Journal of Hydraulic Research Vol. 50, No. 4 (2012), pp. 400–408
http://dx.doi.org/10.1080/00221686.2012.695041

Research paper

Hydraulic design of A-type Piano Key Weirs

MARCELO LEITE RIBEIRO, Dr., Engineer, *Laboratory of Hydraulic Constructions (LCH), Ecole Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland.* Now at *Stucky SA, Rue du lac 33, CH-1020 Renens, Switzerland.*
Email: mleiteribeiro@stucky.ch

MICHAEL PFISTER (IAHR Member), Research Associate, *Laboratory of Hydraulic Constructions (LCH), Ecole Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland.*
Email: michael.pfister@epfl.ch (author for correspondence)

ANTON J. SCHLEISS (IAHR Member), Professor, *Laboratory of Hydraulic Constructions (LCH), Ecole Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland.*
Email: anton.schleiss@epfl.ch

JEAN-LOUIS BOILLAT, *formerly Laboratory of Hydraulic Constructions (LCH), Ecole Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland.*

ABSTRACT
Piano Key Weirs (PKWs) are an alternative to linear overflow structures, increasing the unit discharge for similar heads and spillway widths. Thus, they allow to operate reservoirs with elevated supply levels, thereby providing additional storage volume. As they are relatively novel structures, few design criteria are available. Hence, physical model tests of prototypes are required. This study describes comprehensive model tests on a sectional set-up of several A-type PKWs, in which the relevant parameters were systematically varied. Considering data of former studies, a general design equation relating to the head–discharge ratio is derived and discussed. The latter is mainly a function of the approach flow head, the developed crest length, the inlet key height, and the transverse width. To extend its application range, case study model tests were analysed to provide a design approach if reservoir approach flow instead of channel flow is considered.

Keywords: Capacity; discharge; flood; Piano Key Weir; spillway

1 Introduction

The Piano Key Weir (PKW) is a further development of the Labyrinth Weir. It was mainly elaborated by Hydrocoop (France), in collaboration with the Laboratory of Hydraulic Developments and Environment of the University of Biskra, Algeria, and the National Laboratory of Hydraulic and Environment of Electricité de France (EDF-LNHE Chatou). Schleiss (2011) and Lempérière *et al.* (2011) present historical reviews on the evolution from Labyrinth Weirs to PKWs.

Two advantages of PKWs as compared with Labyrinth Weirs are:

(1) Reduced structural footprint allowing the installation on top of existing gravity dams (Lempérière and Ouamane 2003).
(2) High discharge capacity, mainly because the developed crest length corresponds several times to the transverse weir width. The inclined bottom of the keys instead of the horizontal–vertical arrangement of Labyrinth Weirs improves their hydraulic efficiency (Laugier *et al.* 2009, Anderson and Tullis 2011, 2012).

Due to increased flood discharges and strict specifications regarding the dam safety, a large number of existing dams require spillway rehabilitation to improve their hydraulic capacity. The recently developed PKWs are often an interesting option (Laugier 2007, Laugier *et al.* 2009, Leite Ribeiro *et al.* 2009). Considerable efforts have so far been made to understand their hydraulic behaviour. Tests performed on scale models as well as numerical simulations contributed to increased knowledge. However, the hydraulics of PKWs is still not completely understood hence most PKW prototype projects are designed

Revision received 16 May 2012/Open for discussion until 28 February 2013.

ISSN 0022-1686 print/ISSN 1814-2079 online
http://www.tandfonline.com

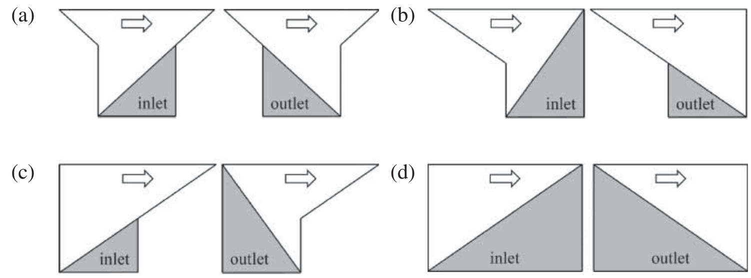

Figure 1 PKW types (a) A, (b) B, (c) C, (d) D (modified from Lempérière *et al.* 2011)

using physical models (Laugier 2007, Cicero *et al.* 2011, Dugue *et al.* 2011, Erpicum *et al.* 2011b). Although the flow over a PKW is highly three-dimensional, Erpicum *et al.* (2011a) present a simplified one-dimensional numerical modelling for preliminary designs with an accuracy of $\pm 10\%$. The model is based on cross-section-averaged equations of mass and momentum conservation with only the upstream discharge as boundary condition.

Lempérière and Ouamane (2003) were the first to present systematic PKW tests of type A and B (Fig. 1), proposing a rough design criterion. Ouamane and Lempérière (2006) extended their 2003 study for different dimensionless parameters. Results highlight the relevance of the ratio between the developed crest length L and the transverse width W. Furthermore, they discuss the positive effect of an upstream deflector and the satisfactory behaviour regarding floating debris passage. Lempérière *et al.* (2011) summarize different types of PKWs which have been studied by Hydrocoop since 1998. These were classified according to the presence or absence of overhangs (Fig. 1). In type A, the up- and downstream overhangs are identical. Types B and C include only up- or downstream overhangs. Although type D has inclined bottoms, it does not contain overhangs.

The standard notation as defined by Pralong *et al.* (2011) is used herein (Fig. 2), with $B =$ streamwise length, $P =$ vertical height, $T_s =$ thickness, and $R =$ parapet wall height. Furthermore, subscript i refers to the inlet key, i.e. the key that is filled with water for a reservoir surface at the PKW crest elevation, and subscript o to the outlet key, i.e. the 'dry' key for the latter reservoir level.

Machiels *et al.* (2011d) analysed the flow characteristics of PKW type A. They observed the presence of a critical flow section that 'advances from the downstream crest to the inlet for increasing heads generating an undular free surface downstream of the critical section', limiting the hydraulic capacity. To improve the latter, they suggest increasing the inlet width W_i, the upstream overhang B_o, and the height P_i. With extended upstream overhangs, type A tends to a similar geometry as type B, so that their results agree with those of Lempérière and Ouamane (2003). By comparing PKWs of type A and D, Anderson and Tullis (2011) confirm that the presence of the overhangs has a positive effect on the PKW discharge capacity. Upstream overhangs increase the inlet flow area and wetted perimeter, which results in lower energy losses.

Leite Ribeiro *et al.* (2011) presented a detailed study on the effect of various dimensionless parameters, e.g. the relative developed crest length L/W, the relative key widths W_i/W_o, the ratio of the vertical to the horizontal shape P_i/W_i, and the vertical dam height relative to the PKW height P_d/P_i on the type A discharge capacity. For bottom slopes from 0.3 to 0.6 ($V:H$), the discharge is directly proportional to P_i. Machiels *et al.* (2011c) suggest that for bottom slopes beyond this range, increasing P_i hardly affects the capacity. For $W_i/W_o > 1$, the discharge efficiency is increased, in agreement with Machiels *et al.* (2011a), and Leite Ribeiro *et al.* (2012a), who recommend $W_i/W_o \cong 1.5$.

Concerning the hydraulic design of PKWs, few general design criteria are available. A methodology for the preliminary design, mainly based on hydraulic model tests of prototype structures, was given by Leite Ribeiro *et al.* (2012a). A discharge increase factor of PKWs is proposed with dimensionless charts versus the energy head, W_i and P_i. A similar approach is applied by Machiels *et al.* (2011b). Leite Ribeiro *et al.* (2011) proposed an empirical equation for computing the discharge increase factor based on the dimensionless terms L/W, W_i/W_o, P_d/W_i and H/P_i. However, this equation is complex and not structured according to physical phenomena. Kabiri-Samani and Javaheri (2012) present a more global approach for the discharge coefficient. The present study re-analyses the data of Leite Ribeiro and Machiels, and a general, simplified and physically based approach is suggested.

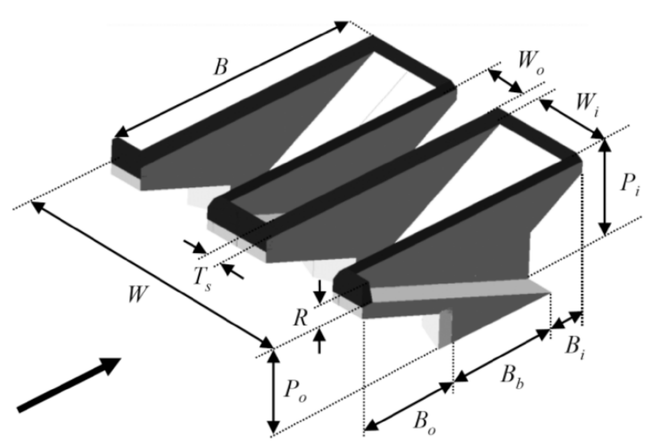

Figure 2 Notation of PKW according to Pralong *et al.* (2011), with B as streamwise length, W as transverse width, P as vertical height, T_s as wall thickness, and R as parapet wall height

2 Experimental set-up

Systematic physical model tests were conducted at the Laboratory of Hydraulic Constructions (LCH) of Ecole Polytechnique Fédérale de Lausanne (EPFL) in a straight rectangular channel 40 m long, 2 m wide, and 1 m high. The test section was reduced in width to $W = 0.5$ m and in length to some 3 m, with a sufficiently long parallel approach flow reach. The model was set up to conduct basic research tests but was not related to a prototype case study, corresponding to a sectional model including 1.5 PKW units (Fig. 3a). To exclude an effect of the unit number, preliminary tests were conducted with up to 3 units and $W \leq 1.0$ m (Fig. 3b), whose results were analogues so that the selected number of units was sufficient with 1.5 (Leite Ribeiro *et al.* 2012b). Different arrangements with similar up- and downstream overhangs were tested, according to type A, including an upstream rounded nose (Fig. 3a). The thickness of all side walls was $T_s = 0.02$ m, and the overflow crest shape of the latter is half-circular. All tests were conducted for free overfall conditions.

In total, 380 model tests were conducted, with 49 different PKW geometries. The basic parameter variation included $1.50\,\text{m} \leq L \leq 3.50\,\text{m}$, $0.33\,\text{m} \leq B \leq 1.00\,\text{m}$, $0.10\,\text{m} \leq W_i \leq 0.20\,\text{m}$, $0.10\,\text{m} \leq W_o \leq 0.20\,\text{m}$, $0.10\,\text{m} \leq P_i \leq 0.28\,\text{m}$, $0.10\,\text{m} \leq P_o \leq 0.28\,\text{m}$, $0.00\,\text{m} \leq P_d \leq 0.62\,\text{m}$, $0.07\,\text{m} \leq B_i \leq 0.40\,\text{m}$, $0.07\,\text{m} \leq B_o \leq 0.40\,\text{m}$, $0.00\,\text{m} \leq R \leq 0.06\,\text{m}$, and $0.02\,\text{m} \leq H \leq 0.27\,\text{m}$. The parameters $T_s = 0.02$ m and $W = 0.50$ m were kept constant. Here, H = total approach flow energy head above the PKW crest and P_d = channel height below the PKW foot, i.e. below P_o to investigate the effect of the approach flow velocity. The bottom slopes of the inlet key $(P_i - R)/(B_i + B_b)$ and of the outlet key $(P_o - R)/(B_o + B_b)$ were between 0.34 and 0.84. The model discharge was varied between $0.013\,\text{m}^3/\text{s} \leq Q \leq 0.220\,\text{m}^3/\text{s}$. Expressed in relative terms, the parameter variation included values of $3.0 \leq L/W \leq 7.0$, $0.1 \leq H/P_i \leq 2.8$, $1.5 \leq B/P_i \leq 4.6$, $0.7 \leq P_i/P_o \leq 1.4$, and $0.5 \leq W_i/W_o \leq 2.0$.

Since the derivation of the head–discharge equation as a function of the relevant parameters was the focus of the study, the latter values were measured in the model using a point gauge to 0.5 mm reading accuracy. The water levels were taken laterally in the channel in stagnant water, away from the width reduction where the effect of the velocity head was absent and H identical to the total head. The discharge was measured with a magnetic inductive flow meter to 0.5%-full-span, equivalent to 1.25 l/s.

Scale effects on PKWs were so far rarely discussed, so that the rules of sharp-crested weirs were applied herein. Machiels *et al.* (2011d) report a specific 'low-head behaviour' regarding the transition from the clinging to the leaping nappe. The viscosity and the surface tension of water are fluid properties which cannot be scaled, so that scale effects occur for small overflow depths on weir crests. Hager (2010) mentions critical values for $H < 0.05$ m, and Novak *et al.* (2010) state $H < 0.03$ m, so that values below 0.05 m were not considered to develop the equations presented herein. Furthermore, the approach flow velocity upstream of the weir has an effect on its capacity if $H > 0.5(P_i + P_d)$ (Vischer and Hager 1999). These data were also excluded, resulting finally in 304 unaffected tests. Although these were performed for research purposes, the model dimensions were defined corresponding to characteristic prototypes with a geometrical scale factor of $\lambda \cong 15$. As for the air transport, which is observed on PKWs for large discharges, e.g. Pfister and Hager (2010) recommend maximum scale factors in this range to limit a significant underestimation of the latter.

3 Data analysis

3.1 *Normalization*

The discharge Q_S over a linear sharp-crested (subscript S) weir serves as reference, given as

$$Q_S = C_S W \sqrt{2g} H^{3/2} \tag{1}$$

with $C_S = 0.42$ as the discharge coefficient (Hager and Schleiss 2009). Two approaches may be chosen to derive the PKW (subscript P) discharge Q_P: via Eq. (1) with $C_S \to C_P$ (Ouamane and Lempérière 2006, Anderson and Tullis 2011, Machiels *et al.* 2011d, Kabiri-Samani and Javaheri 2012) or via a comparison with sharp-crested weirs in terms of a relative discharge increase

(a)

(b)


Figure 3 Model view from (a) upstream for 1.5 PKW units, (b) downstream for 3 PKW units

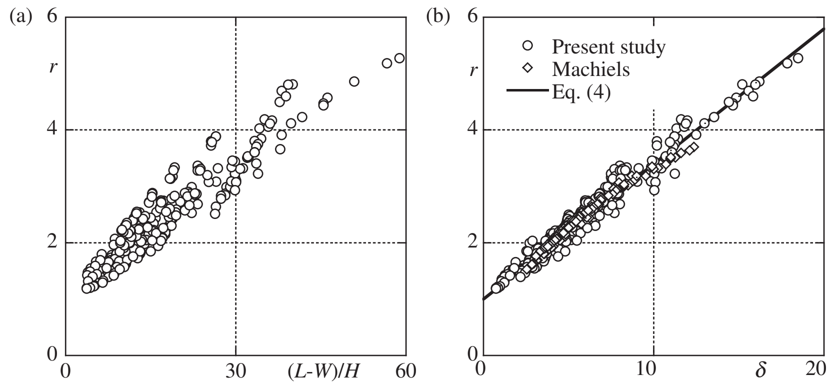

Figure 4 Test data of r versus (a) $(L-W)/H$, (b) δ. Notation in Fig. 2

ratio (Leite Ribeiro *et al.* 2012a). The second approach is selected herein as it represents the effective developed crest length L as compared with a linear weir width W which is therefore better physically based (Falvey 2003, Schleiss 2011). The discharge Q_P as measured in the model is then compared with the theoretical value for a linear sharp-crested weir of width W, by keeping H constant. As C_S strictly applies for frontal approach flow conditions and C_P includes both, frontal and lateral approach flows, the effect of these coefficients is *a priori* unknown. The discharge increase ratio r is defined as

$$r = \frac{Q_P}{Q_S} = \frac{C_P L\sqrt{2g}H^{3/2}}{C_S W\sqrt{2g}H^{3/2}} \approx f\left(\frac{L}{W}\right) \tag{2}$$

relating the ratio of the PKW discharge to that of a linear sharp-crested weir for identical H. As PKWs spill higher discharges per width W than equivalent linear sharp-crested weirs, $r > 1$, particularly for small H.

3.2 *Primary effects*

The values of r are given as a function of $(L-W)/H$ in Fig. 4(a), excluding for the moment the other parameters. The data essentially collapse, indicating that $(L-W)/H$ is a dominant term, and that the effect of the other parameters is relatively small. A further data analysis indicates in addition that (1) parameters P_i and W have a relevant effect, and (2) W, P, B_o (here equivalent to B_i) and R have a minor effect. A pragmatic normalization regarding the PKW efficiency is thus

$$\delta = \left(\frac{(L-W)P_i}{WH}\right)^{0.9} \tag{3}$$

Equation (3) was validated with the data of Machiels *et al.* (2011a) (Fig. 4b). They tested seven A-type PKW model geometries varying the key bottom slopes by modifying $P_i = P_o$. Tests with scale effects as described above were ignored, and a maximum key bottom slope of 0.7 was considered. The range of validity of Eq. (3) is not extended by the additional data set. In contrast, the key bottom slope was limited to 0.7. For steeper key bottoms, the accuracy of Eq. (3) decreases and the predicted values exceed those measured. For extremely steep slopes, the PKW approaches geometrically a rectangular Labyrinth Weir, which has typically a reduced discharge capacity as compared with PKWs (Blancher *et al.* 2011).

As shown in Fig. 4(b), the measured r collapse with a trend line if normalized with δ as

$$r = 1 + 0.24\delta \tag{4}$$

Here, $r(\delta = 0) = 1$ ($L = W$ or small P_i combined with large H) is similar to a linear sharp-crested weir. The capacity of a PKW increases as compared with the linear sharp-crested weir if providing in particular long L and high P_i. Equation (4) is limited to $0 < \delta < 20$, and all tests considered herein included a range of $1.2 \leq r \leq 5.3$. The coefficient of determination between the measured values and Eq. (4) is $R^2 = 0.964$ for the present data and $R^2 = 0.975$ for Machiels *et al.* (2011a). Furthermore, the maximum error between the measured and computed values of r is $\pm 17\%$, including the data of Machiels *et al.* (2011a). The normalized root-mean-square deviation (NRMSD) between measured and computed values is 0.021.

3.3 *Secondary effects*

Equation (4) represents a pragmatic approach, yet small effects of the secondary parameters were observed as reported by Leite Ribeiro *et al.* (2012a) or Machiels *et al.* (2011d), so that they were considered in a further data analysis. Four correction factors to Eq. (4) resulted, including the inlet width relative to the outlet key W_i/W_o, the ratio of inlet to outlet heights P_i/P_o, the relative overhang length $(B_i + B_o)/B$, and the relative parapet wall height R_o/P_o. The motivation for this secondary analysis relates to an advantage of PKWs, i.e. their high discharge capacity for small heads H. Small variations of H may result in a significant reservoir volume or a slight increase in dam height.

The relative width of the inlet key determines the unit discharge approaching its crest. For relatively large W_i, the flow has laterally more space thereby reducing losses, with a slightly

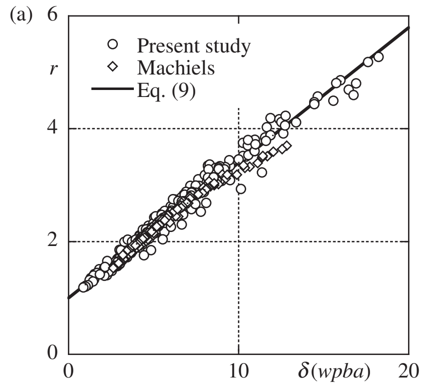

Figure 5 (a) Measured r versus $\delta(wpba)$, (b) comparison between computed (Eq. 9) and measured r values, (—) perfect agreement, (– – –) ±10% error

increased efficiency (Le Doucen *et al.* 2009). The data analysis indicates a small effect of W_i/W_o on r, so that a first correction factor is

$$w = \left(\frac{W_i}{W_o}\right)^{0.05} \tag{5}$$

Its range is $0.97 \leq w \leq 1.04$ for the present data and those of Machiels *et al.* (2011a) for $0.5 \leq W_i/W_o \leq 2.0$. Relatively wide inlet keys thus generate a marginally higher discharge for the same H than small values.

The height ratio P_o/P_i has a small effect on the PKW discharge capacity. It turned out that Eq. (4) slightly underestimates the effective discharge for large P_o/P_i, whereas the reverse was observed for small P_o/P_i. Accordingly, the second correction factor reads

$$p = \left(\frac{P_o}{P_i}\right)^{0.25} \tag{6}$$

The range tested in the present and Machiels *et al.*'s (2011a) investigation is $0.72 \leq P_o/P_i \leq 1.38$ for which $0.92 \leq p \leq 1.08$. Note, however, that P_i is also included in δ so that *a priori* relatively large values of P_i are efficient, whereas the effect of large P_o is small.

The effect of the overhang lengths B_o and B_i is linked to the effect of L, so that an increase in L implicitly also *increases* $(B_o + B_i)$. As a consequence, relatively large overhangs increase the discharge capacity of a PKW (Anderson and Tullis 2011). The basic equation, however, slightly overestimates this effect, so that the third correction factor includes a negative exponent as

$$b = \left(0.3 + \frac{B_o + B_i}{B}\right)^{-0.50} \tag{7}$$

The range tested in the present and Machiels *et al.*'s (2011a) investigations is $0.4 \leq (B_o + B_i)/B \leq 0.8$ for which $0.95 \leq b \leq 1.20$. Note that all considered PKWs were symmetrical regarding B_o and B_i, representing a limitation of the herein developed equations.

Parapet walls are known to slightly increase the capacity of PKWs (Leite Ribeiro *et al.* 2012). The data analysis indicates that the fourth correction factor is

$$a = 1 + \left(\frac{R_o}{P_o}\right)^2 \tag{8}$$

The range tested in the present and Machiels *et al.*'s (2011a) investigations is $0 \leq R_o/P_o \leq 0.22$ for which $1 \leq a \leq 1.05$. Note that $R_i \leq R_o$ in the tested set-ups. The presence of parapet walls on the outlet keys appears efficient, while those on the inlet key hardly improve the discharge capacity.

To include the secondary effects, Eq. (4) is completed with the correction factors, so that

$$r = 1 + 0.24\delta(wpba) \tag{9}$$

The same range of validity applies as for Eq. (4). The measured data are shown in Fig. 5(a), normalized with $\delta(wpba)$. Note the small difference between the Figs. 4(b) and 5(a). A statistical analysis indicates a slightly better performance of Eq. (9) as compared with the pragmatic and simplified approach of Eq. (4). In particular, the number of outliers was reduced. The coefficient of determination between the measured values and the prediction according to Eq. (9) is $R^2 = 0.976$ for the present data and $R^2 = 0.975$ for Machiels *et al.* (2011a). Furthermore, maximum errors of $+18$ and -11% occur between measured and computed values of r, including the data of Machiels *et al*. The NRMSD between measured and computed values is 0.018.

A comparison between r_C computed (subscript C) using Eq. (9) and r_M measured (subscript M) is shown in Fig. 5(b). Few points lay outside of the ±10% range of confidence. Taking into account that the measurement accuracy is also on the order of few percents leads to the conclusion that the basic hydraulic characteristics relating to the head–discharge relation of PKWs are satisfactorily described.

4 Case studies

Several prototype PKWs currently exist. As few general design guidelines are available, these structures are typically model-tested prior to erection to guarantee an adequate performance.

Table 1 Parameters of PKW case studies in model dimensions; notation Fig. 2

Name	L (m)	W (m)	W_i (m)	W_o (m)	P_i (m)	P_o (m)	R (m)	B_b (m)	λ (−)	Lab.
St. Marc	2.568	0.519	0.115	0.082	0.165	0.165	0.000	0.403	30	LCH
Gloriettes 4	2.894	0.545	0.092	0.058	0.110	0.110	0.000	0.325	30	LCH
Gloriettes 7	3.103	0.555	0.045	0.040	0.075	0.075	0.015	0.190	30	LCH
Etroit	2.595	0.388	0.090	0.058	0.135	0.135	0.017	0.410	30	Sogreah

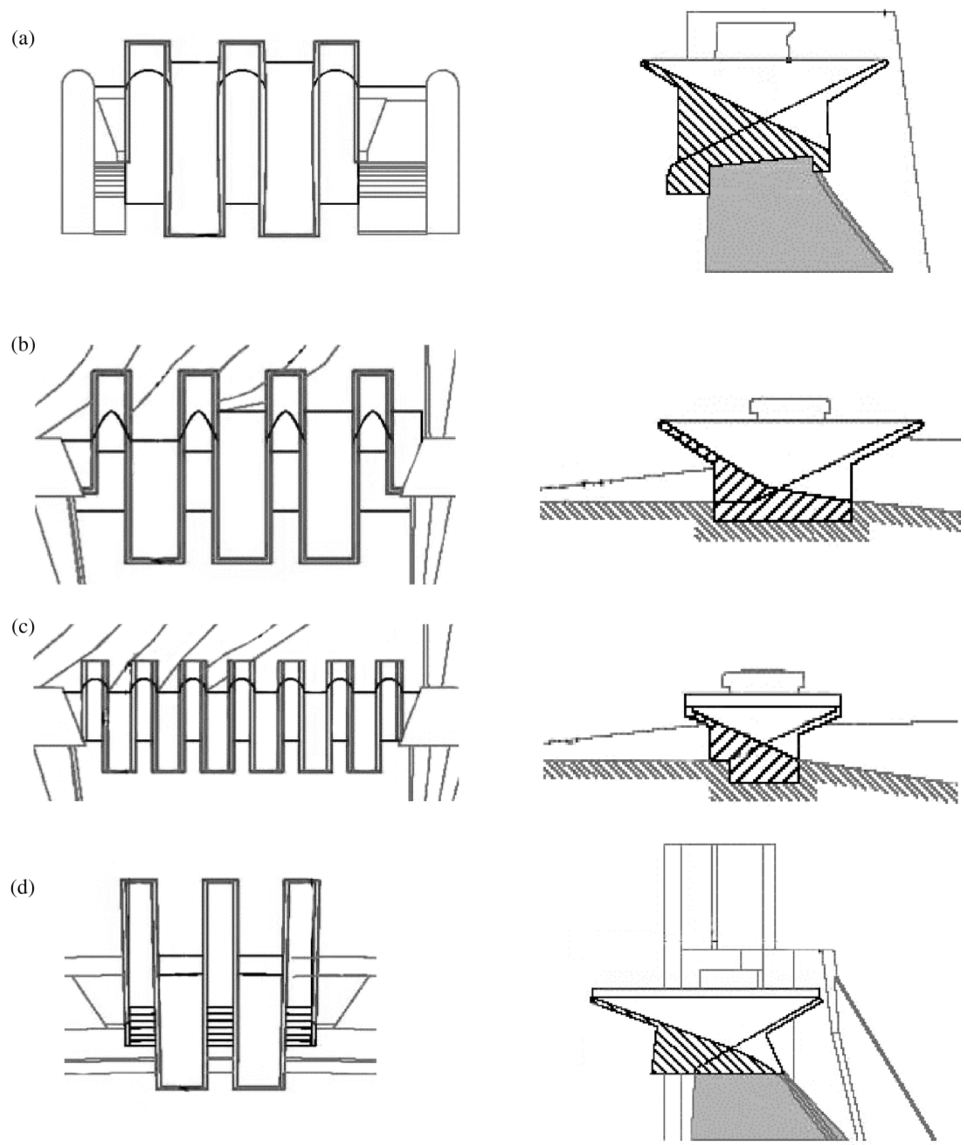

Figure 6 Overview of PKW case studies, with left plan view, right longitudinal section, for (a) St. Marc, (b) Gloriettes 4, (c) Gloriettes 7, (d) Etroit (Leite Ribeiro *et al.* 2009)

Four of these model studies were taken as references to discuss the herein presented equations, with similar geometries as the present set-up representing 'straight' A-type PKWs (with a straight transverse axis not curved in the plan). They are thus real cases with 'reservoir' inflow instead of basic research models using a sectional 'channel' approach flow. The main parameters of these case studies are listed in Table 1 as tested in the models, whereas the effectively built dimensions may slightly differ (e.g. Vermeulen *et al.* 2011). A plan view as well as a longitudinal section of each model is shown in Fig. 6.

Again, tests with $H > 0.5(P_i + P_d)$ and $H > 0.05$ m were considered. Then, only few points remain, leading to preliminary results only. Other model studies are available, but they often comprise small heads below the herein assumed limit of scale effects. Applying the normalization $\delta(wpba)$ of Eq. (9) results in an overestimation of r, which is explained by the effect of the distal weir ends, as the case studies are of reservoir type. For linear standard weirs, the effective width is typically reduced to predict an accurate discharge. To consider this effect for PKWs, the effective width may be reduced in analogy. A preliminary

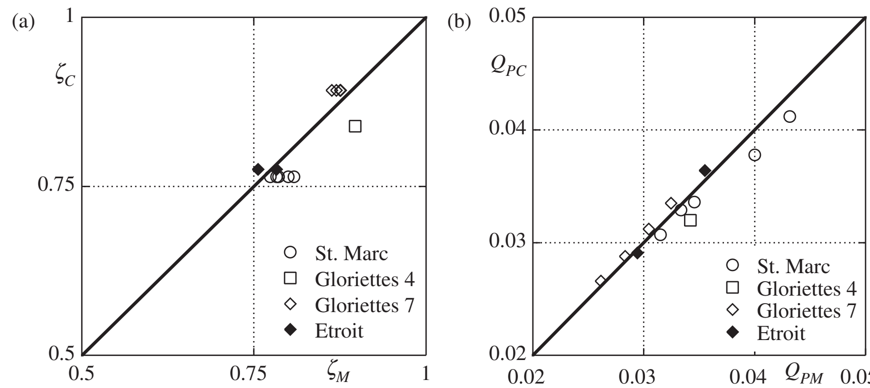

Figure 7 Comparison between computed and measured values (a) ζ, (b) Q_P, including distal effect, (—) perfect agreement

analysis indicates that r may be computed using Eq. (9), but with a reduction factor ζ in Eq. (2)

$$Q_P = \zeta r Q_S \tag{10}$$

with ζ related to the effective weir width, if reservoir inflow applies. The case studies indicate that

$$\zeta = 1 - \left(\frac{1.5W_o}{W}\right) \tag{11}$$

The parameter W_o is considered as outlet keys are located in the present examples near the distal weir ends. For narrow PKWs with only three cycles, for example, $\zeta \cong 0.75$ thus pointing at a significant effect of the distal weir ends for reservoir approach flow. The measured and computed values of ζ are compared in Fig. 7(a), and the computed and measured discharges in Fig. 7(b). As only few points are available, no details of the accuracy of the proposed equation are provided.

5 Discussion

In Eq. (9) $r(\delta = 0) = 1$ (Eq. 4), meaning that PKWs work similar to linear sharp-crested weirs. This is the case if $L = W$. In parallel, an operation mode close to that of sharp-crested weirs is given if H is large, so that the PKW structure becomes negligible. As stated, Eq. (4) is limited to $0 < \delta < 20$, to avoid $\delta(H \to 0) \to \infty$. Beside this, $r(P \to 0) \to 1$ and $P_d > 0$ m, indicating that the structure tends to a linear broad-crested weir. Their discharge coefficient, with the present relative weir lengths as basis, is between 0.33 and 0.35 (Hager and Schwalt 1994), i.e. close to $C_S = 0.42$. These cases are, however, beyond the herein applied ranges of validity as well as design recommendations, and thus of theoretical interest.The exponents of Eq. (9) taking into account the sub-equations allow for identifying the hydraulically relevant dimensions of PKWs, considering the tested parameter ranges. In Table 2, the first column gives the dimensionless term to discuss, the second its test range, the third the exponent, and the fourth column applies the exponent on the values of column 2. These values finally are equivalent to the factors of an individual term to compute r. The most relevant term P_i/H represents the hydraulic criterion, indicating that the PKW 'efficiency' reduces with increasing head. The relative crest length $(L - W)/W$ is furthermore highly relevant, as it increases the 'efficiency'. Finally, the terms W_i/W_o, P_o/P_i, $(B_o + B_i)/B$, and R_o/P_o affect the 'efficiency' only slightly, so that their variation marginally affects r. According to Vermeulen *et al.* (2011) the side wall angle, crest profile shape, and the wall thickness further affect the PKW capacity. The set-up of the basic equation allows adding further correction factors to w, p, b, and a.

Scale effects occur on PKW models, similar to free overfall models. As no precise limits are available so far, a conservative approach excluding tests with $H < 0.05$ m was chosen to limit the effects of viscosity and surface tension. Note that the latter also affects prototype flows at small heads. The so-called 'clinging nappe' flow type for very small model discharges (Machiels *et al.* 2009) will appear different on the prototype, also due to increased flow aeration.

Table 2 Relevance of individual terms of Eq. (9) and sub-equations, including data of present study and of Machiels *et al.* (2011a)

Term	Test range		Exponent	Range incl. exponent	
$(L - W)/W$	From 2.00	To 6.00	0.90	From 1.87	To 5.02
P_i/H	From 0.32	To 17.77	0.90	From 0.36	To 13.33
W_i/W_o	From 0.50	To 2.00	0.05	From 0.97	To 1.04
P_o/P_i	From 0.72	To 1.38	0.25	From 0.92	To 1.08
$(B_o + B_i)/B$	From 0.40	To 0.80	−0.50	From 0.95	To 1.20
R_o/P_o	From 0.00	To 0.22	2.00	From 1.00	To 1.05

6 Conclusions

The head–discharge relation of A-type PKWs with a half-circular crest was systematically investigated in two sectional physical model test series, varying the relevant parameters in typical ranges. It was assured that the downstream conditions had no effect on the head–discharge relation. The conclusions following from the data analysis are:

- A general equation of the head–discharge relation of A-type PKWs is provided, expressed as discharge increase ratio. The latter refers to the relative discharge increase from the PKW as compared with the linear sharp-crested weir.
- Primary and secondary parameters were identified. The primary parameters having a significant effect on the capacity are the relative developed crest length and the relative head. The secondary parameters of small but not negligible effect include the ratio of the inlet and outlet key widths, the ratio of the inlet and outlet key heights, the relative overhang lengths, and the relative height of the parapet walls.
- The physical model represents a sectional channel set-up, ignoring the distal effect of a reservoir type approach flow. To compensate this simplification, additional case study model tests including reservoirs were considered to estimate the latter effect, proving a reduction factor.
- Limitations for the present study are provided.

Acknowledgements

The model investigation was supported by EDF, France. The authors thank Mr Frédéric Laugier for the support and the excellent collaboration.

Notation

a = correction factor (–)
B = streamwise length (m)
b = correction factor (–)
C = discharge coefficient (–)
c = reliability coefficient (–)
g = gravity acceleration (m^2/s)
H = total approach flow head (m)
L = developed crest length (m)
P = vertical height (m)
p = correction factor (–)
Q = discharge (m^3/s)
R = height of parapet wall (m)
r = discharge increase ratio (–)
T_s = side wall thickness (m)
W = transversal width (m)
w = correction factor (–)
δ = normalization (–)
λ = scale factor (–)
ζ = reduction factor (–)

Subscripts

b = basis
C = computed value
i = inlet key
o = outlet key
M = measured model value
P = Piano Key Weir
S = sharp-crested weir

References

Anderson, R.M., Tullis, B. (2011). Influence of Piano Key Weir geometry on discharge. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 75–80, CRC Press, Boca Raton, FL.

Anderson, R.M., Tullis, B. (2012). Comparison of Piano Key and rectangular Labyrinth Weir hydraulics. *J. Hydraulic Eng.* 138(4), 358–361.

Blancher, B., Montarros, F., Laugier, F. (2011). Hydraulic comparison between Piano Key Weirs and labyrinth spillways. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 141–150, CRC Press, Boca Raton, FL.

Cicero, G.M., Menon, J.M., Luck, M., Pinchard, T. (2011). Experimental study of side and scale effects on hydraulic performances of a Piano Key Weir. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 167–172, CRC Press, Boca Raton, FL.

Dugué, V., Hachem, F., Boillat, J.-L., Nagel, V., Roca, J.-P., Laugier, F. (2011). PKWeir and flap gate spillway for the Gage II Dam. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 35–42, CRC Press, Boca Raton, FL.

Erpicum, S., Machiels, O., Archambeau, P., Dewals, B., Pirotton, M. (2011a). 1D numerical modeling of the flow over a Piano Key Weir. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 151–158, CRC Press, Boca Raton, FL.

Erpicum, S., Nagel, V., Laugier, F. (2011b). Piano Key Weir design of Raviege dam. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 43–50, CRC Press, Boca Raton, FL.

Falvey, H.T. (2003). *Hydraulic design of labyrinth weirs*. ASCE Press, Reston, VA.

Hager, W.H. (2010). *Wastewater hydraulics*, ed. 2. Springer, Berlin.

Hager, W.H., Schwalt, M. (1994). Broad-crested weir. *J. Irrig. Drain. Eng.* 120(1), 13–26.

Hager, W.H., Schleiss, A.J. (2009). *Constructions hydrauliques, Ecoulements stationnaires* [Hydraulic structures, steady flow]. *Traité de Génie Civil*. Presses Polytechniques et Universitaires Romandes, Lausanne, Switzerland.

Kabiri-Samani, A., Javaheri, A. (2012). Discharge coefficient for free and submerged flow over Piano Key weirs. *J. Hydraulic Res.* 50(1), 114–120.

Laugier, F. (2007). Design and construction of the first Piano Key Weir (PKW) spillway at the Goulours dam. *Intl. J. Hydropower Dams* 13(5), 94–101.

Laugier, F., Lochu, A., Gille, C., Leite Ribeiro, M., Boillat, J.-L. (2009). Design and construction of a labyrinth PKW spillway at St-Marc Dam, France. *Intl. J. Hydropower Dams* 15(5), 100–107.

Le Doucen, O., Leite Ribeiro, M., Boillat, J.L., Schleiss, A.J., Laugier, F. (2009). Etude paramétrique de la capacité des PK-Weirs [Parametric study of the discharge capacity of PK-Weirs]. *Modèles physiques hydrauliques – outils indispensables du* XXIe *siècle* (CD-ROM). SHF, Lyon F.

Leite Ribeiro, M., Bieri, M., Boillat, J.-L., Schleiss, A., Delorme, F., Laugier, F. (2009). Hydraulic capacity improvement of existing spillways – design of piano key weirs. 23rd *ICOLD Congress*, Brasilia (Q90, R43), 25–29.

Leite Ribeiro, M., Bieri, M., Boillat, J. L., Schleiss, A.J., Singhal, G., Sharma, N. (2012a). Discharge capacity of Piano Key Weirs. *J. Hydraulic Eng.* 138(2), 199–203.

Leite Ribeiro, M., Boillat, J.-L., Schleiss, A.J. (2011). Experimental parametric study for hydraulic design of PKWs. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 183–190, CRC Press, Boca Raton, FL.

Leite Ribeiro, M., Pfister, M., Boillat, J.-L., Schleiss, A.J., Laugier, F. (2012b). Piano Key Weirs as efficient spillway structure. 24th *ICOLD Congress* Kyoto (Q94, R13), 1–10.

Lempérière, F., Ouamane, A. (2003). The Piano Keys weir: A new cost-effective solution for spillways. *Intl. J. Hydropower Dams* 10(5), 144–149.

Lempérière, F., Vigny, J.P., Ouamane, A. (2011). General comments on Labyrinth and Piano Key Weirs: The past and present. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 17–24, CRC Press, Boca Raton, FL.

Machiels, O., Erpicum, S., Archambeau, P., Dewals, B.J., Pirotton, M. (2009). Large scale experimental study of piano key weirs. 33rd *IAHR World Congress*, Vancouver, Canada.

Machiels, O., Erpicum, S., Archambeau, P., Dewals, B.J., Pirotton, M. (2011a). Influence of the Piano Key Weir height on its discharge capacity. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 59–66, CRC Press, Boca Raton, FL.

Machiels, O., Erpicum, S., Archambeau, P., Dewals, B.J., Pirotton, M. (2011b). Piano Key Weir preliminary design method – application to a new dam project. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 199–206, CRC Press, Boca Raton, FL.

Machiels, O., Erpicum, S., Dewals, B.J., Archambeau, P., Pirotton, M. (2011c). Influence of the alveoli slopes on the discharge capacity of Piano Key Weirs. 34th *IAHR World Congress*, Brisbane, Australia.

Machiels, O., Erpicum, S., Dewals, B.J., Archambeau, P., Pirotton, M. (2011d). Experimental observation of flow characteristics over a Piano Key Weir. *J. Hydraulic Res.* 49(3), 359–366.

Novak, P., Guinot, V., Jeffrey, A., Reeve, D.E. (2010). *Hydraulic modelling: An introduction*. Spon Press, London.

Ouamane, A., Lempérière, F. (2006). Design of a new economic shape of weir. Proc. Int. Symp. *Dams in the Societies of the 21st Century*, Barcelona, 463–470.

Pfister, M., Hager, W.H. (2010). Chute aerators. I: Air transport characteristics. *J. Hydraulic Eng.* 136(6), 352–359.

Pralong, J., Vermeulen, J., Blancher, B., Laugier, F., Erpicum, S., Machiels, O., Pirotton, M., Boillat, J.L., Leite Ribeiro, M., Schleiss, A.J. (2011). A naming convention for the piano key weirs geometrical parameters. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 271–278, CRC Press, Boca Raton, FL.

Schleiss, A.J. (2011). From labyrinth to piano key weirs: A historical review. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 3–15, CRC Press, Boca Raton, FL.

Vermeulen, J., Laugier, F., Faramond, L., Gille, C. (2011). Lessons learnt from design and construction of EDF first Piano Key Weirs. Proc. Int. Conf. *Labyrinth and Piano Key Weirs* Liège B, 215–224, CRC Press, Boca Raton, FL.

Vischer, D., Hager, W.H. (1999). *Dam hydraulics*. Wiley, Chichester, UK.