

Discussion on: “Experimental observations of saltwater up-coning” by A. D. Werner, D. Jakovovic, and C. T. Simmons, 2009. Journal of Hydrology 373, 230-241

Authors:

Adrian D. Werner^{1,2*}

Danica Jakovovic^{1,2}

D. A. Barry³

Craig T. Simmons^{1,2}

Hong Zhang⁴

Contact Details:

¹National Centre for Groundwater Research and Training, Flinders University, GPO Box 2100, Adelaide, SA 5001, Australia

²School of the Environment, Flinders University, GPO Box 2100, Adelaide, SA 5001, Australia

³Laboratoire de technologie écologique, Institut d’ingénierie de l’environnement, Station 2, Ecole polytechnique fédérale de Lausanne (EPFL), 1015 Lausanne, Switzerland

⁴Griffith School of Engineering, Griffith University, Gold Coast Campus, QLD 4222, Australia

*Corresponding author

Adrian Werner, Phone: +618 8201 2710, email: adrian.werner@flinders.edu.au

Accepted for publication in: Journal of Hydrology

Recently, Zhang et al. (2012) highlighted that analytical solutions for steady critical withdrawal can be used to characterise the saltwater up-coning experiments of Werner et al. (2009). In summary, Zhang et al. (2012) made the following points: (1) a critical pumping rate for the Werner et al. (2009) experimental set-up can be determined and, based on this, the experiments can be characterised as subcritical, critical or supercritical pumping cases, (2) the shapes of the plumes in the four experiments of Werner et al. (2009) accord with the criticality classification of Zhang et al. (2012), (3) Werner et al.'s (2009) Experiment 3 is better predicted by the formulation of Zhang et al. (1997) than by Dagan and Bear's (1968) analytical approximation, and (4) the plume widths reported by Werner et al. (2009) are not for the full-plume widths. Since the appearance of Werner et al. (2009), and subsequent to the submission of Zhang et al. (2012), further work has appeared, viz. Jakovovic et al. (2011, 2012), which report numerical modelling of the Werner et al. (2009) experiments and additional laboratory testing. These offer new information on the Werner et al. (2009) experiments (i.e., that was not available to Zhang et al. (2012) at the time of their comment), and offer important insights into many of the points raised by Zhang et al. (2012). Thus, here we take the opportunity to re-evaluate the four remarks of Zhang et al. (2012).

In brief, Jakovovic et al. (2011) simulated the Werner et al. (2009) laboratory experiments to assess the numerical reproducibility of the experimentally observed transient rise in saltwater plumes under pumping bores. Jakovovic et al. (2012) undertook additional laboratory testing and numerical modelling to explore further the double-peaked up-coning of one of the Werner et al. (2009) experiments that was not reproducible from previous modelling attempts.

First, it is worthwhile reiterating briefly the up-coning criticality concepts raised by Zhang et al. (2012), who focussed their attention on steady conditions. Previous studies that mention up-coning criticality consider one or both of the following definitions: (1) the critical pumping rate is that for which the underlying salt water will just reach the well (e.g., Zhang et al., 1997), and (2) the critical height of up-coning relates to the position of the plume apex at which the shape of up-coning changes from convex to concave (Bear, 1979). In the past, the critical pumping rate has typically been analysed and described considering that the freshwater-saltwater interface is sharp. In that case, changes in the shape of up-coning only occur in the transition from subcritical to critical pumping. In other words, subcritical pumping is consistent with a steady convex fresh/saltwater interface. As the (steady) pumping rate is increased, the interface abruptly becomes concave, corresponding to saltwater breakthrough to the pumping well. An important question remains: To what extent is the sharp interface assumption of Zhang et al. (2012) – which is the point of departure of their analysis – affected by the gradual transition from saltwater to freshwater?

Second, we mention that the analytical results of Zhang et al. (1997) were based on the assumptions: (1) an aquifer that extended without bound laterally, (2) that the fresh/saltwater interface was fixed at two points equidistant from the pumping well and (3) that the phreatic surface was fixed and horizontal. Any laboratory experiment will involve conditions for which these assumptions are approximations. Zhang et al. (2012) already noted that the experimental conditions of Werner et al. (2009) differ from the theoretical basis of the theory of Zhang et al. (1997), so we do not reconsider that issue here.

Werner et al. (2009) highlighted that criticality definitions by Bear and Dagan (1964), Bear (1979) and Bower et al. (1999) do not appear to apply to their laboratory observations, in

terms of both plume stability and plume shape. The flux-based definition of criticality developed by Zhang et al. (1997), and applied by Zhang et al. (2012) to the experiments of Werner et al. (2009), matched reasonably well the laboratory results, in terms of late-stage plume apex heights and plume shapes. In addition to the assumptions listed above, we note that, necessarily, Zhang et al. (2012) assumed steady conditions existed in the experiments. Independent of this, the unresolved issue of dispersive up-coning is perhaps of most interest. Groundwater of significant salinity enters the pumping well in Experiment 4 of Werner et al. (2009), despite that Zhang et al.'s (2012) criticality analysis indicates subcritical conditions due to the low pumping rate and high freshwater-saltwater density difference. A similar result (salinity-impacted wells despite stable up-coning conditions) was observed by Reilly and Goodman (1987). We remark also that Experiment 5 of Jakovovic et al. (2012) can also be characterised as a subcritical case according to the theory of Zhang et al. (1997). However, the plume reached the pumping well in this experiment, causing significant salinity in the extracted water.

The shape of up-coning plumes also requires discussion, especially given that the transient numerical modelling of Jakovovic et al. (2011, 2012) offers further insight into plume shapes and the associated convex-to-concave transitions. Generally, the dispersive simulation results of Jakovovic et al. (2011, 2012) show that for all laboratory experiments (i.e., Experiments 1-4 of Werner et al., 2009, and Experiment 5 by Jakovovic et al., 2012), the shape of salinity contours changed from convex at early stages to concave later in the experiments. The definition of up-coning shapes (i.e., convex or concave) depends on whether the interface is viewed from above or below. Werner et al. (2009) and Zhang et al. (2012) adopt the former perspective and we adopt the same here, although it should be noted that on one occasion, Werner et al. (2009) mistakenly referred to an interface transition from concave to convex in

their description of Experiment 1. Consistent with the gradual salinity transition across the freshwater-saltwater interface, the numerical simulation results of Jakovovic et al. (2011) show that, simultaneously, low concentration salinity contours can have a concave shape (indicating supercritical up-coning; Zhang et al., 2012), while high salinity contours have the convex shape associated with subcritical conditions. As such, up-coning criticality is concentration-dependent, as opposed to sharp-interface criticality (assumed by Zhang et al. 2012). Further research is needed to identify better the salinities associated with critical states of up-coning, following the initial work of Reilly and Goodman (1987) that compares dispersive and sharp-interface representations of up-coning. Reilly and Goodman (1987) found that the 50% salinity contour was a reasonable approximation of the predicted position of a sharp interface in their investigation. The suggestion by Zhang et al. (2012) that Experiment 4 has a convex shape (based on the laboratory photographs of Werner et al. (2009)) is valid if the double peaks that formed in the latter stages of the experiment are neglected. However, Jakovovic et al. (2012) demonstrated that adsorption caused the dye tracer to lag the salinity plumes, especially in Experiment 4 (and Experiment 5 of Jakovovic et al., 2012), and hence saltwater plumes were in fact more concave than the shapes produced by the dye tracer. The results of the adsorption analyses by Jakovovic et al. (2012) thereby introduce new information regarding the laboratory experiments, and therefore the applicability of the Zhang et al. (1997) criticality definitions to the Werner et al. (2009) experiments needs to be addressed in accordance with this.

New conclusions can be drawn from the combined results of Zhang et al. (2012), Werner et al. (2009) and Jakovovic et al. (2011, 2012). All of the laboratory experiments and numerical simulations of Werner et al. (2009) or Jakovovic et al. (2011, 2012) obtained concave-shaped up-coning plumes, despite some of these being designated as subcritical using the sharp-

interface theory of Zhang et al. (1997). The concavity in plume shape was only obtained for the low salt concentrations in some cases. This is consistent with the theory of Zhang et al. (1997), which includes the density difference in its prediction of critical flux. Nonetheless, it may be the case that sharp-interface interpretations of criticality that rely on the development of steady-state conditions do not adequately characterise the laboratory experiments of Werner et al. (2009) and Jakovovic et al. (2012). We undertook further numerical simulation to test this notion, using an extended version of Experiment 5 of Jakovovic et al. (2012), involving a 10-d period of pumping (i.e., twice as long as the original experiment). Figure 1 presents the salinity distribution after 10 d, at which time the 50% salinity contour is clearly concave and in contact with the well, causing a significant influx of saline groundwater. Clearly, steady-state, sub-critical conditions are not obtained. There is evidence in Figure 1 that the side inflow ports start to impact on the plume shape near the side boundaries at 10 d, and hence the results are representative of the experimental conditions.

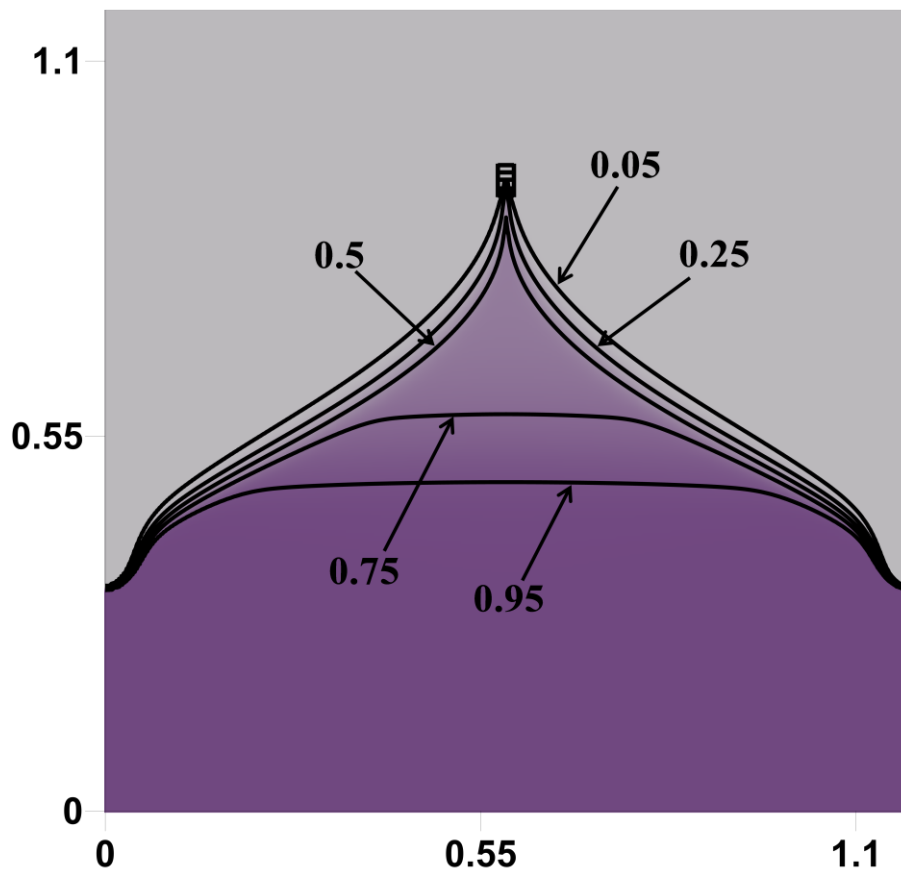


Figure 1. Simulated relative salinity contours for a 10-d extended version of Experiment 5 from Jakovovic et al. (2012).

The modelling for Figure 1, and the original laboratory experiments and numerical modelling by Werner et al. (2009) and Jakovovic et al. (2011, 2012), were unable to generate stable (subcritical) plumes, despite that two of the cases are classified as subcritical using the theory of Zhang et al. (1997). Therefore, it is likely that experimental nuances of the laboratory sand-tank that was used by Werner et al. (2009) and Jakovovic et al. (2011, 2012) preclude steady-state up-coning, and therefore, the steady criticality definitions of Zhang et al. (1997) do not strictly apply to the laboratory experiments. Numerical testing of up-coning under idealised field-scale conditions is warranted to test further the applicability of the Zhang et al. (1997) criticality definitions under real-world situations. Such an analysis of field-scale up-coning should evaluate whether the dispersive nature of up-coning processes can preclude the formation of stable plumes, as was observed in the laboratory experiments described above, considering a range of field situations.

Zhang et al. (2012) present a prediction of the Experiment 3 interface position. The results of Zhang et al. (2012) compare more closely to the Jakovovic et al. (2011) numerical results for Experiment 3, relative to the Dagan and Bear (1968) prediction. This is expected, given that the perturbation approximation of Dagan and Bear (1968) is valid only for the early stages of the experiments. However, we recall that the Experiment 3 plume position given by Zhang et al. (2012) is for a steady-state condition, rather than a transient snapshot. We suggest that Experiment 3 was yet to reach steady state, and that this likely contributed to the differences in the comparison between Zhang et al.'s (1997) prediction and the laboratory results. Experiments 1 and 2 of Werner et al. (2009) were closer to a steady-state situation, while Experiment 4 (and Experiment 5 from Jakovovic et al., 2012) was still rising at the termination of the experiment.

We reconfirm that in Table 4 of Werner et al. (2009), full-plume widths at the plume half-depth (i.e., at halfway between the plume apex and the initial interface position) were reported and not half-plume widths as Zhang et al. (2012) suggest. This is supported by Jakovovic et al. (2011) numerical simulation results of the full-plume widths that were a reasonable match to the experimental plume widths reported by Werner et al. (2009). The simulated 50% salinity contours were considered the closest match to plume widths obtained from laboratory photography of Experiments 1 and 2, whereas the simulated 25% salinity contour was a closer match to the plume widths obtained from laboratory photographs of Experiment 3. Further research is needed to better quantify the salinities associated with plume widths obtained from laboratory photography.

The numerical simulations of Jakovovic et al. (2011, 2012) reproduced reasonably well the laboratory observations of Werner et al. (2009) and Jakovovic et al. (2012), thereby demonstrating advantages of numerical modelling, which led to important insights into up-coning processes and experimental nuances (e.g., adsorption of the dye tracer) that would have not been characterised properly from physical experiments and sharp-interface analyses alone. The characterisation of criticality should be reassessed, by considering a concentration-dependent criticality condition, whereby pumping precludes the interaction of a specific salinity contour with the well. Three-dimensional modelling of this problem is needed, especially for application to the field. This would, we suggest, lead to better understanding of saltwater intrusion to wells under the (pseudo-)radial flow conditions that are expected in the field, as opposed to the two-dimensional laboratory experiments discussed here.

References

- Bear J., 1979. *Hydraulics of Groundwater*. McGraw-Hill Book Company, New York, USA.
569 pp.
- Bear J., Dagan G., 1964. Some exact solutions of interface problems by means of the hodograph method. *Journal of Geophysical Research* 69 (8), 1563-1572.
- Bower J.W., Motz L.H., Durden D.W., 1999. Analytical solution for determining the critical condition of saltwater upconing in a leaky artesian aquifer. *Journal of Hydrology* 221 (1-2), 43-54.
- Dagan G., Bear J., 1968. Solving the problem of local interface upconing in a coastal aquifer by the method of small perturbations. *Journal of Hydraulic Research* 6 (1), 15-44.
- Jakovovic D., Werner A.D., Simmons C.T., 2011. Numerical modelling of saltwater upconing: Comparison with experimental laboratory observations. *Journal of Hydrology* 402 (3-4), 261-273.
- Jakovovic D., Post V.E.A., Werner A.D., Männicke O., Hutson J.L., Simmons C.T., 2012. Tracer adsorption in sand-tank experiments of saltwater up-coning. *Journal of Hydrology* 414-415, 476-481.
- Reilly T.E., Goodman A.S., 1987. Analysis of saltwater up coning beneath a pumping well. *Journal of Hydrology* 89 (3-4), 169-204.

Werner A.D., Jakovovic D., Simmons C.T., 2009. Experimental observations of saltwater up-coning. *Journal of Hydrology* 373 (1-2), 230-241.

Zhang H., Hocking G.C., Barry D.A., 1997. An analytical solution for critical withdrawal of layered fluid through a line sink in a porous medium. *Journal of the Australian Mathematical Society, Series B* 39 (2), 271-279.

Zhang H., Barry D.A., Hocking G.C., 2012. Comment on “Experimental observations of saltwater up-coning”. *Journal of Hydrology* 422-423, 81-83.