# Fixed-order LPV Controller Design for Rejection of a Sinusoidal Disturbance with Time-varying Frequency

Zlatko Emedi and Alireza Karimi

***Abstract*— A new method for the design of fixed-order Linear Parameter Varying (LPV) controllers with polytopic representation for LTI plants is proposed. The stability constraints for the closed-loop system are presented through a set of Linear Matrix Inequalities (LMIs). An additional set of LMIs guarantees $H_\infty$ performance for the weighted closed-loop sensitivity function. The method is successfully applied to the problem of controller design for the rejection of a sinusoidal disturbance with time-varying frequency.**

## I. INTRODUCTION

Disturbance rejection is a practical problem appearing in many engineering applications. Performance of the system can be strongly deteriorated by the presence of a disturbance. The disturbance is often periodic and can be expressed as a combination of several sinusoidal signals. Typical examples of systems with periodic disturbances are hard drives ([1]), optical disk drives ([2]), helicopter rotor blades ([3]) and active noise control systems ([4]), to name just a few. In [5] it is pointed out that most periodic disturbance rejection methods are based on

- use of the internal model principle, and
- use of the "phase-locked" loop structure from communications systems.

The internal model principle states that asymptotic rejection of a disturbance is ensured by inserting the disturbance model into the controller. In the case of a sinusoidal disturbance, this model depends on the sinusoid's frequency. If the disturbance frequency varies over time, the disturbance model changes over time and controller is no longer Linear Time Invariant, but belongs to the class of Linear Parameter Varying controllers.

The notion of LPV system in general comes from many control applications in which the real nonlinear plant is approximated by a linear system around an operating point. Then, the well developed control techniques for linear systems are used to control the nonlinear plant. However, when the operating point changes considerably the controlled system's performance is degraded. In order to achieve good performance throughout the entire operating region, but still use linear system techniques, the class of Linear Parameter Varying systems has been defined ([6], [7]). LPV systems can be thought of as a weighted combination of linear models, each valid at a specific operating point. The weightings are a function of the operating point which is, in turn, a function of certain scheduling parameters. These scheduling parameters can either be endogenous signals, such as the system's states or outputs, or exogenous signals, which cause the dynamics to change as a function of time according to the trajectories of these signals. In a similar manner, the class of LPV controllers can be defined.

In [8] a sinusoidal disturbance rejection approach based on the internal model principle is developed using Youla-Kucera parameterization. The internal model is inserted into the controller directly by adjusting the parameters of the Q polynomial. The order of designed controller is equal to the order of the plant model plus the order of Q polynomial, which is related to the disturbance model. The closed-loop performance is influenced only through the closed-loop pole placement and other types of performance cannot be treated in this framework.

The LPV controller design method developed in [9] guarantees $H_\infty$ performance and closed-loop stability for every permitted value of the disturbance frequency. This method is based on the LPV gain scheduling technique described in [10]. The plant model is augmented by the disturbance model and weighting functions to obtain the generalized plant for which we can present $H_\infty$ performance and stability constraints as a set of LMIs. The main limitation of this approach is that fixed-order controller design in this framework leads to the addition of a non-convex rank constraint.

In this paper, we present a method for the design of fixed-order LPV controllers for LTI plants with guaranteed $H_\infty$ performance and stability for all values of scheduling parameters from a polytope. The important merit of the proposed method is that the order of the controller is an input value to the design process, so the tradeoff between performance and controller complexity can easily be adjusted. In the paper, special attention is given to the problem of rejection of a sinusoidal disturbance with a time-varying frequency.

The rest of this paper is organized as follows. Section 2 describes the main idea of the paper. The LPV controller parameterization considered in this approach and robust stability conditions for all fixed values of the scheduling parameter derived as a set of LMIs are presented in Section 3. In Section 4, LMIs that guarantee $H_\infty$ performance for all fixed values of scheduling parameter are derived. The efficiency of the proposed method is presented using simulation results in Section 5, and conclusions in Section 6.

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## II. The Main Idea

As a starting point for the development of the LPV controller design method described in this paper, the resemblance between parametric uncertainty and scheduling parameters will be exploited. Nonetheless, it is important to bear in mind the fundamental difference between uncertainty , which is generally constant and unknown, and scheduling parameters, which are time-varying and measurable. If the plant model has a parametric uncertainty description, a robust LTI controller will guarantee performance and stability for every model from the uncertainty set, i.e. for every allowable value of the uncertain parameters. On the other hand, if we are designing an LPV controller for an LTI plant, we have to ensure performance and stability for every allowable value of the scheduling parameter. So, for a given parametric uncertainty/scheduling parameter set and for the same performance constraint, these two very different problems can be linked through a common mathematical representation. For this reason, ideas developed in [11] and [12], where a method for $H_\infty$ controller design for systems with polytopic uncertainty is described, will be adapted for the problem described here.

In [12], plants with a polytopic uncertainty description are treated:

$$\mathscr{G} = \left(\sum_{i=1}^{q} \lambda_i N_i\right)\left(\sum_{i=1}^{q} \lambda_i M_i\right)^{-1}, \qquad (1)$$

where $\lambda_i \geq 0$, $\sum_{i=1}^{q} \lambda_i = 1$ and $q$ is the number of the polytope's vertices. Transfer functions $N_i$ and $M_i$ are coprime and belong to $\mathscr{RH}_\infty$, the set of all proper stable rational transfer functions with a bounded infinity norm. Controller being designed can be represented as $K = XY^{-1}$, with $X, Y \in \mathscr{RH}_\infty$.

As a basis for the characterization of the sought-after controllers, the following theorem is used.

*Theorem 1:* [12] The set of all stabilizing controllers for the polytopic system defined in (1) is given by

$$\left\{K = XY^{-1} | M_i Y + N_i X \in \mathscr{S}, i = 1, \ldots, q\right\}, \qquad (2)$$

where $\mathscr{S}$ denotes the convex set of all Strictly Positive Real (SPR) transfer functions.

The main gain that comes from the polytopic representation of the plant is that by ensuring the stability and $H_\infty$ performance for every vertex of the polytope, the same is guaranteed for every model inside the polytope.

In this paper we consider a SISO LTI plant $G$ given by its rational transfer function representation:

$$G = NM^{-1},$$

where coprime transfer functions $N$ and $M$ belong to $\mathscr{RH}_\infty$. We will suppose that the scheduling parameter vector $\theta$, coming for example from the time-varying disturbance model, belongs to the polytope

$$\theta = \sum_{i=1}^{q} \lambda_i \theta_i. \qquad (3)$$

The class of LPV controllers that can be treated by our approach is characterized by the polytopic representation

$$X(\lambda) = \sum_{i=1}^{q} \lambda_i X_i \qquad Y(\lambda) = \sum_{i=1}^{q} \lambda_i Y_i, \qquad (4)$$

where $X_i = X(\theta_i)$ and $Y_i = Y(\theta_i)$ belong to $\mathscr{RH}_\infty$. This representation covers a wide class of dependencies of the controller on the scheduling parameters. The following theorem parameterizes polytopic LPV controllers stabilizing the closed-loop system for every value of scheduling parameter vector $\theta$.

*Theorem 2:* The set of all stabilizing polytopic LPV controllers for the LTI plant $G = NM^{-1}$ is given by:

$$\mathscr{K} : \left\{K = X_i Y_i^{-1} \text{ for } i = 1, \ldots, q \,| F_i \in \mathscr{S}\right\}, \qquad (5)$$

where $F_i = MY_i + NX_i$.

*Proof:* We use the same line of thought from the proof of Theorem 1 in [12].

*Sufficiency:* First, from Theorem 1 we can conclude that the closed-loop system for every vertex controller is stable. Then, we obtain the convex combination of the transfer functions $F_i$ as

$$\begin{aligned} F(\lambda) &= \sum_{i=1}^{q} \lambda_i (MY_i + NX_i) \\ &= M\left(\sum_{i=1}^{q} \lambda_i Y_i\right) + N\left(\sum_{i=1}^{q} \lambda_i X_i\right) \\ &= MY(\lambda) + NX(\lambda). \end{aligned} \qquad (6)$$

The transfer function $F(\lambda)$ is also SPR since the sum of SPR transfer functions weighted by nonnegative weights is SPR. Hence, the plant is stabilized by every controller from the polytope $K(\lambda) = X(\lambda)Y^{-1}(\lambda)$.

*Necessity:* Assume that there exists a polytopic LPV controller stabilizing the LTI plant $G$ given by its vertices $K_i^* = X_i^*(Y_i^*)^{-1}$ that does not satisfy $F_i \in \mathscr{S}$. However, a polytope of stable characteristic polynomials with vertices $c_i$ can be constructed from the plant $G$ and the vertex controllers $K_i^*$. For such a polynomial polytope it has been shown [13] that the phase difference between its elements is less than $\pi$. So, according to Theorem 2.1 of [14] (for discrete-time systems, for continuous-time systems Theorem 3.1 of the same paper) there always exists a polynomial or transfer function $d$ such that $c_i/d$ is SPR for $i = 1, \ldots, q$. As a result, there exists a transfer function

$$L = (MY_i^* + NX_i^*)^{-1} c_i/d$$

such that $(MY_i^* + NX_i^*)L$ is SPR for $i = 1, \ldots, q$. Note that $L$ does not depend on $i$ because the numerator of $(MY_i^* + NX_i^*)$ is equal to $c_i$ and cancels it out in the expression for L. Finally, the polytopic LPV controller

$$K(\lambda) = \left(\sum_{i=1}^{q} \lambda_i X_i\right)\left(\sum_{i=1}^{q} \lambda_i Y_i\right)^{-1}$$

belongs to $\mathscr{K}$ taking $X_i = X_i^* L$ and $Y_i = Y_i^* L$. ■

## III. Convex set of stabilizing LPV controllers

Our first goal is to propose the parameterization of LPV controllers for which the stability of the closed-loop system is guaranteed for every controller lying in the polytope described by (4). To do that, a suitable controller structure must be chosen. Using the fact that every controller in the polytope should depend affinely on the scheduling parameters, vertex controllers can be represented in the form

$$X_i(\theta_i, z) = x(\theta_i)^T \phi(z), Y_i(\theta_i, z) = y(\theta_i)^T \phi(z), \quad (7)$$

where $x(\theta_i)$ and $y(\theta_i)$ are the vector of the controller parameters affine with respect to the scheduling parameters. A good choice of basis function vectors $\phi$ are orthonormal basis functions such as Kautz, Laguerre or generalized orthonormal functions [15].

The SPRness condition in 5 can be represented as a set of constraints in the frequency domain:

$$Re\{M(e^{-j\omega})Y_i(e^{-j\omega}) + N(e^{-j\omega})X_i(e^{-j\omega})\} > 0, \\ \forall \omega \in [0, \omega_N], \text{ for } i = 1, \ldots, q \quad (8)$$

where $Re(\cdot)$ represents the real part of a complex number, and $\omega_N$ is the Nyquist frequency of the system. To solve this problem, frequency gridding is necessary. As a result, constraint violation between the grid frequencies could occur.

The SPR condition in (5) can alternatively be presented using the KYP lemma. For the discrete-time systems (similarly for continuous-time systems), the KYP lemma states that the transfer function $F_i(z) = C_i(zI - A)^{-1}B + D_i$ belongs to $\mathscr{S}$ if and only if there exists a matrix $P_i = P_i^T > 0$ such that

$$\begin{bmatrix} A^T P_i A - P_i & A^T P_i B - C_i^T \\ B^T P_i A - C_i & B^T P_i B - (D_i + D_i^T) \end{bmatrix} < 0. \quad (9)$$

If we choose $(A, B, C_i, D_i)$ as a controllable canonical realization of $F_i = MY_i + NX_i$, then the controller and the plant parameters which are in the numerator of $F_i$ appear only in $C_i$ and $D_i$, so the inequality (9) becomes an LMI with respect to $P_i$, $C_i$ and $D_i$. Matrices $A$ and $B$ are the same for all the vertices because of the properties of controllable canonical form and the fact that denominators of all transfer functions $F_i$ are the same (where also the fact that all the transfer functions $X_i$ and $Y_i$ have the same poles is used). Unknown controller parameter vectors $x(\theta_i)$ and $y(\theta_i)$, which appear in matrices $C_i$ and $D_i$, are found as a feasible point of the LMI constraints.

## IV. $H_\infty$ performance constraints

For fixed values of the parameter $\lambda$, we can discuss the $H_\infty$ performance considering the weighted closed-loop transfer functions. The motivation for this lies in the above-mentioned problem of sinusoidal disturbance rejection. If we take, for example, that the frequency of the sinusoid is fixed over some period, our aim during that time is to reject the disturbance, but also not to amplify the noise too much in other frequencies. To ensure this, we could design the controller by shaping the frequency response of the output sensitivity function $S$ using the performance filter $W_1$. The output sensitivity function $S$ is defined as the transfer function from the output disturbance to the output of the closed-loop system and is given by the expression

$$S = (1 + GK)^{-1} = MY(MY + NX)^{-1}.$$

The following should be ensured for $S$:

$$\|W_1 S\|\infty = \left\| \frac{W_1 MY}{MY + NX} \right\|_\infty < \gamma, \quad (10)$$

where $\gamma$ is a bound on the $H_\infty$ norm of the weighted output sensitivity function. As we see, the controller parameters appear both in the numerator and denominator of the transfer function $W_1 S$, so the application of Bounded Real Lemma on the state-space representation of the weighted sensitivity function would result in a nonconvex problem. To convexify this performance constraint, the relation between the Bounded Real Lemma and the Positive Real Lemma can be employed ([16], [17]).

In [11] it is shown that Inequality (10) is satisfied if and only if the following stands:

$$H = \frac{(MY + NX) - \gamma^{-1} W_1 MY}{(MY + NX) + \gamma^{-1} W_1 MY} \in \mathscr{S}. \quad (11)$$

Therefore, the set of all controllers for which the inequality $\|W_1 S\|\infty < \gamma$ is satisfied is given by

$$\mathscr{K}_{s_\infty} : \{K = XY^{-1} | H \in \mathscr{S}\}.$$

To enable the calculation of the controller parameters the set $\mathscr{K}_{s_\infty}$ will be represented via LMIs. Let $H$ in (11) be defined as the ratio of two coprime transfer functions

$$H = H_n / H_d,$$

where $H_n$ and $H_d$ are given by

$$H_n = (MY + NX) - \gamma^{-1} W_1 MY \quad (12)$$

$$H_d = (MY + NX) + \gamma^{-1} W_1 MY. \quad (13)$$

Then, the set of all stabilizing controllers that ensure Inequality (10) is given by:

$$\mathscr{K}_{s_\infty} : \{K = XY^{-1} | H_n \text{ and } H_d \text{ are CL-SPR}\},$$

where CL-SPR stands for Common Lyapunov - SPR. We say that two transfer functions are CL - SPR if they satisfy the inequality of KYP lemma with the same Lyapunov matrix $P$ [11].

Using the fact that transfer functions $H_n$ and $H_d$ have the same denominators, their controllable canonical realizations can be represented as $(A, B, C_n, D_n)$ and $A, B, C_d, D_d$, respectively. Then, the condition that the transfer functions $H_n$ and $H_d$ are CL-SPR is expressed using the following set of matrix inequalities:

$$\begin{bmatrix} A^T P A - P & A^T P B - C_n^T \\ B^T P A - C_n & B^T P B - (D_n + D_n^T) \end{bmatrix} < 0, \quad (14)$$

$$\begin{bmatrix} A^T P A - P & A^T P B - C_d^T \\ B^T P A - C_d & B^T P B - (D_d + D_d^T) \end{bmatrix} < 0. \quad (15)$$

As this design approach is for controllers with a polytopic structure, the LMIs have to be adapted in a similar manner to the stability constraints. First, for the controller representing the polytope vertex $i$, the transfer functions $H_{ni}$ and $H_{di}$ are defined by

$$H_{ni} = (MY_i + NX_i) - \gamma^{-1} W_1 MY_i,$$

$$H_{di} = (MY_i + NX_i) + \gamma^{-1} W_1 MY_i.$$

Their controllable canonical representations will be labeled as $(A, B, C_{ni}, D_{ni})$ and $(A, B, C_{di}, D_{di})$, respectively, with $A$ and $B$ the same for all the vertices (similar explanation as in the case of the stability constraints (9)).

By writing Inequalities (14) and (15) for every $H_{ni}$ and $H_{di}$ we get that the $H_\infty$ performance is guaranteed for the closed-loop system with every polytope vertex controller:

$$\begin{bmatrix} A^T P_i A - P_i & A^T P_i B - C_{ni}^T \\ B^T P_i A - C_{ni} & B^T P_i B - (D_{ni} + D_{ni}^T) \end{bmatrix} < 0, \quad (16)$$

$$\begin{bmatrix} A^T P_i A - P_i & A^T P_i B - C_{di}^T \\ B^T P_i A - C_{di} & B^T P_i B - (D_{di} + D_{di}^T) \end{bmatrix} < 0. \quad (17)$$

But, if this is satisfied, all the transfer functions $H_{ni}$ and $H_{di}$ are SPR. By taking the convex combination of all the transfer functions $H_{ni}$, we get the new SPR transfer function

$$H_n(\lambda) = \sum_{i=1}^{q} \lambda_i H_{ni} = \sum_{i=1}^{q} \lambda_i [(MY_i + NX_i) - \gamma^{-1} W_1 MY_i],$$

which can be rewritten as

$$H_n(\lambda) = M \sum_{i=1}^{q} \lambda_i Y_i + N \sum_{i=1}^{q} \lambda_i X_i - \gamma^{-1} W_1 M \sum_{i=1}^{q} \lambda_i Y_i. \quad (18)$$

This means that for any controller from the polytope $K(\lambda)$ the transfer function $H_n(\lambda)$ is SPR. In a similar manner, we can conclude that for the controller $K(\lambda)$, the transfer function $H_d(\lambda)$ is SPR. Therefore, for any controller belonging to the polytope (4), the $H_\infty$ constraint in (10) is satisfied.

*Remark 1:* As we can notice, from the fact that the transfer functions $H_{ni}$ and $H_{di}$ are SPR we can obtain that the transfer function $H_i = H_{ni} + H_{di}$ is SPR too. But, from the expressions for $H_{ni}$ and $H_{di}$ we get that $H_i = 2(MY_i + NX_i)$ is also SPR. Then, according to Section II, the closed-loop system is stabilized for every controller from the polytope. It practically means that if we set LMIs (16) and (17) as design constraints, we don't need a separate set of LMIs defined by (9) for stability.

*Remark 2:* The results can be extended to the case of an uncertain plant model with polytopic uncertainty. Let the polytopic plant model be given by

$$G(\eta) = N(\eta) M^{-1}(\eta),$$

where $N(\eta) = \sum_{j=1}^{p} \eta_j N_j$, $M(\eta) = \sum_{j=1}^{p} \eta_j M_j$, $\eta_j \geq 0$, $\sum_{j=1}^{p} \eta_j = 1$, and $N_j$ and $M_j$ belong to $\mathscr{RH}_\infty$. If we have to design an LPV controller, given by (4), for such a system, we would have to set LMI (9) (or (16) and (17) for both stability and performance) as a design constraint for every possible combination of vertices of two polytopes, i.e. by ensuring the SPRness of

$$F_{i,j} = M_j Y_i + N_j X_i$$

for $i = 1, \ldots, q$ and $j = 1, \ldots, p$. It is easy to show, by taking convex combinations of these $pq$ LMIs, that the stability (and performance) of any model from the plant uncertainty polytope will be guaranteed by application of any controller from the controller LPV polytope.

*Remark 3:* All of the results can easily be extended to the continuous-time case. The only difference is that the LMIs of the KYP lemma should be expressed for continuous-time systems.

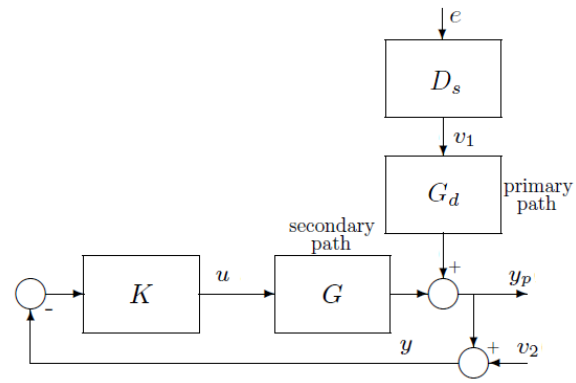


Fig. 1. Block diagram of the active suspension system

## V. Simulation results

As a test system, the model of the active suspension system in the Control Systems Department in Grenoble (GIPSA - lab) was used. A detailed description of the system can be found in [18]. The proposed method was used to design a controller capable of rejecting a sinusoidal disturbance with a time-varying frequency. The disturbance frequency is known to lie in the interval between $45$ and $105$Hz, and the sampling frequency for both data acquisition and control is set to $800$Hz. The block diagram of the active suspension system is shown in Figure 1.

The sinusoidal disturbance $v_1(t)$ can be represented as a white noise $e$ filtered through the disturbance model $D_s$. The transfer function $G_d$ between the disturbance input and the open-loop system output $y_p(t)$ is called the primary path. The measured output affected by the measurement noise is denoted as $y(t)$ and it is fed back to the controller. The secondary path denotes the transfer function $G$ between the output of controller $u(t)$ and the system output in the open loop. Both the primary (red line on Figure 2) and the secondary path (blue line) contain several high-resonant modes in the disturbance frequency region, as can be observed on the amplitude Bode diagram of the identified test model.

By the application of the Internal Model Principle (IMP) the controller is parameterized as a function of the disturbance frequency. Note that the denominator of the discrete-time model (with $T_s$ as the sampling period) of the sinusoidal

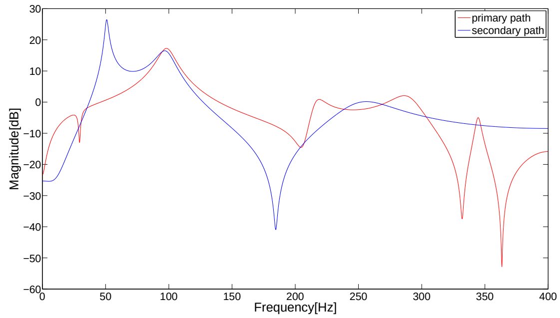


Fig. 2. Frequency response of the primary and secondary paths

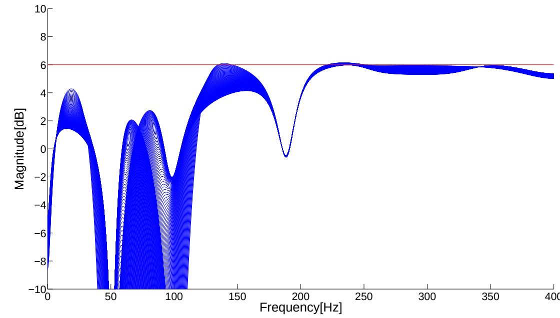


Fig. 3. Joint sensitivity plot for a fine grid of $\theta$

disturbance with frequency $\omega$ is given by

$$D_s(z,\omega) = z^2 - 2cos(2\pi T_s \omega)z + 1.$$

For a given disturbance frequency interval $[\omega_1, \omega_2]$, it can be rewritten as a linear function of one parameter $\theta \in [\theta_1, \theta_2]$, with $\theta_1 = -2cos(2\pi T_s \omega_1)$ and $\theta_2 = -2cos(2\pi T_s \omega_2)$. So, $\theta$ enters the denominator of the controller affinely. We will label the controller for $\theta = \theta_1$ as $K_1(z) = X_1(z)Y_1^{-1}(z)$ and that for $\theta = \theta_2$ as $K_2(z) = X_2(z)Y_2^{-1}(z)$, where $Y_i(z) = Y_f(z)(z^2 + \theta_i z + 1))$. Then the stabilizing controller for any $\theta \in [\theta_1, \theta_2]$ that incorporates $z^2 + \theta z + 1$ in the denominator is given by

$$K(z,\lambda) = (\lambda X_1(z) + (1-\lambda)X_2(z)) \times (\lambda Y_1(z) + (1-\lambda)Y_2(z))^{-1}, \quad (19)$$

where $\lambda = (\theta_2 - \theta)(\theta_2 - \theta_1)^{-1}$.

Due to the “waterbed effect”, we have to allow amplification at other frequencies to have strong attenuation at the disturbance frequency and still preserve the stability of the closed-loop system. To guarantee that the noise at other frequencies won’t be strongly amplified, the performance constraint $\|S\|_\infty < 6$dB is set using the performance filter $W_1 = 0.5$. The value of 6dB is a general practical recommendation [19]. For both polytope vertices (for the limiting frequencies of 45 and 105Hz) two appropriate LMIs are set to ensure the performance. For defining the constraints Yalmip [20] is used as a Matlab interface. The chosen semidefinite programming (SDP) solver is SDPT3 [21].

For the sake of simplicity, the denominators of the transfer functions $X_1(z)$, $X_2(z)$, $Y_1(z)$ and $Y_2(z)$ are set to have all poles at $0.2$ (the sensitivity of the design approach to this choice is not high). The numerators of these transfer functions, as well as two Lyapunov matrices $P_1$ and $P_2$, represent the optimization variables in this problem (bearing in mind that $Y_1(z)$ and $Y_2(z)$ have a fixed part $D_s(z,\theta)$ for $\theta \in \{\theta_1, \theta_2\}$). The exact problem to be solved is given as $\|W_1(z)S(z,\theta)\|_\infty < \gamma$, for $\theta \in \{\theta_1, \theta_2\}$ and fixed $\gamma$. If our SDP problem can be solved for $\gamma = 1$ it means that the desired performance level can be obtained. The optimal $\gamma$ is the minimal value for which the problem can be solved. To find the optimal $\gamma$, the bisection algorithm is used.

As a representative solution to the given problem a 10-th order controller is chosen. The optimal $\gamma$ obtained is $1.03$ (for the purpose of comparison, for the 6-th order controller the optimal $\gamma$ equals $1.16$, and for the 12-th order one the optimal $\gamma$ is $0.95$). Solving the problem takes around 5 seconds per iteration of the bisection algorithm. Figure 3 depicts superimposed Bode amplitude plots of the transfer function $S$ for a fine grid of disturbance frequencies $\theta$ between 45 and 105Hz (a grid step of 0.5Hz was used). The upper part of the graph satisfies the performance constraint. In the lower part asymptotic rejection in the area of disturbance frequencies can be observed, which comes from the presence of the disturbance model in the denominator of the LPV controller.

To illustrate the controller’s performance, simulations were performed on the plant model. Since the objective of this paper is controller design for the rejection of a sinusoidal disturbance with varying but known frequency, estimation of the disturbance frequency was omitted in the simulation. Different indirect adaptation schemes applicable to this problem can be found in the literature ([19]). For the purpose of simulation, the disturbance frequency was directly fed to the controller, which corresponds to the situation when it is possible to measure the disturbance (or a signal correlated to it) and directly obtain the disturbance frequency.

The initial disturbance frequency was set to 45Hz, and was then modified every 2 seconds via a step change to 105Hz, 75Hz, 60Hz and 90Hz, in that order. In Figure 4, the open-loop simulation response to such a disturbance is represented in red. The blue plot in Figure 5 depicts the closed-loop response to the same disturbance. Both plots are shown on the same scale, so the asymptotic rejection can easily be noticed. For a closer view of the transient after the disturbance frequency changes, the open-loop and closed-loop responses from $2.0$ to $2.1$ seconds are superimposed in Figure 6. The closed-loop transient after each frequency change is rather short and the peak value is less than in the open-loop case.

## VI. Conclusions

In this paper a fixed-order LPV controller design method for LTI plants is described, with a focus on the problem of rejection of a frequency-varying sinusoidal disturbance. Two

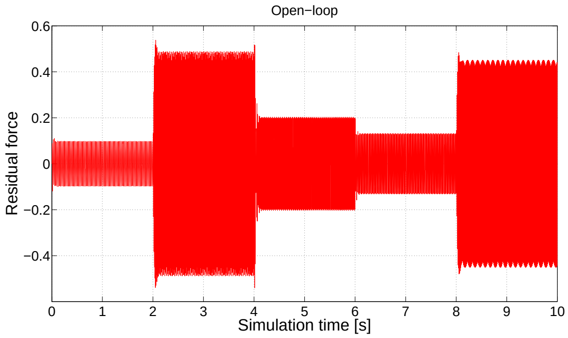


Fig. 4. Open-loop response to step changes in the disturbance frequency

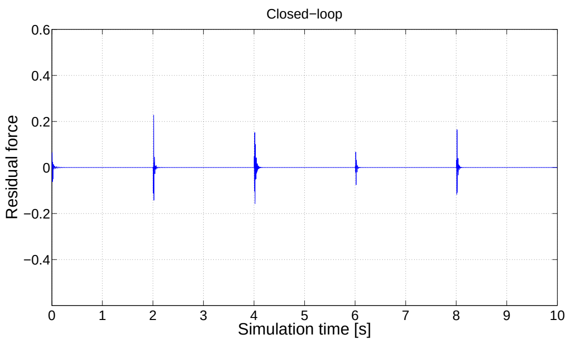


Fig. 5. Closed-loop response to step changes in the disturbance frequency

different sets of LMIs are proposed which ensure stability of the system for all values of the scheduling parameter (i.e. the disturbance frequency) and desired $H_\infty$ performance. Simulations results show that sensitivity function shaping ensures good performance for fixed values of the scheduling parameter, and that stability of the closed-loop system is preserved during the scheduling parameter variations. However, the method cannot the guarantee of the closed-loop system stability during fast scheduling parameter variations. Although this does not pose a practical problem for this application, from a theoretical point of view it is interesting to study the global stability of the system. Other issues that will be addressed in future work are the rejection of multiple sinusoidal disturbances and uncertainty in the scheduling parameter.

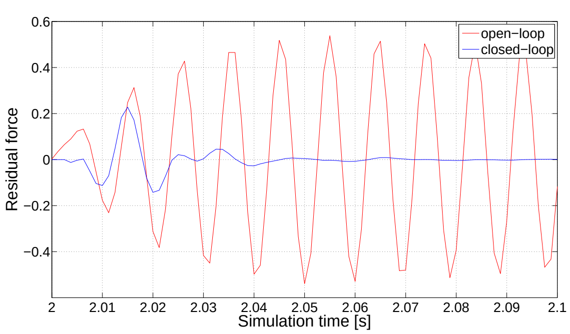


Fig. 6. Comparison of the open-loop and closed-loop response

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