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Influence of geometric, strain and size effects on bond in structural concrete

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ABSTRACT: Design formulations for calculation of bond strength in structural concrete members usually include a number of parameters such as the stress acting in the reinforcement, the concrete cover and the diameter of the reinforcing bars. These parameters are actually necessary to account for geometric, strain and size effect that govern many bond issues and that have been experimentally verified. Unfortunately, most design approaches are still based on empirical formulas that do not lead to a clear understanding why the different phenomena have to be taken into account. In this paper, a mechanical explanation of such effects is presented with reference to the various potential bond failure modes. Analogies to other related problems (such as spalling of concrete cover in arch-shaped members) are finally discussed

1 INTRODUCTION

Bond is an instrumental mechanism in structural concrete, allowing the transfer of forces between concrete and reinforcement bars or tendons. It thus governs many phenomena from first loading (cracking, tension–stiffening) to failure (lap splices, ductility). Bond is activated under various actions (pure tension, pull–out, push–in...) and may develop various failure modes (shearing of concrete between ribs, splitting failures…). Its behaviour depends on a wide set of geometrical parameters (short/long embedment-length, small/large concrete cover, small/large bar diameter ...) as well as on mechanical parameters of reinforcing bars (elastic and hardening moduli, yield strength ...) and of the concrete (compressive and tensile strength...). Also, structural context (defining for instance the confinement level) play a significant role. Although research on bond is as ancient as structural concrete itself and some consistent mechanical explanations have been provided to explain its features, most design expressions are still based on empirical approaches. Research works summarizing current knowledge and design models can be consulted elsewhere (*fib* 2000, *fib*, 2012).

In 2010, first complete draft of Model Code 2010 (*fib* 2010a,b) was published, constituting a state-of-the-art design approach with respect to bond. The design provisions given in MC2010 cover most significant influences known in bond. They are explained and thoroughly grounded in a specific document (*fib*, 2012) providing justification of the failure modes considered and how are they accounted for. In spite of the fact that the expressions are inspired on mechanical analogies, the design formulas and constitutive laws are still empirical. They have been derived through consideation of governing parameters and by fitting of them through available test data.

According to MC2010, two governing failure modes are distinguished: failure of specimens with sufficient concrete cover (referred to as pull-out failures) and failure of specimens with insufficient concrete cover (referred to as splitting failures), see Figure 1. This distinction is adopted most times for investigation of bond problems as it allows accounting for cases where tensile strength of concrete cover is governing (splitting failures) or not (pull-out failures).

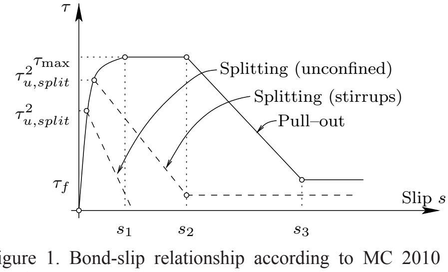

Figure 1. Bond-slip relationship according to MC 2010 (*fib* 2010b).

MC2010 proposes to correct the bond-slip relationship to account for a number of situations such as: the influence of longitudinal and transversal cracking, yielding of the reinforcement, transverse stresses, cyclic and fatigue loading and creep effects.

The bond-slip law proposed by MC2010 aims thus at reproducing the average behaviour of reinforcement in a realistic manner. This allows its use for investigation of serviceability limit state behaviour (crack width and spacing). However, at ultimate (splice lengths), a design formula is provided accounting for both pull-out and splitting failures. This design equation has been obtained by adapting the average bond strength to satisfy a 5% characteristic value and by introducing partial safety factors.

The steel stress (f_{stm}) that can be developed by bond is given in Eq. (6.1-12) of the code for members with long embedment lengths:

$$f_{stm} = 54\left(\frac{f_c}{20}\right)^{0.25}\left(\frac{20}{\phi_s}\right)^{0.2}\left(\frac{l_b}{\phi_s}\right)^{0.55}\left[\left(\frac{c_{\min}}{\phi_s}\right)^{0.33}\left(\frac{c_{\max}}{c_{\min}}\right)^{0.1} + 8K_{tr}\right] \quad (1)$$

Where f_c [MPa] refers to the concrete compressive strength (15 MPa < f_c < 110 MPa), ϕ_s [mm] to the bar diameter, l_b [mm] to the bonded length, $c_{\min}$ and $c_{\max}$ to the minimum and maximum concrete cover (or bar half-spacing) and K_{tr} refers to the confinement reinforcement ratio. The expression has been adjusted for ordinary ribbed reinforcing bars respecting a set of detailing rules. Bars with different bond indexes should require a specific fit of the formula.

It can be noted that this formula can be rewritten as a function of the average bond stress acting in the bar (τ) as:

$$\frac{\tau}{\sqrt{f_c}} = 3\left(\frac{20}{f_c}\right)^{0.25}\left(\frac{20}{\phi_s}\right)^{0.2}\left(\frac{\phi_s}{l_b}\right)^{0.45}\left[\left(\frac{c_{\min}}{\phi_s}\right)^{0.33}\left(\frac{c_{\max}}{c_{\min}}\right)^{0.1} + 8K_{tr}\right] \quad (2)$$

In this expression, the MC2010 formula is shown to include a material-strength factor $(20/f_c)^{0.25}$ (accounting for the fact that as the strength of concrete increases, its brittleness also does) and a size-effect factor $(20/\phi_s)^{0.2}$ (with reduced bond strength for larger diameter sizes). Term $(\phi_s/l_b)^{0.45}$ is a factor accounting for the combined effect of the strains and distribution of bond stresses along the development length (considering the concentrations of bond stresses at the development length). Finally, the latter term in parenthesis, accounts for geometric concrete cover effects as well as by and transverse confinement effects.

In this paper, these phenomena (strain effects, transverse tension) will be investigated on the basis of a simple hypothesis on the affinity of the strain profiles, showing their influence and deriving a set of expressions for bond-related problems. These expressions are aimed at providing a more clear understanding to designers of the meaning and influence of the various parameters on bond behaviour.

2 GEOMETRIC AND STRAIN EFFECTS IN PULL-OUT AND PUSH-IN BOND BEHAVIOUR

Assuming sufficient concrete cover (more than approximately three times the bar diameter) splitting failures are not governing (Schenkel 1998). Bond behaviour is thus governed (after loss of chemical adhesion) by local crushing of concrete in contact with the ribs and by the opening of conical cracks. For short embedment lengths (lower than approximately five times the bar diameter, Fir. 2a-b), the bond stresses developed at the bar interface are rather constant and the behaviour is mostly dependent on the bond index (geometry of the ribs) and on the concrete strength.

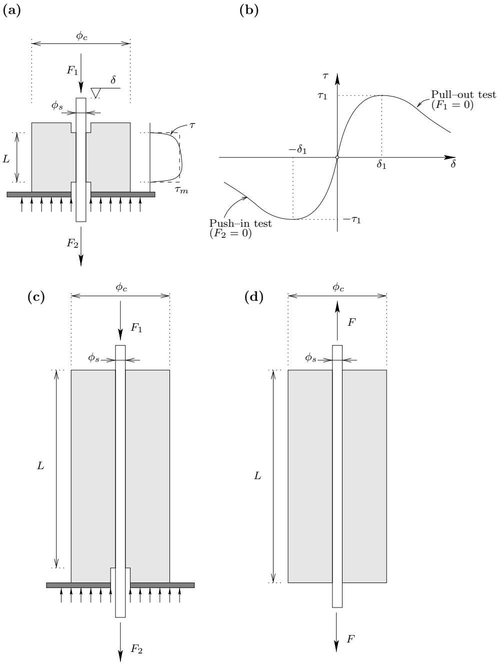

Figure 2. Bond specimens with sufficient concrete cover: (a) short pull-out specimen; (b) corresponding bond-slip law; (c) long pull-out (F_1 = 0) or push-in (F_2 = 0) specimen; and (d) tension tie.

For larger embedment lengths or for tension ties (see Figs. 2c-d), bond stresses may vary along the bar according to the local bond-slip law $\tau(\delta)$. Such cases can be solved by integrating the differential equilibrium equation (see Fig. 3). In a general manner this equation can be written as:

$$\frac{d\sigma_s}{dx} = -\frac{4\tau}{\phi_s} \quad (3)$$

Since the bond stress (τ) is a function of the relative slip (δ) and the steel stress (σ_s) is a function of the steel strain (ε_s), integration of the previous equation requires first to differentiate it (by considering δ as a function of ε_s if concrete strains are neglected) and then to solve a second-order differential equation. Obtaining closed-form solutions of the second-order differential equation is only possible for a number of simplified analytical laws (Marti et al. 1998, *fib* 2000).

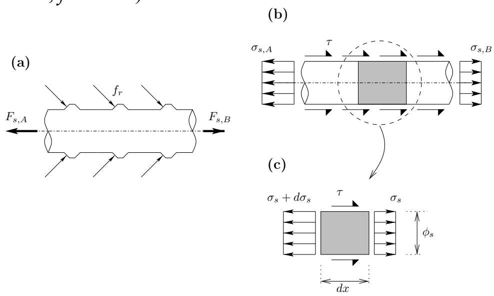

Figure 3. Equilibrium of forces in the steel bar: (a) acting forces; (b) equivalent stresses; and (c) differential element.

A more general approach was proposed by Fernández Ruiz et al. (2007a), assuming perfect affinity between the slip curves at different load levels, see Figure 4. This allows assuming $\tau = \tau(\varepsilon_s)$ and then solving Eq. (2) a first-order differential equation (which allows assuming a wide range of bond-slip laws, including laws as the one proposed in MC2010).

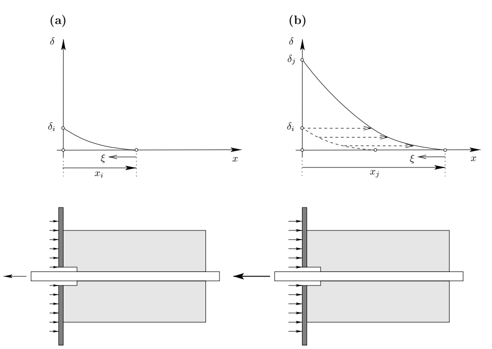

Figure 4. Relative slips along the axis of the bar at: (a) low load level; and (b) higher load level.

The advantage of this approach is furthermore that, since the bond law is expressed in terms of the strains of the bar, the influence of the strains on the bond stresses (strain effect of the reinforcement) can be directly introduced through a strain-effect factor (K_b):

$$\tau = \tau(\varepsilon_s) \cdot K_b(\varepsilon_s) \tag{4}$$

In Figure 5 it can be noted that the strain-effect factor (K_b) reduces the bond stresses significantly after yielding in tension (as demonstrated experimentally by Shima, 1987) and increases bond stresses after straining in compression (as proposed by Hoyer for prestressing tendons).

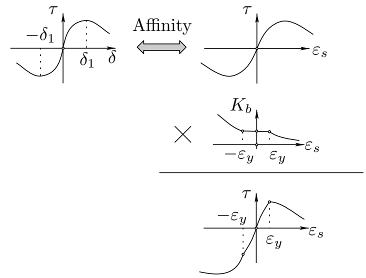

Figure 5. τ-δ and τ-ε_s laws for obtained with the affinity hypothesis and including the effect of the strains at the bar.

Figure 6 compares the results of this approach to the test results obtained by Shima and to the results obtained through a FEM analysis of the bond stresses considering the actual rib geometry as detailed in Fernández Ruiz et al (2007b), refer to Figure 7.

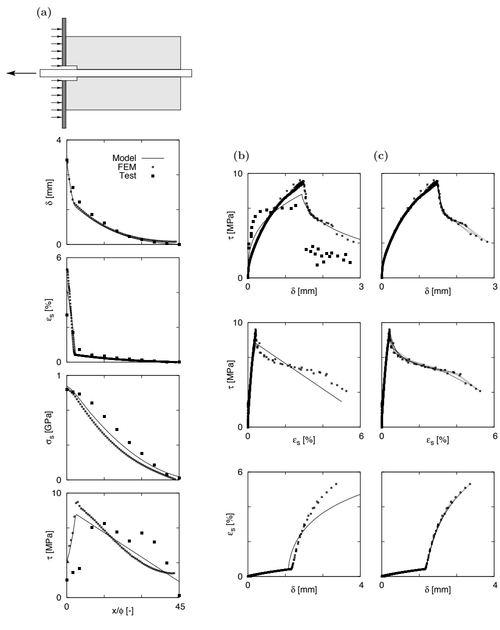

Figure 6. Comparison of the results obtained for the test specimen SD70 from Shima (1987) with the FEM and analytical models: (a) longitudinal slip, strain, stress and bond distributions along the axis of the bar at the last load step; (b) relationship between the bond stresses, slip and axial strains in the bar; and (c) same results obtained with the FEM model for the different load steps.

Details of the FEM model used can be found elsewhere (Fernández Ruiz et al. 2007b). Further applications of this FEM modelling approach to bond splitting failures are detailed in a separate paper of this proceeding (Prieto et al. 2012).

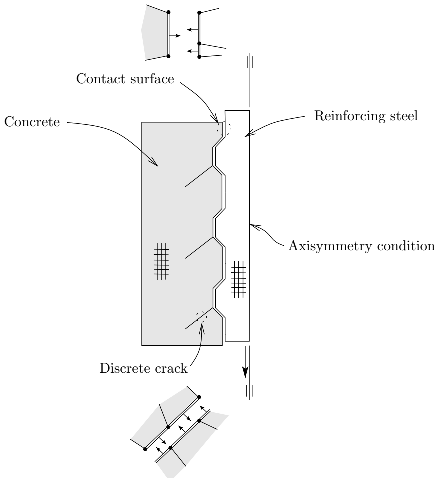

Figure 7. FEM model used to reproduce bond problems.

The pertinence of the increase of bond stresses after yielding in compression and of the affinity hypothesis (bond stresses expressed as a function of reinforcement strains, $\tau = \tau(\varepsilon_s)$) is confirmed in Figure 8 where a numerical simulation using FEM of the long pull-out test of Shima (1987) was performed but reversing the side where the displacement was imposed (long push-in test). The figure plots the results for various levels of load (prior and after yielding of the reinforcement).

Applications of these hypotheses to other bond-related problems as the post-yield tension stiffening and rebar rupture in concrete members can be consulted elsewhere (Lee et al. 2011).

With respect to geometric effects in pull-out or push-in specimens, they are associated to local punching of conical surfaces (*fib* 2010b) and can also be investigated on the basis of the hypothesis of affinity of the bond-reinforcement strain curves (Fernández Ruiz et al. 2007). The decrease of the bond stresses can be introduced and integrated directly in the first-order differential equation (Eq. (2)) because it depends only on the location along the axis of the bar (x). This influence is incorporated in the bond law by means of a strength reduction factor named λ (Fig. 9), locally reducing bond stresses.

Bond stresses can thus be calculated according to the following expression (including both strain effects and local punching of concrete):

$$\tau = \tau(\varepsilon_s) \cdot K_b(\varepsilon_s) \cdot \lambda(x/\phi_s) \tag{5}$$

According to Fernández Ruiz et al. (2007a), a good estimate of the phenomenon is obtained by using a strength reduction factor defined by the following expression:

$$\lambda(x/\phi_s) = 1 - \exp(-x/\phi_s) \tag{6}$$

Closed-form solutions for tension ties according to this formulation and leading to good agreement to test results have been reported elsewhere (Fernández Ruiz et al. 2007a).

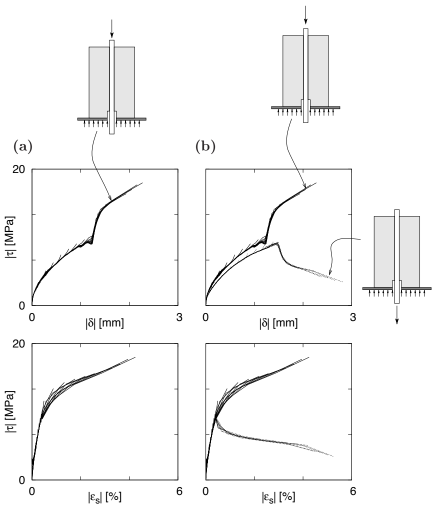

Figure 8. Numerical results obtained with a FEM analysis of the test specimen SD70 (Shima 1987) for a push--in test at different load steps: (a) relationship between the bond stresses, slip and axial strains in the bar; (d) comparison of the FEM pull-out and push-in test results.

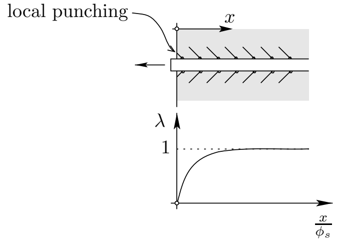

Figure 9. Local punching of outer ribs and coefficient λ for a long anchored bar.

3 GEOMETRIC AND STRAIN EFFECTS IN SPLITTING FAILURES

When concrete cover is insufficient (less than approximately three times the bar diameter) splitting failure may become governing. Bond strength is thus dependent on the effective cover thickness and on the tensile strength of concrete. Tepfers (1973) already investigated these cases by assuming the equilibrium between the developed conical bursting stresses and a tension ring developed in the concrete, refer to Figure 10. Thus, the tensile stresses in the concrete can be calculated as a function of the bond stresses by adopting a suitable value of the angle of the struts.

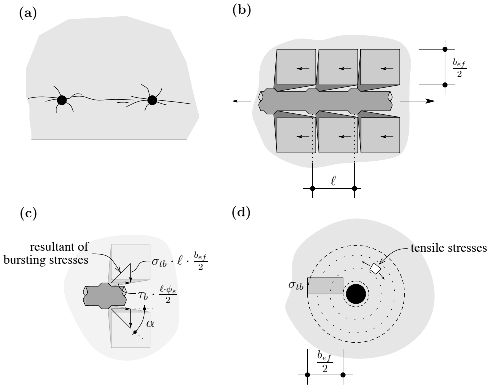

Figure 10. Bond splitting phenomenon: (a) spalling of concrete cover due to splitting (radial) cracks; (b) tension rings and bursting stresses around reinforcement bar; (c) equilibrium of longitudinal forces; and (d) transverse tensile stresses (assuming a constant value of the stress in the tension ring).

Other than in lap splices, splitting failures are also governing in arch-shaped members such as cut-and-cover tunnels, pipes or silos, where bending moments lead to deviation forces in the reinforcement which act in combination with the tensile stresses of the tension rings, see Figure 11 (Fernández Ruiz et al. 2010). This can lead to global (Fig. 11c) or local (Fig. 11d) spalling of the concrete cover induced by the combined splitting and deviation tensile stresses. The tensile transverse stress can be calculated by equilibrium conditions as:

$$\sigma_t = \sigma_s \frac{\pi/4 \cdot \phi_s^{\,2}}{b_{ef} \cdot R} + \tau \cdot \frac{\tan\alpha \cdot \phi_s}{b_{ef}} \tag{7}$$

where σ_t and σ_s refer to the transverse tensile stress of the tension ring and to the longitudinal reinforcement stress respectively, R to the radius of curvature of the reinforcing bars, α to the angle of the struts. (Fig. 10c) and b_{ef} to the effective width activated by the tension ring/deviation forces (Figs. 10b-d).

The value of the effective width where the tension ring develops (b_{ef}) depends on the geometry of the reinforcing bars (clear spacing) and on the concrete cover. In order to account both for global (Fig. 11c) and local (Fig. 11d) failure modes due to splitting stresses, it can be estimated as:

$$b_{ef} = \min(s - \phi_s; 6\phi_s; 4c_{\min}) \tag{8}$$

where s refers to the spacing of the bars and $c_{\min}$ to the minimum concrete cover.

Figures 12 and 13 show the geometry, loading conditions and crack patterns at failure of a test campaign performed by the authors on six arch-shaped beams subjected to pure bending. Failures occurred in all cases by spalling of the concrete cover due to combined deviation forces and splitting stresses (Fernández Ruiz et al. 2010).

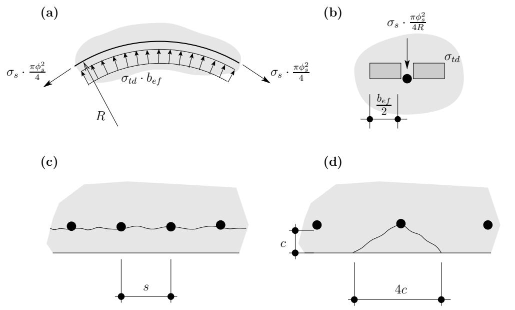

Figure 11. Spalling of concrete cover: (a) equilibrium of deviation forces of a curved reinforcement; (b) tensile stresses due to deviation forces (assuming a constant tensile stress in concrete); (c) global spalling of concrete cover; and (d) local spalling of concrete cover.

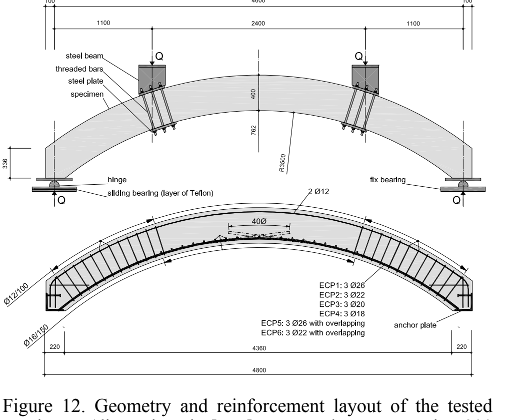

Figure 12. Geometry and reinforcement layout of the tested specimens (dimensions in [mm], rectangular cross-section 300 mm width).

Calculation of the splitting strength can be performed consistently using the hypothesis of affinity. This treatment has proved to be very convenient as both the bond forces and the deviation forces are calculated as a function of the steel strains (including elastic and plastic regimes). Details of the application of the affinity hypothesis and of its implementation for these cases can be found elsewhere (Fernández Ruiz et al. 2010).

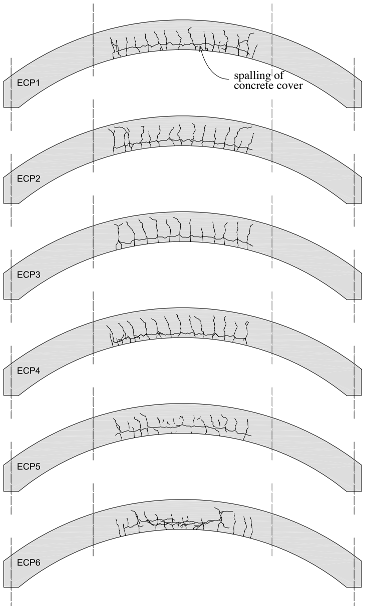

Figure 13. Cracking pattern at failure of tested specimens (Fernández Ruiz et al. 2010), all failures by spalling of concrete cover.

For practical purposes of arch-shaped members, solving the general equation obtained through the hypothesis of affinity is however not necessary. Instead, it can be reformulated using the following format (typically adopted in code provisions):

$$k \cdot f_{ct} \cdot b_{ef} \geq q_{tr,d} \tag{9}$$

Where k is a strength reduction factor accounting for the strain effect of the longitudinal reinforcement on the tensile strength of concrete.

For usual cases, adopting a value prior to bar yielding k_{el} = ¼ leads to safe estimates of the spalling strength (Fernández Ruiz et al. 2010). For a more refined estimate of the strength reduction factor, its value can be calculated assuming that at bar yielding ($q_{tr,d} = f_{yd} \cdot \pi \cdot \phi_s^2/(4R)$), no spalling of the concrete cover occurs. The following expression thus results (derived on the basis of the affinity assumptions, Fernández Ruiz et al. 2010):

$$k_{el} = 0.8 - 2.5 \frac{\phi_s}{b_{ef}} \kappa_b \tag{10}$$

Where κ_b is a parameter whose value is equal to $\sqrt{2}$ in case of lap splices and to 1 otherwise. It can be noted that factor κ_b accounts for the interaction between bond action and deviation forces in spalling failures, which has shown to have a significant influence on the strength and deformation capacity of RC members (Fernández Ruiz et al. 2010).

Calculation of the theoretical strength reduction factor for available test data is shown in Figure 14. The strain effect on the strength is notable and it decreases the splitting strength for increasing values of the bar strain.

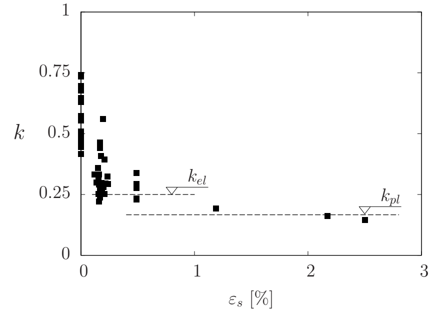

Figure 14. Comparison between measured and predicted failure loads for tests of figure 9 and others taken from the scientific literature (Fernández Ruiz et al. 2010) as a function of the longitudinal strain of the bar (ε_s).

After bar yielding, and although the deviation forces remain approximately constant, the splitting stresses increase significantly due to the wedge effect of the steel ribs. This implies that spalling failures are also possible in the plastic regime, which has been confirmed experimentally for arch-shaped beams (Fernández Ruiz et al. 2010) and may govern the redistribution of internal forces necessary to develop a plastic mechanism for a vaulted construction. In this regime, the value of the strength reduction factor has to be reduced to k_{pl} = 1/6 if safe estimates of the spalling strength are to be obtained, see Figure 14. Detailed analysis using the hypothesis of affinity is also possible in this regime (Fernández Ruiz et al. 2010).

4 SIZE EFFECTS IN SPLITTING FAILURES

Size effect in bond is a topic that has been widely investigated both for smooth and ribbed bars. A consistent approach and a discussion on the state-of-the –art to this problem can be consulted in *fib* 2000 and in Bamonte & Gambarova (2007).

With respect to ribbed bars, size effect is clearly present in splitting failures (insufficient concrete cover) since the strength is governed by the tensile strength of concrete (Bazant & Sener 1988). It however also influences the strength of well-confined specimens as experimentally demonstrated by Bamonte & Gambarova (2007). This can be explained by the local punching of concrete conical surfaces parallel to bursting struts (dependent on the bar diameter, Fernández Ruiz et al 2007) and by the larger damage introduced in the concrete for larger bar ribs.

5 CONCLUSIONS

This paper presents an investigation on bond transfer actions and on the role of various geometrical and mechanical influences. Its main conclusions are:

1. Strain effects are relevant after yielding of the reinforcement bars. This is due to the localization of strains in the plastic region which locally decreases bond stresses (for lateral contraction in tension) or increases (for lateral expansion in compression)
2. Bond in long specimens (where stresses vary along the reinforcement bar) can be consistently investigated by assuming perfect affinity between the bond-reinforcement strain curves
3. Premature splitting failures can occur in arch-shaped members where tensile splitting stresses due to bond are potentially increased with the deviation forces of the curved reinforcement
4. Splitting strength of arch-shaped members is thus dependent of the strains in the reinforcement (strain effect) and can be investigated assuming perfect affinity between the bond-reinforcement curves. Alternatively, simplified design methods based on strength reductions factors of the tensile strength of concrete are possible and practical for design in practical applications

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