Robust joint reconstruction of misaligned images using semi-parametric dictionaries

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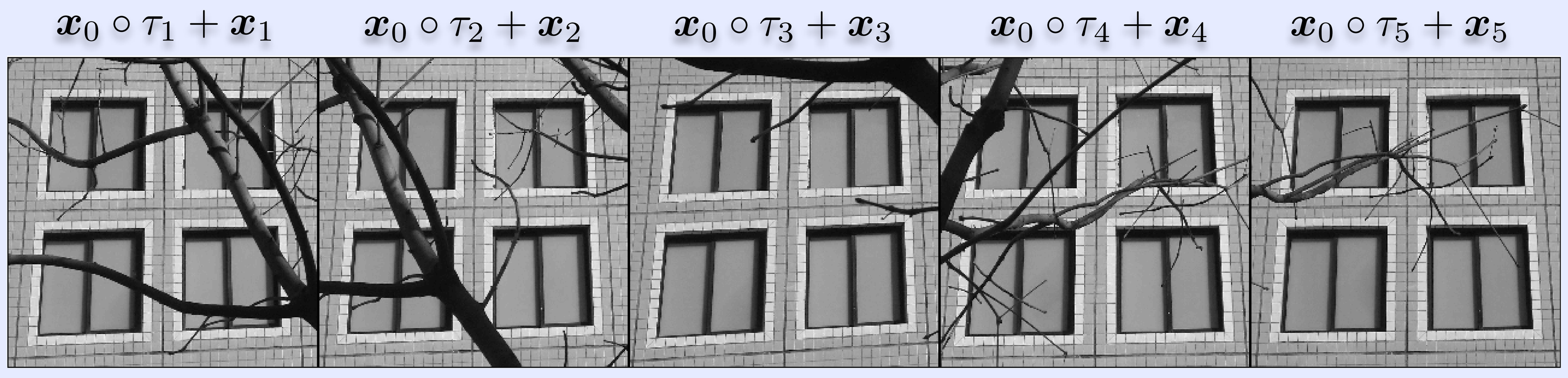
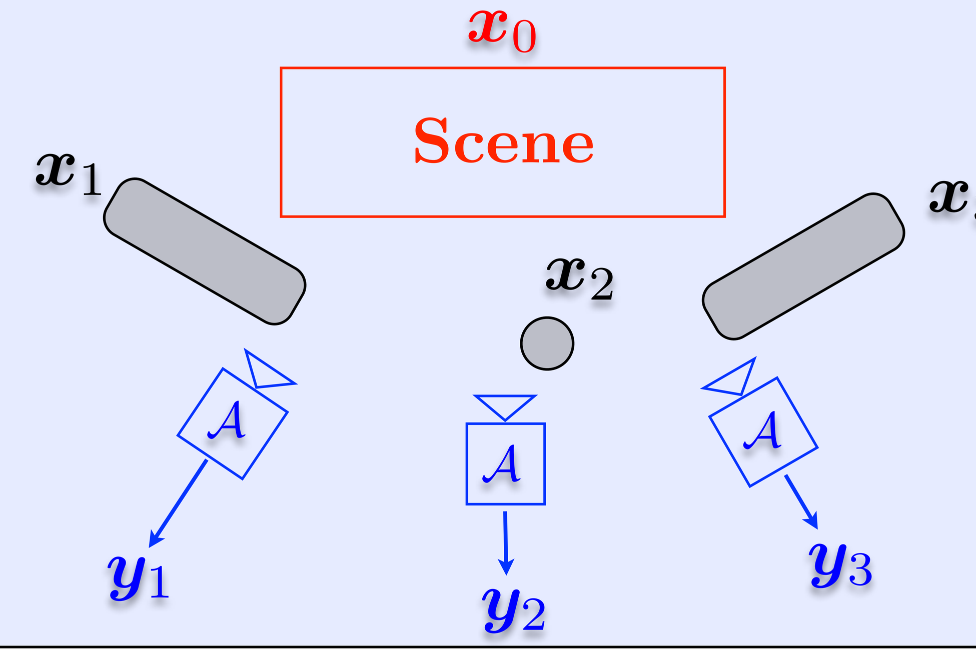
Motivation

- A scene $\boldsymbol{x}_0 \in \mathbb{R}^n$ is observed from l point-of-views, providing l noisy measurement vectors $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_l \in \mathbb{R}^m, m \leqslant n$.
- The scene is partly occluded by some objects $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_l \in \mathbb{R}^n$.
- The scene undergoes geometric transformations $\tau_1, \ldots, \tau_l$ that depend on the position of the observer.



Observed images (dataset available in [1]).

- The observation system is modeled by a linear operator $\mathsf{A} \in \mathbb{R}^{m \times n}$.
- The goal is to reconstruct $\boldsymbol{x}_0$ from the observations $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_l$, without the knowledge of the geometric transformations $\tau_1, \ldots, \tau_l$.



[1] Peng et al., CVPR, pp. 763-770, 2010.

Problem formulation

- The transformations belong to a transformation group represented by p parameters: $\theta_j \in \mathbb{R}^p, \forall j \in \{1, \ldots, l\}$.
- We denote $\mathsf{S}(\theta_j) \in \mathbb{R}^{n \times n}$ the interpolating matrix such that $\boldsymbol{x}_0 \circ \tau_j = \mathsf{S}(\theta_j)\boldsymbol{x}_0$.
- The measurement model satisfies:

$$\underbrace{\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_l \end{bmatrix}}_{\boldsymbol{y}} = \underbrace{\begin{bmatrix} \mathsf{AS}(\theta_1) & \mathsf{A} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathsf{AS}(\theta_l) & 0 & \ldots & \mathsf{A} \end{bmatrix}}_{\tilde{\mathsf{A}}(\theta)} \underbrace{\begin{bmatrix} \boldsymbol{x}_0 \\ \vdots \\ \boldsymbol{x}_l \end{bmatrix}}_{\boldsymbol{x}} + \begin{bmatrix} \boldsymbol{n}_1 \\ \vdots \\ \boldsymbol{n}_l \end{bmatrix}.$$

- The inverse ill-posed problem is regularized by assuming that the scene $\boldsymbol{x}_0$ and the occluding objects $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_l$ are sparse in a wavelet basis $\mathsf{W} \in \mathbb{R}^{n \times n}$. The decomposition of $\boldsymbol{x}_j$ in W is denoted $\boldsymbol{\alpha}_j \in \mathbb{R}^n$.
- We want to solve the following non-convex problem:

$$\min_{\boldsymbol{\alpha}, \boldsymbol{z}, \theta} \|\boldsymbol{\alpha}\|_1 + \kappa \|\mathsf{A}(\theta)\,\boldsymbol{\alpha} - \boldsymbol{z}\|_2^2 \ \text{ s.t. } \begin{cases} \|\boldsymbol{y}_1 - \boldsymbol{z}_1\|_2 \leqslant \epsilon_1 \\ \vdots \\ \|\boldsymbol{y}_l - \boldsymbol{z}_l\|_2 \leqslant \epsilon_l \end{cases} \text{ and } \theta \in \mathcal{T},$$

where $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_0, \ldots, \boldsymbol{\alpha}_l]^{\mathsf{T}}, \boldsymbol{z} = [\boldsymbol{z}_0, \ldots, \boldsymbol{z}_l]^{\mathsf{T}}$, and $\mathsf{A}(\theta) = \tilde{\mathsf{A}}(\theta)\underbrace{[\mathsf{W}, \ldots, \mathsf{W}]}_{\times(l+1)}{}^{\mathsf{T}}$.

Method

- We define $\mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{z}, \theta) = \|\boldsymbol{\alpha}\|_1 + \kappa \|\mathsf{A}(\theta)\,\boldsymbol{\alpha} - \boldsymbol{z}\|_2^2 + i_{\mathcal{B}(\boldsymbol{y},\epsilon)}(\boldsymbol{z}) + i_{\mathcal{T}}(\theta)$, where $i_{\mathcal{T}}$ is the indicator function of the set $\mathcal{T}$, and $i_{\mathcal{B}(\boldsymbol{y},\epsilon)}$ the indicator function of the set $\mathcal{B}(\boldsymbol{y}, \epsilon) = \{\boldsymbol{z} = \{\boldsymbol{z}_j\}_{1 \leqslant j \leqslant l} : \|\boldsymbol{y}_j - \boldsymbol{z}_j\|_2^2 \leqslant \epsilon_j\}$.
- Initializations: set $k = 0$, $\boldsymbol{\alpha}^0 = \boldsymbol{0} \in \mathbb{R}^{(l+1)n}$, $\boldsymbol{z}^0 = \boldsymbol{y}, \theta^0 \in \mathcal{T}$, choose $0 < \lambda_{\min} \leqslant \lambda_{\boldsymbol{z}}, \lambda_\theta, \{\lambda_{\boldsymbol{\alpha}}^k\}_{k \in \mathbb{N}} \leqslant \lambda_{\max}$.
- Repeat:
  1) $(\boldsymbol{\alpha}^{k+1}, \boldsymbol{z}^{k+1}) \in \operatorname{argmin}_{\boldsymbol{\alpha}, \boldsymbol{z}} \mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{z}, \theta^k) + \lambda_z \|\boldsymbol{z} - \boldsymbol{z}^k\|_2^2 + \lambda_{\boldsymbol{\alpha}}^k \|\boldsymbol{\alpha} - \boldsymbol{\alpha}^k\|_1$.
  2) Find $\theta^{k+1} \in \mathcal{T}$ such that: $\kappa \|\mathsf{A}(\theta^{k+1})\,\boldsymbol{\alpha}^{k+1} - \boldsymbol{z}^{k+1}\|_2^2 + \lambda_\theta \|\theta^{k+1} - \theta^k\| \leqslant \kappa \|\mathsf{A}(\theta^k)\,\boldsymbol{\alpha}^{k+1} - \boldsymbol{z}^{k+1}\|_2^2$.
  3) $k \leftarrow k + 1$.

  until *convergence* or $k \geqslant k_{\max}$.

[2] Attouch et al., Mathematics of Operations Research , vol. 35(2) , pp. 438-457, 2010.
[3] Attouch et al., J. Mathematical programming, 2011.

- The algorithm is similar to the method studied by Attouch et al. in [2-3] for solving non-convex problems. We have replaced the cost-to-move function $\lambda_{\boldsymbol{\alpha}}^k \|\boldsymbol{\alpha} - \boldsymbol{\alpha}^k\|_2^2$ by $\lambda_{\boldsymbol{\alpha}}^k \|\boldsymbol{\alpha} - \boldsymbol{\alpha}^k\|_1$. For the considered problem, this leads to much better results in practice.
- By construction $\mathcal{L}$ is not increasing. Indeed, for all $k \geqslant 0$,

$$\mathcal{L}(\boldsymbol{\alpha}^{k+1}, \boldsymbol{z}^{k+1}, \theta^{k+1}) + \lambda_{\boldsymbol{z}} \|\boldsymbol{z}^{k+1} - \boldsymbol{z}^k\|_2^2 + \lambda_\theta \|\theta^{k+1} - \theta^k\|_2^2 + \lambda_{\boldsymbol{\alpha}}^k \|\boldsymbol{\alpha}^{k+1} - \boldsymbol{\alpha}^k\| \leqslant \mathcal{L}(\boldsymbol{\alpha}^k, \boldsymbol{z}^k, \theta^k).$$

- We also have:

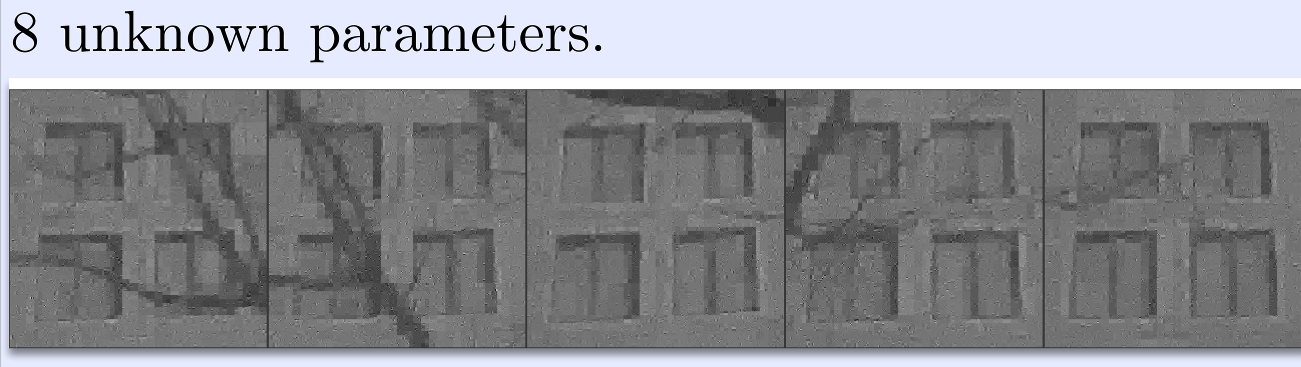
$$\sum_{k=0}^{\infty} \|\boldsymbol{z}^{k+1} - \boldsymbol{z}^k\|_2^2 + \|\theta^{k+1} - \theta^k\|_2^2 + \|\boldsymbol{\alpha}^{k+1} - \boldsymbol{\alpha}^k\|_1 < \infty.$$

Illustration

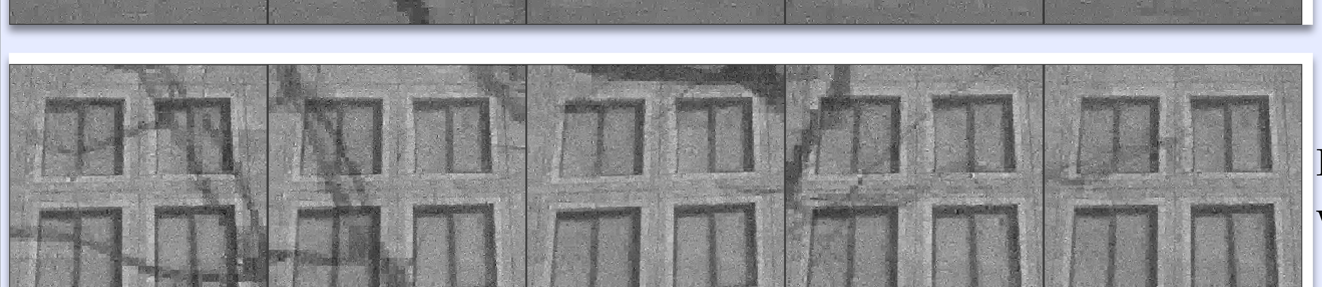
- $m = 0.1n$ measurements per image obtained with the spread spectrum technique [4].

[4] Puy et al., EURASIP Journal on Advances in Signal Processing, vol. 2012(6), 2012.

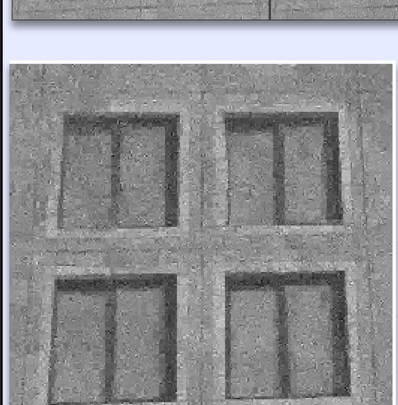
- The transformations are assumed to be homographies modeled by 8 unknown parameters.



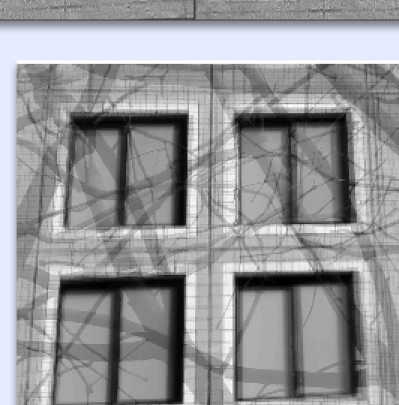
Reconstructions obtained without taking into account correlations between the measurements.



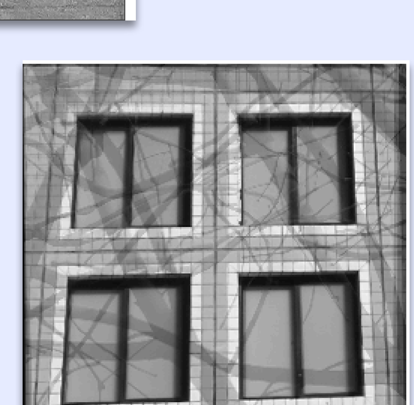
Reconstructions obtained with the proposed method.



Estimated background image $\boldsymbol{x}_0$.



Superposed unregistered images.



Superposed registered images with estimated parameters.

Simulation results

- Simulations for several number of measurements and noise levels using the following 5 images.

Castle-R20 dataset available at cvlab.epfl.ch (Strecha et al., CVPR, 2008).

Image quality vs. Nb. measurements

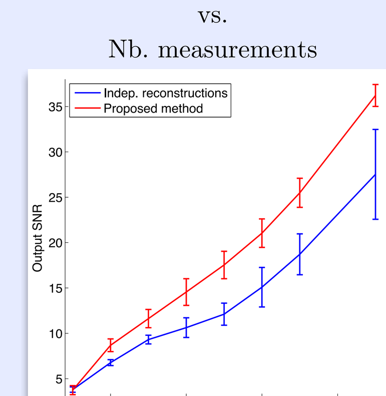
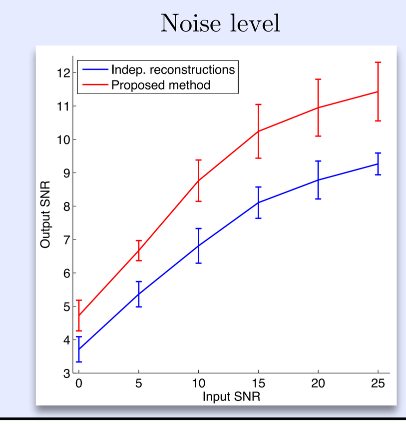
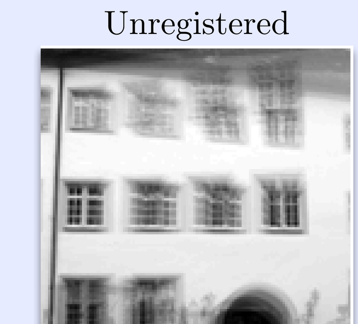


Image quality vs. Noise level



Unregistered



Registered

Conclusions & Perspectives

- We presented a method using semi-parametric dictionaries for joint reconstruction of misaligned images.
- The method estimates correctly the parameters of the semi-parametric dictionary as well as the background $\boldsymbol{x}_0$ and foreground images $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_l$.
- This method may have interests in, e.g., cardiac MR imaging.