THE FUKUSHIMA INVERSE PROBLEM

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Abstract

Knowing what amount of radioactive material was released from Fukushima in March 2011 is crucial to understand the scope of the consequences. Moreover, it could be used in forward simulations to obtain accurate maps of deposition. But these data are often not publicly available, or are of questionable quality. We propose to estimate the emission waveforms by solving an inverse problem. Previous approaches rely on a detailed expert guess of how the releases appeared, and they produce a solution strongly biased by this guess. If we plant a nonexistent peak in the guess, the solution also exhibits a nonexistent peak. We propose a method based on sparse regularization that solves the Fukushima inverse problem blindly. Together with the atmospheric dispersion models and worldwide radioactivity measurements our method correctly reconstructs the times of major events during the accident, and gives plausible estimates of the released quantities of Xenon.

Index Terms— Fukushima, Daiichi, nuclear power plant, explosion, inverse problems, FLEXPART, dispersion

1. INTRODUCTION

Nuclear power plants (NPP) provide an abundant, relatively cheap, and carbon-neutral source of energy. However, they also introduce a possibility, albeit a very remote one, of a major accident. A nuclear accident is defined by the International Atomic Energy Agency as an event having lethal consequences, environmental effects such as large radioactivity releases, and producing long-lasting facility defects such as core melts. Level 7 on the International Nuclear Events Scale is defined as a major release of radioactive material with widespread health and environmental effects requiring implementation of planned and extended countermeasures [1]. Two accidents have reached this level—Chernobyl in April 1986 and Fukushima Daiichi in March 2011.

The principal consequence of NPP accidents is the release of radioactive material. Transported through the atmosphere, it eventually gets widely spread, polluting the environment for centuries at a large scale. The exposure to the radioactive material causes cancer, teratogenesis, cognitive decline, and heart disease [2]. Thus it is imperative to monitor the radioactive contamination of soil, waters and atmosphere. Unfortunately, the contamination can only be accurately measured at a limited number of survey sites due to the cost of scientific grade equipment. This suggests the need for numerical simulations of atmospheric dispersion [3,4].

But getting accurate concentration and deposition values through simulations requires the knowledge of the source term—we should know how much radioactive material was released at what times. Its precise estimate is essential to properly estimate the contamination and take risk reducing measures. However, these data are often not publicly available or it is unknown. In particular, some works [4,5] challenge the data released by the Japanese government about the Fukushima accident. An option is to calculate the source term based on spatio-temporal samples of the concentration, that is, by solving an inverse problem.

We propose to estimate the source term by inverting the atmospheric dispersion. This would provide us with an estimate of how much radioactive material was released, and with a firm starting point for understanding the scope of the pollution through dispersion simulation. We summarize our effort as follows,

Problem 1 (Fukushima Inverse Problem). Given the measurements \( y \) collected at the survey sites, and the model \( A \), recover the temporal variation of the radioactive material release \( x \).

The main contribution of this paper is a solution to the Fukushima inverse problem formulated as a convex program. We use worldwide radioactivity measurements and weather data from March 2011 on. Unlike previous approaches, the proposed formulation correctly estimates the explosion and venting times based on Xenon emissions, without involving an expert-knowledge-based initial guess. The estimated release magnitudes match earlier expert estimates.

2. MODELING AND PRIOR ART

Numerical simulations relate the deposition at locations of interest with the amount of material emitted by the source. The link is established through atmospheric dispersion models. These models frequently assume a linear relationship,

\[
y(\xi, t) = \int_0^\tau A(\xi, t, \tau) x(\tau) \, d\tau
\]

where \( y(\xi, t) \) is the measured concentration of the material at the location \( \xi \in \mathbb{R}^2 \) at time \( t \), \( x(\tau) \) is the amount of material emitted at time \( \tau \), and \( A(\xi, t, \tau) \) is the spatio-temporal kernel of atmospheric dispersion. In practice, we discretize the source term, and compute the deposition values at a finite number of locations and
focus on estimating the amount of material released in this specific
time interval. But ideally, we should be able to blindly reconstruct
the timing of releases. This is valuable since it would permit us to
detect radioactive material releases that otherwise went unnoticed.
A blind reconstruction would also strongly suggest that the correct
temporal reconstruction does not result from the a priori guess.

In the discrete formulation (2), \( y \) contains the concentration
measurements of radioactive Xenon-isotopes in air at stations from
the Comprehensive Nuclear-Test-Ban Treaty Organization (CTBT0)
monitoring network. This network comprises 25 stations equipped
with very sensitive radioactive Xenon detectors. The detectors
typically have three stages. Two to concentrate and purify the gas
sample, and the third one to measure the activity of the final gas
sample. The duration of the sampling and purification process limits
the number of samples per day to between one and three, depending
on the detector model [12]. These systems allow to measure
\( ^{133}\text{Xe} \) to an accuracy of 0.1 mBq m\(^{-3}\) [4]. Measurements from 15
CTBT0 and 2 non-CTBT0 stations are eventually used to solve the
inverse problem. The locations of these stations are shown in Figure
2(C). Interestingly, measurements from the CTBT0 station located
in Japan could not be used because the levels of Xenon were over
the highest detectable level of the system, saturating the detectors.
All the measurements were corrected for radioactive decay. For
additional information on the pre-processing of the data see [4].

On the right hand side in (2), the source term \( x \) contains the rate
of release of Xenon in Bq s\(^{-1}\), at the location of Fukushima Dai-
ichi NPP between March 10 and March 16. Temporal resolution is
three hours and three different ejection heights are considered. Dif-
f erent heights, 0.5 m, 50-300 m, and 300-1000 m, are necessary
since the atmospheric transport of particles depends substantially
on the altitude of the source [4]. Finally, the model matrix \( A \) describes
every measurement as a linear combination of source terms. The
coefficients of \( A \) are computed using FLEXPART. The number
of measurements is relatively low, so the coarse discretization is
necessary. The sensitivity matrix has dimensions 858 \( \times \) 120. Since it
is an overdetermined system, the first idea is to find the least-squares
solution,

\[
\hat{x} = \arg \min \| Ax - y \|^2. 
\]

Minimization (3) has a closed form solution given in terms of the
Moore-Penrose pseudoinverse of \( A \), \( \hat{x} = A^\dagger y = (A^\top A)^{-1}A^\top y \). However, applying the pseudoinverse does not do a good job. This
happens for 2 reasons: 1) the matrix has a huge condition number
and 2) the matrix likely exhibits a sizeable model mismatch, and
there is noise in the measurements (although negligible in compar-
ison with the model mismatch). We might attempt to fix the condi-
tioning issues by the Tikhonov regularization, but the result would
still be unsatisfactory. We need a good model for the source term,
and assuming that it minimizes the Euclidean norm of the measure-
ments has no justification.

The dispersion modeling community developed specific meth-
ods to cope with these nuisances. Typically, they aid the estima-
tion by forming an expert-knowledge-based guess of the result. In
the Fukushima case this comprises knowledge about when the acci-
dents took place, what specific parts of the power plant were affected
at what times, and the general expert knowledge about the nuclear
power plant technology. The minimization then involves terms fa-
voring the a priori solution, and an additional smoothing term with
a second discrete derivative. If we have an estimate of uncertainties
of different components in the guess, and similarly for observations
(since they are obtained by techniques of different quality), we may
also include these in the solution. Denoting by \( x_a \) the initial guess,
this leads to the following convex program,

$$\hat{x} = \arg\min \|W_y(AX - y)\|^2_2$$

$$+ \lambda \|W_u(x - x_o)\|^2_2 + \varepsilon \|D_2(x - x_o)\|^2_2,$$  

(4)

where $D_2$ is the discrete second derivative, $W_y W_y = C_y^{-1}$ and $W_u W_u = C_u^{-1}$; if $C_y$ and $C_u$ are the covariance matrices of the observations and of the initial guess.

Estimation using (4) was performed in [4]. The authors obtain good results with this technique but the caveat is that it is difficult to assess it, since it is very close to the a priori solution (admitting that the initial guess may be very good). To exemplify the point, in Figure 1(A) we plot the solution to (4) with the initial guess used by the authors. The corresponding measurements were generated by plugging their initial guess into the model, and then adding noise (SNR=10 dB) to simulate the model mismatch. Now we relocate a peak and rerun the experiment. The result is shown in Figure 1(B). The solution now perfectly follows the relocated peak, and the old peak appears smoothed out. In conclusion, the method is useful, but very sensitive to the initial solution. If we account for this through “small” $W_u$ (corresponding to large uncertainties), then we drift towards the Tikhonov regularization, since “small” $W_u$ makes the a priori knowledge term small or negligible. But Tikhonov was shown not to perform well in [4]. In summary, what we seek is a method to correctly detect the emission times without the a priori expert knowledge. It would be wrong to just dismiss the available expert estimate, but a good method must give plausible results even without it.

4. INGREDIENTS OF A GOOD SOLUTION

We show that a successful solution to Problem 1 comprises three ingredients. One of the critical steps is data pre-processing. This is often encountered in the machine learning world. We identified the following key ingredients,

1) Choosing the proper regularization,
2) Cleaning the matrix,
3) Incorporating the natural constraints on the solution (such as positivity).

1) Proper regularization. Xenon ejection waveforms (estimates) in Figure 1(A) reveal that it was released in short bursts (some materials like Cesium are released during more extended periods). The temporal variation of the Xenon emission exhibits several peaks and many small elements. Therefore, a proper regularizations should favor signals with many zeros and a few large elements. This is in contrast with what is known about the regularization based on $\ell_2$ norms, such as Tikhonov. These favor many small/moderate elements. We should find the solution yielding something close to the observed measurements, but with as many zero elements as possible. Since minimizing the number of non-zeros is not tractable, we use the standard relaxation based on the $\ell_1$ norm. The solution is termed basis pursuit denoising [13],

$$\hat{x} = \arg\min \|Ax - y\|^2 + \lambda \|x\|_1,$$  

(5)

2) Cleaning the matrix. Figure 2(A) shows the matrix used in [4]. We notice many very small elements (note that the color bar is in logarithmic scale). That is, many matrix rows have zero or negligible norms, meaning that these sensors do not contribute to the result w.r.t. rows with larger norms. Same figure shows the corresponding distribution of the measuring stations around the world. The small-norm rows deteriorate the solution of the inverse problems. This can be interpreted through a very unfavorable condition number of the model matrix A. After removing rows with very small norms, we are left with the matrix shown in Figure 2(B). It is evident from the color map that the remaining rows have much narrower dynamics. Even so, the elements still differ by several orders of magnitude.

It is interesting to see that cleaning the matrix removes two southernmost stations, as indicated in Figure 2. The CTBTO website features a fascinating video explaining that the equator was eventually the material got transported to the southern hemisphere but in very small amounts.

3) Incorporating the natural constraints. The ingredients 1) and 2) already give a reasonable result. We can find a regularization parameter $\lambda$ in the program (5) that gives $\hat{x}$ with correct release times. But changing the regularization parameter yields a very different solution and it is not clear how to properly choose $\lambda$. Furthermore, both solutions shows unrealistic negative values. An obvious solution is to enforce the non-negativity constraint to get a new convex
We have successfully estimated the times and quantities of radioactive Xenon emissions from the Fukushima power plant following the March 2011 earthquake. The complete solution involves sparsity promoting regularizations, as the observed Xenon emission waveforms consist of a few peaks and many very small elements. We have shown that standard approaches from the dispersion modeling community lack of stability w.r.t. their a priori guess. Future work involves testing of the sensitivity of our approach to model mismatch, revisiting the results of the tracer-experiment verification of FLEXPART, and repeating the experiment for Cesium.

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8. REFERENCES


