Global scrape-off layer electromagnetic fluid turbulence simulations

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Outline

Introduction

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  - Non linear saturation mechanism
  - Electromagnetic global mode onset
  - Resistive global mode onset

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Motivation

- For the development of fusion, it is essential to understand:
  - Instabilities relevant to tokamak scrape-off layer (SOL)
  - Mechanisms responsible for SOL turbulent transport regimes
  - Steady-state profiles and their typical scale lengths
SOL turbulence: interplay between $\beta$, $\nu$, and $\omega_*$

\[ \alpha_{\text{MHD}} = q^2 \beta_e R / L_p \]

\[ \alpha_d = \frac{2^{3/4}}{2\pi q} \nu^{-1/2} \left( \frac{R}{L_p} \right)^{1/4} \]

Resistivity increases

$\nu$ increases

Beta increases

Figure: LaBombard et al., *Nucl Fusion* 45, 1658 (2005)

Our goal: understanding SOL turbulence dependence on $\beta$ and $\nu$
Scrape off-layer turbulence: a challenge for modeling

- Large fluctuations respect to background density
- No separation between equilibrium and turbulent scales
- Parallel losses due to presence of open field lines
Our approach: 3D global fluid electromagnetic simulations

- SOL *global* power balance between
  - Plasma outflow from the core
  - Perpendicular turbulent fluxes
  - Parallel losses at the limiter
- No separation between fluctuations and background quantities
- Gradient length $L_p = -p/\nabla p$ is a simulation result
  - Turbulence drive $(R/L_p)$ is a *priori* unknown
Model equations

Drift-reduced Braginskii equations in $\hat{s} - \alpha$ geometry, $T_i \ll T_e$

\begin{align*}
\frac{\partial_t n}{n} &= -\frac{R}{B} [\phi, n] + \frac{2}{B} \left[ \hat{C}(p_e) - n\hat{C}(\phi) \right] - \nabla_\parallel \left( n\nu || e \right) + S_n \\
\frac{\partial_t \omega}{\omega} &= -\frac{R}{B} [\phi, \omega] + \frac{2B}{n} \hat{C}(p_e) - \nu || i \nabla_\parallel \omega + \frac{B^2}{n} \nabla_\parallel j || + \frac{B}{3n} \hat{C}(G_i) \\
\frac{\partial_t \chi}{\chi} &= -\frac{R}{B} [\phi, \nu || e] - \nu || e \nabla_\parallel \nu || e + \frac{mi}{me} \left\{ -\nu \nu || e + \nabla_\parallel \nu || e - \frac{1}{n} \nabla_\parallel p_e - 0.71n\nabla || T_e - \frac{2}{3n} \nabla || G_e \right\} \\
\frac{\partial_t \nu || i}{\nu || i} &= -\frac{R}{B} [\phi, \nu || i] - \nu || i \nabla_\parallel \nu || i - \frac{1}{n} \nabla_\parallel p_e - \frac{2}{3n} \nabla_\parallel G_i \\
\frac{\partial_t T_e}{T_e} &= -\frac{R}{B} [\phi, T_e] - \nu || e \nabla_\parallel T_e + \frac{4}{3} \frac{T_e}{B} \left[ \frac{7}{2} \hat{C}(T_e) + \frac{T_e}{n} \hat{C}(n) - \hat{C}(\phi) \right] + S_{Te} \\
&\quad + \frac{2}{3} T_e \left[ 0.71 \nabla_\parallel \nu || i - 1.71 \nabla_\parallel \nu || e + 0.71 \left( \frac{\nu || i - \nu || e}{n} \right) \nabla || n \right]
\end{align*}

with ancillary equations $\omega = \nabla_\perp^2 \phi$, $\chi = \nu || e + \frac{mi}{me} \frac{\beta_e}{2} \psi$, $j || = n \left( \nu || i - \nu || e \right)$, $\nabla_\perp^2 \psi = j ||$.

Parallel gradient $\nabla || a = \hat{b}_0 \cdot \nabla a + \frac{\beta_e}{2} \frac{R}{B} [\psi, a]$, and curvature operator $\hat{C}(a) = \frac{B}{2} \left( \nabla \times \hat{b}_0 \right) \cdot \nabla a$.

BCs consistent with sheath physics are imposed at the limiter.
Non-linear simulations

- Simulations carried out using the GBS code
- Typical simulation parameters

\[
L_Y = 2\pi a = \{400, 800\} \rho_s, \quad R_0 = \{500, 1000\} \rho_s \\
\nu = 5 \times 10^{-3} - 0.1, \quad m_i/m_e = 200 \\
\beta_e = 10^{-5} - 3 \times 10^{-3} \\
\hat{s} = 0, \quad q = 4 \\
[nx, ny, nz] = \{128, 256, 32\}, \{128, 512, 64\}
\]

- Typically, in steady state we recover \( R/L_p \approx 10 - 20 \)
  - \( \alpha_{\text{MHD}} = q^2 \beta_e R/L_p \approx 0 - 0.7 \)
Non-linear turbulence driven by RBMs
Scanned parameter space $\alpha_{\text{MHD}}$ vs $\alpha_d$

High $\nu$, $\beta_e$ lead to enhanced transport
Equilibrium pressure gradient decreases with high $\beta_e$

Figure: $n_0$ for $\alpha_{\text{MHD}} = 3 \times 10^{-3}$ (left) and $\alpha_{\text{MHD}} = 0.7$ (right)

- $R/L_p$ can be computed from toroidal, time average of $p_e$
- In this case $R/L_p$ decreases $\sim 30\%$ at high $\beta_e$
Turbulence changes with high $\beta_e$

Figure: $n_1 = (n - n_0)/n_0$ for $\alpha_{\text{MHD}} = 3 \times 10^{-3}$ (left) and $\alpha_{\text{MHD}} = 0.7$ (right)

- Density fluctuations of order of $O(1)$
- Radially elongated eddies
- $k_\theta \rho_s$ decreases with increasing $\beta_e$
Turbulent spectrum changes with high $\beta_e$

Perturbed spectra shift to lower $k_\theta \rho_s$, $n$ (global modes)

- At low $\beta_e$, wide spectrum with $k_\theta \rho_s \approx 0.1$ dominant
- As $\beta_e$ is increased dominant mode acquires smaller $n$, $k_\theta \rho_s$

![Graphs showing $k_\theta \rho_s$ and $n$ spectrum changes](image)

**Figure**: $k_\theta \rho_s$ (left) and $n$ (right) spectrum
Linear theory does not fully explain global mode onset

- $R/L_p \approx 16$ extracted from GBS non-linear simulation
- Linear growth rates obtained with linear stability code
  - Resistive ballooning modes unstable in electrostatic limit
  - Low $n$ ideal ballooning mode become unstable at $\alpha_{\text{MHD}} \approx 0.25$
  - However, low $n$ modes are still linearly sub-dominant

$\gamma_{\text{lin}} [c_0/R]$ vs $n$
Non linear saturation mechanism

We expect turbulence to saturate when its drive \((R/L_p)\) is removed

\[
\frac{\partial p_{e1}}{\partial r} \sim \frac{\partial p_{e0}}{\partial r} \rightarrow p_{e1} \sim p_{e0}/(L_p k_r)
\]

Estimating \(\phi_1 \sim \gamma_{\text{lin}}/(k_\theta k_r)\) and \(k_r \sim \sqrt{k_\theta/L_p}\) the radial flux is

\[
\Gamma_r = \left\langle p_{e1} \frac{\partial \phi}{\partial \theta} \right\rangle \sim \frac{p_{e0} \gamma_{\text{lin}}}{L_p k_r^2} \sim \frac{p_{e0}}{k_\theta} \gamma_{\text{lin}}
\]

Global modes can drive large flux even if \(\gamma_{\text{lin}}\) is small

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Dominant mode number can be estimated with $\gamma_{\text{lin}}/n$

- Dominant $n$ decreases as $\beta_e$ increases

$$\Gamma \sim \frac{\gamma_{\text{lin}}}{n}$$
dominated by $n \sim 1$ for $\alpha_{\text{MHD}} \gtrsim 0.25$
Γ \sim \gamma_{\text{lin}}/n \text{ matches dominant } n \text{ in GBS simulations}

Downward trend can be explained by saturation mechanism
Similar effect can be shown for $\nu$

- RBM unstable at high $\nu$
  \[ \gamma_{\text{lin}}^2 \sim \frac{2R}{L_p} - \gamma_{\text{lin}} \frac{k_\parallel^2}{k_\perp^2 \nu} \rightarrow k_\perp^2 > \frac{k_\parallel^2}{\nu \sqrt{2R/L_p}} \]

- Lower $n \rightarrow$ global mode
Conclusions

- Global 3-D, E+M simulations of SOL turbulence
  - Recovered $R/L_p \sim 10–20$, turbulent flux dominated by RBMs
  - Large fluctuation amplitude $n_1/n_0 \sim O(1)$
  - Radially extended modes with $k_r \sim \sqrt{k_\theta/L_p}$

  3D non-linear global model required

- Dominant toroidal mode number vs. $\beta_e$ or $\nu$ consistent with flux estimate due to gradient removal mechanism $\Gamma_r \sim \gamma/k_\theta$
- Shift toward global instability observed at high $\beta_e$ and $\nu$

  Enhanced transport for high $\nu$, $\alpha_{\text{MHD}} \gtrsim 0.25$