Proceedings of Meetings on Acoustics

Volume 19, 2013 http://acousticalsociety.org/

ICA 2013 Montreal
Montreal, Canada
2 - 7 June 2013

Noise
Session 2aNSa: Transportation Noise

2aNSa2. Pass-by noise acoustic sensing for estimating speed and wheelbase length of two-axle vehicles.

Patrick Marmaroli*, Jean-Marc Odobez, Xavier Falourd and Herve Lissek

***Corresponding author's address: Laboratory of Electromagnetism and Acoustics (LEMA), Swiss Federal Institute of Technology (EPFL), Lausanne, 1015, Vaud, Switzerland, patrick.marmaroli@epfl.ch**

This paper focuses on acoustic road traffic monitoring and looks, more specifically, into the problem of joint speed and wheelbase length estimation of two-axle vehicles as they pass by. It is known that both front and rear axle trajectories may be observed using a cross-correlation based processing in conjunction with a well designed two-element microphone array placed on the roadside. This is mainly due to the broadband nature of the tyre/road noise which makes two peaks appear in, one per axle, in the correlation function. In a former work, we proposed to conduct this double-peak-tracking problem using a specific particle filter that model road vehicles as bimodal sound sources (bimodal particle filter). After a brief theoretical recall of the method, this paper stresses on the recent preliminary results we obtained from simulation and in-situ experiments.

Published by the Acoustical Society of America through the American Institute of Physics

 [DOI: 10.1121/1.4800396]
Received 22 Jan 2013; published 2 Jun 2013

I. INTRODUCTION

Road Traffic Monitoring (RTM) plays a key role in ensuring road safety, regulating the traffic, improving the reactivity of rescue teams or enabling the future infrastructure investments to be optimized. Equipments dedicated to RTM are numerous and have been investigated through many comparative technical studies in the last decade, in terms of cost, performances, ease of use and so on [1–3]. Amongst existing techniques, passive acoustic ones present the advantage of being non-intrusive, safe for health (no wave emission) and of multiple usage meaning that different kinds of information can be extracted from the same observation depending on the associated processing algorithm. Thus, taking advantage of the power of modern-day computing, a large community of acoustic researchers are working on the challenge of equalling, or even outperforming, the performance of active and/or intrusive technologies with passive ones.

There has been a growing interest in passive acoustic-based systems for vehicle monitoring since the mid 1990s, comprising vehicle detection [4–7], vehicle classification [8], traffic density estimation [9–14], speed estimation [15–20] and also energy consumption estimation using sound [21]. In this paper, we investigate a particle filtering-based technique for jointly estimating speed and wheelbase length of two-axle vehicles as they pass by. The audio signal, commonly known as “pass-by noise”, is acquired with two microphones placed on the roadside. What is filtered is the cross-correlation between the two recordings. Under certain measurement conditions, front and rear axle trajectory over time are clearly distinguishable using cross-correlation, making both their distance and speed estimable.

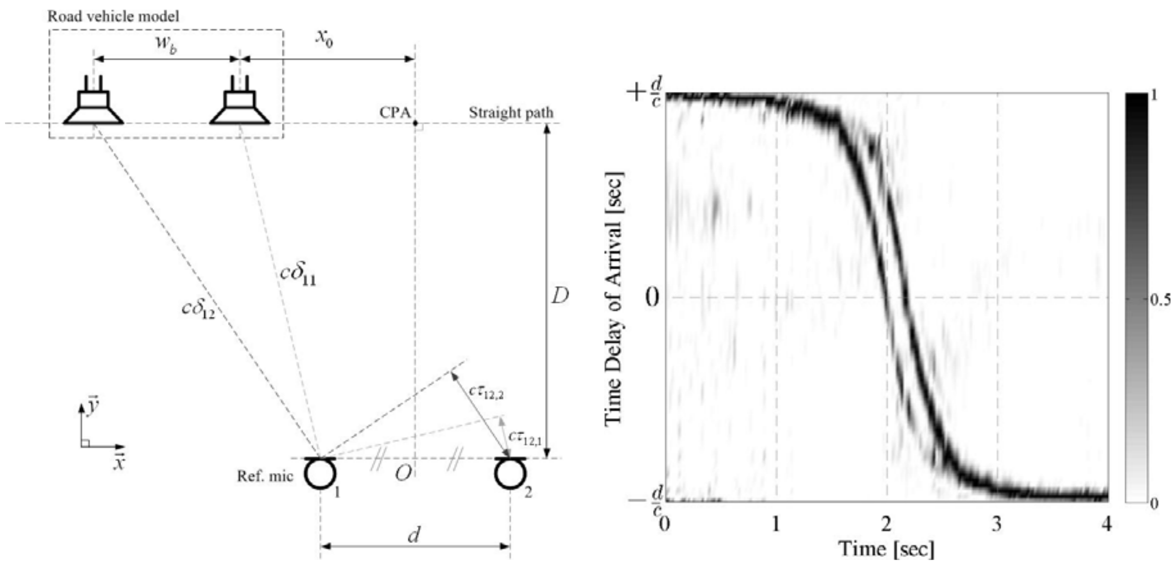
Automatic and acoustic-based procedures dedicated to wheelbase estimation are rarely proposed in the literature. Yet, wheelbase length is an important feature for vehicle classification since it is related to the vehicle mensurations. At the best of our knowledge, the only antecedents works focusing on wheelbase estimation through acoustic sensing are due to V. Cevher *et al.* [20]. Authors developed a wave-pattern recognition-based algorithm allowing the joint speed and wheelbase estimation using a one-channel pass-by recording acquired on the roadside. Engine, tyre, exhaust and air turbulence noises were meticulously modeled. Tyre/road noise directionality, interferences between tyres, microphone directionality and frequency response, were also taken into account. In a totally opposite philosophy, we limit our model to the minimum *a-priori* knowledge of two-axles. This choice is mainly motivated by our experience of real world signals that may be strongly affected by interfering noises or other vehicles in the monitored area. In such cases, resorting to a too precise model may limit the practical applicability of the algorithm. Secondly, the simpler the model, the larger the potentiality to extend it for other applications is.

II. SIGNAL MODEL AND OBSERVATION FUNCTION

The scenario of interest is illustrated in Fig. 1(a). We limit the study to road vehicles that can be acoustically modeled by two static monopoles radiating stochastic and identically distributed sounds separated by a wheelbase length wb in the x-y plane. In practice, cars of vans moving at 50 km/h or more, without much acceleration, fit such a model well since tyre/road noise is predominant over mechanical noise at such speeds. The audio signal is acquired by a two-element microphone array with known spacing d, placed in parallel to the lane, at a distance D to the vehicle's Closest Point of Approach (CPA); x_0 denotes the distance between the front rear and the CPA. In what follows, digital audio signal are partitioned in short frames (30-40 ms each) within vehicle is supposed to be static, so that the two signals belonging to the same frame are modeled as:

$$y_1(t) = s_1(t-\delta_{11}) + s_2(t-\delta_{12}) + n_1(t), \quad (1)$$

$$y_2(t) = s_1(t-\delta_{11}-\tau_{12,1}) + s_2(t-\delta_{12}-\tau_{12,2}) + n_2(t), \quad (2)$$



(a) (b)

FIGURE 1: a. bimodal sound source model of a two-axle road vehicle, wavefronts are acquired by a microphone array placed in parallel to the road lane. b: typical CCTS of a road vehicle pass-by (about 50 km/h).

where s_1, respectively s_2, is the sound wave produced by the front, respectively rear, tyre/road interaction; δ_{11}, respectively δ_{12}, is the time of flight of sound to travel from the front, respectively rear, axle to the first microphone; $\tau_{12,1}$, respectively $\tau_{12,2}$, is the Time Delay Of Arrival (TDOA) of the front, respectively rear, sound wave between the two sensors; n_1, respectively n_2, is an additive noise due to the first, respectively second, channel of the acquisition device, it is considered as a stochastic, stationary, zero-mean Gaussian signal, uncorrelated both with the signals and noise at other sensor.

The recordings are partitioned in frames in which the generalized cross-correlation function with phase transform (GCC-PHAT) [22] is applied. The concatenation of these correlation measurements gives a cross-correlation time series (CCTS) with two dimensions: TDOA versus time. As an example, Fig. 1(b) depicts such a PHAT-CCTS corresponding to a vehicle which passes by at nearly 50 km/h. Two traces are clearly distinct between 1.5 second and 2.5 second, that is when the vehicle is in front of the array. The slope of both traces is related to the vehicle speed, and their space is related to the wheelbase length.

III. TRACKING METHODOLOGY

Due to the outdoor conditions, CCTS are frequently corrupted because of interfering noises (environmental, industrial, pedestrian...) or because of the presence of multiple vehicles in the monitoring zone (crossings). One solution consists in dissociating "good" from "bad" traces in the CCTS by discriminating those following a well-established dynamical model from the others. This is the strong idea brought by the Bayesian theory, forming the basis of most tracking algorithms, and that we propose to apply in the traffic flow monitoring context.

Particle filtering (PF) is a successful Bayesian-based technique to recursively estimate hidden states of non-linear, non-Gaussian dynamical systems [23]. To summarize, one particle is composed of a state value, *i.e.* an hypothesis, and an associated weight, *i.e.* the *probability* that this hypothesis is true regarding the observation. Recursively, each particle is propagated by following an *a priori* dynamical model disturbed by stochastic noise and the associated weights are updated according to the observation. The more the state of a particle matches with the observa-

	Actual states	*A priori* states	Initial stand. dev.	Dynamical noise
Simulation	x_0 = -3 m	$\mu_{x,0}$ = -3 m	$\sigma_{x,0}$ = 0.1 m	$\sigma_x = \sigma_{x,0}/200$
	y_0 = 3.5 m	$\mu_{y,0}$ = 3.5 m	$\sigma_{y,0}$ = 0.1 m	$\sigma_y = \sigma_{y,0}/200$
	$\dot{x}$ = 50 km/h	$\mu_{\dot{x},0}$ = 20 km/h	$\sigma_{\dot{x},0}$ = 20 km/h	$\sigma_{\dot{x}} = \sigma_{\dot{x},0}/200$
	wb = 2.5 m	$\mu_{wb,0}$ = 1.5 m	$\sigma_{wb,0}$ = 0.4 m	$\sigma_{wb} = \sigma_{wb,0}/400$

TABLE 1: Parameters of the bimodal particle filtering used for simulation tests.

tion, the heavier the weight associated to this particle, and the more this particle is duplicated in favor of the lighter ones. The number of particles is defined by the operator and stays constant during all the observations. Four main elements are important in defining a PF:

- the state model, that is, the abstract representation of the object we are interested in;
- the dynamical model governing the temporal evolution of the state;
- the likelihood model measuring the adequacy of the data given the proposed configuration of the tracked object;
- a proposal distribution that role is to propose new configurations in high likelihood regions of the state space.

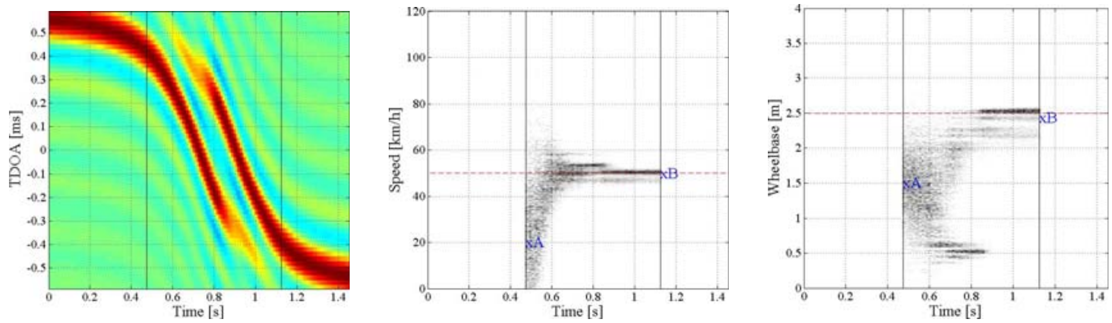
In the current case, we use the standard bootstrap filter, in which the dynamical model is used as proposal. Target, dynamical and likelihood model used in what follows are those defining the bimodal particle filter (BPF) of Marmaroli *et al.* in [24]. To summarize, the target model is composed of four states: abscissa, ordinate, speed and wheelbase length; the dynamical model supposes the target runs at a constant speed during the observation; and the likelihood model is built from the projection of the target states onto the CCTS.

IV. SIMULATION

The BPF of [24] is assessed through an *in-silico* experiment using statistical parameters of Table 1. Fig. 2(a) depicts the CCTS on which particles are launched. This CCTS is build from an analytical expression of the GCC-PHAT and considering the actual states values of Table 1. Fig. 2(b) and Fig. 2(c) depict the distributions of the particles as a function of time for speed and wheelbase states respectively. Only the part between the two black lines constitutes the observation here. At $t = 0$ (first black line), speed and wheelbase states are drawn from the Gaussian distribution $\mathcal{N}(\mu_{\dot{x},0}, \sigma_{\dot{x},0})$ and $\mathcal{N}(\mu_{wb,0}, \sigma_{wb,0})$ respectively. For demonstration purpose, the *a priori* speed and wheelbase, $\mu_{\dot{x},0}$ and $\mu_{wb,0}$ - denoted by blue crosses A - are clearly below their actual values - denoted by red dashed lines. After a few iterations, particles converge properly towards their respective target values. One possible way to build an estimate therefore simply consists in computing the mean of the particle distribution at the end of the tracking (second black line). This is precisely what the blue crosses B in Fig. 2(b) and Fig. 2(c) denote.

Results

The performance of this scenario is performed over N_{test} = 100 runs. For speed, we obtained a global error of -1.1 km/h and a global standard deviation of 1.7 km/h. For wheelbase, we obtained a global error of -17 cm with a global standard deviation of 20 cm. In this example, the performance is promising knowing that the *a priori* values were quite far from the actual ones (-30



(a) observation (b) state *speed* (c) state *wheelbase*

FIGURE 2: Typical example of a tracking result applied to speed estimation. The observation likelihood function is delimited by the two vertical black lines on the CCTS (a). (b) represents the evolution of the speed state histogram with a false *a priori* starting.

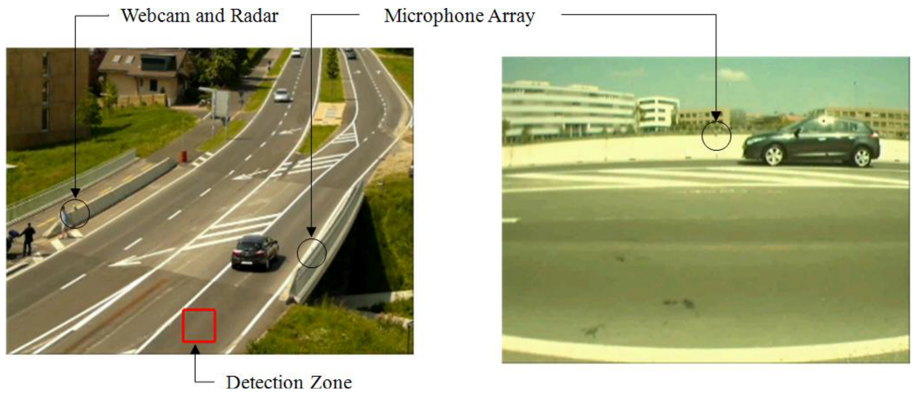
km/h and -100 cm for speed and wheelbase respectively). The repeatability of the speed estimate is very good (1.7 km/h of standard deviation only). The relative standard deviation achieved by the wheelbase length estimator is larger but stays below 10% of the true value.

Looking attentively at Fig. 2(a), one can note that the wheelbase information is strongly expressed when the vehicle is close to its CPA only, namely between 0.7 seconds and 0.9 seconds approximatively in this example. This is a rather short time interval for the particles to converge. On the other hand, the information on speed is always present during the observation. This explains in part why best estimates are obtained for speed than for wheelbase length, and also why particles for speed, Fig. 2(b), converge quicker than particles for wheelbase, Fig. 2(c).

Influence of the parameters

According to Table 1, the operator needs to adjust at least 12 parameters. This number is actually much more important in practice since the number of particles, frame size, distance to the road, inter-sensor distance and so on had to be optimized also. As highlighted by Lichtenauer *et al.* [25] and Abbott *et al.* [26], research works focusing on how the observation and/or statistical parameters affect the tracking performance are rare. Inspired by these two pioneering papers, some *in-silico* tests were carried out in order to assess the influence of three different parameters on speed estimation. They are the number of particles, the difference between *a priori* target position and actual one at initialisation, and the difference between the *a priori* distance to the tyres and the actual one. Results obtained from these tests are discussed in more detail in [27]. They are summed up below.

The number of particles (N_p) It is known that the estimation accuracy of the posterior increases [28] and the risk of loss of tracking decreases [29] as the number of particles (N_p) increases. On the other hand the complexity of the algorithm, and consequently the computation time, increases linearly with N_p [30], so that the practitioner should properly adjust N_p by considering both the execution time and tracking performance in the light of the available CPU ressources. As expected by the theory, we observed that the execution time of the BPF evolves linearly with the number of particles. But mean errors and standard deviations of estimates follow an asymptotic behavior and remains constant as N_p increases. This asymptotic behavior is due to the dynamical noise injected at each iteration, forcing particles to explore states around the tracked mode even if this latter is very sharp. One can conclude that above a certain threshold, increasing the number of particles is not determinant for the particle filtering behavior. This is reminiscent of observations made by Burguera *et.al* in [31].



(a)

FIGURE 3: Views of the two cameras (top and side), location of the microphone array and radar are highlighted by black circles.

***A priori* initial position ($\mu_{x,0}$ vs. x_0).** The knowledge of the initial abscissa is a very critical point. Since our observations, the error on speed increases quickly as soon as the *a priori* initial position of the front axle is a few centimeters away the actual one. In particular, we observed an overestimation of the speed when the target position is underestimated ($\mu_{x,0} < x_0$) and inversely.

***A priori* distance to the tyres ($\mu_{y,0}$ vs. D)** On the ground, D can be roughly measured using a measuring tape or a laser range finder but this value actually varies from several tens of centimeters as the distance from the roadside is different for each motorist. We observed that an underestimation of the distance to the tyres ($\mu_{y,0} < D$) involves poorer results on speed estimation than with an overestimation ($\mu_{y,0} > D$). If D is underestimated, particles initially follow a horizontal line on the CCTS inducing a quick loss of the observation. After some iterations, particle resampling is not ruled by the CCTS and the particles simply follow their initial model quite independently from the observation, which results in a large variance of the estimates because of the stochastic nature of the process. If D is overestimated, the model is incorrect again. But the trajectory of particles will intersect the traces, giving the possibility to the particles to track the traces again and returning a better final result.

V. PRELIMINARY EXPERIMENTS

A real audio recording database was collected on 25^{th} May 2012 at the Route Cantonale of Ecublens, near the EPFL campus, Switzerland (Lat. 46°31'0.28"N, Long. 6°33'50.41"E). Microphones were disposed on the roadside at a height of 84 cm, an average distance of D = 2.5 m to the vehicles closest wheels, and with an inter-sensor distance d of 20 cm. The array was situated between a traffic roundabout (120 meters upstream) and a traffic light (345 meters downstream). Vehicle speed ranged between 50 km/h to 75 km/h. The location was free from reverberation, the nearest building being distant of 30 meters. The day was warm and windless, and the sky was generally clear.

The audio sampling rate was 51.2 kHz, the quantification was 24 bits. A standardized traffic counter, Viacount II, was placed on the opposite shoulder. The Viacount II provides speed (in km/h), direction (sign of the speed) and length (in number of reflected pulses) of vehicles. The scene was continuously filmed by two cameras, one placed on the road side near the radar to get a view of the sides of all the vehicles and another placed on the balcony of a nearby building to get a more global view of the scene. Both devices produced video at 30 frames per second. Fig. 3(a) depicts the two views provided by cameras and the location of the microphone array and radar.

	A priori states	Initial stand. dev.	Dynamical noise
Experiment	$\mu_{x,0}$ = -7 m	$\sigma_{x,0}$ = 1 m	σ_x = 0.001 m
	$\mu_{y,0}$ = 2.5 m	$\sigma_{y,0}$ = 0.1 m	σ_y =0.001 m
	$\mu_{\dot{x},0}$ = 60 km/h	$\sigma_{\dot{x},0}$ = 10 km/h	$\sigma_{\dot{x}}$ = 0.5 km/h
	$\mu_{wb,0}$ = 2.25 m	$\sigma_{wb,0}$ = 0.2 m	σ_{wb} = 0.025 m

TABLE 2: Parameters of the bimodal particle filtering used for real experiments.

Only the right-hand traffic lane is considered in this experiment, namely the lane where a black vehicle is present on Fig. 3(a). An home-made detection algorithm was implemented to return the apparition time of each new vehicle in this lane through "successive image differences" considering pixels within the red square of Fig. 3(a).

Presented results in this paper come from a recording of 240 seconds. During this time, 22 cars and 2 motorbikes were detected. The brand and model of each vehicle was identified manually using the movies so that their actual wheelbase length in meter is also known in addition to their speed and time of apparition.

Results and discussion

The bimodal particle filter of [24] has been applied on the CCTS of each pass-by with the parameters summarized in Table 2. Statistical performances are established over 200 runs. Promising results have been obtained: the absolute difference between actual and estimated wheelbase length, respectively speed, is below 30 cm for 91% of the two-axle detected vehicles (motorbikes excluded), respectively below 5 km/h for 75% of all the vehicles.

We observed that the greatest errors are essentially due to the detection step. The adopted detection strategy consisted here in waiting that the vehicle was completely out of the red square of Fig. 3(a) to launch the particles. As a consequence, the actual position of the front axle at initialisation with respect to the array depends on the length of the vehicle. Adjusting, at hand, the initial *a-priori* position of a target for fitting with its actual position always permitted to significantly improve the performance for smaller or larger vehicles than expected ones (moto, long van). This observation is reminiscent to what have been predicted by simulation about the influence of a difference between *a priori* and actual target position at initialisation (Section IV).

VI. CONCLUSION

In this paper, we presented and assessed a microphone array-based technique aiming at estimating speed and wheelbase of road vehicles that pass by. The method is valid for two-axle vehicles. The main assumption is that the pass-by noise is mainly composed of tyre/road noise during the observation. The microphone array can be composed of two microphones only. The trajectography of each axle is observed through the concatenation of successive cross-correlation measurements applied on short frames. This observation is filtered over time by a Bayesian filter specifically designed for tracking the two traces of the observation (bimodal particle filter). Promising preliminary results through simulation and real data have been obtained and presented. The influence of some parameters that had to be adjusted by the operator has been assessed. Further research will be mainly focused in understanding the influence of the processing parameters on the algorithm performance (*e.g.* number of particles, false initial state, length of frames, distance to the road etc.). It is also expected to improve the detection step since being today the weak link of the whole procedure.

References

[1] L. Klein, M. Mills, and D. Gibson, “Traffic detector handbook: Third edition - volume i”, Technical Report, Federal Highway Administration (2006).

[2] L. E. Y. Mimbela, L. A. Klein, P. Kent, J. L. Hamrick, K. M. Luces, and S. Herrera, “A summary of vehicle detection and surveillance technologies used in intelligent transportation systems”, Technical Report, Funded by the Federal Highway Administration’s Intelligent Transportation Systems Joint Program Office, produced by The Vehicle Detector Clearinghouse (2000).

[3] M. Hallenbeck and H. Weinblatt, “Equipment for collecting traffic load data”, Technical Report, Transportation Research Board of the National Academies (2004).

[4] S. Chen, Z. Sun, and B. Bridge, “Automatic traffic monitoring by intelligent sound detection”, in *Proceedings of the IEEE Conference on Intelligent Transportation System (ITSC)*, 171–176 (1997).

[5] J. F. Forren and D. Jaarsma, “Traffic monitoring by tire noise”, in *Proceedings of the IEEE Conference on Intelligent Transportation Systems (ITSC)*, 177–182 (1997).

[6] E. Brockmann, B. Kwan, and L. Tung, “Audio detection of moving vehicles”, in *Proceedings of IEEE International Conference on Systems, Man, and Cybernetics.*, volume 4, 3817–3821 (1997).

[7] J. P. Kuhn, B. C. Bui, and G. J. Pieper, “Acoustic sensor system for vehicle detection and multi-lane highway monitoring”, (1998).

[8] H. C. Choe, R. E. Karlsen, G. R. Gerhart, and T. J. Meitzler, “Wavelet-based ground vehicle recognition using acoustic signals”, Proceedings of SPIE **2762**, 434–445 (1996).

[9] S. Chen, Z. Sun, and B. Bridge, “Traffic monitoring using digital sound field mapping”, IEEE Transactions on Vehicular Technology **50**, 1582–1589 (2001).

[10] K. Kodera, A. Itai, and H. Yasukawa, “Approaching vehicle detection using linear microphone array”, in *Proceedings of International Symposium on Information Theory and Its Applications (ISITA)*, 1–6 (2008).

[11] C. Kwak, M. Kim, K. Kim, S. Hong, and K. Kim, “Robust in-situ vehicle detection algorithm with acoustic transition bandpass filter”, (2009).

[12] N. Shimada, A. Itai, and H. Yasukawa, “A study on using linear microphone array-based acoustic sensing to detect approaching vehicles”, in *Proceedings of International Symposium on Communications and Information Technologies (ISCIT 2010)*, 182–186 (2010).

[13] B. Barbagli, L. Bencini, I. Magrini, G. Manes, and A. Manes, “A real-time traffic monitoring based on wireless sensor network technologies”, Proceedings of the 7^{th} International Wireless Communications and Mobile Computing Conference (IWCMC) 820–825 (2011).

[14] V. Tyagi, S. Kalyanaraman, and R. Krishnapuram, “Vehicular traffic density state estimation based on cumulative road acoustics”, IEEE Transactions on Intelligent Transportation Systems **13**, 1156–1166 (2012).

[15] J. C. Hassab, B. W. Guimond, and S. C. Nardone, “Estimation of location and motion parameters of a moving source observed from a linear array”, The Journal of Acoustical Society of America **70**, 1054–1061 (1981).

[16] J. Towers and Y. Chan, "Passive localization of an emitting source by parametric means", in *International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, volume 5, 2791–2794 (1990).

[17] C. Couvreur and Y. Bresler, "Doppler-based motion estimation for wide-band sources from single passive sensor measurements", in *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, volume 5, 3537–3540 (1997).

[18] R. López-Valcarce, C. Mosquera, and F. Pérez-González, "Estimation of road vehicles speed using two omnidirectional microphones: a maximum likelihood approach", EURASIP Journal on Applied Signal Processing **8**, 1059–1077 (2004).

[19] O. Duffner, N. O'Connor, N. Murphy, A. Smeanton, and S. Marlow, "Road traffic monitoring using a two-microphone array", in *Audio Engineering Society, Convention 118*, 6355 (2005).

[20] V. Cevher, R. Chellappa, and J. McClellan, "Vehicle speed estimation using acoustic wave patterns", IEEE Transactions on Signal Processing **57**, 30–47 (2009).

[21] A. Can, L. Dekoninck, M. Rademaker, T. V. Renterghem, B. D. Baets, and D. Botteldooren, "Noise measurements as proxies for traffic parameters in monitoring networks", Science of The Total Environment **410-411**, 198–204 (2011).

[22] C. Knapp and G. Carter, "The generalized correlation method for estimation of time delay", IEEE Transactions on Acoustics, Speech and Signal Processing **24**, 320–327 (1976).

[23] A. Doucet, N. de Freitas, and N. Gordon, *Sequential Monte Carlo Methods in Practice* (Springer) (2001).

[24] P. Marmaroli, J.-M. Odobez, X. Falourd, and H. Lissek, "A bimodal sound source model for vehicle tracking in traffic monitoring", in *Proceedings of the 19th European Signal Processing Conference (EUSIPCO)*, 1327–1331 (2011).

[25] J. Lichtenauer, M. Reinders, and E. Hendriks, "Influence of the observation likelihood function on particle filtering performance in tracking applications", in *Proceedings of the Sixth IEEE International Conference on Automatic Face and Gesture Recognition, 2004*, 767–772 (2004).

[26] J. T. Abbott and T. L. Griffiths, "Exploring the influence of particle filter parameters on order effects in causal learning", in *Proceedings of the 33rd Annual Conference of the Cognitive Science Society* (2011).

[27] P. Marmaroli, "Bimodal sound source tracking applied to road traffic monitoring", Ph.D. thesis, Ecole Polytechnique Fédéral de Lausanne (2013).

[28] V. Cevher and J. McClellan, "Fast initialization of particle filters using a modified metropolis-hastings algorithm: mode-hungry approach", in *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, volume 2, 129–132 (2004).

[29] M. Bolic, S. Hong, and P. M. Djuric, "Performance and complexity analysis of adaptive particle filtering for tracking applications", in *Proceedings of the 36th Asilomar Conference on Signals Systems and Computers*, volume 1, 853–857 (IEEE) (2002).

[30] F. Gustafsson, "Particle filter theory and practice with positioning applications", IEEE Magazine in Aerospace and Electronic Systems **25**, 53–82 (2010).

[31] A. Burguera, Y. González, and G. Oliver, *Advances in sonar technology*, chapter Mobile robot localization using particle filters and sonar sensors, 213–232 (Sergio Rui Silva) (2009).