EFFECTS OF AN ASYMMETRIC CHAMBER ON THE BEAM COUPLING IMPEDANCE

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Abstract

The wake function of an accelerator device appears to have a constant term if the geometry of the device is asymmetric or when the beam passes off axis in a symmetric geometry. Its contribution can be significant and has to be taken into account. In this paper a generalized definition of the impedance/wake is presented to take into account also this constant term. An example of a device where the constant term appears is analyzed. Moreover, the impact of a constant wake on the beam dynamics is discussed and illustrated by a HEADTAIL simulation.

INTRODUCTION

The wake functions can be expanded into a power series in the offset of source and test particle [1]. Usually the generalized transverse wakes are written as a sum of a driving and a detuning term [2]:

$$\begin{aligned} W_x^{gen}(z) &= W_x^{driv}(z)x_0 + W_x^{det}(z)x \\ W_y^{gen}(z) &= W_y^{driv}(z)y_0 + W_y^{det}(z)y \end{aligned} \tag{1}$$

where (x_0, y_0) and (x, y) define respectively the transverse offset of source and test particle and z is the distance between the two (z positive means that we are looking at the effect on a test particle that is at a distance z behind the source particle). In this approximation the knowledge of the four frequency dependent terms W_x^{driv}, W_x^{det}, W_y^{driv}, W_y^{det} fully characterizes the impedance of the device.

A NEW GENERALIZED FORMALISM

The above definitions hold only for a device with left/right and top/bottom symmetry and centered beam. Assuming now that a transverse effect is also possible between the source and test particles on their nominal orbits:

$$\begin{aligned} W_x^{gen}(z)\big|_{x=x_0=0} &= A_x(z) \\ W_y^{gen}(z)\big|_{y=y_0=0} &= A_y(z) \end{aligned} \tag{2}$$

and Eq.(1) has to be rewritten as:

$$\begin{aligned} W_x^{gen}(z) &= A_x(z) + W_x^{driv}(z)x_0 + W_x^{det}(z)x \\ W_y^{gen}(z) &= A_x(z) + W_y^{driv}(z)y_0 + W_y^{det}(z)y \end{aligned} \tag{3}$$

where (x_0, y_0) and (x, y) are the offsets with respect to the nominal orbit and driving and detuning wakes must be defined with respect to it:

$$\begin{aligned} W_x^{driv}(z) &= \frac{W_x^{gen}(z) - A_x(z)}{x_0}\Big|_{x=0} \\ W_x^{det}(z) &= \frac{W_x^{gen}(z) - A_x(z)}{x}\Big|_{x_0=0} \\ W_y^{driv}(z) &= \frac{W_y^{gen}(z) - A_y(z)}{y_0}\Big|_{y=0} \\ W_y^{det}(z) &= \frac{W_y^{gen}(z) - A_y(z)}{y}\Big|_{y_0=0} \end{aligned} \tag{4}$$

Similarly, we can apply the same concept to define the generalized impedances, which are the Fourier transforms of the wakes:

$$\begin{aligned} Z_x^{gen}(f) &= a_x(f) + Z_x^{driv}(f) + Z_x^{det}(f) \\ Z_y^{gen}(f) &= a_y(f) + Z_y^{driv}(f) + Z_y^{det}(f) \end{aligned} \tag{5}$$

What we call the "generalized (total) wake field" physically represents the wake calculated off axis generated by a source beam that is off axis by the same amount.

3-D SIMULATIONS: AN EXAMPLE OF APPLICATION

Using one of the standard outputs of CST PS [3] we can simulate the longitudinal and transverse wake potentials.

The CST PS wake field solver allows defining the position of the source beam (it is assumed to have only longitudinal size) and that of the test particle (observation point). Therefore, we can single out the different contributions.

To simulate the constant wake (Eq. 2) both the source bunch and the test particle have to be placed on the nominal orbit. To simulate the driving impedance, we only need to displace the source beam and calculate the wake potential on axis. The result has to be normalized using Eq. (4). To simulate the detuning impedance, only the test particle needs to be displaced. As for the driving, the wake has to be normalized using Eq. (4). Offsetting both the source beam and the test particle, it is possible to simulate the generalized impedance and, therefore, verify the Eq. (3)—thereby checking if the abovementioned assumptions are valid. In structures with left/right and top/bottom symmetries the constant terms are exactly

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zero. In order to show the importance of the constant term we will analyze an example of interest where the symmetry is broken.

C-shaped Kicker Magnets

An example of asymmetric structure of interest is the SPS C-shaped kicker magnet sketched in Fig. 1. This structure is not symmetric with respect to the plane yz. It is important to underline that the constant term is independent of the offset. In order to compare its contribution with respect to the driving term it is possible to define a threshold displacement below which the constant term is larger than the driving one:

$$x_{th} = \frac{A_x}{W_x^{driv}} \tag{6}$$

Figure 2 shows x_{th} as function of the frequency for an SPS MKP module. From the plot it is evident that the constant term has a very strong impact at low frequencies while at high frequency (above few hundred MHz) its contribution becomes negligible. Figure 3 shows all the wake contributions of the C-shaped kicker magnet. The different terms were singled out following the procedure described in the previous section.

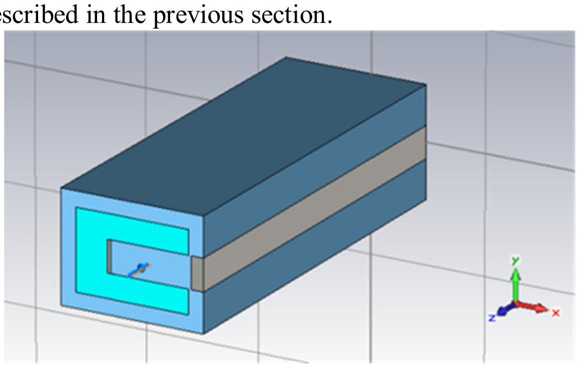

Figure 1: Simplified model of a kicker magnet: Vacuum in blue, ferrite in light green and Perfect Electric Conductor (PEC) in gray.

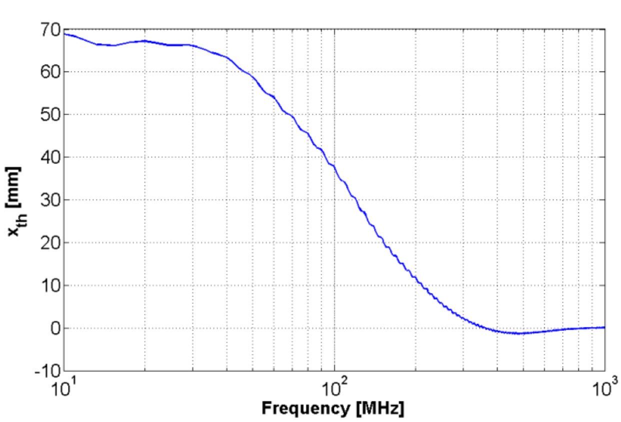

Figure 2: Horizontal offset of threshold for the real part of the horizontal driving impedance of an SPS MKP module.

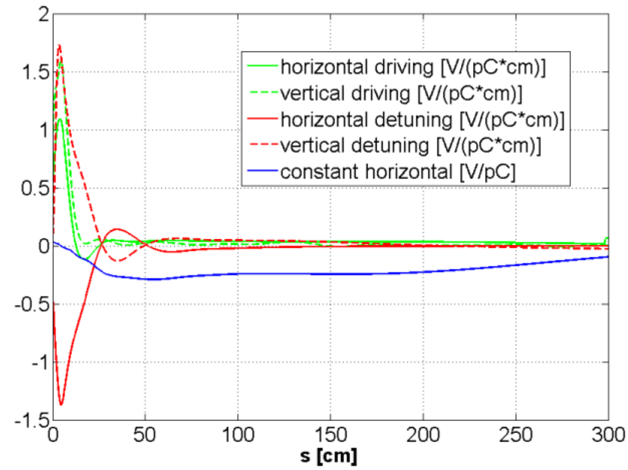

Figure 3: CST 3-D Electro-Magnetic simulations of the horizontal (full lines) and vertical (dashed lines) wake potentials for an SPS-MKP kicker magnet using a Gaussian bunch profile with an rms bunch length of 1.5 cm. The constant term is in blue, the driving terms in green and the detuning in red.

Constant Term Due to Offset Beam

A constant term also appears when the beam passes off axis in a symmetric geometry. In general we can define the constant term as the transverse impedance when both source and test particle are on the nominal beam axis. If the nominal beam axis is in the geometrical centre of the structure the constant term appears only for asymmetric geometry.

Effect of the Constant Terms on the Beam Dynamics

To study the impact of the constant wakefield on the beam dynamics, we write down the Hamiltonian for the betatron motion under the influence of a purely constant wakefield as defined in Eq. (2) following the conventions of [4] as

$$\begin{aligned} H &= \frac{p_x^2}{2} + \left(\frac{\omega_\beta}{\sqrt{2}c}\right)^2 x^2 - \frac{r_0}{T_0\gamma c}\bar{A}(z)x \\ &= \frac{p_x^2}{2} + \frac{1}{2}\left(\frac{\omega_\beta}{c}\right)^2\left(x - \left(\frac{c}{\omega_\beta}\right)^2 \frac{r_0\bar{A}(z)}{T_0\gamma c}\right)^2 \\ &\quad - \frac{1}{2}\left(\frac{r_0\bar{A}(z)}{T_0\gamma\omega_\beta}\right)^2 \end{aligned} \tag{7}$$

where r_0 is the classical particle radius and T_0 is the revolution period. Equation (7) resembles the Hamiltonian of a shifted harmonic oscillator with shift in the neutral position and in the energy.

It is important to note that the constant term in Eq. (7) is written in terms of the wake function $A(z)$ as

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$$\overline{A}(z)=\int_z^\infty \lambda(z')A(z-z')dz' \tag{8}$$

where λ is the line density of the bunch. The constant wakefield term corresponds to an additional dipole field and leads to a slice dependent orbit distortion. For a given slice this distortion depends on the zeroth statistical moment of the preceding slices and as a result, each slice will follow a different closed orbit. This may be regarded as the transverse equivalent of the potential well distortion found in the longitudinal plane.

When a beam is injected on the design orbit, slices are given an initial kick with respect to the distorted orbit. This causes slices along the bunch to rotate around the distorted neutral positions. In transverse phase space, these slices are strongly offset from the centre of rotation causing them to sweep an enlarged area.

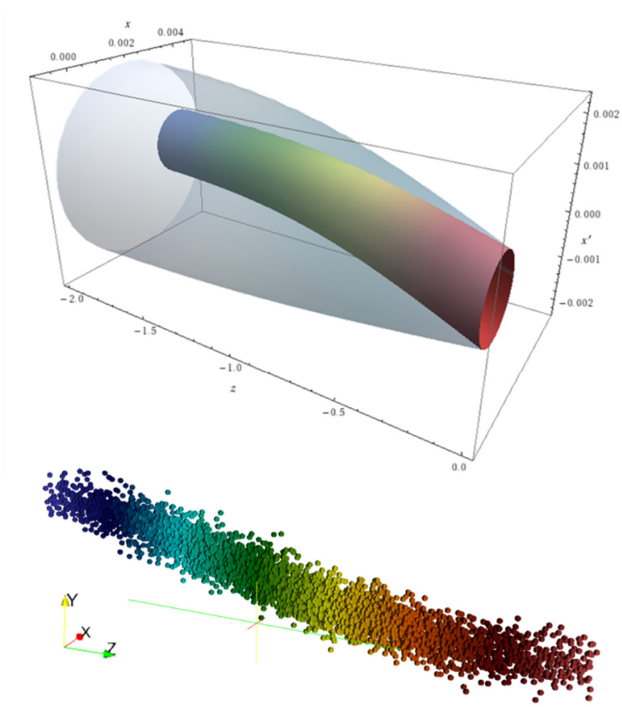

Figure 4: Top: the bunch is represented as the uniform coloured tube. The enlarged phase space volume swept by the bunch is bordered by the grey transparent surface. The distorted neutral positions follow the axis of this phase space volume. Bottom: The same representation of the bunch obtained from a HEADTAIL multi-particle simulation. The axes map as $x \to x,\ y \to x',\ z \to z$.

Figure 4 compares the horizontal phase spaces from an analytical calculation using the Hamiltonian in Eq. (7) (in order to simplify the calculation we have assumed $A(z)=\overline{A}(z)$ and we have used a Gaussian longitudinal bunch profile) and from a HEADTAIL [5] simulation of an SPS beam using only the constant wakefield shown in Fig. 3 in order to reveal the effects resulting purely from this term.

Particles towards the tail of the bunch are performing oscillations around a distorted orbit; which results in the bunch having a flapping tail. The transverse phase space looks increased as the slices towards the tail of the bunch are strongly offset.

As synchrotron motion takes place, provided that $\omega_s \ll \omega_\beta$, for each slice the neutral position changes adiabatically as particles move longitudinally along the bunch. Therefore, the action for all particles in a slice is preserved while its centre of rotation shifts from the neutral position at the tail towards the neutral position at the head and vice versa. Thus, the bunch changes from having a flapping tail to having a flapping head and back within one synchrotron period. The emittance oscillates with the betatron frequency together with an additional modulation at the synchrotron frequency.

CONCLUSION

A new generalized formalism accounting also for the constant term using the same sets of simulation described in [6] has been presented. The impedance behavior of an SPS-MKP kicker magnet (see Fig. 1) has been investigated to quantify the effect of the constant term (see Figs. 2 and 3). The impact of the constant term on the beam dynamics has been formalized and identified as the transverse equivalent of the potential well distortion. The resulting effect has been illustrated by calculation, as well as by a HEADTAIL simulation.

REFERENCES

[1] S. Heifets, A. Wagner, B. Zotter, "Generalized Impedances and Wakes in Asymmetric Structures", SLAC/AP110, Stanford, USA, 1998.

[2] A. Burov and V. Danilov, "Suppression of Transverse Bunch Instabilities by Asymmetries in the Chamber Geometry", Phys. Rev. Lett. 82 (1999) 2286.

[3] www.cst.com

[4] A. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, Wiley, 1993.

[5] G. Rumolo and F. Zimmermann, "Practical user guide for HEADTAIL", SL-Note-2002-036-AP, 2002.

[6] C. Zannini et al., "Electromagnetic Simulations of Simple Models of Ferrite Loaded Kickers", CERN-BE-Note-2010-006, 2010.