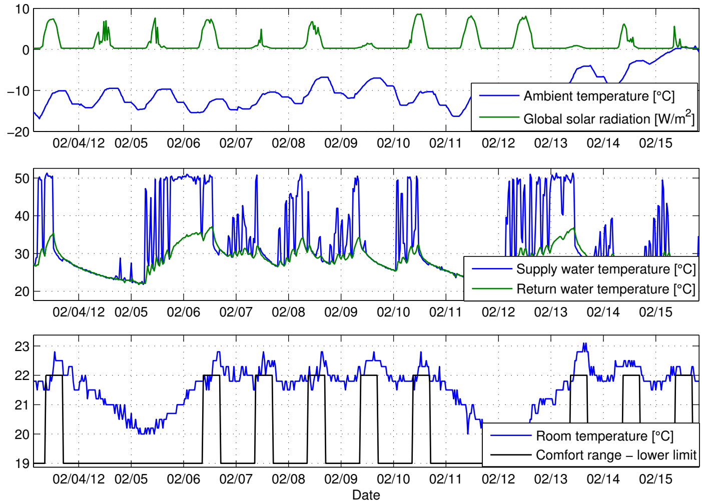



Fig. 1: Oscillations of supply water temperature recorded during MPC operation

in order to return to a "normal operation" of the building. Unfortunately, in the case of our pilot building, the optimal solution that was exercised caused major comfort violations during the first day after the Christmas holiday. Hence, there is a need for definition of comfort requirements that have to fulfilled at any cost.

The paper is further structured as follows: *i)* Section 3. analyzes state-of-the art, what FHOCP formulations are typically used for buildings. These formulations are assessed from the point of view of recursive feasibility, sensitivity to oscillations, etc. *ii)* Section 4. proposes a new FHOCP formulation that addresses the aforementioned issues. *iii)* Section 5. presents a case study where the proposed FHOCP is validated. *iv)* Finally, Section 6. outlines other directions for designing practically robust FHOCP.

## 3. Existing Model Predictive Control Formulations for Buildings

In this section, we present some of the typical optimal control problem formulations for buildings. We restrict ourselves to deterministic, non-hybrid and centralized MPC formulations because such formulations are the most widely used ones in practice. More advanced variants of MPC (e.g. stochastic or distributed) are far more complex to analyze and are left for future investigations.

We assume that the models of the buildings are linear time invariant (LTI) with heat fluxes as inputs and zone temperatures as outputs. The models have the following form:

$$x_{k+1} = Ax_k + Bu_k + Vv_k, \qquad y_k = Cx_k + Du_k + Wv_k, \tag{2}$$

where $v_k \in \mathbb{R}^s$ is the vector of disturbances, $y_k \in \mathbb{R}^p$ is the vector of system outputs and , $x_k \in \mathbb{R}^n$ is the vector of system states. Real matrices $A, B, C, D, V, W$ are so called system matrices and are of appropriate dimensions.

We will start from the formulations that have appeared in the literature. Pros and cons for each of the formulation will be given. Each formulation eliminates some drawbacks of the previous one. In the next section, we then present a new formulation that we deem to be so far the most suitable formulation for buildings (both from the point of view of the quantities being optimized and a practical viewpoint).

***Minimization of delivered energy and satisfaction of the constraints:*** This formulation was reported by [19, 20, 5, 21]. The cost function contains a single term standing for the minimization of the delivered energy while the thermal comfort is guaranteed by means of hard constraints on the system outputs, i.e. zone temperatures.

$$\min_{u} \sum_{k=0}^{N_u-1} |R_k u_k|_1 \tag{MPC1}$$

$$\text{subject to: linear dynamics Eq. (2),} \qquad x_0 = x_{init}, \qquad G_k u_k \leq h, \qquad \underline{r}_k \leq y_k \leq \overline{r}_k.$$

The matrices $G_k, h_k$ define time varying polytopic constraints on system inputs and states, while $\underline{r}_k$ and $\overline{r}_k$ stand for the time varying reference trajectory for the system outputs. Initial state $x_{init}$ is a parameter of the optimization and is provided by means of Kalman filter or full state measurement at each time-step.

Although the presented control strategy was advertised as a “new control strategy suitable for MPC for buildings” [20], from our experience, this optimal control problem formulation as is cannot be used in the practice. The most obvious drawback is the lack of recursive feasibility: if the initial state implies any comfort violation, then the optimization problem will be infeasible and the controller cannot work anymore. Feasibility issues are usually handled with the aid of the so-called slack variables on system states and system outputs. Hard constraints are imposed only on the system inputs.

***Trade-off between energy consumption and set-point tracking error:*** An alternative simple MPC formulation that tackles feasibility issues was presented in [4, 9, 22, 23, 24, 14] and has following form[2]:

$$\min \sum_{k=0}^{N_u-1} |R_k u_k|_1 + \sum_{k=0}^{N_y-1} |Q_k(y_k - r_k)|_2^2 \tag{MPC2}$$

$$\text{subject to: linear dynamics Eq. (2),} \qquad x_0 = x_{init}, \qquad G_k u_k \leq h,$$

Here, $r$ is the set-point which is to be tracked. Although this formulation has the form that is typically used in process industry [6], it is not suitable for buildings. According to standards defining indoor thermal comfort, operative temperature[3] should lie within certain temperature range. Forcing the temperature to follow

[2]Some authors use without any reasoning quadratic norm for penalization of input energy instead of one norm

[3]Operative temperature is defined as the average of the air temperature and the mean radiant temperature (i.e. usually computed as area weighted mean temperature of the surrounding surfaces)

a single set-point curtails the freedom of the controller and may result in a higher energy consumption. In addition, typical effects of MPC regulation like a night-time pre-cooling or pre-heating are suppressed. A similar formulation with the aid of slack variables can add the desired freedom to the MPC controller.

***Trade-off between energy consumption and comfort range violations:*** Slack variables are additional decision variables that are weighted only in situations when some quantity, which the slack variable is imposed on, reaches certain bound. They are useful especially in situations when the objective is to keep system outputs within a certain range and the slacks penalize the violation of the range. The following formulation was presented in [15, 16, 25, 2, 26].

$$\min \sum_{k=0}^{N_u-1} |R_k u_k|_1 + \sum_{k=0}^{N_y-1} |Q_k(y_k - z_k)|_2^2 \tag{MPC3}$$

$$\text{subject to: linear dynamics Eq. (2),} \qquad x_0 = x_{init},$$
$$G_k u_k \leq h, \qquad \underline{r}_k \leq z_k \leq \overline{r}_k$$

In this optimal control problem setup $z_k \in \mathbb{R}^p$ is the slack variable on the zone temperature. The advantages of such a formulation has already been discussed; however, the use of the 1-norm to weight the system inputs is a major disadvantage. As it is well known, the solution of a linear program lies on a vertex of the polytopic constraint set. If the constraints are not very tight, a bang-bang control profile is obtained. This behavior is undesirable in closed loop operation in the presence of model mismatch because then the control actions can lead to a highly oscillatory behavior (see Fig. 1). Unpleasant oscillations can be suppressed by introducing hard constraints on the maximum rate of change of the input signals. But what if, accidentally, there is a strong need to heat up the building and to use the maximum capacity of the heating system immediately? This problem as well as other aforementioned issues are handled in the optimal control problem formulation given in the following section.

## 4. Practical Aspects Motivated Formulation

In this section, we introduce a new MPC formulation that is motivated by practical aspects. The aims of this formulation are (i) suppress oscillation appearing in receding horizon due to minimization of the 1-norm of the input signal, (ii) minimize sensitivity of the controller to the model mismatch and imperfect disturbance predictions while making use of minimal additional energy, (iii) guarantee recursive feasibility, (iv) respect thermal comfort limits defined by standard norms e.g. ISO 7730 and guarantee that significant comfort range violations do not occur, (v) does not increase the numerical complexity of the problem significantly.

The proposed formulation has the following form:

$$\min \sum_{k=0}^{N_u-1} \left(|R_k u_k|_1 + \delta \text{smooth}(k)\right) + \sum_{k=0}^{N_y-1} \left(|Q_k(y_k - z_k)|_2^2 + |Q_k^c(y_k - z_k^c)|_2^2\right) \tag{MPC4}$$

subject to: linear dynamics Eq. (2), $\quad x_0 = x_{init}, \quad u_{last} = \{u_{-1}, u_{-2}, \ldots\}$

$$G_k u_k \leq h, \qquad \underline{r}_k \leq z_k \leq \overline{r}_k, \qquad \underline{r}_k^c \leq z_k^c \leq \overline{r}_k^c$$

Here, $z_k \in \mathbb{R}^p$ and $z_k^c \in \mathbb{R}^p$ are slack variables and together with $\underline{r}_k, \overline{r}_k$ define comfort constraints that can be violated from time-to-time, while $\underline{r}_k^c, \overline{r}_k^c$ define comfort constraints that cannot be violated at any cost. These comfort constraints give the MPC controller sufficient freedom to operate the building in an energy-efficient way.

It is expected that system inputs and outputs are scaled to a similar range of values and that $\max(R_k, \delta_k, Q_k) \ll Q_k^c$.

Recursive feasibility of this formulation is guaranteed as there are no hard constraints imposed on system states nor system outputs.

Finally, the objective of the smoothing term is to suppress oscillations in receding horizon as well as on prediction horizon. Here it is important that there is the term $u_{last}$ holding information about past system inputs that were computed by MPC. Based on these values, we can easily smooth the receding horizon progress of the input signal. We propose following variants of smoothing terms:

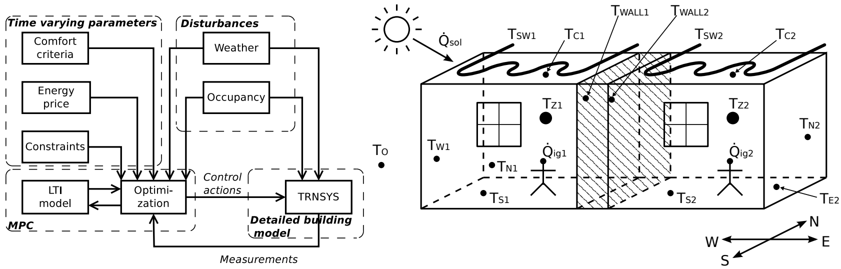
- MPC4a: $\text{smooth}(k) = |Zu_k|_2^2$, i.e. the problem is regularized in such a way that not only the one norm of the input signal is minimized, but also quadratic norm is minimized. Here, $Z$ is an appropriate weighting matrix.
- MPC4b: $\text{smooth}(k) = |u_k - u_{k-1} - p_k|_2^2$ and one additional constraint is introduced $\underline{\Delta u} \leq p_k \leq \overline{\Delta u}$ for $k = 1 \ldots N_u$. Here $\underline{\Delta u}, \overline{\Delta u}$ are minimum/maximum values allowed for the input change not to be penalized, $p$ is a slack variable and thus the square of the inner term regularizes the optimization task.
- MPC4c: $\text{smooth}(k) = |u_{k-2} - 2u_{k-1} + u_k|_2^2$, i.e. minimization of curvature of the input signal. Here, it is required to know two of the past inputs.

In the following section, we will compare the three proposed smoothing terms to the presented MPC formulations without any smoothing term.

## 5. Case Study: Validation of the Proposed MPC Formulation

For validation of the proposed MPC formulation, we will use a TRNSYS simulation environment. Schematically, the simulation setup is depicted in Fig. 2a. In the core, there is a detailed TRNSYS model sharing the same disturbance profiles (occupancy and weather for Prague, Czech Republic) as the MPC part which is composed of an optimization block that uses linear time-invariant (LTI) model for performing the numerical optimization. Time-varying parameters (e.g. variable energy price or reference trajectories etc.) are required by the MPC block. The setup is designed in such a way that the problems with model mismatch causing oscillations may appear. Disturbance prediction errors are not considered here.

The building under investigation, schematically outlined in Fig. 2b, was constructed in TRNSYS environment using Type56. It is a medium weight office building with two zones separated by a concrete wall and with thermo-active building systems (TABS) controlled separately. Both zones have the same




(a) Simulation environment (b) A scheme of the modeled building

Fig. 2: Simulation setup

Table 1: Performance comparison of MPC formulations. In relative comparisons, the results are always compared to the MPC3.

| | Delivered energy [kWh/month] | | Comfort violations [Kh/month] | |
|---|---|---|---|---|
| | Aboslute | Relative | Absolute | Relative |
| MPC1 | – | – | – | – |
| MPC2 | 5555 | -111 % | 0 | -100 % |
| MPC3 | 2631 | 0 % | 3.22 | 0 % |
| MPC4a | 1946 | 26 % | 3.43 | 6 % |
| MPC4b | 2355 | 10 % | 4.61 | 43 % |
| MPC4c | 1885 | 28 % | 4.45 | 38 % |

dimensions ($5 \times 5 \times 3\,\mathrm{m}$) and the south oriented walls of the zones include a window ($3.75\,\mathrm{m}^2$). A detailed description of the building is given in [27].

The LTI model Eq. (2) of the system was identified using grey box technique adopted from [15, Section 3.1.2] and verification of its accuracy is given in [27].

The performance of the presented MPC formulations were validated on one month simulations within the simulation environment. For evaluation of thermal comfort, ISO 7730 class B was used. The results of all formulations are summarized in Table 1 and Fig. 3.

As already noted, MPC1 does not guarantee recursive feasibility. This was confirmed by a simulation that crashed at simulation time $T_s = 16\,\mathrm{h}$. The state of the LTI model ended up out of the allowed range, and hence the optimization problem became infeasible.

The objective of MPC2 is to track a set point – in our case, the average of the lower and upper comfort limits. This fact caused a significant increase in energy consumption. In addition, oscillations described above were observed (due to space limitation, simulation results for MPC2 are not reported in Fig. 3).

Formulation MPC3 is taken as a baseline for all comparisons in the Table 1. From Fig. 3, it can be seen that the oscillations occurring on the CTU building appears also here and especially over the weekend (1/7 and 1/8) when there is a long setback. In such a situation, solution is either to heat at the maximum possible level or to do nothing. Such behavior naturally increases the long-term energy consumption.

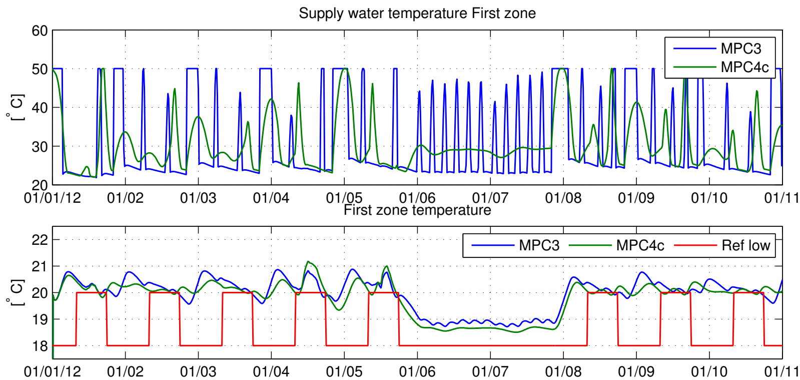



Fig. 3: Comparison of timeseries

On the contrary, MPC4 in all variants achieves better results in terms of energy consumption with comparable amount of comfort violations. MPC4b reaches a slightly higher consumption than the other formulations with smoothing terms. In this case, the amplitude of the oscillations is suppressed, not the oscillations as such.

## 6. Conclusions and Remarks

In this paper, we analyzed existing MPC problem formulations for buildings. We mentioned pros and cons for each of them and based on this analysis and past experience we proposed a new MPC formulation that (i) is not oscillatory (in both open- and closed-loop operation) due to smoothing terms introduced in the cost function, (ii) is sufficiently robust to disturbance predictions and model inaccuracies, (iii) guarantees recursive feasibility of the optimization problem, (iv) respects user-defined comfort limits in such a way that it is high probable that high comfort violations do not occur, (v) does not increase significantly the energy consumption, (vi) does not increase the numerical complexity of the problem significantly – the problem stays in the same class of convex optimization problems, (vii) is able to capture small and high comfort violations, thereby ensuring that high comfort violations do not occur at any cost. A disadvantage of the proposed algorithm is the increased number of tuning parameters. Typically, there are two weighting coefficients (the matrices $Q$ and $R$); the proposed formulation has three. Tuning of the third, smoothing, variable is essential for achieving the benefits described above; an improperly tuned smoothing term can either lead to too oscillatory or too smooth (and hence energy-inefficient) behavior.

Finally, the proposed MPC problem formulations were validated within a TRNSYS simulation environment, showing that the introduced smoothing terms can significantly contribute to the robustness of the MPC for buildings.

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