Stochastic Simulations for DREAM4

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1 ODE model (DREAM3)

\[
\begin{align*}
\frac{dx_i}{dt} &= F_{RNA}^i(x, y) = m_i \cdot f_i(y) - \lambda_{RNA}^i \cdot x_i \\
\frac{dy_i}{dt} &= F_{Prot}^i(x, y) = r_i \cdot x_i - \lambda_{Prot}^i \cdot y_i
\end{align*}
\]

where \( m_i \) is the maximum transcription rate, \( r_i \) the translation rate, \( \lambda_{RNA}^i \) and \( \lambda_{Prot}^i \) are the mRNA and protein degradation rates, and \( f_i(\cdot) \) is the so-called input function of gene \( i \). The input function computes the relative activation of the gene, which is between 0 (the gene is shut off) and 1 (the gene is maximally activated), given the transcription-factor (TF) concentrations \( y \).

2 SDE model (DREAM4)

\[
\begin{align*}
\frac{dx_i}{dt} &= F_{RNA}^i(x, y) + \sigma_{RNA}^i \cdot \eta_i \\
\frac{dy_i}{dt} &= F_{Prot}^i(x, y) + \sigma_{Prot}^i \cdot \zeta_i
\end{align*}
\]

where each \( \eta_i \) and \( \zeta_i \) is an independent Gaussian white noise with zero mean and unit variance. \( \sigma_{RNA}^i \) and \( \sigma_{Prot}^i \) represent the amplitude (standard deviation) of the noise.

3 Numerical simulation of SDEs

For notational simplicity, we consider here only a single equation and not a system of equations. Equations (3) and (4) are of the following, general form (note that we use the Stratonovich scheme)

\[
\begin{align*}
\frac{dX_t}{dt} &= F(X_t)dt + G(X_t)dW_t \\
\frac{dX_t}{dt} &= F(X_t)dt + G(X_t) \circ dW_t \\
F_t &= F - \frac{1}{2} G^t G
\end{align*}
\]

where \( dW_t \) is a Wiener process. The Itô scheme is defined by (5) and the equivalent Stratonovich scheme is given by (6) and (7). In (3) and (4) the amplitude of the noise \( G(X_t) \) is a constant \( \sigma_i \).
We propose to use the Milstein scheme for the numerical integration, which is better than the basic Euler-Marumaya method, but still easy to implement.\textsuperscript{1,3,5,6}

Given \( X(n) = X_n \), the value at the next discrete time point \( X_{n+h} \) is approximated by

\[
X_{n+h} = X_n + F(X_n)h + G(X_n)\Delta W_n + \frac{1}{2} G'(X_n) G(X_n)[\Delta W_n]^2
\]

\( \Delta W_n = [W_{t+h} - W_t] \sim \sqrt{h}N(0,1) \)

where \( h \) is the step size.

References


