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Metastable de Sitter vacua in N=2 to N=1 truncated supergravity

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Abstract

We study the possibility of achieving metastable de Sitter vacua in general N=2 to N=1 truncated supergravities without vector multiplets, and compare with the situations arising in N=2 theories with only hypermultiplets and N=1 theories with only chiral multiplets. In N=2 theories based on a quaternionic manifold and a graviphoton gauging, de Sitter vacua are necessarily unstable, as a result of the peculiar properties of the geometry. In N=1 theories based on a Kähler manifold and a superpotential, de Sitter vacua can instead be metastable provided the geometry satisfies some constraint and the superpotential can be freely adjusted. In N=2 to N=1 truncations, the crucial requirement is then that the tachyon of the mother theory be projected out from the daughter theory, so that the original unstable vacuum is projected to a metastable vacuum. We study the circumstances under which this may happen and derive general constraints for metastability on the geometry and the gauging. We then study in full detail the simplest case of quaternionic manifolds of dimension four with at least one isometry, for which there exists a general parametrization, and study two types of truncations defining Kähler submanifolds of dimension two. As an application, we finally discuss the case of the universal hypermultiplet of N=2 superstrings and its truncations to the dilaton chiral multiplet of N=1 superstrings. We argue that de Sitter vacua in such theories are necessarily unstable in weakly coupled situations, while they can in principle be metastable in strongly coupled regimes.

1 Introduction

Realizing a de Sitter vacuum which is at least metastable and where supersymmetry is spontaneously broken, as required by particle phenomenology and cosmological observation, has proven to be very difficult within the context of string theory, and no completely convincing setup realizing such a vacuum has been singled out so far. It is however by now well understood that a major source of obstruction to metastability stems just from the peculiar general structure of supergravity, which represents the general framework for a low-energy effective description of string theory. To make further progress in discriminating string models, it would then be highly desirable to have a complete understanding of the restrictions arising within supergravity on the possibility of achieving such a viable vacuum. Ideally, this should moreover encompass not only the case of minimal supersymmetry, which is directly interesting for model building, but also the various cases of extended supersymmetry, which partly reflect some of the additional special features displayed by models with a higher-dimensional origin.

The analysis of the conditions under which de Sitter vacua may be at least metastable in supergravity is complicated by the fact that the general form of the mass matrix for the scalar fluctuations around such a vacuum depends in a rather intricate way on the free parameters in the Lagrangian. Studying the full mass matrix and translating the requirement of its positivity into necessary and sufficient constraints on these parameters is then unfortunately possible only on a model by model basis. However, one may specialize this kind of analysis to a restricted set of particularly dangerous modes, and try in this way to derive some constraints that are only necessary and not sufficient but more general and useful. More precisely, one may try to exploit the restrictions on the structure of the kinetic energy, and ignore instead the details of the source of potential energy. To do so, one should focus on those scalar modes for which the mass happens to be constrained independently of the precise form of the potential, as a result of the assumed spontaneous breaking of supersymmetry and possibly also of some of the internal symmetries. The a priori most dangerous modes are those belonging to multiplets for which a supersymmetric mass term is either forbidden or strongly constrained. These include all the real scalar partners of the would-be Goldstino fermions associated to broken supersymmetry [1, 2, 3], as well as those of the would-be Goldstone bosons associated to broken internal symmetries [4], which we shall simply call sGoldstini and sGoldstones. Indeed, such modes have masses that are controlled by supersymmetry or internal symmetry breaking effects, whose form is very constrained.

The situation in N=1 supergravity is well understood. For theories with only chiral multiplets, based on a Kähler manifold and an arbitrary superpotential, the only dangerous modes are the two sGoldstini. The positivity of their average mass implies a simple and sharp necessary condition on the Kähler manifold: this should admit points and directions for which the sectional curvature is larger than a certain critical value depending on the vacuum energy and the gravitino mass [2, 3] (see also [5, 6]). For theories with chiral and vector multiplets, based on a Kähler manifold with isometries gauged by vector bosons and a gauge invariant superpotential, the dangerous modes are not only the two sGoldstini but also all the sGoldstones. The positivity of the average mass of the two sGoldstini implies again a condition on the Kähler manifold: this should admit points and directions for which the sectional curvature is larger than a certain critical value depending now also on the gauging data [4]. On the other hand, the positivity of the masses of the sGoldstones does not seem to imply any simple and general constraint, although examples are known where they can lead to instabilities on their own. The situation in the rigid limit is also well understood, and has been discussed in [7] and also [8], where it was argued that the lightest scalar in the theory is in general a combination of sGoldstini and sGoldstones.

The case of N=2 supergravity is more complex and only partly understood. For theories with only hypermultiplets, based on a quaternionic manifold with an isometry gauged by the graviphoton, the only dangerous modes are the four sGoldstini, out of which one is absorbed by the graviphoton in a Higgs mechanism giving it mass and only three represent physical scalar modes. Their average mass turns out to be given by a completely universal value, which depends only on the vacuum energy and the gravitino mass and turns out to be negative, and as a result metastability is impossible to achieve: there is always at least one sGoldstino that has a negative square mass [9]. In theories with only vector multiplets and Abelian gaugings, based on a special-Kähler manifold with commuting isometries gauged by vector bosons, the dangerous modes are only the two sGoldstini, since there is no way the Abelian symmetries can be broken. Their average mass turns again out to be given by a completely universal value, which depends as before only on the vacuum energy and the gravitino mass and turns out to be negative, and as a result metastability is again impossible to achieve: there is always at least one sGoldstino with a negative square mass [10]. For more general theories involving hyper and vector multiplets and/or non-Abelian gaugings, the situation is more complicated and the dangerous modes are in principle not only the sGoldstini but also all the sGoldstones. No simple general necessary condition for metastability has been derived so far for this case, but a few particular examples of models admitting metastable de Sitter vacua are known [11, 12], and a non-trivial criterion is thus expected to emerge. The situation in the rigid limit is slightly better but still only partly understood. In this limit, it has been possible to rederive the above no-go theorems in a simpler way and generalize the computation of the average mass of the sGoldstini to the case of theories with only vector multiplets but generic non-Abelian gaugings, showing that in that case it can be positive if there are Fayet-Iliopoulos terms for Abelian factors [7]. Moreover, it has been argued in [13] that the N=2 supersymmetry algebra does not allow for a consistent non-linear realization whenever the theory possesses an $SU(2)_R$ symmetry, that is whenever Fayet-Iliopoulos terms are absent.

The cases of N=4 and N=8 supergravities are qualitatively different and again only partly understood. The main novelty in these cases is that the scalar manifold is completely fixed to be a definite coset space, and the only freedom one has is to gauge suitable isometries with the graviphotons and/or vector bosons. The dangerous modes are a priori the various sGoldstini and all the sGoldstones. A systematic analysis of the average sGoldstino mass was performed respectively in [14] and [15], and showed that its positivity puts quite strong constraints on the gauging. No similar study was performed so far for the sGoldstone masses. On the other hand, all the known examples of de Sitter vacua in such theories happen to be unstable (see for instance [16, 17] and [18, 19, 20]) and it is conceivable that one might eventually be able to prove a no-go theorem. The situation in the rigid limit is in this case totally trivial. The N=4 theory requires a flat scalar manifold and its potential does not allow supersymmetry breaking critical points. The N=8 theory, on the other hand, has a totally empty rigid limit.

The aim of this paper is to initiate a similar systematic study of the conditions for the metastability of de Sitter vacua in supergravities where the amount of supersymmetry is reduced through a consistent truncation. This situation is in fact directly realized in several string constructions, and may thus provide a more realistic and representative framework to study the situation for string-derived models. The crucial new aspect arising in such a context is that one may start from an unstable de Sitter vacuum of the mother theory and obtain a metastable de Sitter vacuum in the daughter theory, provided the original tachyon is projected out by the truncation. This possibility was explored in [21] for the case of N=8 to N=4 truncations, where the instability was found to always persist, and for the case of N=4 to N=2 truncations, where the instability was shown to disappear in some particular cases. Here we would like to study in some generality the basic case of N=2 to N=1 truncations. For concreteness, we shall restrict to N=2 theories with only hypermultiplets truncated to N=1 theories which then involve only chiral multiplets. In this simplest situation, we already know that any de Sitter vacuum of the mother N=2 theory is necessarily unstable and involves at least one tachyon, while de Sitter vacua in the N=1 daughter theory can be metastable and free of tachyons. Moreover, the sGoldstino masses and the consistency conditions for the truncation are controlled by purely geometric quantities, and it should therefore be possible to characterize in simple and general terms the possibility of starting with a vacuum possessing a single tachyonic sGoldstino and projecting out this mode through a truncation.

The paper is organized as follows. In sections 2 and 3 we briefly review the metastability conditions emerging in N=1 theories with only chiral multiplets and N=2 theories with only hypermultiplets. In section 4 we derive the metastability conditions in truncations of N=2 theories with only hypermultiplets to N=1 theories with only chiral multiplets. In section 5 we study in detail the simplest case of N=2 theories with a single hypermultiplet, based on a generic quaternionic space with at least one isometry, for which there exists a general explicit description as a Przanowski-Tod space, and describe two different kinds of truncations to N=1 theories with a single chiral multiplet. In section 6 we illustrate our results with a few specific examples of this type, which are directly relevant for the low-energy effective description of the universal hypermultiplet of string models. In section 7 we present our conclusions. In appendix A, we summarize the relevant details about the geometry of Przanowski-Tod spaces.

2 N=1 supergravity with chiral multiplets

In N=1 supergravity with n chiral multiplets, the geometry of the scalar manifold that controls the kinetic terms through its metric $g_{i\bar{\jmath}}$ is restricted to be Kähler, and the source of potential V is represented by an arbitrary holomorphic superpotential. In units where $\kappa = 1$, the part of the Lagrangian describing the n complex scalar fields ϕ^i takes the following form:

$$\mathcal{L} = -g_{i\bar{\jmath}}\,\partial^\mu\phi^i\partial_\mu\phi^{\bar{\jmath}} - V\,. \tag{2.1}$$

Let us first review the properties of the geometry. The holonomy group is $U(n)$. The vielbein is written as e_i^a and its conjugate is $e_{\bar{\imath}}^{\bar{a}} = (e_i^a)^*$. The indices $i, \bar{\imath} = 1, \cdots, n$ refer to the manifold while the indices $a, \bar{a} = 1, \cdots, n$ refer to the tangent space. The tangent space metric is simply $g_{a\bar{b}} = \delta_{a\bar{b}}$. The manifold metric is then given by:

$$g_{i\bar{\jmath}} = \delta_{a\bar{b}}e_i^a e_{\bar{\jmath}}^{\bar{b}}\,. \tag{2.2}$$

There is one complex structure satisfying $J^i{}_k J^k{}_j = -\delta^i_j$. It is given by

$$J_{i\bar{\jmath}} = i\delta_{a\bar{b}}e_i^a e_{\bar{\jmath}}^{\bar{b}} = ig_{i\bar{\jmath}}\,. \tag{2.3}$$

The $U(n)$ connection Γ_i is determined by the torsion-free constraint on the vielbeins, which reads $\nabla_i e_j^a = 0$ in terms of a covariant derivative including both this connection and the Christoffel one. Its curvature two-form can be parametrized in terms of a tensor $R_{a\bar{b}c\bar{d}}$ which has to be symmetric in its holomorphic and antiholomorphic indices but is otherwise arbitrary: $R_{i\bar{\jmath}}^{a\bar{b}} = e_i^c e_{\bar{\jmath}}^{\bar{d}} R^{a\bar{b}}{}_{c\bar{d}}$. The Riemann curvature tensor with flat indices is then simply $R_{a\bar{b}c\bar{d}}$, while its version with only curved indices is instead given by

$$R_{i\bar{\jmath}p\bar{q}} = e_i^a e_{\bar{\jmath}}^{\bar{b}} e_p^c e_{\bar{q}}^{\bar{d}} R_{a\bar{b}c\bar{d}}\,. \tag{2.4}$$

The Ricci and scalar curvatures are given by the contractions $R_{i\bar{\jmath}} = -g^{p\bar{q}} R_{i\bar{\jmath}p\bar{q}}$ and $R_{\rm sca} = g^{i\bar{\jmath}} R_{i\bar{\jmath}}$. The Weyl curvature is instead controlled by the traceless part of $R_{i\bar{\jmath}p\bar{q}}$. Finally, one has the standard Ricci decomposition

$$R_{i\bar{\jmath}p\bar{q}} = \frac{2}{(n+1)(n+2)} g_{i(\bar{\jmath}}\, g_{p\bar{q})} R_{\rm sca} - \frac{2}{n+2} \big(g_{i(\bar{\jmath}} R_{p\bar{q})} + g_{p(\bar{q}} R_{i\bar{\jmath})} \big) + C_{i\bar{\jmath}p\bar{q}} \,. \quad (2.5)$$

Let us next describe the properties of the superpotential W that is used to generate a potential. The only restriction is that it should be holomorphic. It is then convenient to introduce the quantities

$$L = e^{K/2} W \,, \quad N_i = e^{K/2} \big(W_i + K_i W \big) \,. \quad (2.6)$$

These satisfy

$$\nabla_{\bar{\jmath}} L = 0 \,, \quad \nabla_i L = N_i \,. \quad (2.7)$$

Moreover:

$$\big[\nabla_i, \nabla_{\bar{\jmath}} \big] L = -g_{i\bar{\jmath}} L \,, \quad \big[\nabla_i, \nabla_{\bar{\jmath}} \big] N_p = R_{i\bar{\jmath}p\bar{q}} \bar{N}^{\bar{q}} - g_{i\bar{\jmath}} N_p \,. \quad (2.8)$$

The scalar potential then takes the following form:

$$V = \bar{N}^i N_i - 3|L|^2 \,. \quad (2.9)$$

Under supersymmetry transformations, the gravitino and the chiralini transform as $\delta\psi_\mu = iL\gamma_\mu\epsilon + \cdots$ and $\delta\chi^a = N^a\epsilon + \cdots$, where $N_a = U^i_a N_i$. The supersymmetry breaking scale is thus the norm of N_i, $M_{\rm susy} = |N|$, while the gravitino mass is given by the norm of L, $m_{3/2} = |L|$. The Goldstino is $\chi = N_a \chi^a$, while the complex sGoldstino is $\phi = N_i \phi^i$.

The mass matrix of the scalars at a stationary point where $\nabla_i V = 0$ is given by $m^2_{i\bar{\jmath}} = \nabla_i \nabla_{\bar{\jmath}} V$ and $m^2_{ij} = \nabla_i \nabla_j V$. This is not the physical mass matrix, but the only additional thing to take into account is the non-trivial metric $g_{i\bar{\jmath}}$. The physical masses in the subspace defined by the complex direction $n^i = \bar{N}^i/|N|$ are given by $m^2_\pm = m^2_{i\bar{\jmath}} n^i n^{\bar{\jmath}} \pm |m^2_{ij} n^i n^j|$. A straightforward computation gives

$$m^2_\pm = R\, \bar{N}^k N_k + 2|L|^2 \pm \big| \Delta\, \bar{N}^k N_k - 2\, \bar{L}^2 \big| \,, \quad (2.10)$$

where

$$R = -\frac{R_{i\bar{\jmath}p\bar{q}} \bar{N}^i N^{\bar{\jmath}} \bar{N}^p N^{\bar{q}}}{(\bar{N}^k N_k)^2} \,, \quad \Delta = \frac{\nabla_i \nabla_j \nabla_k L\, \bar{N}^i \bar{N}^j \bar{N}^k}{(\bar{N}^w N_w)^2} \,. \quad (2.11)$$

This shows that the average of the masses of the two real sGoldstini is controlled by the sectional curvature R of the scalar manifold in the plane defined by the two conjugate vectors $(N, \bar{N})$, while their splitting is controlled by the quantity Δ depending on the third derivative of the superpotential along the direction N.

In terms of the scalar potential V and the gravitino mass $m_{3/2}$, one finally finds that the average sGoldstino mass $m^2_{\rm avr} = \frac{1}{2}(m^2_+ + m^2_-)$ is simply given by:

$$m^2_{\rm avr} = R\,V + \big(3R + 2\big)\, m^2_{3/2}\,. \tag{2.12}$$

This quantity defines by construction an upper bound on the mass squared of the lightest scalar. In order to have a metastable supersymmetry breaking vacuum with $V > 0$, one then needs the sectional curvature R to satisfy the following bound [2, 3, 5]:

$$R > -\frac{2}{3 + V/m^2_{3/2}} > -\frac{2}{3}\,. \tag{2.13}$$

This represents a necessary condition for the existence of metastable de Sitter vacua on the choice of Kähler manifold. Indeed, the scalar manifold needs to admit at least one special point and one complex direction along which the sectional curvature is larger than the critical value $-\frac{2}{3}$. One can then easily show that once this condition is satisfied, it is always possible to arrange for the two sGoldstino masses to be degenerate and equal to $m^2_{\rm avr}$ while all the other states have larger square masses, by suitably tuning the superpotential. This means that for a given Kähler manifold the necessary and sufficient condition for the existence of metastable de Sitter vacua for some choice of the covariantly holomorphic section is that there exist a point and a complex direction such that $R > -\frac{2}{3}$.

3 N=2 supergravity with hypermultiplets

In N=2 supergravity with n hypermultiplets, the geometry of the scalar manifold that controls the kinetic terms through its metric g_{IJ} is restricted to be quaternionic [22], and the only possible source of potential is the gauging of a triholomorphic isometry through the graviphoton. We follow [23, 24], and introduce an arbitrary real and negative parameter λ in terms of which the negative scalar curvature of the scalar manifold is parametrized as $\mathcal{R}_{\rm sca} = 8n(n+2)\lambda$. [1] In units where $\kappa = 1$, the part of the Lagrangian describing the $4n$ real scalar fields q^I and the graviphoton $\mathcal{A}_\mu$ then takes the following form:

$$\mathcal{L} = -\frac{1}{8}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \lambda\,\mathsf{g}_{IJ} D^\mu q^I D_\mu q^J - \mathcal{V}\,. \tag{3.1}$$

The covariant derivative is defined as $D_\mu q^I \equiv \partial_\mu q^I + \mathcal{A}_\mu k^I(q)$, where k^I is the Killing vector generating the isometry, and the field strength reads $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$.

[1] The relation to the parameter ν of [25] is $\nu = 2\lambda$.

Let us first recall the main properties of the geometry. The holonomy group is $SU(2) \times SP(2n)$. The vielbein can be written as $\mathcal{U}_I^{A\alpha}$ and satisfies the reality condition $(\mathcal{U}_I^{A\alpha})^* = \mathcal{U}_{IA\alpha}$. The index $I = 1, \cdots, 4n$ refers to the manifold while the indices $A = 1, 2$ and $\alpha = 1, \cdots, 2n$ refer to the tangent space. The tangent space metric is given by $g_{A\alpha B\beta} = \epsilon_{AB} c_{\alpha\beta}$, where ϵ_{AB} and $c_{\alpha\beta}$ denote the usual Levi-Civita and symplectic tensors, which are used in the standard way to raise and lower each type of flat sub-index. The manifold metric is then given by:

$$g_{IJ} = \epsilon_{AB} c_{\alpha\beta} \mathcal{U}_I^{A\alpha} \mathcal{U}_J^{B\beta} \,. \tag{3.2}$$

There are three complex structures $\mathcal{J}^{xI}{}_J$, with $x = 1, 2, 3$, satisfying the algebra $\mathcal{J}^{xI}{}_K \mathcal{J}^{yK}{}_J = -\delta^{xy} \delta^I_J + \epsilon^{xyz} \mathcal{J}^{zI}{}_J$. Denoting the usual Pauli matrices by σ^{xB}_A, they are given by:

$$\mathcal{J}^x_{IJ} = i \sigma^x_{AB} c_{\alpha\beta} \mathcal{U}_I^{A\alpha} \mathcal{U}_J^{B\beta} \,. \tag{3.3}$$

The $SU(2) \times SP(2n)$ connection can be decomposed as $\Gamma^{A\alpha}_{IB\beta} = \omega^A_{IB} \delta^\alpha_\beta + \Delta^\alpha_{I\beta} \delta^A_B$, where $\omega^A_{IB} = \frac{i}{2} \sigma^{xA}{}_B \, \omega^x_I$ and $\Delta^\alpha_{I\beta}$ correspond to the $SU(2)$ and $SP(2n)$ parts. It is determined by the torsion-free constraint on the vielbeins, which reads $\nabla_I \mathcal{U}_J^{A\alpha} = 0$ in terms of a covariant derivative including both this connection and the Christoffel one. The curvature two-form correspondingly reads $\mathcal{R}^{A\alpha}_{IJB\beta} = K^A_{IJB} \delta^\alpha_\beta + \Sigma^\alpha_{IJ\beta} \delta^A_B$, where $K^A_{IJB} = \frac{i}{2} \sigma^{xA}{}_B \, K^x_{IJ}$ and $\Sigma^\alpha_{IJ\beta}$ correspond to $SU(2)$ and $SP(2n)$ parts. The general form that these are allowed to take can be parametrized in terms of a completely symmetric but otherwise arbitrary tensor $\mathcal{W}_{\alpha\beta\gamma\delta}$: $K^{AB}_{IJ} = i\lambda \, \sigma^{xAB} \mathcal{J}^x_{IJ}$ and $\Sigma^{\alpha\beta}_{IJ} = \epsilon_{CD} \, \mathcal{U}^{C\gamma}_{[I} \mathcal{U}^{D\delta}_{J]} (2\lambda \, \delta^\alpha_{(\gamma} \delta^\beta_{\delta)} + \mathcal{W}^{\alpha\beta}{}_{\gamma\delta})$. [2] It follows that the Riemann curvature tensor with flat indices takes the form

$$\mathcal{R}_{A\alpha B\beta C\gamma D\delta} = 2\lambda \big(\epsilon_{AB} \epsilon_{CD} c_{\alpha(\gamma} c_{\beta\delta)} + \epsilon_{A(C} \epsilon_{BD)} c_{\alpha\beta} c_{\gamma\delta} \big) + \epsilon_{AB} \epsilon_{CD} \mathcal{W}_{\alpha\beta\gamma\delta} \,. \tag{3.4}$$

Its version with only curved indices is instead given by:

$$\mathcal{R}_{IJPQ} = \lambda \big(g_{I[P} g_{JQ]} + \mathcal{J}^x_{IJ} \mathcal{J}^x_{PQ} + \mathcal{J}^x_{I[P} \mathcal{J}^x_{JQ]} \big) + \mathcal{W}_{IJPQ} \,, \tag{3.5}$$

where

$$\mathcal{W}_{IJPQ} = \epsilon_{AB} \epsilon_{CD} \, \mathcal{U}_I^{A\alpha} \mathcal{U}_J^{B\beta} \mathcal{U}_P^{B\gamma} \mathcal{U}_Q^{D\delta} \mathcal{W}_{\alpha\beta\gamma\delta} \,. \tag{3.6}$$

The Ricci and scalar curvatures are completely fixed by the constant λ, independently of the tensor $\mathcal{W}_{IJPQ}$ which turns out to have vanishing contractions, and read $\mathcal{R}_{IJ} = 2(n+2)\lambda \, g_{IJ}$ and $\mathcal{R}_{\rm sca} = 8n(n+2)\lambda$. The Weyl curvature, on the

[2] Our curvature is defined in the standard way and corresponds to twice that of [23, 24]. The relation with the symbols used in [23, 24] is $K^x_{IJ} = 2\Omega^x_{IJ}$, $\Sigma^\alpha_{IJ\beta} = 2\mathbb{R}^\alpha_{IJ\beta}$, $\mathcal{W}_{\alpha\beta\gamma\delta} = 2\Omega_{\alpha\beta\gamma\delta}$. Moreover, we use the standard conventions for the normalization of differential forms.

other hand, does depend on the tensor $\mathcal{W}_{IJPQ}$. Finally, one can also write a Ricci decomposition of the curvature. Since the space is Einstein, this simply reads:

$$\mathcal{R}_{IJPQ} = \frac{4\lambda(n+2)}{4n-1} g_{I[P} g_{JQ]} + \mathcal{C}_{IJPQ} \,. \tag{3.7}$$

Notice that in general the first and second parts of (3.5) and (3.7) cannot be separately identified, and the tensor $\mathcal{W}_{IJPQ}$ does therefore not exactly coincide with the Weyl curvature $\mathcal{C}_{IJPQ}$. The reason for this is that thanks to the existence of the three complex structure one can actually construct linear combinations of the first three terms in (3.5) which have vanishing trace. It is straightforward to verify that the two pieces in (3.5) and (3.7) only separately coincide in the minimal case $n = 1$, which we shall study in some detail later.

Let us next describe the properties of the Killing vector k^I that is used to generate a potential for the scalars through a gauging with the graviphoton $\mathcal{A}_\mu$. First of all, k^I has to satisfy the usual Killing equation $\nabla_{(I} k_{J)} = 0$. Moreover, it has to be triholomorphic and thus admit a triplet of real Killing prepotentials $\mathcal{P}^x$, such that $\nabla_I \mathcal{P}^x = 2\lambda\, \mathcal{J}^x_{IJ} k^J$ and $k_I = -\frac{1}{6\lambda}\, \mathcal{J}^x_{IJ} \nabla^J \mathcal{P}^x$. It is then convenient to introduce the following quantities:

$$\mathcal{P}^x \,, \;\; \mathcal{N}^I = 2 k^I \,. \tag{3.8}$$

These satisfy

$$\nabla_I \mathcal{P}^x = \lambda\, \mathcal{J}^x_{IJ} \mathcal{N}^J \,. \tag{3.9}$$

Moreover:

$$\big[\nabla_I, \nabla_J\big] \mathcal{P}^x = 2\lambda\, \epsilon^{xyz} \mathcal{J}^y_{IJ} \mathcal{P}^z \,, \;\; \big[\nabla_I, \nabla_J\big] \mathcal{N}_P = \mathcal{R}_{IJPQ} \mathcal{N}^Q \,. \tag{3.10}$$

The scalar potential then takes the following form:

$$\mathcal{V} = -\lambda\, \mathcal{N}^I \mathcal{N}_I - 3 \mathcal{P}^x \mathcal{P}^x \,. \tag{3.11}$$

Under supersymmetry transformations, the gravitino and the hyperini transform as $\delta\psi^A_\mu = \frac{1}{2} \mathcal{P}^x \sigma^{xAB} \gamma_\mu \epsilon_B + \cdots$ and $\delta\xi^\alpha = \sqrt{-\lambda}\, \mathcal{N}^{A\alpha} \epsilon_A + \cdots$, where $\mathcal{N}^{A\alpha} = \mathcal{U}^{A\alpha}_I \mathcal{N}^I$. The supersymmetry breaking scale is thus linked to the norm of $\mathcal{N}_I$, $M_{\rm susy} = \sqrt{-\lambda}\, |\mathcal{N}|$, while the gravitino masses are given by the norm of $\mathcal{P}^x$, $m_{3/2} = |\mathcal{P}|$. The two independent Goldstini are $\xi^A = \mathcal{N}^A_\alpha \xi^\alpha$, while the corresponding four real sGoldstini are $q^{AB} = \mathcal{N}^{AB}_I q^I$, where $\mathcal{N}^{AB}_I = \mathcal{U}^{A\alpha}_I \mathcal{N}^B_\alpha = \frac{1}{2} \epsilon^{AB} \mathcal{N}_I + \frac{i}{2} \sigma^{xAB} (\mathcal{J}^x \mathcal{N})_I$. We then see that the singlet sGoldstino $q = \mathcal{N}_I q^I$ is actually just the would-be Goldstone mode absorbed by the graviphoton in the Higgs mechanism giving it mass, while the triplet sGoldstini $q^x = (\mathcal{J}^x \mathcal{N})_I q^I$ are instead physical modes.

The mass matrix of the scalars at a stationary point, where $\nabla_I \mathcal{V} = 0$, is given by $m^2_{IJ} = -\frac{1}{2\lambda} \nabla_I \nabla_J \mathcal{V}$. This is not yet the physical mass matrix, but having included the factor of $-\frac{1}{2\lambda}$ to partly compensate the non-canonical normalization of the kinetic terms, the only thing that is left is the effect of the non-trivial metric g_{IJ}. The physical masses along the direction $n^I = \mathcal{N}^I/|\mathcal{N}|$ and in the subspace of directions $n^{xI} = (\mathcal{J}^x\mathcal{N})^I/|\mathcal{N}|$ are then simply given by $m^2 = m^2_{IJ}n^In^J$ and $m^{2xy} = m^2_{IJ}n^{xI}n^{yJ}$. After a straightforward computation, one finds that [9]

$$m^2 = 0\,, \quad m^{2xy} = -\big(\mathcal{R}^{xy} - 3\lambda\,\delta^{xy}\big)\mathcal{N}^I\mathcal{N}_I - 4\big(\mathcal{P}^x\mathcal{P}^y - \delta^{xy}\mathcal{P}^z\mathcal{P}^z\big)\,. \tag{3.12}$$

where

$$\mathcal{R}^{xy} = \frac{\mathcal{R}_{IJPQ}\mathcal{N}^I(\mathcal{J}^x\mathcal{N})^J\mathcal{N}^P(\mathcal{J}^y\mathcal{N})^Q}{(\mathcal{N}^R\mathcal{N}_R)^2}\,. \tag{3.13}$$

This shows that the mass of the singlet sGoldstino vanishes, in agreement with the fact that it actually corresponds to the would-be Goldstone mode associated to the gauged isometry, while the mass submatrix of the triplet sGoldstini is entirely controlled by the bisectional curvatures $\mathcal{R}^{xy}$ of the scalar manifold in the two planes defined by the conjugate vectors $(\mathcal{N},\mathcal{J}^x\mathcal{N})$ and $(\mathcal{N},\mathcal{J}^y\mathcal{N})$.

Now, using the general form (3.5) of the Riemann tensor on a quaternionic manifold, one finds that the bisectional curvatures (3.13) take the following general form in terms of $\mathcal{W}_{\alpha\beta\gamma\delta}$:

$$\mathcal{R}^{xy} = 2\lambda\,\delta^{xy} - \frac{\mathcal{W}_{\alpha\beta\gamma\delta}\mathcal{N}^{A\alpha}\mathcal{N}^{B\beta}\mathcal{N}^{C\gamma}\mathcal{N}^{D\delta}}{(\mathcal{N}^{E\epsilon}\mathcal{N}_{E\epsilon})^2}\,\sigma^x_{AB}\sigma^y_{CD}\,. \tag{3.14}$$

A crucial property of the diagonal elements of this matrix, which correspond to the sectional curvatures in the three planes defined by the conjugate vectors $(\mathcal{N},\mathcal{J}^x\mathcal{N})$, is that their average is completely universal and fixed by the Ricci curvature. Indeed, when contracting (3.14) with $\frac{1}{3}\delta_{xy}$ the term involving $\mathcal{W}_{\alpha\beta\gamma\delta}$ drops out, as can be seen by using the identity $\sigma^x_{AB}\sigma^x_{CD} = -2\,\epsilon_{A(C}\epsilon_{BD)}$, and one finds:

$$\mathcal{R}_{\rm avr} = \frac{1}{3}\delta_{xy}\mathcal{R}^{xy} = 2\lambda\,. \tag{3.15}$$

As a consequence of this result, the average square mass $m^2_{\rm avr} = \frac{1}{3}\delta_{xy}m^{2xy}$ of the three non-trivial sGoldstini is simply $m^2_{\rm avr} = \lambda\,\mathcal{N}^I\mathcal{N}_I + \frac{8}{3}\mathcal{P}^x\mathcal{P}^x$, or equivalently [9]:

$$m^2_{\rm avr} = -\,\mathcal{V} - \frac{1}{3}m^2_{3/2}\,. \tag{3.16}$$

This represents by construction an upper bound on the square mass of the lightest scalar. We then deduce that it is impossible to have a metastable supersymmetry breaking vacuum with $\mathcal{V} > 0$, no matter which quaternionic manifold one chooses and which isometry one gauges, since in such a situation the right-hand side of eq. (3.16) is negative and therefore there must be at least one tachyon.

4 N=2 to N=1 truncations

Let us now consider the truncation of a generic N=2 theory with n hypermultiplets to an N=1 theory with n chiral multiplets. For the geometry, this means that we start from a generic quaternionic manifold and then select a Kähler submanifold, [3] while for the potential this means that we start from the gauging of a triholomorphic isometry and get a restricted superpotential out of it. We will follow the general discussion of [28, 29].

Let us first consider the reduction of the geometry. The $4n$ real scalar fields must split into $2n$ tangent and $2n$ normal real fields. We can correspondingly decompose the curved index as $I \to I_\parallel, I_\perp$. The truncation then acts by setting

$$q^{I_\perp}| = 0\,, \tag{4.1}$$

while $q^{I_\parallel}|$ are real coordinates on the Kähler submanifold. The holonomy must reduce from $SU(2) \times SP(2n)$ to $U(n)$. More precisely, since $SU(2) \supset U(1)$ and $SP(2n) \supset U(n)$, the $U(1)$ part can arise from a linear combination of factors coming from $SU(2)$ and $SP(2n)$, while the $SU(n)$ part can only come from $SP(2n)$. We can correspondingly decompose the flat indices as $A \to 1,2$ and $\alpha \to a, \bar{a}$. The truncation then acts by setting

$$\mathcal{U}^{2a}_{I_\parallel}| = (\mathcal{U}^{1\bar{a}}_{I_\parallel})^*| = 0\,, \tag{4.2}$$

while $\mathcal{U}^{1a}_{I_\parallel}| = (\mathcal{U}^{2\bar{a}}_{I_\parallel})^*|$ are vielbeins for the Kähler submanifold which are complex but not necessarily compatible with its complex structure. From the reduction of the torsion-free equations for the vielbeins onto the submanifold, one then deduces that one should have:

$$\omega^1_{I_\parallel}| = \omega^2_{I_\parallel}| = 0\,, \quad \Delta^a_{I_\parallel \bar{b}}| = 0\,. \tag{4.3}$$

Moreover, Frobenius' theorem dictates an involution condition, which amounts to requiring that the curvatures of the connections that have been set to zero are also zero. For the $SU(2)$ part this is automatic, but for the $SP(2n)$ part this requires an additional condition, which reads:

$$\mathcal{W}_{abc\bar{d}}| = 0\,. \tag{4.4}$$

In such a situation, the metric automatically splits into a block diagonal form, and the first block is identified with the metric of the submanifold, modulo a normalization factor dictated by the different normalizations of the kinetic terms in the N=2

[3] It has been shown that the maximal dimension of Kähler submanifolds of a quaternionic Kähler manifold of dimension n is $n/2$ [26], therefore we are considering the maximal case here. See also [27] for a recent discussion on methods to obtain Kähler manifolds from quaternionic manifolds through quotients.

and N=1 Lagrangians. Similarly, the reduced vielbein splits in tangent and normal components, and the former can be identified with the vielbein on the submanifold, again modulo a normalization factor. Furthermore, the first two complex structure vanish, $\mathcal{J}^1_{I_\parallel J_\parallel}| = \mathcal{J}^2_{I_\parallel J_\parallel}| = 0$, while the third one is proportional to the complex structure of the submanifold. More precisely, one finds:

$$g_{I_\parallel J_\parallel} = -2\lambda\, \mathcal{g}_{I_\parallel J_\parallel}|\,,\;\; e^a_{I_\parallel} = \sqrt{-2\lambda}\,\mathcal{U}^{1a}_{I_\parallel}|\,,\;\; J_{I_\parallel J_\parallel} = -2\lambda\, \mathcal{J}^3_{I_\parallel J_\parallel}|\,. \tag{4.5}$$

The $U(n)$ connection $\Gamma^a_{I_\parallel\, b}$ can be identified with a definite linear combination of the reduced $SU(2)$ and $SP(2n)$ connections: $\Gamma^a_{I_\parallel\, b} = \omega^3_{I_\parallel}|\delta^a_b + \Delta^a_{I_\parallel\, b}|$. The curvature two-form is then similarly given by $R^a_{I_\parallel J_\parallel b} = K^3_{I_\parallel J_\parallel}|\delta^a_b + \Sigma^a_{I_\parallel J_\parallel b}|$. Finally, the reduced Riemann tensor is proportional to the curvature tensor of the submanifold:

$$R_{I_\parallel J_\parallel P_\parallel Q_\parallel} = -2\lambda \mathcal{R}_{I_\parallel J_\parallel P_\parallel Q_\parallel}|\,. \tag{4.6}$$

The curvature on the submanifold can be more explicitly expressed in terms of the symmetric tensor characterizing the original quaternionic geometry. More precisely, what matters is the suitably rescaled component with two holomorphic and two antiholomorphic tangent-space indices, defined as:

$$Y_{a\bar b c\bar d} = -\frac{1}{2\lambda}\mathcal{W}_{a\bar b c\bar d}|\,. \tag{4.7}$$

The curvature two-form then reads $R^{a\bar b}_{I_\parallel J_\parallel} = -\frac{i}{2} J_{I_\parallel J_\parallel}\delta^{a\bar b} + e^c_{[I_\parallel}\, e^{\bar d}_{J_\parallel]}\big(\delta^a_c\delta^{\bar b}_{\bar d} + 2\, Y^{a\bar b}{}_{c\bar d}\big)$. The curvature tensor with flat indices correspondingly takes the simple form:

$$R_{a\bar b c\bar d} = \delta_{a(\bar b}\,\delta_{c\bar d)} + Y_{a\bar b c\bar d}\,. \tag{4.8}$$

Finally, to write its version with curved indices, it is convenient to switch to complex coordinates that are compatible with the complex structure, $I_\parallel \to i, \bar\imath$, in which $e^a_i \neq 0$ but $e^a_{\bar\imath} = 0$. One then finds:

$$R_{i\bar\jmath p\bar q} = g_{i(\bar\jmath}\, g_{p\bar q)} + Y_{i\bar\jmath p\bar q}\,, \tag{4.9}$$

where now:

$$Y_{i\bar\jmath p\bar q} = e^a_i e^{\bar b}_{\bar\jmath} e^c_p e^{\bar d}_{\bar q}\, Y_{a\bar b c\bar d}\,. \tag{4.10}$$

We see that the Ricci and the scalar curvatures have a universal part plus a contribution from the contractions $Y_{a\bar b} = -\delta^{c\bar d} Y_{a\bar b c\bar d}$ and $Y_{\rm sca} = \delta^{a\bar b} Y_{a\bar b}$: $R_{i\bar\jmath} = -\frac{1}{2}(n+1) g_{i\bar\jmath} + Y_{i\bar\jmath}$ and $R_{\rm sca} = -\frac{1}{2}n(n+1) + Y_{\rm sca}$. The Weyl part of the curvature is instead controlled by the traceless part of the tensor $Y_{a\bar b c\bar d}$. From this we see that the curvature of the Kähler submanifold is a priori arbitrary, since the tensor $Y_{a\bar b c\bar d}$ characterizing the curvature of the original quaternionic manifold is also arbitrary in principle. In particular, there is a priori no restriction on the value of the sectional curvature along a complex direction within the submanifold.

Let us next consider the reduction of the gauging. The graviphoton is of course set to zero by the truncation: $\mathcal{A}_\mu| = 0$. In the situation where the graviphoton is used to gauge an isometry on the quaternionic manifold, consistency imposes some restrictions on the Killing vector k^I and the related Killing prepotentials $\mathcal{P}^x$. More precisely, open finds that one should have $\mathcal{P}^3| = 0$, while $\mathcal{P}^1|$ and $\mathcal{P}^2|$ can be non-vanishing, and similarly that $k^{I_\parallel}| = 0$ while $k^{I_\perp}|$ can be non-vanishing:

$$\mathcal{P}^3| = 0\,, \quad k^{I_\parallel}| = 0\,. \tag{4.11}$$

One can then see that the complex combination $\mathcal{P}^2| - i\mathcal{P}^1|$ can be identified with the covariantly holomorphic section of the truncated theory:

$$L = \mathcal{P}^2| - i\mathcal{P}^1|\,. \tag{4.12}$$

The reduced fermionic shifts split into vanishing components $\mathcal{N}^{1a}| = (\mathcal{N}^{2\bar{a}})^*| = 0$ and non-vanishing components $\mathcal{N}^{2a}| = (\mathcal{N}^{1\bar{a}})^*| \neq 0$. The later can be identified with the fermionic shifts of the truncated theory, modulo a normalization factor: [4]

$$\bar{N}^a = \sqrt{-2\lambda}\,\mathcal{N}^{2a}\,. \tag{4.13}$$

With this identification, one then finds that the reduced potential coincides with the potential of the truncated theory:

$$V = \mathcal{V}|\,. \tag{4.14}$$

Finally, it turns out that the third derivative of the holomorphic section, which enters in the expression for the mass matrix and controls in particular the splitting of sGoldstino masses, can also be more explicitly expressed in terms of the symmetric tensor characterizing the original quaternionic geometry. More precisely, what matters in this case is the suitably rescaled component with four holomorphic tangent-space indices, defined as:

$$Z_{abcd} = -\frac{1}{2\lambda}\mathcal{W}_{abcd}|\,. \tag{4.15}$$

Using the fact that $\mathcal{N}^{A\alpha}$ and $\bar{N}^a$ are defined in terms of a Killing vector, on which one can then act with two covariant derivatives to produce a Riemann tensor, it is straightforward to show that

$$\nabla_a\nabla_b\nabla_c L = -Z_{abcd}\bar{N}^c\,. \tag{4.16}$$

[4]Notice that by using the reality condition satisfied by $\mathcal{U}_I^{A\alpha}$, one finds that $N^{\bar{a}} = -\sqrt{-2\lambda}\,\mathcal{N}^{1\bar{a}}$. As a check, one may verify that the second relation in eq. (3.10) correctly reduces to the second relation in eq. (2.8) when restricted to the Kähler submanifold.

We may now check what happens to the sGoldstino mass matrix of the N=2 theory after the truncation and compare this with the general form of the sGoldstino mass matrix for N=1 theories. After a straightforward computation, one finds that

$$\mathcal{R}^{xy}| = 2\lambda\delta^{xy} - \lambda\big(\sigma_{11}^{(x}\sigma_{22}^{y)} + 2\,\sigma_{12}^{x}\sigma_{12}^{y}\big)Y - \frac{\lambda}{2}\sigma_{22}^{x}\sigma_{22}^{y}Z - \frac{\lambda}{2}\sigma_{11}^{x}\sigma_{11}^{y}\bar{Z}\,, \tag{4.17}$$

$$\mathcal{P}^{x}| = \frac{i}{2}\big(\sigma_{11}^{x}L + \sigma_{22}^{x}\bar{L}\big)\,, \tag{4.18}$$

where

$$Y = -\frac{Y_{a\bar{b}c\bar{d}}\bar{N}^a N^{\bar{b}}\bar{N}^c N^{\bar{d}}}{(\bar{N}^e N_e)^2}\,,\quad Z = -\frac{Z_{abcd}\bar{N}^a\bar{N}^b\bar{N}^c\bar{N}^d}{(\bar{N}^e N_e)^2}\,. \tag{4.19}$$

From this we deduce that the reduced mass matrix $\mathscr{m}^{2xy}|$ is block diagonal, and recalling that $|\mathcal{N}|^2 = -\frac{1}{\lambda}|N|^2$ we see that its non-vanishing entries are $(\hat{x},\hat{y} = 1,2)$:

$$\mathscr{m}^{2\hat{x}\hat{y}}| = \begin{pmatrix} \big(-1+Y-\mathrm{Re}Z\big)|N|^2+4(\mathrm{Re}L)^2 & \mathrm{Im}Z\,|N|^2+4\,\mathrm{Re}L\,\mathrm{Im}L \\ \mathrm{Im}Z\,|N|^2+4\,\mathrm{Re}L\,\mathrm{Im}L & \big(-1+Y+\mathrm{Re}Z\big)|N|^2+4(\mathrm{Im}L)^2 \end{pmatrix}, \tag{4.20}$$

$$\mathscr{m}^{233}| = \big(-1-2Y\big)|N|^2+4|L|^2\,. \tag{4.21}$$

The three eigenvalues of this mass matrix are easily found to be

$$\mathscr{m}_{\pm}^2| = \big(-1+Y\big)|N|^2+2|L|^2 \pm \big|Z|N|^2-2\bar{L}^2\big|\,, \tag{4.22}$$

$$\mathscr{m}_{\rm pro}^2| = \big(-1-2Y\big)|N|^2+4|L|^2\,. \tag{4.23}$$

The first two eigenvalues correspond to the two sGoldstini of the N=1 truncated theory: $m_{\pm}^2 = \mathscr{m}_{\pm}^2|$. Indeed, they take the expected form

$$m_{\pm}^2 = R|N|^2 + 2|L|^2 \pm \big|\Delta|N|^2 - 2\bar{L}^2\big|\,, \tag{4.24}$$

in terms of the holomorphic sectional curvature R and the third covariant derivative of the section Δ evaluated along the supersymmetry breaking direction, which as a consequence of eqs. (4.9) and (4.16) are related to Y and Z as follows:

$$R = -1+Y\,,\quad \Delta = Z\,. \tag{4.25}$$

The third eigenvalue instead corresponds to the remaining original sGoldstino that is projected out from the N=1 theory: $m_{\rm pro}^2 = \mathscr{m}_{\rm pro}^2|$. Summarizing, the average mass $m_{\rm avr}^2 = \frac{1}{2}(m_+^2 + m_-^2)$ of the two surviving sGoldstini and the mass $m_{\rm pro}^2$ of the projected sGoldstino are then given by the following simple expressions in terms of V and $m_{3/2}^2$:

$$m_{\rm avr}^2 = R\,V + \big(3R+2\big)m_{3/2}^2\,, \tag{4.26}$$

$$m_{\rm pro}^2 = -\big(2R+3\big)V - \big(6R+5\big)m_{3/2}^2\,. \tag{4.27}$$

As a check, we can verify that these satisfy the sum rule of N=2 theories:

$$\frac{2}{3}\, m^2_{\rm avr} + \frac{1}{3}\, m^2_{\rm pro} = -\, V - \frac{1}{3}\, m^2_{3/2}\,. \tag{4.28}$$

The above analysis provides the link between the metastability condition for N=2 theories and the one for N=1 theories, and shows what may happen in a truncation. [5] As in the case of general N=1 theories, the quantity $m^2_{\rm avr}$ defines by construction an upper bound to the square mass of the lightest scalar, and in order to have a metastable supersymmetry breaking vacuum with $V > 0$, one needs the sectional curvature R to satisfy the bound

$$R > -\frac{2}{3 + V/m^2_{3/2}} > -\frac{2}{3}\,. \tag{4.29}$$

This represents a necessary condition for the existence of metastable de Sitter vacua on the geometry of the Kähler submanifold. In this case, however, this condition is only necessary and no-longer sufficient. This is because it may now no-longer be possible to suitably tune the splitting between the two sGoldstino masses, since the superpotential is no-longer arbitrary. More precisely, the quantity Δ controlling this splitting now also has a geometrical expression, and its value may be restricted. In fact, the mass splitting $\Delta m^2 = \frac{1}{2}(m_+^2 - m_-^2)$ between the two surviving sGoldstini is given by the following simple expression in terms of V, $m^2_{3/2}$, and the phase δ of the section L:

$$\Delta m^2 = \big|e^{2i\delta} Z V + \big(3 e^{2i\delta} Z - 2\big) m^2_{3/2}\big|\,. \tag{4.30}$$

In order for this to be smaller that the average mass, so that none of the two sGoldstini is tachyonic, the quantity $e^{2i\delta} Z$ should satisfy the following bound:

$$\bigg|e^{2i\delta} Z - \frac{2}{3 + V/m^2_{3/2}}\bigg| < R + \frac{2}{3 + V/m^2_{3/2}}\,. \tag{4.31}$$

This represents another necessary condition for the existence of metastable de Sitter vacua, this time on the geometry of the space complementary to the Kähler submanifold. Indeed, the possible values of Z are already constrained by the knowledge of the quantities Z_{abcd}, even if one treats $\bar N^a$ as an arbitrary vector. Of course, the fact that this vector is actually related to a Killing vector then further constrains the result. In other words, some crucial restriction on the ability of tuning the superpotential to make the sGoldstino mass splitting sufficiently small already descends from the form of the geometry of the original quaternionic manifold, independently of the knowledge of which isometries this may admit.

[5] A similar analysis was previously performed in [7] in the rigid limit, where N=2 theories can be viewed as particular N=1 theories even without performing any truncation.

5 Models with one hypermultiplet

Let us now study the problem in the simplest possible context of N=2 theories with one hypermultiplet and a graviphoton gauging. For this we need to consider a generic quaternionic manifold of minimal dimension four possessing at least one isometry. Such a theory can then be truncated to an N=1 theory with one chiral multiplet, which is again the simplest possible case in this family, by selecting a suitable Kähler submanifold.

5.1 Przanowski-Tod spaces

Four-dimensional quaternionic manifolds possessing at least one isometry are well known and their most general realization goes under the name of Przanowski-Tod spaces [30, 31]. With a suitable choice of real local coordinates $q^I = r, u, v, t$ and with overall normalization corresponding to a scalar curvature $\mathcal{R}_{\rm sca} = 24\lambda$, the line element takes the following general form:

$$ds^2 = -\frac{1}{4\lambda r^2}\Big(f dr^2 + f e^h (du^2 + dv^2) + f^{-1}(dt + \Theta)^2\Big)\,. \tag{5.1}$$

This depends on a single function h of the three variables r, u, v, which must satisfy the three-dimensional Toda equation:

$$h_{uu} + h_{vv} + (e^h)_{rr} = 0\,. \tag{5.2}$$

The function f is then related to the function h by

$$f = 2 - r h_r\,. \tag{5.3}$$

Finally, the 1-form Θ is determined, modulo an irrelevant exact form, by the following equation, whose integrability is guaranteed by the Toda equation:

$$d\Theta = \big(f_u\, dv - f_v\, du\big) \wedge dr + (f e^h)_r\, du \wedge dv\,. \tag{5.4}$$

In this general parametrization, the manifest isometry acts as a constant real shift on the variable t.

In appendix A, we give a detailed account of the geometry of these Przanowski-Tod spaces. We describe the most general way of parametrizing the vielbeins and explain how one may then compute explicitly the $SU(2)$ and $SP(2)$ connections as well as their curvatures. We present explicit results in two different parametrizations, which allow us to analyze two different kinds of truncations.

5.2 Lagrangian and masses

The N=2 theory with a single hypermultiplet spanning a Przanowski-Tod space has a kinetic term for the four real fields $q^I = r, u, v, t$ with the following metric:

$$g_{IJ} = -\frac{1}{4\lambda r^2}\begin{pmatrix} f+f^{-1}\Theta_r^2 & f^{-1}\Theta_r\Theta_u & f^{-1}\Theta_r\Theta_v & f^{-1}\Theta_r \\ f^{-1}\Theta_u\Theta_r & fe^h+f^{-1}\Theta_u^2 & f^{-1}\Theta_u\Theta_v & f^{-1}\Theta_u \\ f^{-1}\Theta_v\Theta_r & f^{-1}\Theta_v\Theta_u & fe^h+f^{-1}\Theta_v^2 & f^{-1}\Theta_v \\ f^{-1}\Theta_r & f^{-1}\Theta_u & f^{-1}\Theta_v & f^{-1} \end{pmatrix}, \tag{5.5}$$

The inverse of this metric, which will be relevant below, is easily computed and takes the following form:

$$g^{IJ} = -4\lambda r^2\begin{pmatrix} f^{-1} & 0 & 0 & -f^{-1}\Theta_r \\ 0 & f^{-1}e^{-h} & 0 & -f^{-1}e^{-h}\Theta_u \\ 0 & 0 & f^{-1}e^{-h} & -f^{-1}e^{-h}\Theta_v \\ -f^{-1}\Theta_r & -f^{-1}e^{-h}\Theta_u & -f^{-1}e^{-h}\Theta_v & f+f^{-1}(\Theta_r^2+e^{-h}(\Theta_u^2+\Theta_v^2)) \end{pmatrix}. \tag{5.6}$$

We notice that in order for the kinetic energy to be positive the metric should to be positive definite, and for this we need the function f to be positive: $f > 0$.

We now want to generate a potential through the gauging of an isometry with the graviphoton. For this, we may use the isometry that universally occurs in Przanowski-Tod spaces. In the coordinates that we have used, this amounts to a simple shift of the t coordinates, and the corresponding Killing vector has a single constant non-vanishing component with overall normalization corresponding to the coupling constant: $k^t = g$. The shift vector is then given by

$$\mathcal{N}^I = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 2g \end{pmatrix}. \tag{5.7}$$

It is straightforward to verify that the Killing vector k^I is triholomorphic. The form of the corresponding Killing prepotentials $\mathcal{P}^x$ depends on the choice of parametrization, but their norm is always given by

$$\sqrt{\mathcal{P}^x\mathcal{P}^x} = \frac{g}{2r}. \tag{5.8}$$

By gauging this isometry with the graviphoton one obtains a potential energy given by (3.11), which yields in this case [32]

$$\mathcal{V} = \frac{g^2}{r^2}\left(\frac{1}{f} - \frac{3}{4}\right). \tag{5.9}$$

The supersymmetry breaking scale $M_{\rm susy} = |\mathcal{N}|$ and the gravitino mass $m_{3/2} = |\mathcal{P}|$ are instead given by

$$m_{3/2} = \frac{g}{2r}\,, \quad M_{\rm susy} = \frac{g}{f^{1/2}\,r}\,. \tag{5.10}$$

Let us now study the properties of a generic supersymmetry breaking vacuum in this theory and compare them with the general results discussed in the previous sections. We first notice that $\mathcal{V}$ does not depend on t, reflecting the fact that this is the would-be Goldstone boson associated to the shift symmetry. The dependence of $\mathcal{V}$ on r, u, v, on the other hand, is constrained by the fact that f is related to a function h that has to satisfy the Toda equation. The stationarity condition $\mathcal{V}_I = 0$ is trivially satisfied for the field t but yields three non-trivial equations that must be satisfied on the vacuum for the fields r, u, v:

$$f_r = -2f^2 r g^{-2}\mathcal{V}\,, \quad f_u = 0\,, \quad f_v = 0\,. \tag{5.11}$$

At such a point, the mass matrix $\mathpzc{m}^2_{IJ} = -\frac{1}{2\lambda}\mathcal{V}_{IJ}$ is then found to be:

$$\mathpzc{m}^2_{IJ} = -\frac{g^2}{2\lambda r^2 f^2}\begin{pmatrix} \xi - f_{rr} & -f_{ru} & -f_{rv} & 0 \\ -f_{ur} & -f_{uu} & -f_{uv} & 0 \\ -f_{vr} & -f_{vu} & -f_{vv} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\,, \tag{5.12}$$

where

$$\xi = -\frac{3f^3}{2r^2}\big(r^2 g^{-2}\mathcal{V} - 4r^4 g^{-4}\mathcal{V}^2\big)\,. \tag{5.13}$$

We may now compute the quantity $\mathpzc{m}^2_{\rm avr}$ as simply $\frac{1}{3}$ of the trace of this matrix computed with the inverse metric (5.6), that is $\mathpzc{m}^2_{\rm avr} = \frac{1}{3}\mathpzc{g}^{IJ}\mathpzc{m}^2_{IJ}$. One finds:

$$\mathpzc{m}^2_{\rm avr} = \frac{2g^2}{3f^3}\big(\xi - f_{rr} - e^{-h} f_{uu} - e^{-h} f_{vv}\big)\,. \tag{5.14}$$

But using the definition of f and the Toda equation satisfied by h, it is straightforward to show that $f_{rr} + e^{-h} f_{uu} + e^{-h} f_{vv} = -2h_{rr} + rh_r\big(3h_{rr} + h_r^2\big)$. Using then the form of the potential in terms of f and thus h as well as the stationarity conditions, one easily verifies that this further implies the following result:

$$f_{rr} + e^{-h} f_{uu} + e^{-h} f_{vv} = \frac{f^3}{8\,r^2}\big(1 + 48r^4 g^{-4}\mathcal{V}^2\big)\,. \tag{5.15}$$

It then follows that

$$\mathpzc{m}^2_{\rm avr} = -\frac{g^2}{12r^2}\big(1 + 12r^2 g^{-2}\mathcal{V}\big)\,. \tag{5.16}$$

Finally, recalling the form of the potential and the gravitino mass, this can be brought into the form that has been shown to hold for the average sGoldstino mass in any theory with only hypermultiplets and a graviphoton gauging, namely

$$m^2_{\rm avr} = -\,\mathcal{V} - \frac{1}{3}\, m^2_{3/2}\,. \tag{5.17}$$

As already said, this result implies that for any supersymmetry breaking de Sitter vacuum at least one of the scalars is tachyonic and makes the vacuum unstable.

5.3 Truncation of the first kind

A first possibility for selecting a Kähler submanifold of Przanowski-Tod spaces consists in keeping the coordinates $q^{I_\parallel} = r, t$ and discarding the coordinates $q^{I_\perp} = u, v$. A parametrization of the original space that is suitable to study this kind of truncation is described in appendix A.2. With the help of the explicit results reported there, and using the short-hand notation $z = u + iv$, we can study very explicitly under which conditions the truncation is consistent. The condition $\mathcal{U}^{21}| = 0$ is automatically satisfied (since $b| = 0$). Moreover, since $d\Theta| = 0$ we can locally choose that $\Theta| = 0$ in the surviving vielbein (given by $a|$). We then see that the conditions $\omega^1| = \omega^2| = 0$ are also automatically satisfied, while the condition $\Delta^1{}_{\bar 1}| = 0$ (where the values $1, \bar 1$ of the flat indices correspond to the values $1, 2$) implies that $f_z| = 0$. The involution condition $\mathcal{W}_{111\bar 1}| = 0$ is then also automatically satisfied. The only condition for the truncation to be consistent is thus that:

$$f_z| = 0\,. \tag{5.18}$$

One may now compute the relevant quantities characterizing this truncation of the geometry, by using the results collected in appendix A.2 and the definitions of section 4. The line element $ds^2 = -2\lambda\, d\mathcal{s}^2|$ is

$$ds^2 = \frac{1}{2r^2}\Big(f|\, dr^2 + f^{-1}|dt^2\Big)\,. \tag{5.19}$$

The $U(1)$ connection $\Gamma^1{}_1 = \omega^3| + \Delta^1{}_1|$ reads

$$\Gamma^1{}_1 = \frac{2f^2 + i\big(4f - f^2 + 2rf_r\big)}{4rf^2}\bigg|\, dt\,. \tag{5.20}$$

The Kähler form $J = -2\lambda \mathcal{J}^3|$ and the two distinct quantities $Y_{1\bar 1 1\bar 1} = -\frac{1}{2\lambda}\mathcal{W}_{1\bar 1 1\bar 1}|$ and $Z_{1111} = -\frac{1}{2\lambda}\mathcal{W}_{1111}|$ controlling the curvature are finally given by

$$J = \frac{1}{2r^2}\, dr \wedge dt\,, \tag{5.21}$$

$$Y_{1\bar 1 1\bar 1} = \frac{r^2\big(rfh_{rrr} - fh_r^2 + 2f_r^2\big)}{f^3}\bigg|\,, \tag{5.22}$$

$$Z_{1111} = -\frac{r^2 e^{-h} f_{\bar z\bar z}}{f^2}\bigg|\,. \tag{5.23}$$

The kinetic term of the truncated theory is controlled by the metric on the selected submanifold. More explicitly, this metric and its inverse are given by the following expressions in terms of the real coordinates $q^{I_\parallel} = r, t$:

$$g_{I_\parallel J_\parallel} = \frac{1}{2r^2} \begin{pmatrix} f| & 0 \\ 0 & f^{-1}| \end{pmatrix} \,, \quad g^{I_\parallel J_\parallel} = 2r^2 \begin{pmatrix} f^{-1}| & 0 \\ 0 & f| \end{pmatrix} \,. \tag{5.24}$$

It should then be emphasized that the positivity of the kinetic term implies the condition $f| > 0$.

To generate a potential, one may now try to gauge the manifest shift symmetry that arises in any Przanowski-Tod space. But unfortunately this turns out to be inconsistent with the type of truncation studied in this subsection. Indeed, the tangent and orthogonal components of the Killing vector are given by $k^r = 0$, $k^t = g$ and $k^u = 0$, $k^v = 0$, and with the adopted parametrization the Killing prepotentials are found to be given by $\mathcal{P}^1 = 0$, $\mathcal{P}^2 = 0$ and $\mathcal{P}^3 = g/(2r)$. We then see that $k^{I_\parallel}| \neq 0$ and also $\mathcal{P}^3| \neq 0$, unless g vanishes. In this kind of truncation, the gauging of the shift symmetry is therefore impossible. The only way to generate a potential would then be to rely on some additional isometry that might arise in specific cases. To describe in full generality such a possibility, one may consider the particular subset of Przanowski-Tod spaces which admit a second commuting isometry. The general spaces with this property are also well known and go under the name of Calderbank-Pedersen spaces [33]. They can be parametrized in terms of a simpler potential defined through a function that depends only on two of the coordinates and now satisfies a linear partial differential equation. We will however not attempt to describe in full generality the theories that one can get in this way.

5.4 Truncations of the second kind

A second possibility for selecting a Kähler submanifold of Przanowski-Tod spaces consists in keeping the coordinates $q^{I_\parallel} = r, v$ and discarding instead the coordinates $q^{I_\perp} = u, t$. A parametrization of the original space that is suitable to analyze this kind of truncation is described in appendix A.3. With the help of the explicit results reported there, we can as before study under which conditions the truncation is consistent. The condition $\mathcal{U}^{21}| = 0$ (which is $b| = 0$) implies that $\Theta| = 0$. In order for this to be locally possible, one then needs to have $d\Theta| = 0$. Since $d\Theta| = f_u|\, dv \wedge dr$, we see that this implies the condition $f_u| = 0$. The conditions $\omega^1| = \omega^2| = 0$ and $\Delta^1{}_{\bar{1}}| = 0$ (where again the values $1, \bar{1}$ of the flat indices correspond to the values $1, 2$) now require the stronger condition $h_u| = 0$ to hold true. The involution condition $\mathcal{W}_{111\bar{1}}| = 0$ is then also automatically satisfied. The only condition for the truncation to be consistent is thus that:

$$h_u| = 0 \,. \tag{5.25}$$

One may now as before compute the relevant quantities characterizing this truncation of the geometry, by using the results collected in appendix A.3 and the definitions of section 4. The line element $ds^2 = -2\lambda d\mathsf{s}^2|$ is

$$ds^2 = \frac{1}{2r^2}\Big(f|\,dr^2 + fe^h|dv^2\Big)\,. \tag{5.26}$$

The $U(1)$ connection $\Gamma^1{}_1 = \omega^3| + \Delta^1{}_1|$ reads

$$\Gamma^1{}_1 = \frac{e^{-h/2}\big(if_v\big)}{2f}\bigg|\,dr + \frac{e^{h/2}\big(2f + i\big(f^2 - f - rf_r\big)\big)}{2rf}\bigg|\,dv\,. \tag{5.27}$$

The Kähler form $J = -2\lambda\mathcal{J}^3|$ and the two distinct quantities $Y_{1\bar{1}1\bar{1}} = -\frac{1}{2\lambda}\mathcal{W}_{1\bar{1}1\bar{1}}|$ and $Z_{1111} = -\frac{1}{2\lambda}\mathcal{W}_{1111}|$ controlling the curvature are finally given by

$$J = \frac{fe^{h/2}}{2r^2}\bigg|\,dr\wedge dv\,, \tag{5.28}$$

$$\begin{aligned} Y_{1\bar{1}1\bar{1}} = \frac{r^2}{2f^3}\Big[&-\Big(rf\big(2h_{rrr} + 3h_rh_{rr} + h_r^3\big) - fh_r^2 + 2f_r^2\Big) \\ &+ e^{-h}\Big(f\big(2f_{vv} - f_vh_v\big) - 2f_v^2\Big)\Big]\Big|\,, \end{aligned} \tag{5.29}$$

$$\begin{aligned} Z_{1111} = \frac{r^2}{2f^3}\Big[&\Big(rf\big(2h_{rrr} - 3h_rh_{rr} - h_r^3\big) - 3fh_r^2 + 6f_r^2\Big) \\ &+ 2ie^{-h/2}\Big(f\big(fh_{rv} + 2f_{rv}\big) - 3\big(fh_r + 2f_r\big)f_v\Big) \\ &+ e^{-h}\Big(f\big(2f_{vv} - f_vh_v\big) - 6f_v^2\Big)\Big]\Big|\,. \end{aligned} \tag{5.30}$$

The kinetic term of the truncated theory is controlled by the metric on the selected submanifold. More explicitly, this metric and its inverse are given by the following expressions in terms of the real coordinates $q^{I_\parallel} = r, v$:

$$g_{I_\parallel J_\parallel} = \frac{1}{2r^2}\begin{pmatrix} f| & 0 \\ 0 & fe^h| \end{pmatrix}\,,\quad g^{I_\parallel J_\parallel} = 2r^2\begin{pmatrix} f^{-1}| & 0 \\ 0 & f^{-1}e^{-h}| \end{pmatrix}\,. \tag{5.31}$$

Once again, it should be recalled that the positivity of the kinetic term implies the condition $f| > 0$.

To generate a potential, one may again try to gauge the shift symmetry that arises in any Przanowski-Tod space. Fortunately, this turns out to be consistent with the type of truncation studied in this subsection. Indeed, the tangent and orthogonal components of the Killing vector are given by $k^r = 0$, $k^v = 0$ and $k^u = 0$, $k^t = g$, and with the adopted parametrization the Killing prepotentials are found to be given by $\mathcal{P}^1 = g/(2r)$, $\mathcal{P}^2 = 0$ and $\mathcal{P}^3 = 0$. We then see that $k^{I_\parallel}| = 0$ and also $\mathcal{P}^3| = 0$, independently of the value of g. In this kind of truncation, the gauging of the shift symmetry is therefore possible, and we shall now study the form of the resulting potential. Of course, one may also try to gauge other additional isometries whenever these arise, like for instance in Calderbank-Pedersen spaces, but we shall not study this possibility. Coming back to the gauging of the manifest shift symmetry of t in any Przanowski-Tod space, we see that this induces the following non-trivial result for the section $L = \mathcal{P}^2| - i\mathcal{P}^1|$:

$$L = -i\frac{g}{2r}\,. \tag{5.32}$$

The corresponding scalar potential of the truncated theory is then a function of the real fields r and v, which is just the restriction of the N=2 potential (5.9):

$$V = \frac{g^2}{r^2}\bigg(\frac{1}{f} - \frac{3}{4}\bigg)\bigg|\,. \tag{5.33}$$

The gravitino mass and the supersymmetry breaking scale are instead given by

$$m_{3/2} = \frac{g}{2r}\,,\quad M_{\rm susy} = \frac{g}{f^{1/2}r}\bigg|\,. \tag{5.34}$$

Let us finally study the properties of the stationary points of the above potential for the truncated theory and investigate the possibility of getting metastable de Sitter vacua. The stationarity conditions $V_{I_\parallel} = 0$ with respect to the two fields r, v imply respectively the following conditions on the vacuum:

$$f_r| = -2f^2 r g^{-2} V\big|\,,\quad f_v| = 0\,. \tag{5.35}$$

At such a point, the mass matrix $m^2_{I_\parallel J_\parallel} = V_{I_\parallel J_\parallel}$ is then found to be

$$m^2_{I_\parallel J_\parallel} = \frac{g^2}{r^2 f^2}\begin{pmatrix} \xi - f_{rr} & -f_{rv} \\ -f_{vr} & -f_{vv}\end{pmatrix}\bigg|\,, \tag{5.36}$$

where now

$$\xi = -\frac{3f^3}{2r^2}\big(r^2 g^{-2} V - 4r^4 g^{-4} V^2\big)\Big|\,. \tag{5.37}$$

To compute the two physical masses, one needs to take into account the non-trivial metric, which is given by (5.31). The simplest way to do so is to first raise an index of the mass matrix with the inverse metric to compute ${m^{2I_\parallel}}_{J_\parallel}$, and then derive the two physical masses as the eigenvalues of this matrix: $m^2_\pm = \text{eigenvalues}\big({m^{2I_\parallel}}_{J_\parallel}\big)$. By doing so one arrives at the following expression, which can be verified to match the general result (4.24) with the relations (4.25) after using the stationarity condition and the Toda equation:

$$m^2_\pm = \frac{g^2}{f^3}\Big[\big(\xi - f_{rr} - e^{-h} f_{vv}\big) \pm \sqrt{\big(\xi - f_{rr} + e^{-h} f_{vv}\big)^2 + 4\,e^{-h} f_{rv}^2}\,\Big]\Big|\,. \tag{5.38}$$

The necessary and sufficient conditions for these masses $m^2_\pm = m^2_{\rm avr} \pm \Delta m^2$ to be positive are that $m^2_{\rm avr} > 0$ and $\Delta m^2 < m^2_{\rm avr}$. This requires that $(f_{rr} + e^{-h} f_{vv})| < \xi$

and $f_{rv}^2| < \big(\xi - f_{rr}\big) f_{vv}$. Equivalently, one needs the two diagonal elements of the mass matrix to be positive and the off-diagonal element to be smaller than their geometrical meaning absolute value, which means:

$$f_{rr}| < \xi \,, \;\; f_{vv}| < 0 \,, \;\; |f_{rv}|| < \sqrt{(f_{rr} - \xi) f_{vv}} \,\Big| \,. \tag{5.39}$$

We also note that the mass $m_{uu}^2 = -\frac{1}{2\lambda} \mathcal{V}_{uu}|$ that the non-trivially projected mode would have in the N=2 theory is given by $m_{uu}^2 = \frac{1}{2\lambda} g^2 r^{-2} f^{-2} f_{uu}|$. Taking into account that $g^{uu} = -4\lambda\, r^2 f^{-1} e^{-h}|$, the physical mass of this mode is found to be:

$$m_{\rm pro}^2 = -\frac{2g^2}{f^3} e^{-h} f_{uu} \bigg| \,. \tag{5.40}$$

As a check, we notice that the average of all the three masses m_+^2, m_-^2 and $m_{\rm pro}^2$ is given by $\frac{2}{3} m_{\rm avr}^2 + \frac{1}{3} m_{\rm pro}^2$ and matches (5.14), which was shown to take the universal negative value $-\mathcal{V} - \frac{1}{3} m_{3/2}^2$. We then know that $m_{\rm avr}^2 = -\frac{3}{2}\, V - \frac{1}{2} m_{3/2}^2 - \frac{1}{2} m_{\rm pro}^2$, and achieving $m_{\rm avr}^2 > 0$ thus requires that $m_{\rm pro}^2 < -3\, V - m_{3/2}^2$. We can verify that this bound on the square mass of the projected state indeed follows from the conditions (5.39) ensuring the positivity of the square masses of the retained states, as a consequence of the relation (5.15). Taking the reversed logic, we can say that this bound on $m_{\rm pro}^2$ represents an alternative form of the condition for $m_{\rm avr}^2$ to be positive, which exploits the fact that h must satisfy the Toda equation. More precisely, this condition takes the following form, which shows in a very simple and direct way that metastable Sitter vacua are forbidden whenever the function f does not depend on the variable u, no matter how clever the dependence on r and v is:

$$f_{uu}| > \frac{f^3}{8r^2} e^h \big(1 + 12 r^2 g^{-2} V\big) \bigg| \,. \tag{5.41}$$

We finally conclude that in this truncation it is possible for metastable de Sitter vacua to exist, but only for suitable choices of h and thus f. For example, to achieve the simplest and most interesting situation of a metastable vacuum with $V \simeq 0$, one needs to find a point where $f \simeq \frac{4}{3}$, then $f_r| \simeq 0$, $f_v| = 0$, $f_u| = 0$ and finally $f_{rr}| \lesssim 0$, $f_{vv}| \lesssim 0$, $|f_{rv}|| \lesssim \sqrt{f_{rr} f_{vv}}|$, $r^2 e^{-h} f_{uu}| \gtrsim \frac{8}{27}$, $f_{ru}| = 0$, $f_{vu}| = 0$.

5.5 Canonical Kähler coordinates

The two types of truncations described in the previous subsections define N=1 theories. These have been parametrized by using a pair of real coordinates $q^{I_\parallel}$, but it should in principle be possible to find a canonical complex coordinate ϕ, and then deduce a real Kähler potential $K(\phi, \bar\phi)$ and a holomorphic superpotential $W(\phi)$. More precisely, this complex coordinate should be such that the Kähler form J and the line element ds^2 on the submanifold take the canonical forms

$$J = i\, g_{\phi\bar\phi}\, d\phi \wedge d\bar\phi \,, \;\; ds^2 = 2\, g_{\phi\bar\phi}\, d\phi \otimes d\bar\phi \,. \tag{5.42}$$

Once this coordinate has been found, one can derive the Kähler potential and the superpotential from the following relations:

$$g_{\phi\bar{\phi}} = \nabla_\phi \nabla_{\bar{\phi}} K \,, \quad L = e^{K/2} W \,. \tag{5.43}$$

In practice, however, finding the explicit coordinate change from the real variables $q^{I_\parallel}$ to the canonical complex coordinate ϕ, and then deriving the forms of K and W, is not so easy. Indeed, this involves solving a differential equation, and it does not seem to be possible to find a closed form solution for a completely general Przanowski-Tod space parametrized in terms of the real potential h. On the other hand, there also exists a different parametrization of such spaces where complex coordinates are used from the beginning and a different potential satisfying a slightly different differential equation is involved [34]. Here we will not attempt to discuss any further the general derivation of the canonical coordinate ϕ and the explicit forms of K and W, since as we saw this is not needed to study the physical properties of the truncated model. Instead, we shall describe how this works in some simple examples in next section.

6 Specific examples

To illustrate the results derived in previous section for a generic N=2 theory with a single hypermultiplet truncated to an N=1 theory with a single chiral multiplet, let us now describe a few explicit examples. For this purpose, we shall consider some simple classes of Przanowski-Tod spaces, based on explicit solutions of the Toda equation for h, and study the implementation of the two kinds of truncations that we have described in general terms. We will mostly focus on those cases which are relevant for the description of the low-energy effective dynamics of the so-called universal hypermultiplet of N=2 superstrings, which after truncation maps to the universal dilaton of N=1 superstrings.

Let us provide some further details on the application to ten-dimensional superstrings compactified on a rigid Calabi-Yau manifold, following [32, 35]. We start from the type IIA superstring and focus our attention on the universal sector of scalar fields that always arise as a consequence of the existence of a Kähler two-form and a holomorphic three-form which are harmonic. This sector contains the dilaton ϕ and the axion σ originating from the metric and the Kalb-Ramond antisymmetric field in the NSNS sector, and the two scalars χ and φ originating from the three-index antisymmetric field in the RR sector. We can then identify the Przanowski-Tod coordinates r, u, v, t in the following way in terms of these fields: $r = e^\phi$, $u = \chi$, $v = \varphi$, $t = \sigma$. In this way, the manifest symmetry shifting t in the Przanowski-Tod framework corresponds to a symmetry shifting the field σ, which can indeed be argued to be a good symmetry of the theory under some mild assumptions. Moreover, the gauging of this symmetry and the associated potential can naturally emerge in the string context as the result of a non-vanishing flux for the RR three-form over the Calabi-Yau. Let us then see how the two kinds of truncations we described in previous section act in this framework. The first truncation we considered retains r, t and discards u, v. It therefore corresponds to a kind of heterotic truncation. The second projection we considered retains r, v and discards u, t. It therefore corresponds to an orientifold truncation. This is very similar to what discussed in [36].

6.1 Exact metrics depending on a single coordinate

The simplest class of Przanowski-Tod spaces is defined by a function h that only depends on r and is a solution of the simplified Toda equation $(e^h)_{rr} = 0$. The function f then also depends only on r, and the form Θ has a simple universal expression. The most general solution of this type can in fact by studied exactly and turns out to be parametrized by a single real constant c. Restricting the coordinates to $r > \max\{-c, -2c\}$ in such a way that the metric is real and positive definite, and keeping an arbitrary real and positive integration constant r_0 which could be reabsorbed by a rescaling of the coordinates u and v, one finds

$$h = \log \frac{r+c}{r_0}\,, \quad f = \frac{r+2c}{r+c}\,, \quad \Theta = \frac{1}{2r_0}\big(u\,dv - v\,du\big)\,. \tag{6.1}$$

In the limit $c \to 0$ this space reduces to the $SU(1,2)/(U(1)\times SU(2))$ coset manifold. Choosing $r_0 = 1$, this corresponds to taking $h = \log r$, $f = 1$, $\Theta = \frac{1}{2}\big(u\,dv - v\,du\big)$. In the limit $c \to +\infty$ this space reduces to the $SO(1,4)/SO(4)$ coset manifold. Choosing $r_0 = c$ to simplify things, this corresponds to taking $h = 0$, $f = 2$, $\Theta = 0$. In the limit $c \to -\infty$ this space remains as a complicated non-symmetric space, because the coordinate r cannot be kept finite.

It was show in [37, 38, 39] that the above space describes the low-energy effective theory of the universal hypermultiplet if one ignores non-perturbative effects arising from two-branes and five-branes. The constant c parametrizes the perturbative quantum corrections to the classical dynamics, and can be vanishing, positive or negative, depending on the topology of the Calabi-Yau manifold. The classical moduli space is thus maximally symmetric, while the perturbative corrections lift some isometries and leave only four of them. However, it is important to emphasize that the above space is truly quaternionic even for finite and large values of c. [6]

The N=2 theory based on the above space has kinetic terms that are controlled by a metric whose inverse has the following non-trivial entries for the fields r, u, v: $g^{rr} = -4\lambda r^2(r+c)/(r+2c)$, $g^{uu} = -4\lambda r^2 r_0/(r+2c)$, $g^{vv} = -4\lambda r^2 r_0/(r+2c)$. The gauging of the symmetry shifting t then produces a potential, but this depends only

[6] See also [40] for a study of the rigid limit of this corrected moduli space.

on r and not on u and v. Its explicit form is:

$$\mathcal{V} = \frac{g^2}{r^2}\left(\frac{r+c}{r+2c} - \frac{3}{4}\right). \tag{6.2}$$

We already argued in full generality that such a potential depending on a single variable cannot admit metastable de Sitter vacua. This is because two of the three sGoldstini are unavoidably massless and actually correspond to the Goldstone modes of the additional spontaneously broken isometries acting as shifts on u and v, and the third one must then necessarily be tachyonic as a consequence of the sum rule (3.16). It is straightforward to verify that this is indeed what happens. For $c > 0$, there is an unstable de Sitter stationary point at $r = (1+\sqrt{5})\,c$ with cosmological constant $\mathcal{V} = \frac{1}{32}(5\sqrt{5}-11)\,g^2c^{-2}$, gravitino mass $m_{3/2}^2 = \frac{1}{32}(3-\sqrt{5})\,g^2c^{-2}$ and physical rescaled masses $\boldsymbol{m}_r^2 = -\frac{1}{16}(7\sqrt{5}-15)\,g^2c^{-2}$, $\boldsymbol{m}_u^2 = 0$, $\boldsymbol{m}_v^2 = 0$, which give as expected $\boldsymbol{m}_{\rm avr}^2 = -\mathcal{V} - \frac{1}{3}m_{3/2}^2$. For $c < 0$ there is no stationary point.

Let us now examine the first kind of N=1 truncation discussed in section 5.3. The consistency condition (5.18) is automatically satisfied. From the expression of the metric in real coordinates, or equivalently from the expression (4.9) for the Riemann tensor, one computes that the sectional curvature is given by $R = -1-(1+2c/r)^{-3}$. For $c > 0$, this decreases from 0 to -1 and finally to -2 when r goes from $-c$ to 0 and then to $+\infty$. For $c < 0$, this increases instead from $-\infty$ to -2 when r goes from $-2c$ to $+\infty$. The canonical complex coordinate can in this case be taken to be $\phi = r + c\log[(r+c)/r_0] + it$, so that $\partial(\phi+\bar\phi)/\partial r = 2(r+2c)/(r+c)$. One then finds $g_{\phi\bar\phi} = \frac{1}{4}r^{-2}(r+c)/(r+2c)$ and $K = -\log\big[(k_0/c)r^2/(r+c)\big]$, where k_0 is an arbitrary real constant related to the ambiguity of Kähler transformations. The inverse relation for r in terms of $\phi+\bar\phi$ can be obtained in terms of the so-called product-logarithm or Lambert function $P(x)$, which is implicitly defined by the relation $P(x)e^{P(x)} = x$. [7] This function is double-valued for $x \in (-1/e, 0)$ and single-valued for $x \in (0, +\infty)$, and the two branches are denoted by $P_0(x)$ and $P_{-1}(x)$. For our purposes, we will however define a single-valued version of $P(x)$ by choosing $P(x) = P_0(x)$ for $x \in (0,+\infty)$ and $P(x) = P_{-1}(x)$ for $x \in (-1/e, 0)$. In terms of this function $P(x)$, one easily finds that $r = c\big[P\big(\frac{c}{r_0}e^{\frac{\phi+\bar\phi+2c}{2c}}\big) - 1\big]$. The Kähler potential then reads:

$$K = -\log\left[k_0\frac{\Big(P\big(\frac{c}{r_0}e^{\frac{\phi+\bar\phi+2c}{2c}}\big)-1\Big)^2}{P\big(\frac{c}{r_0}e^{\frac{\phi+\bar\phi+2c}{2c}}\big)}\right]. \tag{6.3}$$

In the special case of $SU(1,2)/(U(1)\times SU(2))$, which corresponds to the limit $c \to 0$, we choose as before $r_0 = 1$ and set for convenience $k_0 = 2c$. In that case we have $\phi = r + it$ and therefore $r = (\phi+\bar\phi)/2$. The sectional curvature is now constant

[7] This function is usually denoted by $W(x)$, but we shall call it here P(x) to avoid confusion with the superpotential.

and given by $R = -2$. Finally, the Kähler potential is simply $K = -\log\big(\phi + \bar\phi\big)$. In the special case of $SO(1,4)/SO(4)$, which corresponds to the limit $c \to +\infty$, we choose as before $r_0 = c$ and set for convenience $k_0 = 16c^2$. In that case we have $s = 2r + it$ and thus $r = (\phi + \bar\phi)/4$. The sectional curvature is again constant and given by $R = -1$. Finally, the Kähler potential is simply $K = -2\log\big(\phi + \bar\phi\big)$. [8]

For this first kind of N=1 truncation, it is not possible to gauge the symmetry shifting t. One may on the other hand try to gauge one of the other three isometries possessed by the spaces under consideration. We have verified that only two of these isometries can be gauged compatibly with this truncation, and that they both lead to a constant superpotential. The corresponding scalar potential does however not posses any stationary point.

Let us next examine the second kind of N=1 truncation discussed in section 5.4. The consistency condition (5.25) is automatically satisfied. From the expression of the metric in real coordinates, or equivalently from the expression (4.9) for the Riemann tensor, one finds that the sectional curvature is $R = -1 + \frac{1}{2}(1 + 2c/r)^{-3}$. For $c > 0$, this increases from $-\frac{3}{2}$ to -1 and finally to $-\frac{1}{2}$ when r goes from $-c$ to 0 and then to $+\infty$. For $c < 0$, this decreases from $+\infty$ to $-\frac{1}{2}$ when r goes from $-2c$ to $+\infty$. The canonical complex coordinates can in this case be taken to be $\phi = 2\big(\sqrt{r_0(r+c)} - |c|\big) + iv$, so that $\partial(\phi + \bar\phi) = 2\sqrt{r_0/(r+c)}$. One then finds $g_{\phi\bar\phi} = \frac{1}{4}r^{-2}(r+c)/r_0$ and $K = -2\log\big[16k_0 r_0 r\big]$, where k_0 is an arbitrary real constant related to the choice of Kähler frame. The inverse relation for r in terms of $\phi + \bar\phi$ is simply given by $r = \frac{1}{16}r_0^{-1}\big[(\phi + \bar\phi + 4|c|)^2 - 16r_0 c\big]$. The Kähler potential then reads:

$$K = -2\log\Big[k_0\Big(\big(\phi + \bar\phi + 4|c|\big)^2 - 16r_0 c\Big)\Big]\,. \qquad (6.4)$$

In the special case of $SU(1,2)/(U(1) \times SU(2))$ corresponding to the limit $c \to 0$, we choose as before $r_0 = 1$ and take for convenience $k_0 = \frac{1}{4}$. In that case we have $\phi = 2\sqrt{r} + iv$, and thus $r = (\phi + \bar\phi)^2/4$. The sectional curvature is now constant and given by $R = -\frac{1}{2}$. Finally, the Kähler potential is just $K = -4\log\big(\phi + \bar\phi\big)$. [9] In the special case of $SO(1,4)/SO(4)$, which corresponds to the limit $c \to +\infty$, we choose as before $r_0 = c$ and take $k_0 = \frac{1}{8}c^{-1}$. In that case we have $s = r + iv$ and thus $r = (\phi + \bar\phi)/2$. The sectional curvature is constant and given by $R = -1$. Finally, the Kähler potential is simply $K = -2\log\big(\phi + \bar\phi\big)$.

For this second kind of N=1 truncation, it is possible to gauge the symmetry shifting t. The section of the truncated theory then takes the value $L = -ig/(2r)$. Recalling the definition of the complex coordinate ϕ and the form of the Kähler potential that we just derived, and working in terms of the real variable r, one easily finds that $W = -8igk_0 r_0$. Since this is constant, the change to the complex

[8] These results for the two basic coset quaternionic manifolds were also obtained in [41].

[9] This agrees with the general result derived in [42].

coordinate has no effect and one simply finds:

$$W = -8igk_0 r_0\,. \tag{6.5}$$

In the special case of the space $SU(1,2)/(U(1)\times SU(2))$ corresponding to $c\to 0$, with the choices $r_0 = 1$ and $k_0 = \frac{1}{4}$ also used in the study of K, one finds $W = -2ig$. In the special case of the space $SO(1,4)/SO(4)$ corresponding to $c\to +\infty$, with $r_0 = c$ and $k_0 = \frac{1}{8}c^{-1}$, one finds instead $W = -ig$. Notice finally that one may also try to gauge one of the other three isometries possessed by the spaces under consideration. We have verified that two of them can be gauged compatibly with the truncation, and lead to a linear and a quadratic superpotential. In the first case, the scalar potential admits no stationary point, while in the second case, one finds a supersymmetric AdS stationary point. Considering the gauging of a linear combination of the admissible isometries doesn't seem to give any novelty either. We thus stick to the gauging of the symmetry shifting t.

The physics of this simple truncation is very similar to that of the original theory. The kinetic terms are controlled by a metric whose inverse has the following non-trivial entries for the fields r, v: $g^{rr} = 2r^2(r+c)/(r+2c)$, $g^{vv} = 2r^2 r_0/(r+2c)$. The gauging of the symmetry shifting t then produces a potential, but this depends only on r and not on v. This has the same form as in the original theory, since the discarded modes correspond to the would-be Goldstone mode absorbed by the graviphoton and to a flat direction:

$$V = \frac{g^2}{r^2}\left(\frac{r+c}{r+2c} - \frac{3}{4}\right). \tag{6.6}$$

Consequently, the structure of the stationary points of this potential is exactly the same as in the original theory. For $c>0$, there is an unstable de Sitter stationary point at $r = (1+\sqrt{5})\,c$ with $V = \frac{1}{32}(5\sqrt{5}-11)\,g^2c^{-2}$, $m_{3/2}^2 = \frac{1}{32}(3-\sqrt{5})\,g^2c^{-2}$, $R = \frac{1}{2}(\sqrt{5}-4)$ and physical masses $m_r^2 = -\frac{1}{16}(7\sqrt{5}-15)\,g^2c^{-2}$, $m_v^2 = 0$, which give as expected $m_{\rm avr}^2 = RV + (3R+2)m_{3/2}^2$. For $c<0$ there is again no stationary point.

6.2 Approximate metrics depending on three coordinates

A more general class of Przanowski-Tod spaces is defined by a function h that depends not only on r but also on u and v and is a solution of the full Toda equation equation $h_{uu} + h_{vv} + (e^h)_{rr} = 0$. Unfortunately the solutions of such an equation are more difficult to characterize. It has been argued in [32] that one family of such solutions corresponds to a deformation of the one discussed in previous section through terms which are exponentially suppressed for large r. More precisely, the general form of this solution has been shown to be

$$h = \log\left[r + c + \sum_{n=1}^{+\infty}\sum_{m=0}^{+\infty}\kappa_{n,m}(u,v)(r+c)^{1-m/2}e^{-2n\sqrt{r+c}}\right]. \tag{6.7}$$

However, the exact determination of the form that the functions $\kappa_{n,m}(u,v)$ must take in order for this to represent a solution of the Toda equation is a difficult problem, on which some progress has been recently achieved by relying on twistor techniques (see for example [43] for an overview). On the other hand, it is rather straightforward to find approximate solutions of the above form, which solve the Toda equation only to some finite degree of accuracy. This can be done by performing an expansion for large values of r, in such a way that terms with higher and higher values of n and m in (6.7) are more and more suppressed. At leading order in such a large r expansion, one can then take $h \simeq \log(r) + c/r + \kappa(u,v)\, e^{-2\sqrt{r}}$. This approximately satisfies the Toda equation in the limit of large r provided the function κ satisfies the linear equation $\kappa_{uu} + \kappa_{vv} \simeq -\kappa$. We can then take $\kappa(u,v) \simeq A\cos(u+\alpha) + B\cos(v+\beta)$, where A, B, α, β are arbitrary real constants. In this way, we arrive at the following approximate solution:

$$h \simeq \log(r) + \frac{c}{r} + \big(A\cos(u+\alpha) + B\cos(v+\beta)\big)\, e^{-2\sqrt{r}}\,. \tag{6.8}$$

In the context of the effective theory describing the universal hypermultiplet of type II string theory, the infinite sum of exponential corrections in (6.7) corresponds to non-perturbative instanton effects induced by Euclidean two-branes wrapping on three-cycles of the Calabi-Yau [44]. [10] The large-r expansion corresponds instead to a weak-coupling expansion and the three terms in (6.8) correspond respectively to the leading classical contribution, the one-loop quantum correction and the one-instanton quantum correction. It should however be emphasized that the exact u and v dependence dictated by the Toda equation is obtained only by including the infinite series of instanton corrections as in (6.7), while the one-instanton approximation (6.8) only provides an approximate solution to the Toda equation.

The gauging of the symmetry shifting t produces a potential for the N=2 theory based on the above approximate space, which now depends not only on r but also on u and v. Working in the large r approximation and using the expression (6.8) for h, one deduces that $f \simeq 1 + c/r + (A\cos(u+\alpha) + B\cos(v+\beta))\,\sqrt{r}e^{-2\sqrt{r}}$ and $\Theta = \frac{1}{2}(udv - vdu) + (A\sin(u+\alpha) + B\sin(v+\beta))\, r\, e^{-2\sqrt{r}}$. One then finds the following approximate form for the potential [32]:

$$\mathcal{V} \simeq \frac{g^2}{r^2}\bigg(\frac{1}{4} - \frac{c}{r} - \big(A\cos(u+\alpha) + B\cos(v+\beta)\big)\sqrt{r}\, e^{-2\sqrt{r}}\bigg)\,. \tag{6.9}$$

Taken as an exact starting point, this leads to a stationary point with positive Hessian matrix and a value of the potential that can be adjusted by tuning the parameters c, A and B [32]. This is in apparent contradiction with the no-go theorem of [9] reviewed in section 3, which forbids metastable de Sitter vacua. We notice however that while the fields u and v are stabilized by the one-instanton effect

[10] For a similar discussion of the effects of five-branes, see [45, 46].

on its own, the field r is stabilized thanks to a competition between the classical and loop effects and the one-instanton effect. This suggests that at such a minimum the one-instanton approximation for the potential is actually not reliable. Moreover, from the general result $m^2_{\rm avr} \simeq -\mathcal{V} - \frac{1}{3}m^2_{3/2}$, we can infer that in a consistent treatment, where all the corrections are included in such a way to have an exact solution of the Toda equation, a tachyon must unavoidably arise. This is in fact true not only in the weak coupling regime, but for any value of the coupling.

The fact that the one-instanton approximation breaks down at the putative stationary point is supported by noting that the exponential factor $e^{-2\sqrt{r}}$ suppressing non-perturbative higher-instanton corrections would be of the same order as the power factor $1/r$ controlling perturbative higher-loop corrections. This is contrary to the implicit assumptions of [32] when arguing that eq. (6.7) provides an exact solution to the Toda equation. Another way of detecting this problem is to work analytically with the one-instanton approximation to the potential, but to pay attention to discard all the effects that would formally be affected by higher-order corrections on the basis of their scaling with r. Proceeding in this way to study the stationarity condition, one finds that the stationary point now appears to be unstable. It arises for $re^{-2\sqrt{r}} \simeq \frac{1}{2}\frac{1}{A+B}$, $u \simeq -\alpha$ and $v \simeq -\beta$, with cosmological constant $\mathcal{V} \simeq \frac{1}{4}g^2r^{-2}$, gravitino mass $m^2_{3/2} \simeq \frac{1}{4}g^2r^{-2}$, and physical rescaled scalar masses $m^2_r \simeq -g^2r^{-3/2}$, $m^2_u \simeq \frac{A}{A+B}g^2r^{-3/2}$, $m^2_v \simeq \frac{B}{A+B}g^2r^{-3/2}$, which give $m^2_{\rm avr} \simeq 0$. This is now compatible with the general result $m^2_{\rm avr} = -\mathcal{V} - \frac{1}{3}m^2_{3/2}$, since both $\mathcal{V}$ and $m^2_{3/2}$ are of order r^{-2}, which is smaller than the leading order $r^{-3/2}$ that we are allowed to keep for square masses in this approximation. This analysis points again to the conclusion that the metastable de Sitter vacuum that seems to arise from (6.9) is actually fake.

Let us now examine the first kind of N=1 truncation discussed in section 5.3. The consistency condition (5.18) requires that $\alpha = 0$ and $\beta = 0$. One then finds a corrected Kähler potential and kinetic metric. The gauging of the symmetry shifting t is on the other hand not consistent with the truncation, and therefore there cannot be any superpotential and potential from this source.

Let us next consider the second kind of N=1 truncation discussed in section 5.4, following [35]. The consistency condition (5.25) requires that $\alpha = 0$. But for simplicity we shall also restrict to $\beta = 0$. One then finds a corrected Kähler potential and kinetic metric. The gauging of the symmetry shifting t is in this case possible, and yields a non-trivial and corrected superpotential. The scalar potential for the truncated theory is then simply given by the reduction of (6.9) and reads:

$$V \simeq \frac{g^2}{r^2}\bigg(\frac{1}{4} - \frac{c}{r} - B\cos v\sqrt{r}\,e^{-2\sqrt{r}}\bigg)\,. \tag{6.10}$$

At this point, one gets exactly the same situation as in the N=2 theory. The above potential taken as an exact starting point appears to admit a metastable de Sitter vacuum with a cosmological constant that can be adjusted by tuning the parameters c and B [35]. However, such a vacuum in fact lies outside the range of validity of the above approximated potential, for the same reasons as in the N=2 theory, and its existence can therefore not be trusted. In this case, there is no reason why consistently including higher-order corrections should unavoidably lead to an instability in the N=1 theory. Nevertheless, it remains true that when one starts from a weak-coupling regime in the N=2 theory the tachyon that unavoidably arises in such a theory is preserved by the truncation and remains in the N=1 theory. On the other hand, it is conceivable that by starting from a strong-coupling regime in the N=2 theory the tachyon is discarded by the truncation and disappears from the N=1 theory. But there appears to be no way to have computational control over such a situation.

6.3 General lessons

To conclude this section, let us spell out more clearly what are the reasons behind the difficulty in achieving a metastable de Sitter vacuum in a truncation of the universal hypermultiplet sector of superstrings. The main issue is that in order to be in a weakly-coupled situation, where quantum corrections can be treated through an asymptotic expansion, the space in the neighborhood of the vacuum point should be a small deformation of the classical geometry $SU(1,2)/(U(1)\times SU(2))$. More concretely, this means that the value of r should be large, since it corresponds to the inverse coupling. From an N=1 viewpoint, the problem of finding a metastable de Sitter vacuum then maps to first achieving a Kähler submanifold with a large enough curvature and then also a suitable superpotential from a gauging, compatible with the weak-coupling regime. Concerning the Kähler submanifold, we see it represents an obstruction for the first kind of truncation but not for the second. Indeed, starting from the $SU(1,2)/(U(1)\times SU(2))$ quaternionic manifold with scalar curvature 24λ one gets a different $SU(1,1)/U(1)$ Kähler submanifold in the two truncations, with curvature given respectively by -2 and $-\frac{1}{2}$. In situations where the space is deformed by a small amount, one will then get submanifolds which are correspondingly slightly deformed and whose curvature can therefore not depart substantially from the values -2 and $-\frac{1}{2}$. Comparing now with the lower bound $-\frac{2}{3}$ for the existence of metastable de Sitter vacua, we see that this is badly violated in the first truncation but satisfied in the second truncation. Concerning the superpotential, we saw that the gauging of the symmetry shifting t, which is the only symmetry that can always be gauged and has a clear interpretation within the string context, does not do the job. No metastable de Sitter vacuum can therefore arise in these truncations of the universal hypermultiplet in a weak-coupling regime. On the other hand, as already mentioned, it is not excluded that such a vacuum might arise in a strongly coupled regime.

7 Conclusions

In this paper, we characterized the conditions under which a metastable de Sitter vacuum may arise in a generic N=2 to N=1 supergravity truncation and compared these with the known situations of N=2 and N=1 theories, focusing for simplicity on models involving only scalar matter multiplets. In N=2 theories with hypermultiplets based on a quaternionic manifold with a triholomorphic isometry gauged by the graviphoton, metastable de Sitter vacua are excluded due to a sum rule satisfied by the triholomorphic sectional curvatures, independently of the gauged isometry [9]. In N=1 theories with chiral multiplets based on a Kähler manifold and a holomorphic superpotential, metastable de Sitter vacua are instead permitted if the sectional curvature of the manifold can be sufficiently large and the superpotential can be suitably adjusted [2]. In truncations of N=2 to N=1 theories, the possibility of achieving a viable vacuum then requires to start with a de Sitter vacuum of the mother theory that is unstable but leads to a small enough number of tachyonic scalars, and then to arrange that the projection enforcing the truncation eliminates all of these tachyonic scalars in such a way as to leave a metastable de Sitter vacuum for the daughter theory.

The general methodology on which our analysis was based is a systematic study of the form of the mass submatrix for the sGoldstini scalars, which represent the most severe danger for metastability. We first studied the problem in full generality, in terms of the geometry of the initial quaternionic manifold and of the Kähler submanifold selected by the truncation, as well as the isometries. We then performed a detailed study of the simplest case of theories with a single hypermultiplet truncated to theories with a single chiral multiplet, and described two distinct general ways of performing such a truncation. Using the general parametrization of such models in terms of Przanowski-Tod spaces based on a Toda potential h, we then derived more explicitly the conditions that the latter function has to satisfy in order for a metastable de Sitter vacuum to emerge after the truncation. Finally, we illustrated these results with a few explicit examples, which describe to different levels of accuracy the low-energy effective theory of the universal hypermultiplet truncated to a dilaton chiral multiplet in superstring models. This allowed us to argue that in such a context no metastable de Sitter vacuum can emerge even after a truncation, if one assumes a weakly coupled regime.

An other interesting application of the results that we have obtained would be to study models with $n+1$ hypermultiplets based on a so-called dual-quaternionic manifold. These manifolds naturally emerge in the context of type II superstrings compactified on a Calabi-Yau manifold, where they describe the classical moduli space for the universal plus the n additional hypermultiplets occurring when the Calabi-Yau space possesses not only a Kähler-form but n additional harmonic two-forms [37, 38]. They have a geometry that is entirely specified by a holomorphic prepotential, and possess the property of always admitting two distinct submanifolds of complex dimension n, which are respectively special-Kähler and only Kähler. Moreover, it is always automatically possible to consistently truncate the original N=2 theory based on the dual-quaternionic space to N=1 theories based on either of these special-Kähler and Kähler submanifolds [47]. These two distinct truncations correspond from the ten-dimensional point of view to heterotic and orientifold reductions, and represent in some sense the generalization of the two truncations we considered in our study of the universal hypermultiplet. It would then be interesting to study the possibility of achieving a metastable de Sitter vacuum in such truncated theories by relying on a source of potential that consists of a gauging that is compatible with the original N=2 supersymmetry, rather than a generic superpotential that is compatible only with the final N=1 supersymmetry. The first situation is clearly more restrictive and necessarily emerges at the classical level, where a potential can only be inherited from the mother theory through effects like background fluxes. The second situation is instead more flexible and may emerge only at the quantum level, where a potential can also be induced within the daughter theory through effects like instantons or fermion condensates. The conditions for the possible existence of metastable de Sitter vacua can be studied from an N=1 viewpoint in the second case, along the lines of [5, 48], but they should instead be studied from an N=2 viewpoint in the first case, and it would be interesting to know whether this leaves any viable option or not.

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A Geometry of Przanowski-Tod spaces

In this appendix, we give a detailed account of the geometry of Przanowski-Tod spaces. We first explain how to set up the problem and parametrize the vielbein. We then consider two different kinds of parametrizations, which are suited to analyze the two kinds of truncations considered in the paper, and compute for each of these the connections and the curvatures.

A.1 General parametrization

To set up the problem of computing the connection and the curvature, it is convenient to use the language of differential forms. The basic ingredients are the vielbein 1-forms $\mathcal{U}^{A\alpha}$, which satisfy $(\mathcal{U}^{A\alpha})^* = \mathcal{U}_{A\alpha}$. The holonomy is in this case $SU(2) \times SP(2)$, and both indices A and α run over two values. The line element can be rewritten as

$$ds^2 = \mathcal{U}^{A\alpha} \otimes \mathcal{U}_{A\alpha}\,. \tag{A.1}$$

The three hyper-Kähler 2-forms are instead given by

$$\mathcal{J}^x = -\frac{i}{2}\sigma^{x\ B}_{\ A}\mathcal{U}^{A\alpha} \wedge \mathcal{U}_{B\alpha}\,. \tag{A.2}$$

The $SU(2)$ and $SP(2)$ connections $\omega^A_{\ B} = \frac{i}{2}\,\sigma^{xA}_{\ \ \ B}\,\omega^x$ and $\Delta^\alpha_{\ \beta}$ are then determined by the torsion-free constraints:

$$d\mathcal{U}^{A\alpha} + \omega^A_{\ B}\mathcal{U}^{B\alpha} + \Delta^\alpha_{\ \beta}\mathcal{U}^{A\beta} = 0\,. \tag{A.3}$$

Finally, the curvatures of these connections take the general form:

$$K^x = d\omega^x + \frac{1}{2}\epsilon^{xyz}\omega^y \wedge \omega^z = -i\lambda\,\sigma^{x\ B}_{\ A}\mathcal{U}^{A\alpha} \wedge \mathcal{U}_{B\alpha} = 2\lambda\mathcal{J}^x\,, \tag{A.4}$$

$$\Sigma^\alpha_{\ \beta} = d\Delta^\alpha_{\ \beta} + \Delta^\alpha_{\ \gamma} \wedge \Delta^\gamma_{\ \beta} = \lambda\,\mathcal{U}^{A\alpha} \wedge \mathcal{U}_{A\beta} + \frac{1}{2}\mathcal{W}^\alpha_{\ \beta\gamma\delta}\mathcal{U}^{A\gamma} \wedge \mathcal{U}^\delta_A\,. \tag{A.5}$$

In the case under examination, the vielbein is a two-by-two matrix which can be parametrized in terms of only two independent complex elements a and b. This parametrization corresponds to the most general way of recasting the structure group of the tangent space from the $SO(4)$ form, that is natural when viewing the space as a Riemannian manifold, to the $SU(2) \times SP(2)$ form that is natural when viewing the space as a quaternionic manifold. More precisely, one can write:

$$\mathcal{U}^{A\alpha} = \begin{pmatrix} \bar{a} & -\bar{b} \\ b & a \end{pmatrix}\,. \tag{A.6}$$

With this parameterization, the line element reads

$$ds^2 = a \otimes \bar{a} + b \otimes \bar{b} + \text{c.c.}\,, \tag{A.7}$$

and the three complex structures are given by

$$\mathcal{J}^1 = i\big(a \wedge b - \bar{a} \wedge \bar{b}\big)\,, \tag{A.8}$$

$$\mathcal{J}^2 = -\big(a \wedge b + \bar{a} \wedge \bar{b}\big)\,, \tag{A.9}$$

$$\mathcal{J}^3 = i\big(a \wedge \bar{a} + b \wedge \bar{b}\big)\,. \tag{A.10}$$

One can then work out the form of the connections by solving the torsion-free equations (A.3) and compute their curvatures by using the definitions (A.4), (A.5). To do so, one however needs to make a definite choice for a and b, because their exterior derivatives get involved. There are infinitely many possible choices for a and b, based on different complex combinations of the basic differentials $dq^I = dr, du, dv, dt$. In view of discussing a truncation, however, it will be convenient to choose to include in a only the differentials of the preserved coordinates $dq^{i_\parallel}$ and in b only the differentials of the truncated coordinates $dq^{I_\perp}$. We shall now discuss two different kinds of parametrization, which are well suited to study the two truncations analyzed in the paper.

A.2 Explicit parametrization of the first kind

A first possible parametrization, which is well suited to describe truncations where the coordinates $q^{I_\parallel} = r, t$ are kept while the coordinates $q^{I_\perp} = u, v$ are discarded, is based on taking

$$a = \frac{1}{\sqrt{-8\lambda}\, r}\big(f^{1/2}dr + i\, f^{-1/2}(dt + \Theta)\big)\,, \tag{A.11}$$

$$b = \frac{1}{\sqrt{-8\lambda}\, r}(f e^h)^{1/2}\big(du + i\, dv\big)\,, \tag{A.12}$$

The $SU(2)$ and $SP(2)$ connections are determined by solving the torsion-free constraint in the basis of independent forms a, $\bar{a}$, b, $\bar{b}$. Denoting for short $z = u + iv$, one finds

$$\omega^1 = -\frac{e^{h/2}}{r}dv\,,\ \ \omega^2 = -\frac{e^{h/2}}{r}du\,,\ \ \omega^3 = \frac{1}{2r}(dt + \Theta) + \frac{h_v}{2}du - \frac{h_u}{2}dv\,, \tag{A.13}$$

and ($\Delta^2{}_2 = -\Delta^1{}_1$, $\Delta^2{}_1 = -\bar{\Delta}^1{}_2$):

$$\Delta^1{}_1 = i\frac{4f - f^2 + 2rf_r}{4\,rf^2}(dt + \Theta) - i\frac{2f_v + fh_v}{4f}du + i\frac{2f_u + fh_u}{4f}dv\,, \tag{A.14}$$

$$\Delta^1{}_2 = -\frac{e^{-h/2}f_z}{2f^2}\big(fdr + i(dt + \Theta)\big) + \frac{e^{h/2}(f - f^2 + rf_r)}{2\,rf}\big(du + idv\big)\,. \tag{A.15}$$

The hyper-Kähler forms controlling the $SU(2)$ curvature are then found to be

$$\mathcal{J}^1 = -\frac{e^{h/2}}{4\lambda r^2}du \wedge (dt + \Theta) + \frac{fe^{h/2}}{4\lambda r^2}dr \wedge dv\,, \tag{A.16}$$

$$\mathcal{J}^2 = \frac{e^{h/2}}{4\lambda r^2}dv \wedge (dt + \Theta) + \frac{fe^{h/2}}{4\lambda r^2}dr \wedge du\,, \tag{A.17}$$

$$\mathcal{J}^3 = -\frac{1}{4\lambda r^2}dr \wedge (dt + \Theta) - \frac{fe^{h}}{4\lambda r^2}du \wedge dv\,, \tag{A.18}$$

while for the symmetric tensor with flat indices controlling the $SP(2)$ curvature one finds ($\mathcal{W}_{2222} = \overline{\mathcal{W}}_{1111}, \mathcal{W}_{1222} = -\overline{\mathcal{W}}_{1112}$):

$$\mathcal{W}_{1111} = \frac{2\lambda r^2 e^{-h}}{f^3}\Big[f(f_{\bar z\bar z} - f_{\bar z}h_{\bar z}) - 3f_{\bar z}^2\Big], \tag{A.19}$$

$$\mathcal{W}_{1112} = \frac{\lambda r^2 e^{-h/2}}{f^3}\Big[f(fh_{r\bar z} + 2f_{r\bar z}) - 3(fh_r + 2f_r)f_{\bar z}\Big], \tag{A.20}$$

$$\mathcal{W}_{1122} = -\frac{2\lambda r^2}{f^3}\Big[f\big(rh_{rrr} - h_r^2\big) + 2f_r^2 - e^{-h}f_z f_{\bar z}\Big]. \tag{A.21}$$

A.3 Explicit parametrization of the second kind

A second possible parametrization, which is well suited to describe truncations where the coordinates $q^{I_\parallel} = r, v$ are kept while the coordinates $q^{I_\perp} = u, t$ are discarded, can be obtained from the previous one through combined $SU(2)$ and $SP(2)$ transformations both corresponding to a $\pi/2$ rotation of $SO(3)$ around the second axis, and is based on taking

$$a = \frac{1}{\sqrt{-8\lambda}\, r}\big(f^{1/2}dr + i\,(fe^h)^{1/2}dv\big), \tag{A.22}$$

$$b = \frac{1}{\sqrt{-8\lambda}\, r}\big((fe^h)^{1/2}du - i\,f^{-1/2}(dt+\Theta)\big). \tag{A.23}$$

The $SU(2)$ and $SP(2)$ connections are again determined by solving the torsion-free constraint in the basis of independent forms a, $\bar a$, b, $\bar b$. One finds:

$$\omega^1 = -\frac{h_u}{2}dv + \frac{h_v}{2}du + \frac{1}{2r}(dt+\Theta), \quad \omega^2 = -\frac{e^{h/2}}{r}du, \quad \omega^3 = \frac{e^{h/2}}{r}dv, \tag{A.24}$$

and (${\Delta^2}_2 = -{\Delta^1}_1$, ${\Delta^2}_1 = -{\bar\Delta^1}_2$):

$${\Delta^1}_1 = i\frac{e^{-h/2}f_v}{2f}dr + i\frac{e^{h/2}(f^2 - f - rf_r)}{2\,rf}dv + i\frac{e^{-h/2}f_u}{2f^2}(dt+\Theta), \tag{A.25}$$

$$\begin{aligned}{\Delta^1}_2 = &-\frac{e^{-h/2}f_u}{2f}dr + i\frac{fh_u + 2f_u}{4f}dv - \frac{2\,e^{h/2}(f^2 - f - rf_r) + ir(2f_v + fh_v)}{4\,rf}du \\ &- i\frac{f^2 - 4f - 2rf_r + 2ire^{-h/2}f_v}{4\,rf^2}(dt+\Theta).\end{aligned} \tag{A.26}$$

The hyper-Kähler forms read

$$\mathcal{J}^1 = -\frac{1}{4\lambda r^2}dr\wedge(dt+\Theta) - \frac{fe^h}{4\lambda r^2}du\wedge dv, \tag{A.27}$$

$$\mathcal{J}^2 = \frac{e^{h/2}}{4\lambda r^2}dv\wedge(dt+\Theta) + \frac{fe^{h/2}}{4\lambda r^2}dr\wedge du, \tag{A.28}$$

$$\mathcal{J}^3 = \frac{e^{h/2}}{4\lambda r^2}du\wedge(dt+\Theta) - \frac{fe^{h/2}}{4\lambda r^2}dr\wedge dv, \tag{A.29}$$

while for the symmetric tensor with flat indices controlling the $SP(2)$ curvature one finds ($\mathcal{W}_{2222} = \overline{\mathcal{W}}_{1111}, \mathcal{W}_{1222} = -\overline{\mathcal{W}}_{1112}$):

$$\begin{aligned}\mathcal{W}_{1111} = -\frac{\lambda r^2}{f^3}\Big[&3\big(f(rh_{rrr} - h_r^2) + 2f_r^2\big)\\ &+ 2ie^{-h/2}\big(f(fh_{rv} + 2f_{rv}) - 3(fh_r + 2f_r)f_v\big)\\ &+ e^{-h}\big(f(f_u h_u - f_{uu} - f_v h_v + f_{vv}) - 6f_v^2\big)\Big]\,, \end{aligned} \tag{A.30}$$

$$\begin{aligned}\mathcal{W}_{1112} = -\frac{\lambda r^2}{f^3}\Big[&e^{-h/2}\big(f(fh_{ru} + 2f_{ru}) - 3(fh_r + 2f_r)f_u\big)\\ &+ ie^{-h}\big(f(h_u f_v + f_u h_v - 2f_{uv}) + 6f_u f_v\big)\Big]\,, \end{aligned} \tag{A.31}$$

$$\begin{aligned}\mathcal{W}_{1122} = -\frac{\lambda r^2}{f^3}\Big[&-\big(f(rh_{rrr} - h_r^2) + 2f_r^2\big)\\ &+ e^{-h}\big(f(f_u h_u - f_{uu} - f_v h_v + f_{vv}) + 4f_u^2 - 2f_v^2\big)\Big]\,. \end{aligned} \tag{A.32}$$

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