Pushover analysis of multi-storey cantilever wall systems

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SUMMARY:
Mid- to high-rise buildings are often braced by slender reinforced concrete (RC) walls, which are coupled by RC floor diaphragms. In design it is typically assumed that the walls act independently and the design base shear demand is computed neglecting any compatibility forces between the walls. Pushover analysis of systems comprising walls of different lengths have, however, shown that large compatibility forces can develop between walls of different length, which should be considered in design, but also that the magnitude of the computed forces is very sensitive to the modelling assumptions. The paper explores by means of a case study of an eight storey structure with two walls of different lengths the shear forces developing at the base of the wall. It compares and discusses the analysis results from different models including simple hand calculations, a lumped plasticity beam element model and a complex shell element model. It concludes that numerical and analytical approaches which are based on the lumped plasticity model tend to overestimate the shear force demand on the shorter wall.

Keywords: Reinforced concrete walls, slab coupled walls, walls of unequal length, pushover analysis.

1. INTRODUCTION

Reinforced concrete (RC) walls are frequently used as structural systems providing horizontal stiffness and strength to multi-storey buildings in regions of medium to high seismicity. Such walls are expected to undergo inelastic deformations during a design level earthquake, and are typically designed using capacity design principles (Paulay and Priestley, 1992), the objective of which is the formation of a stable plastic flexural mechanism. A key element of capacity design is the estimation of the design shear forces, which must account for all sources that can amplify the shear forces as obtained from the equivalent static force procedure, which is the standard procedure in major codes dealing with the seismic design of new structures that are regular over the height and in plane. Shear amplifications factors in today’s codes are based on the pioneering work of Blakeley *et al.* (1975) and the work by Eibl and Keintzel (Eibl and Keintzel, 1988; Keintzel, 1990). In summary, the amplification factors aim at accounting for the following effects influencing the inelastic response of structures (Priestley and Amaris, 2002):

- Amplified importance of the higher modes on the response of the structure as the structure undergoes inelastic deformations: The inelastic deformations affect mainly the first mode response. As a result, the effective period of the first mode elongates and the spectral accelerations reduce. Moreover, the first mode shear forces are reduced due to the energy dissipated by the first mode.
- Strength in excess of the required design flexural strength (overstrength): The flexural overstrength leads to a nearly proportional increase in base shear demand.

While the latter can easily be accounted for in design, nonlinear time history analysis is theoretically required to quantify the dynamic amplification in structures responding in the inelastic range. In order to avoid the necessity of nonlinear time history analysis during the design process, codes propose simplified equations estimating the dynamic amplification factor as a function of different parameters. The validity of the simplified equations for estimating the dynamic amplification factor implemented in codes has been investigated by a number of recent parametric studies, which examined the effect of the height of the building and the ductility demand on the wall (e.g., Priestley and Amaris, 2002; Rejec *et al.*, 2011), the influence of multiple plastic hinges along the height of the wall (e.g., Rad and Adebar, 2008; Panagiotou and Restrepo, 2009), as well as of lowered shear rigidity due to shear cracking (Rad and Adebar, 2008) on the amplification of the base shear.

In multi-storey cantilever wall systems comprising walls of different lengths a further effect influences the internal force distribution and therefore also the maximum base shear demand on each wall. While all walls are responding in the elastic range, the lateral force distribution among the walls is proportional to their relative lateral stiffness. However, when some walls are elastic while others have already started to form a plastic hinge at their wall base, the base shear is no longer distributed proportional to the elastic stiffness of the walls. At this point, the incremental lateral deformations of the uncoupled walls would not be compatible at the floor levels. In order to make these compatible when the walls are coupled by floor diaphragms, the floor diaphragms must transfer in-plane forces, which might increase the base shear demands on the slender walls when compared to their uncoupled counterparts while the total base shear of the system remains unaffected (Rutenberg, 2004; Beyer, 2005; Rutenberg and Nsieri, 2006). This increase in base shear force in the individual walls is therefore not related to flexural overstrength and is only indirectly related to the dynamic amplification, which influences the height of the resultant of the horizontal inertia forces acting on the wall, which in return affects the forces transferred by the floor diaphragms. The base shear amplification due to compatibility forces can therefore be analysed using pushover analysis with different lateral load patterns (Rutenberg, 2004). For the design of new structures, pushover analysis is not a standard tool and simpler models, which can be analysed by hand, are desirable. Such models for cantilever wall systems were proposed by Paulay and Restrepo (1998) and Rutenberg and Nsieri (Rutenberg, 2004; Rutenberg and Nsieri, 2006). The two models represent limit cases concerning the effect of the coupling on the base shear distribution between the walls. Section 3 presents the two models and outlines the underlying assumptions.

Within the scope of this paper, the two hand-calculation methods are presented and applied to an example structure (Section 3). Next, the results of these methods are compared to the pushover analysis results of two numerical models (Section 4). The first numerical model is a lumped plasticity beam element model and the second is an advanced shell element model with nonlinear material constitutive models for concrete and reinforcement. The results of this shell element model are taken as benchmark results and the results of the hand-calculation methods and the lumped plasticity beam element model are judged based on their agreement with this shell element model. The comparison of the different models is illustrated using as an example an eight storey cantilever wall system composed of two walls of different lengths (Section 2). In the following the term “coupled system” refers to analyses in which equal lateral displacements are enforced on the two walls at the storey levels. The term “uncoupled walls” is used to describe the behaviour of the walls when the effect of the coupling by the floor diaphragms is not considered.

2 EXAMPLE STRUCTURE

A simple example structure is used to illustrate the differences between the analysis approaches, which will be discussed in the following sections. The example structure is planar and has eight storeys (H_w = 8 x 3.0 m = 24 m). It is braced by two RC walls with lengths of 6 m and 4 m, respectively (Figure 2.1). The RC walls are coupled at each storey level by a RC slab. In this study the effect of gravity columns is not considered, i.e., only the walls and the slabs spanning between the walls are considered. Both walls have a width of 0.2 m and were designed according to Eurocode 8 (CEN, 2004). Figure 2.2a shows the moment-curvature relationships of the long wall (l_w = 6.0 m) and the short wall (l_w = 4.0 m). Based on the plastic hinge analysis approach, the flexural force-deformation relationship of the two walls are computed considering the two walls as uncoupled (Priestley *et al.*, 2007; Figure 2.2b). The force-displacement relationships are computed for a uniformly distributed lateral load and the plotted displacement corresponds to the top displacement.

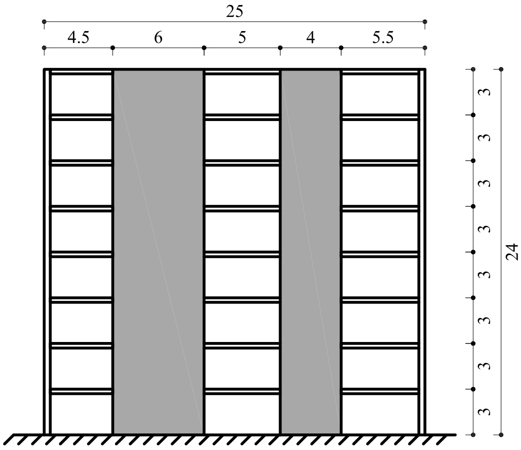


Figure 2.1. Geometry of the planar example structure comprising two walls of different length. All dimensions are in *m*.

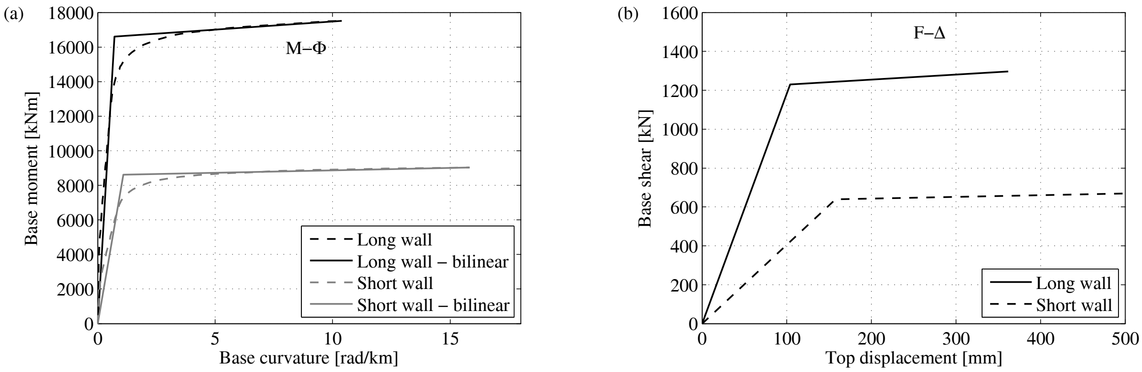


Figure 2.2. (a) Moment-curvature relationship of the individual walls including the corresponding bi-linear approximations. (b) Force-deformation relationship of the individual walls.

3 HAND-CALCULATION METHODS FOR THE ANALYSIS OF CANTILEVER WALL SYSTEMS

This section presents two hand-calculation methods for the analysis of cantilever wall systems. The first model by Paulay and Restrepo (1998) assumes that the coupling is negligible and that the walls can be treated as uncoupled walls subjected to the same top displacement. Rutenberg and Nsieri (Rutenberg, 2004; Rutenberg and Nsieri, 2006), on the other hand, assume that the floor diaphragms are rigid in-plane and that the incremental displacement profile after yielding of the stiffest wall satisfies compatibility requirements. The two models yield very different estimates of the base shear distribution among the walls. The predicted *system* response, however, is approximately the same. Note, that both approaches do not account for the out-of-plane stiffness and strength of the slabs, which would affect the axial forces as well as the bending moments in the walls. Both approaches are therefore valid, only when the walls are spaced by a significant distance, so that the out-of-plane stiffness of the slab element spanning between the walls becomes negligible.

3.1 Approach by Paulay and Restrepo

Paulay and Restrepo (1998) calculate the force deformation relationship of the individual walls based on the plastic hinge method. They estimate the yield curvature of a wall from the yield strain ε_{sy} of the reinforcement and the inverse of the wall length l_w:

$$\phi_y \approx \frac{2\varepsilon_{sy}}{l_w} \tag{3.1}$$

If all walls have the same height and cross sectional shape and if for all walls the same type of reinforcement is used, the yield displacement of each wall can be expressed as a proportionality constant times the inverse of the wall length:

$$\Delta_y \approx C \frac{2\varepsilon_{sy}}{l_w} H_w^2 \propto \frac{1}{l_w} \tag{3.2}$$

Hence, the longer the wall, the smaller is its yield displacement. Paulay and Restrepo postulate that the force-deformation relationships of the individual walls in the coupled system are assumed to be identical to those of the uncoupled walls. The system's force-deformation response can hence be estimated as the sum of the force-deformation responses of the uncoupled walls (see Figure 3.1a).

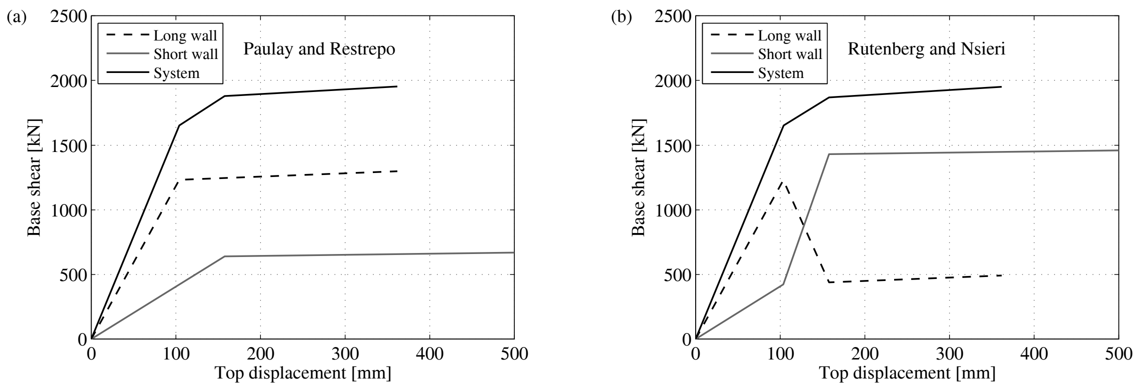


Figure 3.1. Force-deformation relationship of the long and short wall and of the system derived using two analytical models: (a) Paulay and Restrepo, (b) Rutenberg and Nsieri.

3.1 Approach by Rutenberg and Nsieri

Rutenberg and Nsieri developed a simple analytical approach for estimating the compatibility forces which arise between two RC walls once the longer wall has yielded and the shorter is still elastic. Rutenberg and Nsieri formulated the approach in a general manner so that it can be applied to a coupled wall system including any number of walls (Rutenberg, 2004; Rutenberg and Nsieri, 2006). In this section, the method will be presented, however, for a system comprising two walls only, which are referred to as long wall and short wall. Up to yielding of the long wall, both walls behave in the same manner and the system base shear will be distributed between the two walls in proportion to their stiffnesses. Neglecting any post-yield stiffness, the system's base moment and base shear force after the onset of yielding of the long wall can increase by:

$$\Delta M = M_{short,y}\left(1 - \frac{\phi_{long,y}}{\phi_{short,y}}\right) \tag{3.3a}$$

$$\Delta V = \frac{\Delta M}{h_{eff}} \tag{3.3b}$$

where $M_{short,y}$ is the yield moment of the short wall, $\phi_{long,y}$ and $\phi_{short,y}$ are the yield curvatures of the long and short wall, respectively, and h_{eff} is the height of the resultant lateral force. Assuming in-plane floor slab rigidity, neglecting shear deformation and any post-yield stiffness the resulting base shear increments ΔV_i of the two walls after yielding of the long wall, can be estimated from:

$$\Delta V_{short} = \frac{I_{short}}{I_{long} + I_{short}} \left(\Delta V + \frac{\alpha \Delta M}{h} \frac{I_{long}}{I_{short}} \right) \tag{3.4a}$$

$$\Delta V_{long} = \frac{I_{long}}{I_{long} + I_{short}} \left(\Delta V - \frac{\alpha \Delta M}{h} \right) \tag{3.4b}$$

where I_{long} is the moment of inertia of the long wall, I_{short} the moment of inertia of the short wall, ΔV the base shear increment, ΔM the base moment increment and h the storey height. For walls having a uniform stiffness over the height, the factor α can be estimated as:

$$\alpha = 3 - \sqrt{3} = 1.27 \tag{3.5}$$

As an example, the forces transmitted by the floor slabs and the resulting base shear increments in a 2-storey system due to force ΔH acting at the top after the long wall has yielded is shown in Fig. 3.2.

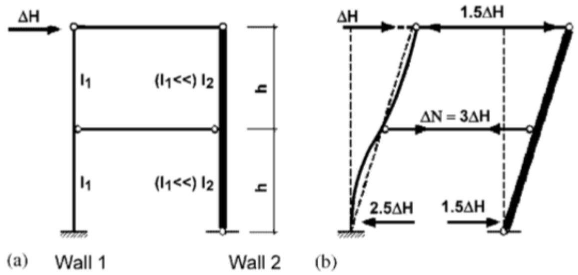


Figure 3.2. Two storey system: (a) Properties and loading, (b) assumed deflected shape and resulting floor forces and base shear forces (Rutenberg, 2004).

In this example the following additional assumptions were made:

- When compared to the short wall, the long wall is considered as infinitely rigid.
- Using the plastic hinge concept, the displacement field of the long wall after the onset of yielding can be described by a rigid body rotation about its base.

Rutenberg and Nsieri note that the underlying assumptions of the analytical model represent an over simplification of the real wall behaviour since, for example, distributed plasticity and shear deformations will affect the actual behaviour.

Herein Rutenberg and Nsieri's method is applied to the eight storey example structure presented in Section 2 and the obtained base shear forces are plotted in Figure 3.1b. Due to the compatibility forces arising after the onset of yielding of the long wall, the base shear force of the long wall drops and the base shear force of the short wall increases. The peak base shear of the short wall in the coupled system is therefore considerably larger than the peak base shear force of the uncoupled short wall.

3.3 Comparison of the results obtained with the two hand-calculation methods

Figure 3.1 compares the force-deformation relationships for the two walls and the system that were obtained with the two hand-calculation methods. The system's force-deformation relationship is not affected by the assumptions concerning the compatibility forces transferred by the RC slabs and therefore both methods yield the same curve. The force-deformation relationships of the individual walls are, however, strongly affected by the assumptions concerning the compatibility forces transferred by the RC slabs. For design, the maximum base shear forces are of particular interest. While the peak value of the long wall is the same for the two models, the model by Rutenberg and Nsieri predicts for the short wall a maximum base shear which is approximately 120% larger than the base shear of the uncoupled wall predicted by Paulay and Restrepo. Such an increase in base shear force would have severe implications on the seismic design of RC wall structures and should therefore be investigated further. As both methods are based on simplifications, the results of the hand-calculation methods will be compared to numerical results (Section 4).

4 COMPARISON OF THE RESULTS OF THE HAND-CALCULATION METHODS TO NUMERICAL RESULTS

To evaluate the results of the hand-calculation methods for estimating the base shear demand on wall structures coupled by slabs presented in Section 3, the example structure will be modelled using two different modelling approaches and the analytical results compared to the numerical ones. The models discussed within the scope of this paper are a lumped plasticity beam element model (Section 4.1) and a shell element model (Section 4.2).

4.1 Analysis of a lumped plasticity beam element model

The example structure was modelled using lumped plasticity beam elements for the two walls (Figure 4.1a). The model was analysed using the finite element programme Seismostruct (SeismoSoft, 2011). The plastic hinges at the wall bases were assigned the rigid-plastic moment-rotation relationships used in the plastic hinge analysis (Section 2, Figure 4.1b). The beam elements represented only the flexural flexibility of the wall elements; the shear flexibility was not accounted for. At the floor levels, the two walls were connected by horizontal rigid links, which had an infinite axial stiffness and therefore imposed equal horizontal displacements on the two walls. The out-of-plane stiffness of the slabs was neglected. Evidently, this represents an over simplification. However, it allows eliminating all parameters related to the slab stiffness and focusing on the influence of the wall deformations on the compatibility forces.

Figure 4.2 compares the results of the lumped plasticity beam element model to the two hand-calculation models introduced in the previous section. The results obtained with the model by Rutenberg and Nsieri correspond very well to the numerical results of the lumped plasticity model. The two models agree particularly well with respect to the shear force distribution between the two walls after the onset of yielding of the long wall. This is not completely unexpected as the model by Rutenberg and Nsieri is also based on the assumption of a lumped plasticity at the base of the walls. In addition, both the analytical and the numerical model neglect the influence of shear flexibility. As outlined in Section 3, the increase in base shear force for the short wall would have significant consequences on the seismic design of RC wall structures. However, before venturing on a modification of design guidelines, we must determine whether the increase in base shear force is "real", i.e., whether it would also be observed in real structures if it could be measured, or it is mainly caused by modelling assumptions which over simplify-reality, such as the lumped plasticity approach. In order to obtain benchmark results in the absence of experimental results, it is necessary to analyse the example structure by means of a sophisticated nonlinear shell element. This is described in the following section.

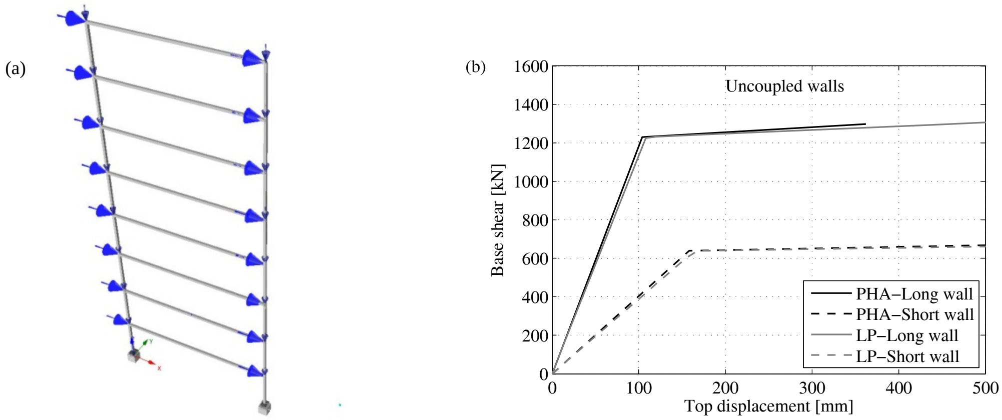


Figure 4.1. (a) Screenshot of the coupled wall system with the lumped plasticity model in Seismostruct. (b) Force-deformation relationships of the single walls obtained: Comparison of the results obtained with the numerical model and the results obtained from the plastic hinge analysis.

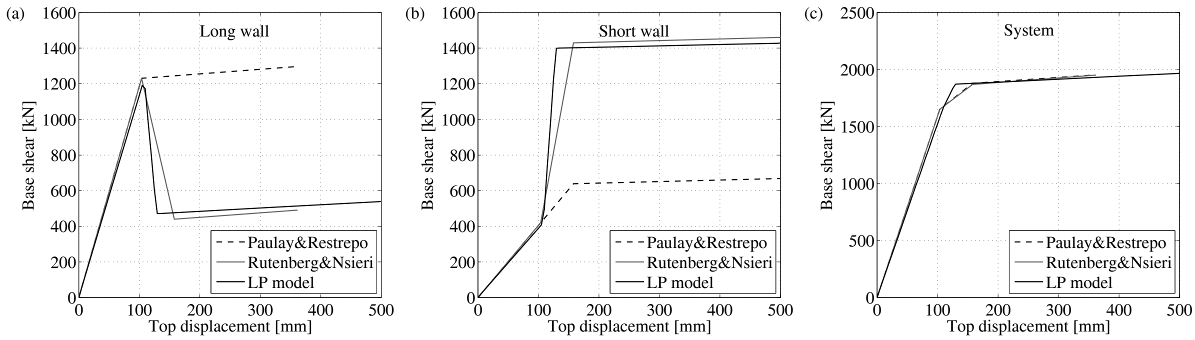
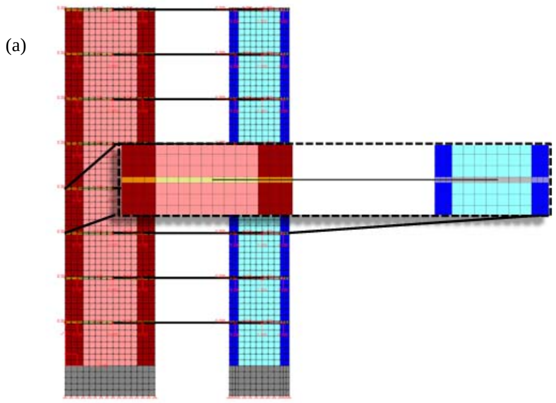


Figure 4.2. Force-deformation relationships of the coupled individual walls and of the system: comparison between numerical results obtained with the lumped plasticity beam element model (LP model) and the two analytical models.

4.2 Analysis of a shell element model

To obtain benchmark results, the example structure was analysed using the finite element program VecTor2 developed by Vecchio and his co-workers at the University of Toronto (Wong and Vecchio, 2002) for the analysis of planar 2D RC structures. It is based on the Modified Compression Field Theory (Vecchio and Collins, 1986) and on the Disturbed Stress Field Theory (Vecchio, 2000). The two walls were modelled using smeared longitudinal and transverse reinforcement. The walls were divided into boundary areas with higher longitudinal reinforcement ratios and web zones with lower longitudinal reinforcement ratios (Figure 4.3a). In the boundary zones, the confining effect of stirrups and hoops was accounted for following the approach by Mander *et al.* (1988). The coupling by the slabs was modelled by imposing equal lateral displacement at the storey heights on the two walls. For this purpose, the two walls were connected with truss elements which had a very large axial stiffness. The foundations of the two walls were modelled as very rigid. The model geometry and mesh are shown in Figure 4.3a. Lateral loading was applied through incremental lateral forces with equal values at every storey level.

Figure 4.3b shows for the uncoupled walls the comparison of the numerical results with the results of the plastic hinge analysis, which approximates the former rather well. The analysis results of the coupled wall system are compared to the results of the two hand-calculation methods in Figure 4.4. The numerical results of the shell element model lie in-between the results obtained with the two hand-calculation methods. Hence, for the short wall, the maximum base shear calculated on the assumption that the compatibility forces are zero (Paulay and Restrepo, 2001) underestimates the base shear force obtained with the shell element model by ~25%. The model by Rutenberg and Nsieri, on the contrary overestimates the maximum base shear force by ~70%. This is of course safe from a designer's point of view. However, for assessment purposes in particular a more accurate estimate might be warranted.



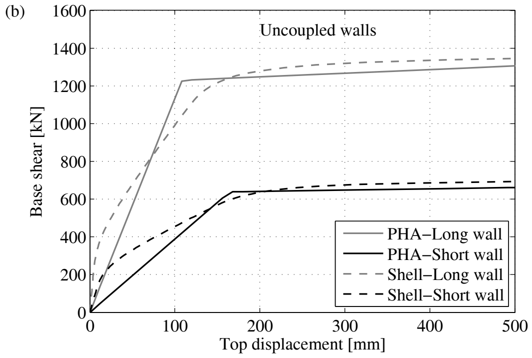


Figure 4.3. (a) Screenshot of the coupled wall system model in VecTor2 including detail for the rigid connection model. (b) Force-deformation relationships of the single walls: Comparison of the results obtained with the numerical model and the results obtained from the plastic hinge analysis.

The comparison of the results obtained with the lumped plasticity beam element model and the shell element model shows that the lumped plasticity model does not capture the shear force distribution of the coupled wall system correctly (Figure 4.5). As for the hand-calculation method by Rutenberg and Nsieri, the compatibility forces transferred as axial forces by the links are overestimated. This, however, does not affect the system's response. Hence, lumped plasticity models seem suitable for estimating the system's response but may lead to unrealistic results concerning the internal force distribution in a statically indeterminate system such as the coupled wall system, since the latter is very sensitive to assumed member stiffnesses and deformation modes.

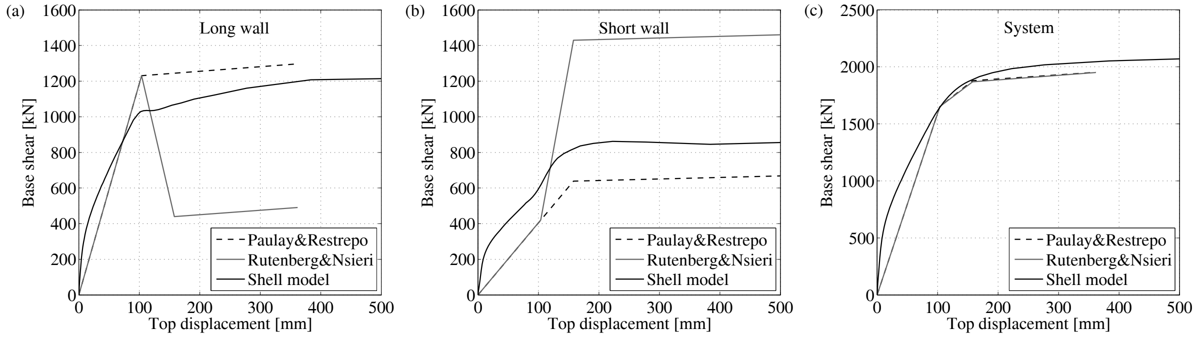


Figure 4.4. Force-deformation relationships of the coupled individual walls and of the system: comparison of numerical results obtained with the shell element model and the two analytical models.

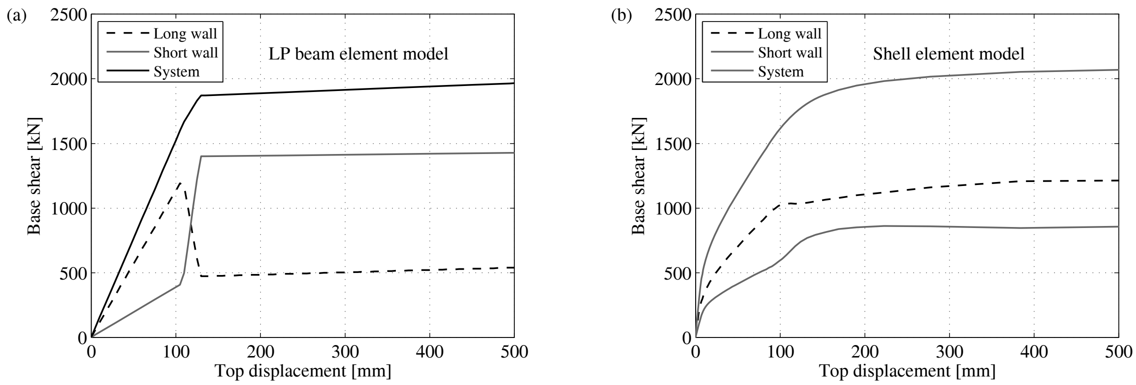


Figure 4.5. Comparison of the force-deformation relationships obtained for the coupled system with the numerical models: (a) lumped plasticity beam element model, (b) shell element model.

5 CONCLUSIONS AND OUTLOOK

Good estimates of the shear force demand on structural elements are a necessity in seismic design. If the shear force demand is underestimated, premature shear failure might lead to an appreciably reduced deformation capacity of the structure. The large base shear forces on the short wall obtained from lumped plasticity models were therefore of concern (Rutenberg, 2004; Beyer, 2005; Rutenberg and Nsieri, 2006). The shear forces result from compatibility forces enforced by the in-plane rigid slab elements which couple the cantilever walls at the storey levels. The compatibility forces arose after yielding of the long wall when the short wall was still elastic and caused redistribution of the base shear forces from the long to the short wall. The objective of this paper was to investigate whether the large resulting base shear forces for the short wall were "real" or whether they were mainly due to the simplifying modelling assumptions.

To examine the shear forces in cantilever wall systems an eight-storey planar structure was analysed, comprising of 6 m and 4 m long walls , i.e., wall lengths ratio =1.5. The base shear distribution was predicted using two hand-calculation methods and two numerical models. Of the latter, a nonlinear shell element model analysed with VecTor2 was considered to yield benchmark results, and the results of the other models were compared to the VecTor2 results. The comparison showed that the analytical and numerical models based on the lumped plasticity approach overestimated the peak shear force in the short wall. Considering the two walls as uncoupled, however, underestimated the peak shear force in the short wall to some extent. In particular, for systems with wall length ratios larger than the one investigated here, neglecting the compatibility forces might lead to unsafe estimates of the design shear forces while the lumped plasticity models will yield conservative estimates.

The system's response was insensitive to the modelling assumptions. Therefore for the numerical analysis of cantilever wall systems, models with lumped plasticity beam elements are recommended when the *system's* response is sought. However, based on the results presented herein for walls with length ratio =1.5, these models appear to yield very conservative estimates of the base shear demand on the short wall, which might be acceptable for the design of new structures but may lead to unwarranted interventions if they are used for seismic assessment purposes. While the analysis of a shell element model is computationally rather expensive, the analysis of simpler numerical models, such as beam models with distributed plasticity, are feasible within the scope of a real design or the assessment of a structure. On-going studies aim therefore at deriving modelling recommendations for cantilever wall structures using distributed plasticity beam elements.

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