Supplementary information item 1: stationary and localized stimulated Brillouin scattering interaction using phase-modulation of pump and signal waves

Let us denote the scalar complex envelopes of the pump and signal waves as $A_p(t,z)$ and $A_s(t,z)$, respectively, where t denotes time and z represents position. The pump wave enters the fiber at $z=0$ and propagates in the positive z direction, whereas the signal wave is launched at $z=L$ and counter-propagates in the negative direction. We assume that the difference between the optical frequencies Ω matches Ω_B of the fiber. The temporal evolution of the acoustic density wave magnitude $\rho(t,z)$ is governed by the differential equation [8, 15]:

(1)
$$\frac{\partial\rho(t,z)}{\partial t}+\frac{1}{2\tau}\rho(t,z)=jg_1A_p(t,z)A_s^*(t,z).$$

The value of the parameter g_1 depends on the speed of sound in the fiber, its electrostrictive coefficient and density [8, 15]. The solution is of the form:

(2)
$$\rho(t,z)=jg_1\int_0^t\exp\left[-\frac{(t-t')}{2\tau}\right]A_p(t',z)A_s^*(t',z)\mathrm{d}t'.$$

The acoustic field scales with the inner product of the two optical complex envelopes, weighed and summed by a moving exponential window. In our work, the two complex envelopes are stationary, stochastic processes that vary on a time scale much faster than τ. The acoustic field magnitude, therefore, is closely related to the cross-correlation of the two envelopes at a given location.

The principle of localizing the SBS interaction is inspired and motivated by the field of radars. Many radar systems transmit long sequences of pseudo-random, short pulses [1]. Upon detection, the received echo sequences are correlated with a reference replica. The correlation collapses the entire energy of the received sequence into a narrow peak, which provides both high ranging resolution and large signal to noise ratio [1]. Following a similar rationale, both the pump and the signal waves used in the SBS interaction are phase-modulated by a common PRBS, with a symbol duration $T << \tau$ [32]:

(3) $$A_p(t, z=0) = A_{p0}\exp\left[j\varphi(t)\right] = A_{p0}\left\{\sum_n \text{rect}\left[(t-nT)/T\right]\exp(j\varphi_n)\right\},$$

(4) $$A_s(t, z=L) = A_{s0}\exp\left[j\varphi(t)\right] = A_{s0}\left\{\sum_n \text{rect}\left[(t-nT)/T\right]\exp(j\varphi_n)\right\}.$$

In equations (3), (4), $\text{rect}(\xi)$ equals 1 for $|\xi| < 0.5$ and zero elsewhere, A_{p0} and A_{s0} are the constant magnitudes of the pump and signal waves, respectively, and $\{\varphi_n\}$ are random phase variables which equal either 0 or π with equal probabilities.

Suppose that the phase modulation is synchronized, so that the two waves are of equal phases at their respective entry points, as in equations (3), (4). We distinguish between the dynamics of $\rho(t,z)$ in two different regions. In the vicinity of $z = L/2$, the pump and signal are correlated, the driving force in equation (1) is of constant value $jg_1 A_{p0} A_{s0}^*$, and $\rho(t,z)$ is allowed to build up to its steady state value of $j2\tau g_1 A_{p0} A_{s0}^*$. The width of the correlation peak is on the order of $\Delta z = \frac{1}{2} v_g T$, where v_g is the group velocity of light in the fiber [32]. In all other locations, the driving force for the acoustic field is randomly alternating in sign, between $\pm jg_1 A_{p0} A_{s0}^*$, on every symbol duration $T << \tau$. The integral of equation (2) thus averages to a zero expectation value, and the SBS interaction outside the correlation peak is largely inhibited [32].